A mathematical model of fluid and gas flow in nanoporous media

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The mathematical modeling of the flow in nanoporous rocks (e.g., shales) becomes an important new branch of subterranean fluid mechanics. The classic approach that was successfully used in the construction of the technology to develop oil and gas deposits in the United States, Canada, and the Union of Soviet Socialist Republics becomes insufficient for deposits in shales. In the present article a mathematical model of the flow in nanoporous rocks is proposed. The model assumes the rock consists of two components: (i) a matrix, which is more or less an ordinary porous or fissurized-porous medium, and (ii) specific organic inclusions composed of kerogen. These inclusions may have substantial porosity but, due to the nanoscale of pores, tubes, and channels, have extremely low permeability on the order of a nanodarcy ($\sim 10^{-21}$ m²) or less. These inclusions contain the majority of fluid: oil and gas. Our model is based on the hypothesis that the permeability of the inclusions substantially depends on the pressure gradient. At the beginning of the development of the deposit, boundary layers are formed at the boundaries of the low-permeable inclusions, where the permeability is strongly increased and intensive flow from inclusions to the matrix occurs. The resulting formulae for the production rate of the deposit are presented in explicit form. The formulae demonstrate that the production rate of deposits decays with time following a power law whose exponent lies between $-1/2$ and -1 . Processing of experimental data obtained from various oil and gas deposits in shales demonstrated an instructive agreement with the prediction of the model.

The US Energy Information Administration estimates that American shale gas resources comprise over 20 trillion cubic meters, which is approximately one-third of the total natural gas reserve (1). China, Canada, Saudi Arabia, Germany, and Australia, among other countries, are also developing strong programs to study and explore the extraction of shale gas, which has led to a massive effort to study the mineralogy of shales. The challenges facing the mathematical modeling of gas flow in shales are insightfully discussed in the work of Silin and Kneafsey (2), who also provided contributions on the multiscale characterization of the structure of shales, ranging from the nanoscale up to the scale of a deposit. Improvements in the mathematical modeling of the flow of fluids and gas in nanoporous geomaterials can create a new branch of subterranean fluid mechanics.

Shales are fine-grained sedimentary rocks containing high-volume fractions of clay minerals and organic matter, including kerogen, which is resistant to hydrochloric and hydrofluoric acids (3). Although there is agreement on the importance of kerogen in the generation of gas in shales, no mathematical model has explicitly incorporated them in the formulation. Therefore, large-scale development of oil and gas deposits in shales, which started in the past several decades, requires a principally new mathematical model of the flow of oil and gas in these rocks. Loucks et al. (4) reported that most of the nanopores in the Barnett shale exist in the discrete grains of the organic matter; additional nanopores were observed in the bedding plane-parallel wisps of organic-rich matter and in association with extremely fine-grained matrix material. A 30–40% porosity in kerogen has been reported for

Barnett shale (2). For the Fayetteville shale at a depth of 700 m, the porosity in kerogen was 40–50%, with nanopores in the range of 10–50 nm (5). The nanoscale size of pores and channels in shales leads to extremely low permeability on the order of a nanodarcy ($\sim 10^{-21}$ m²). This excludes the practical application of classic models of subterranean fluid dynamics, which were always the basis of the technology to develop oil and gas deposits.

In the present report, such a model is presented based on the assumption of a specific nanostructural transformation of rocks under the action of a large pressure gradient. The basic concepts of the model are presented, and a simple example of the flow in a nanostructural filter is considered in detail. The mathematical equations and their solutions are presented and discussed. After that, more sophisticated models of oil and gas flow in the development of deposits in nanoporous rocks are given. The basic result following from the mathematical model, power law decay of production with time, is confirmed by comparison with the data obtained from shale deposits.

Basic Model of the Unsteady Flow in Nanoporous Rock

The equation of mass conservation in a permeable medium is taken in its usual form as

$$\phi \partial_t \rho + \nabla \cdot \mathbf{j} = 0. \quad [1]$$

Here, ρ is the fluid density; ϕ is the porosity, which can be substantial (i.e., 0.3–0.4); $\mathbf{j}$ is the flux density; and t is time. Eq. 1 is complemented by the expression for the flux

$$\mathbf{j} = -\frac{k\rho}{\mu} \nabla p, \quad [2]$$

where p is the pressure and μ is the dynamic fluid viscosity. We assume that the viscosity and porosity are constant. We emphasize that Eq. 2 is not yet the Darcy law, but the introduction of a new rock characteristic, the permeability k . Eq. 2 becomes the ordinary or a generalized Darcy law only when the permeability is defined explicitly as a material property of the rock. For nanoporous rocks, this is not the case, as we will discuss later.

From Eqs. 1 and 2, the relation

$$\phi \partial_t \rho = \nabla \cdot \left(\frac{k\rho}{\mu} \nabla p \right) \quad [3]$$

is obtained. At the beginning of the development of the deposit, the pressure in the matrix decreases rather rapidly; however, due to the extremely low permeability of the inclusions, there appears to be a sharp pressure difference at the inclusion boundaries (Fig. 1). The pressure gradients at close vicinity to

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By making the substitution $\xi^2 = \xi_0^2 q$, the integral can be simplified to give

$$1 = \left(\frac{m}{2(m+1)(m+2)} \right)^{1/m} \int_0^1 \xi_0^{2/m} (1-q)^{1/m} \frac{\xi_0}{2\sqrt{q}} dq \quad [19]$$

$$= \left(\frac{m \xi_0^{m+2}}{2^{m+1}(m+1)(m+2)} \right)^{1/m} B\left(\frac{1}{2}, 1 + \frac{1}{m}\right)$$

where B is the Euler β -function; therefore,

$$\xi_0 = \left(\frac{2(m+1)(m+2)}{m} \right)^{\frac{1}{m+2}} \left[\frac{1}{2} B\left(\frac{1}{2}, 1 + \frac{1}{m}\right) \right]^{-\frac{m}{m+2}}. \quad [20]$$

After $t = t_l$, the stabilization of the filtering begins; this is not a self-similar process. It was computed numerically for some typical values of m , and it was demonstrated that the pressure gradient becomes constant inside the filter with 1% accuracy at $\tau = 0.308, 0.157, 0.101$ for the cases of $m = 0.5, 1.5, 2.5$, respectively. As can be seen from Eq. 15, the time of stabilization of the filtration in the nanoporous filter grows significantly with its thickness.

Oil and Gas Flow in Shales

According to our basic model, we assume that starting from the moment $t = t_0$ of the beginning of the development of the deposit, the inclusions in the rocks are enveloped by the boundary layers of elevated permeability (Fig. 1) through which the fluid is flowing out of the inclusions.

The boundary layers are thin; thus, the flow in these layers can be assumed to be 1D as we explained previously. The pressure p_0 in the matrix is varying slowly; thus, we come to Eqs. 7 and 8, respectively, for the oil and gas flows. The boundary conditions are

$$p(0, t) = p_0, \quad p(\infty, t) = P. \quad [21]$$

Taking the condition at $x = \infty$ is justified by the same argument as the general boundary layer arguments for viscous flows. Therefore, the pressure distribution across the boundary layer in weakly compressible oil flow is described by the same self-similar solution as in the case of the filter, such that

$$p = Pf(\xi), \quad \xi = \frac{x}{[\chi(t - t_0)Pm]^{1/(m+2)}}, \quad [22]$$

where $f(\xi)$ satisfies the same equation (Eq. 13), but with different boundary conditions

$$f(0) = \frac{P_0}{P} = \sigma, \quad f(\infty) = 1. \quad [23]$$

In Fig. 4, results of the numerical computation of the function f and its derivative $df/d\xi$ for the case $\sigma = 0.9$ are shown. As we see, in this case, the function f also has compact support, which can also be proved analytically.

The obtained solution gives the possibility of evaluating the most interesting quantity: the production rate per unit of area, the instantaneous flow from the inclusions to the matrix. We obtain

$$Q = S j_n$$

$$= S \rho_0 u_n = \frac{S \rho_0}{\mu} \left(\frac{df(0)}{d\xi} \right)^{m+1} \frac{P^{m+1}}{[\chi(t - t_0)Pm]^{(m+1)/(m+2)}} \quad [24]$$

$$= \frac{S A \rho_0}{\mu} P^{\frac{2(m+1)}{m+2}} [\chi(t - t_0)]^{\frac{m+1}{m+2}} \left(\frac{df(0)}{d\xi} \right)^{m+1}.$$

Here, S is the specific surface of the inclusion: the surface of the inclusions per unit of volume of the rock.

A similar consideration is appropriate for the gas flow. In this case, the basic equation for the pressure in the boundary layer is Eq. 8 and the boundary conditions are

$$p(0, t) = p_0, \quad p(\infty, t) = P. \quad [25]$$

The solution to Eq. 8 with the boundary conditions in Eq. 25 is also self-similar, although the independent variable is different, given by

$$p = Pf(\xi), \quad \xi = \frac{x}{[\kappa(t - t_0)Pm+1]^{1/(m+2)}}, \quad [26]$$

where the function f in this case satisfies a different differential equation:

$$\frac{\xi}{m+2} \frac{df}{d\xi} + \frac{d}{d\xi} \left[f \left(\frac{df}{d\xi} \right)^{m+1} \right] = 0. \quad [27]$$

The boundary conditions remain the same as in Eq. 23. The boundary value problem of Eqs. 23 and 27 was solved numerically, and the results are presented in Fig. 5A. An interesting observation followed from these calculations (Fig. 5B), which

Fig. 5. (A) Profiles of $df/d\xi'$ for the gas extraction problem for the case of $\sigma = 0.9$ for several different values of m . (B) Profiles of the rescaled permeability show a universality in behavior for different values of m .