Ratchet effect and amplitude dependence of phase locking in a two-dimensional Frenkel-Kontorova model

Cang-Long Wang,¹ Jasmina Tekić,² Wen-Shan Duan,³ Zhi-Gang Shao,⁴ and Lei Yang^{1,5,a)}

¹*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

²*Theoretical Physics Department 020, “Vinča” Institute of Nuclear Sciences, University of Belgrade, P.O. Box 522, 11001 Belgrade, Serbia*

³*College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China*

⁴*Laboratory of Quantum Information Technology, Institute of Condensed Matter Physics, School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China*

⁵*Department of Physics, Lanzhou University, Lanzhou 730000, China*

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We demonstrate the ratchet and phase locking effects in a two-dimensional overdamped Frenkel-Kontorova model with a square symmetric periodic substrate when both a longitudinal dc drive and a circular ac drive are applied. Besides the harmonic steps, the large half integer steps can also clearly be seen in the longitudinal (x) direction. These half integer steps are directly correlated to the appearance of positive and negative ratchet effects in the transverse (y) direction due to the symmetry breaking in the combination of the dc and ac drives. The angle between the net displacement and the longitudinal direction is analytically obtained in a single period of the ac drive. In the examination of the amplitude dependence of the ac drive, the maxima decrease monotonically with the amplitude, while the anomalies occur for the critical depinning force and the harmonic steps due to the spatial symmetry breaking of orbits in the presence of the ac drive. © 2013 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4776226>]

I. INTRODUCTION

The main characteristic of ratchet effects is the ability to transport particles in the absence of any net macroscopic force. The simplest realization is the rectification of an ac driven particle in an asymmetric potential.¹ Explorations of ratchet effects may help us to understand biological motors,² particle segregation,^{2,3} atom transport in optical lattices,⁴ and fluxon motion in superconductors.⁵ The ratchet effects arise due to the symmetry breaking caused by an imposed one-dimensional (1D) asymmetric potential. However, in a two-dimensional (2D) system, there are many new ways in which symmetry can be broken, so it is possible to induce ratchet effects even when the substrate is symmetric.

On the other hand, in the presence of ac and dc drives, phase locking effects occur when the external ac frequency matches the internally generated frequency of the motion of the particle over the periodic potential. Due to the great significance of technical applications of phase locking effects, the amplitude and frequency dependence have particularly been a matter of many theoretical and experimental studies in Josephson-junction arrays (JJA),⁶ charge density waves (CDW),⁷ and superconducting nanowires.⁸ In order to gain an insight into physics of these complex macroscopic many-body systems with competing interactions, attention has been always focused on simple many-body models. Among these models, the dissipative (overdamped) Frenkel-Kontorova (FK) model is one of the simplest but still complex

enough to capture the essence of many physical and biological phenomena.

The 1D FK model represents a chain of harmonically interacting particles subjected to a sinusoidal substrate potential.⁹ Using molecular dynamics simulation, Floría *et al.*^{10–13} have studied in detail the overdamped dynamics of the 1D FK model submitted to the ac and dc drives. They obtained the staircase macroscopic response in the curve for average velocity as a function of the average external driving force $\bar{v}(\bar{F})$. Recently, the influence of the amplitude and frequency of the external ac force on the interference phenomena of the FK model has been studied in the works of Hu and Tekić.¹⁴ However, in the numerous theoretical studies, the ac and dc drives are in the same direction and the motion can be considered 1D. Different effects can arise in a 2D FK model.

In the application of ratchet and interference phenomena, the effect on the dynamics of the many-body system, especially on the transport properties of the 2D driven FK model, has not been investigated any further even if it is of great importance. Further, in the present paper, we study the ratchet effects and the phase locking effects in the 2D overdamped FK model with a square symmetric periodic substrate when both a longitudinal dc drive and a circular ac drive are applied. Contrary to the interference phenomena^{14,15} in the 1D FK model and the ratchet effect of the single particle,^{16,17} the 2D FK model exhibits completely different physics. Besides the harmonic steps, we have also observed the large half integer steps in the longitudinal (x) direction. These half integer steps are associated with the onset of positive and negative ratchet effects in the transverse (y) direction due to the spatial

^{a)} Author to whom correspondence should be addressed. Electronic mail: lyang@impcas.ac.cn.

symmetry breaking when the angle between the net displacement and x direction is $\langle\theta\rangle = \pm \arctan \frac{1}{p}$ (p is odd number). These analytical results are in good agreement with numerical ones. In the examination of amplitude dependence, it was found that the anomalies occur for the critical depinning force (F_c) and the harmonic steps, where the second lobe is smaller than the third one, the fourth lobe is smaller than the fifth one, and the sixth lobe is larger than the fifth one, while for the half integer steps, the maxima decrease monotonically with the amplitude. The phase difference of the oscillations for F_c and the width of harmonic steps is nearly equal, and 180° for F_c and the width of the half integer steps. These phenomena are in good agreement with the ones obtained in the 2D JJA,⁶ where the anomalous amplitude dependence has been observed due to the field-induced vortex super lattice and broken symmetry. The spatial symmetry breaking of the orbits are also obtained in the 2D FK model.

The paper is organized as follows. The model is introduced in Sec. II. Numerical simulation results are presented and analyzed in Sec. III. Finally, Sec. IV concludes the paper.

II. MODEL

We consider a 2D lattice of particles with interaction (the red balls in the upper layer) coupled to a periodic substrate potential with the square symmetry (the gray-scale map in the lower layer),²⁰ as given in Fig. 1. The substrate potential is chosen in the simplest form,^{18–20}

$$V_{sub} = -\frac{E}{2} \cos\left(\frac{2\pi}{a}x\right) \cos\left(\frac{2\pi}{a}y\right). \quad (1)$$

Equation (1) represents the first term of the Fourier series of 2D substrate potential, where E is the height of the substrate potential and the length a defines the lattice constant of the lower layer.

FIG. 1. Schematic view of the 2D FK model. The red balls represent the particles. The substrate potential is shown by the gray-scale map. The letters A-J indicate the possible sites from the initial site O in a single period of ac drive. The eight nearest and next nearest neighbors distribute in the larger and smaller blue circles, respectively.

For the sake of simplicity, we consider an arbitrary particle of (i, j) in the upper layer which interacts with the eight nearest and next nearest neighbors (distributed in the larger and smaller blue circles in Fig. 1, respectively). We also consider the case of the simple harmonic interaction between the particles

$$V_{int} = \sum_{i,j=1}^8 \frac{K}{2} (r_{i,j} - l_0)^2, \quad (2)$$

with a stiffness strength K . The lattice constants of the upper layer are $l_0 = a$ and $\sqrt{2}a$ for the nearest (the smaller circle in Fig. 1) and next nearest (the larger circle in Fig. 1) neighbors, respectively.

We consider the dissipative (overdamped) dynamics of an arbitrary particle with positive vector $\mathbf{r}_{i,j} = (x_{i,j}, y_{i,j})$. The overdamped equation of motion of an arbitrary is given as follows:

$$\eta \dot{\mathbf{r}}_{i,j} = \mathbf{F}_{sub} + \mathbf{F}_{int} + \mathbf{F}_{dc} + \mathbf{F}_{ac}, \quad (3)$$

where the damping constant $\eta = 1$. The assumption of overdamped motion should be valid for vortices in superconductors or JJA as well as for colloidal systems. The force from the substrate $\mathbf{F}_{sub} = -\nabla V_{sub}$. The interaction force from the other particles is $\mathbf{F}_{int} = -\nabla V_{int}$. In our model, an arbitrary particle is driven by both the external dc drive and the external periodic ac drive. The dc drive $\mathbf{F}_{dc} = F_{dc} \hat{\mathbf{x}}$ is applied in the positive longitudinal (x) direction, where F_{dc} is the magnitude of the dc drive. The circular ac drive is applied in both the longitudinal direction and the transverse direction, and is given by

$$\mathbf{F}_{ac} = F_{ac} \sin(2\pi \nu t) \hat{\mathbf{x}} + F_{ac} \cos(2\pi \nu t) \hat{\mathbf{y}},$$

where F_{ac} is the magnitude of the ac drive and ν is the frequency of the ac drive. Note that there is no dc driving component in the transverse (y) direction.

Throughout the paper, we consider the system of size $10a \times 10a$. We fix the lattice constant $a = 1$, the frequency of the ac drive $\nu = 0.2$, the height of the substrate potential $E = 1$, and the stiffness strength of the interatomic interaction $K = 1$. The fourth order Runge-Kutta algorithm with the periodic boundary conditions was implemented to solve the equation of motion numerically. As an initial condition, we place the particles close to the minima of the substrate potential. For different initial placements, the results are identical. In the simulations, the dc drive $\mathbf{F}_{dc}$ is increased from 0 to 1.2 in increments of 0.0001 in the positive x direction, where a time interval of 400τ (τ is the period of the ac force) is used as relaxation time to allow the system to reach the steady state. The longitudinal time averaged velocity $\langle V_x \rangle$ and the transverse time averaged velocity $\langle V_y \rangle$, in particular, the step width and the critical depinning force, are analyzed in the paper.

III. RESULTS

The longitudinal time averaged velocity $\langle V_x \rangle$ as a function of external dc drive is presented for different amplitudes of the ac drive in Fig. 2(a).

FIG. 2. (a) and (b) Longitudinal velocity $\langle V_x \rangle$ and transverse velocity $\langle V_y \rangle$ vs F_{dc} for different F_{ac} , respectively. To clarify, the curves have been systematically shifted in the y direction by 0.2, 0.4, and 0.6, respectively. (c) Angle $\langle \theta \rangle$ in a single period for $F_{ac} = 0.2$.

As we can see from Fig. 2(a), $\langle V_x \rangle$ increases in quantized steps with step weights of $\langle V_x \rangle = n\nu$ (n is an even number). Besides the n th harmonic step, the half integer step with height $\langle V_x \rangle = \frac{p}{2}\nu$ (p is an odd number) is also clearly seen in the x direction. However, in the 1D standard FK model,¹⁴ the half integer steps are so small that they are invisible. The staircase macroscopic response is called Shapiro steps in the JJA language.⁶ When the system is driven by a homogenous periodic force, the competition between two frequency scales (the frequency ν of the external periodic ac force and the characteristic frequency of the motion over the periodic substrate potential driven by the dc force F_{dc}) can result in the appearance of the phase locking effects.

If $\{x_{i,j}, y_{i,j}\}$ is the solution of Eq. (3), then the transformation

$$\sigma_{k,l,m}\{x_{i,j}(t), y_{i,j}(t)\} = \left\{x_{i+k,j}\left(t - \frac{m}{\nu}\right) + l, y_{i,j}(t)\right\} \quad (4)$$

produces another solution, where k , l , and m are integers. The solution is called resonant if it is invariant under the symmetry operation as follows:

$$\sigma_{k,l,m}\{x_{i,j}(t), y_{i,j}(t)\} = \{x_{i,j}(t), y_{i,j}(t)\}. \quad (5)$$

The longitudinal velocity of the resonant solution is given by

$$\langle V_x \rangle = \frac{k\omega + l}{m}\nu, \quad (6)$$

where $\omega = \langle (x_{i+1,j} - x_{i,j}) \rangle$ corresponds to the winding number (the interparticle average distance in the x direction) in

the 1D FK model. In our 2D FK model, $\omega = 1$. When $m = 1$, the resonant solutions and the steps are called harmonic, while when $m > 1$, the steps are called subharmonic ($m = 2$ for fractional or half-integer steps). The analytical results are in good agreement with the ones obtained in Fig. 2(a).

In Fig. 2(b) the transverse time averaged velocity $\langle V_y \rangle$ as a function of the external dc drive is presented for the same amplitudes of ac drive in Fig. 2(a).

Besides the interference phenomena in the x direction, the ratchet effect occurs in the y direction. As we can see, for the harmonic steps in Fig. 2(a), $\langle V_y \rangle = 0$ in Fig. 2(b), while, for the half integer steps in Fig. 2(a), $\langle V_y \rangle$ are nonzero in Fig. 2(b), i.e., positive (+) and negative (−) ratchet effects occur in the y direction which have exactly defined heights of $\Delta\langle V_y \rangle = \pm \frac{\nu}{2}$.

We will now explain the nature of the ratchet effects and the phase locking effects. First, for the harmonic steps in the x direction, no ratchet effects occur in the y direction. That is, the net displacement of an arbitrary particle in a single period $\frac{1}{\nu}$ of the ac drive is in the x direction when the net displacement in the y direction is zero. Although an arbitrary particle has only translated by n unit cells in the x direction after a single period of the ac drive, i.e., an arbitrary particle moves from the site O to I or J (see Fig. 1), the particle is in the same location within its unit cell as it was at the start of the cycle. Thus, the displacement in a single period of the ac drive is n and velocity $\langle V_x \rangle = n\nu$. Second, the chirality of the ac drive breaks a symmetry and the dc drive breaks a reflection symmetry, so that the combined effect is to induce the ratchet effects in the y direction. Finally, when the ratchet effects occur in the y direction, there exists a map for an arbitrary particle which satisfies $(0, 0) \rightarrow (\frac{p}{2}, \pm \frac{1}{2})$ in a single period of the ac drive, from the site O to A (or D), B (or E), or C (or F), as shown in Fig. 1. Thus, $\langle V_x \rangle = \frac{p}{2}\nu$ and $\langle V_y \rangle = \frac{\nu}{2}$ (or $-\frac{\nu}{2}$) for the ratchet effect. Moreover, the angle between the net displacement and the x direction is $\langle \theta \rangle = \arctan\left(\frac{\langle V_y \rangle}{\langle V_x \rangle}\right) = 0^\circ$ for the harmonic steps, while $\langle \theta \rangle = \arctan\left(\frac{\langle V_y \rangle}{\langle V_x \rangle}\right) = \pm \arctan\left(\frac{1}{p}\right)$ (p is odd number) for the half integer steps. The analytical results are in good agreement with the numerical ones shown in Fig. 2(c).

In the examination of dc and ac driven systems, the main interest is always focused on the existence and structural stability of the phase locking phenomena against the changing of the system parameters. As we can also clearly see from Fig. 2(a), the amplitude of the ac drive strongly influences the step width and the critical depinning force.

In Fig. 3 we show the amplitude dependence of the step width for the first two harmonic steps (ΔF_1 and ΔF_2), the first two half integer steps ($\Delta F_{\frac{1}{2}}$ and $\Delta F_{\frac{3}{2}}$), and the critical depinning force F_c . The figures are obtained by extracting width ΔF from the series of response functions $\langle V_x \rangle(F_{dc})$ made for different amplitudes.

- The critical depinning force F_c .

In contrast with the behavior in the 1D FK model with commensurate structure¹⁴ and the single Josephson-junction²¹ where the maxima of the F_c decrease monotonically with the amplitude, things become more interesting once the amplitude of the ac drive is varied in the 2D FK model. As

FIG. 3. The width ΔF and the critical depinning force F_c as a function of ac amplitude, in (a) $\Delta F_{1/2}$, (b) ΔF_1 , (c) $\Delta F_{3/2}$, and (d) ΔF_2 , where F_c are plotted for the sake of comparison with ΔF .

we can see from Fig. 3, the F_c still oscillates with the amplitude, but the anomalies occur for the even lobes, for example, the second maximum is smaller than the third one, the fourth maximum is smaller than the fifth one, and the sixth maximum is larger than the fifth one. Note also that in the region of $0.58 < A < 0.66$, the F_c almost drops to zero.

- The width of the half integer step $\Delta F_{1/2}$ (or $\Delta F_{3/2}$).

As shown in Figs. 3(a) and 3(c), in the amplitude dependence of the $\Delta F_{1/2}$ (or $\Delta F_{3/2}$), the maxima decrease monotonically with the amplitude. Moreover, the phase difference of the oscillations for the F_c and $\Delta F_{1/2}$ (or $\Delta F_{3/2}$) is 180° . These phenomena have also been observed in 2D JJA.⁶ However, for the 1D FK model¹⁴ and the single particle¹⁷ system which describes the vortex pinball in a superconductor, the half integer steps are invisible.

- The width of the harmonic step ΔF_1 (or ΔF_2).

The nonmonotonic decay of the maxima of the harmonic step width is also exhibited by the harmonic steps (Figs. 3(b) and 3(d)). At large amplitudes, the phase of the oscillations for the F_c and the ΔF_1 (or ΔF_2) is nearly equal. However, for the single particle¹⁷ system in a superconductor, the maxima of the harmonic step width decrease monotonically with the amplitude, and the phenomena can also be obtained in the 1D FK model.¹⁴

In order to understand these oscillatory behaviors, we will analyze the orbits of an arbitrary particle for no dc drive and different ac amplitudes. In Fig. 4, the locations of the substrate potential maxima and the orbits of an arbitrary particle

are presented. For the amplitude of ac drive $F_{ac} = 0$, the particle is pinned at a substrate potential minimum, as shown in Fig. 4(a). For the region of $0 < F_{ac} < 0.58$, the particle moves in a circular orbit around the center of the potential minimum, as shown in Fig. 4(b). The radius of the orbit increases with the amplitude. For the region of $0.58 < F_{ac} < 0.66$ (the F_c almost drops to zero), the particle does not follow any particular orbit, as in Fig. 4(c). Stable orbits also occur in high amplitude regions, where 1, 2, 4, 6, 9, and 12 potential maxima are encircled for the first ($0.66 < F_{ac} < 0.96$), second ($0.96 < F_{ac} < 1.12$), third ($1.12 < F_{ac} < 1.38$), fourth ($1.38 < F_{ac} < 1.64$), fifth ($1.64 < F_{ac} < 1.8$), and sixth ($1.8 < F_{ac} < 2$) lobes of the F_c (see Figs. 4(d)–4(i)), respectively. Note that the odd lobes of the F_c correspond to the orbits which encircle 1, 4, and 9 potential maxima, respectively. Moreover, the centers of these orbits are in either the potential minima or maxima. However, the orbits encircle 2, 6, and 12 potential maxima and the centers are neither the potential minima nor maxima for the even lobes of the F_c . The ac force affects the cyclotron motions and breaks the spatial reflection symmetry of the motions. Thus, the anomalous behaviors occur.

A similar phenomena of orbits for particles undergoing circular or cyclotron motion in a periodic array of scatterers also occurs in electron pinball model.²² Another similar system is the vortex pinball model, where stable orbits occur for vortices in superconductors with periodic pinning arrays when the density of the magnetic field is such that there are twice as many vortices as pinning sites.¹⁷

FIG. 4. The substrate potential maxima (red dots), an arbitrary particle (black dot in Fig. 4(a)), and the orbits of the particle (black line) for different amplitudes F_{ac} .

IV. CONCLUSION

As a conclusion, this work is dedicated to the studies of ratchet and interference phenomena in the 2D FK model with a square symmetric periodic substrate when a dc drive and a circular ac drive are applied. We would like to emphasize that the half integer steps in the longitudinal direction are associated with the onset of positive and negative ratchet effects in the transverse direction due to the spatial symmetry breaking. The ratchet effects can be used to manipulate the vortex motion in a superconductor with periodic pinning,⁵ where vortices act as overdamped particles moving in a square periodic array of holes or magnetic dots when each site captures one vortex, and the ac force is provided by an oscillating external magnetic field. Ratchet designs without spatial asymmetry have been successfully achieved in layered superconductors.²³

We have also studied the amplitude dependence of the phase locking effects. It was found that the anomalous oscillations occur for the F_c and harmonic steps, while for the first two half integer steps, the maxima decrease monotonically with the amplitude. Moreover, the stability of the ratchet effects is directly correlated to the stability of the F_c and ΔF_1 (or ΔF_2): the maxima (minima) of the oscillations for the F_c and ΔF_1 (or ΔF_2) correspond to the minima (maxima) of the $\Delta F_{1/2}$ (or $\Delta F_{3/2}$). In addition, to understand these phenomena, the cyclotron motions were presented.

The results could be of great importance for all real systems with overdamped motion and driven by periodic forces such as CDW in solids and JJA that are motivated by fabrica-

tion of synchronization and superconducting devices. In many situations, these systems are closely related to the dissipative dynamics of the Frenkel-Kontorova model. Understanding the ratchet and interference effects in 2D systems and from there building a good theoretical guideline are particularly important due to the technical applications of the ratchet and interference phenomena.

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