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Multiple-hit parameter estimation in monolithic detectors

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Abstract

We examine a maximum-a-priori (MAP) method for estimating the primary interaction position of gamma rays with multiple-interaction sites (hits) in a monolithic detector. In assessing the performance of a multiple-hit estimator over that of a conventional one-hit estimator, we consider a few different detector and readout configurations of a 50-mm-wide square LSO block. For this study, we use simulated data from SCOUT, a Monte-Carlo tool for photon tracking and modeling scintillation-camera output. With this tool, we determine estimate bias and variance for a multiple-hit estimator and compare these with similar metrics for a conventional ML estimator, which assumes full energy deposition in one hit. We also examine the effect of event filtering on these metrics; for this purpose, we use a likelihood threshold to reject signals that are not likely to have been produced under the assumed likelihood model.

Depending on detector design, we observe a 1–12% improvement of intrinsic resolution for a 1-or-2-hit estimator as compared with a 1-hit estimator. We also observe improved differentiation of photopeak events using a 1-or-2-hit estimator as compared with the 1-hit estimator; more than 6% of photopeak events that were rejected by likelihood filtering for the 1-hit estimator were accurately identified as photo peak events and positioned without loss of resolution by a 1-or-2-hit estimator.

I. Introduction

Much of single-photon and positron emission tomography (SPECT and PET) is presently done under the assumption that photons deposit all their energy in one location (hit). To the contrary, many photon-detector interactions result in scatter or secondary emission that result in multiple hits. For PET, in particular, multiple-hit interactions represent the norm. If ignored, multiple hits can blur or bias our estimate of the first-hit position (Fig. 1).

A gamma ray deposits its energy by one of several mechanisms; for sub-MeV photons, energy deposition occurs predominantly via photoelectric and Compton interactions. In the case of Compton scatter, the deposition of incident gamma-ray energy can occur in more than one location. The direction and energy of the scattered gamma ray distributes itself

according to the Compton and Klein-Nishina equations; depending on its characteristic pathlength and detector geometry, the scattered photon may re-interact or escape from the detector (see Fig. 2). Characteristic X-rays and Auger electrons can also play a part in secondary interactions, but their mean free path (often $<100\text{ }\mu\text{m}$) is usually negligible compared to the detector resolution. At present, we consider multiple hits just due to Compton scatter and the photoelectric effect.

Noteworthy positioning methods that do not make the one-hit assumption are [1] the Compton camera, [2,3] Bayesian methods for determining interaction sequence in a system of arrayed-crystal detectors, and [4,5] methods to determine interaction sequence by minimizing the sum of the squared difference of scatter-angle cosines. For these methods, signals from multiple interactions are assumed to be separable, either occurring in isolated detector blocks (i.e. Compton camera), in decoupled detector elements (i.e. discrete crystal arrays), or in well-separated regions of the detector volume. However, our continued interest in and work with monolithic detectors prompts us to examine the more general case of estimating first-hit position when multiple-hit signals are multiplexed by shared detector readouts.

II. Methods

We examine a maximum-a-priori (MAP) method for estimating the first-hit position and incident-photon energy for multiple-hit events. We demonstrate this method in simulation for monolithic detectors and compare estimate bias, estimate variance, and gamma detection efficiency of these methods to that of the conventional approach, which assumes full energy deposition in a single hit.

Note that our focus is on PET imaging. However, estimation of first-hit parameters for SPECT represents a simplified case of the PET example considered here; prior knowledge of the geometric collimator can be used in lieu of additional parameters for a coincident photon.

A. One-hit estimator

We compare results for multiple-hit estimation to results for a MAP estimator of 3D position for a one-hit assumption. Signal centroiding (Anger estimation) is traditionally used to estimate one-hit 2D position. However, centroiding is significantly biased near the detector periphery. Here, we aim to assess the benefit of multiple-hit estimation over just the one-hit assumption.

Escaped secondaries and multiple-hit events can result in signals that are not well represented by a one-hit probability model. Therefore, a threshold on the likelihood of a one-hit ML estimate can be used to reject unlikely events (section 2F). However, any improvement to estimate variance and/or bias resulting from event filtering is at the expense of gamma-ray detection efficiency.

B. Two-hit estimator

The pertinent parameters we are considering for PET are the first-hit 3D-positions, $\mathbf{R}_{j1}$, and the number of hits, K_j , for each of a pair of coincident photons, $j \in \{1,2\}$; the incident-photon energies, ϵ_j , are assumed to be fixed. All other multiple-hit parameters we term *nuisance* parameters. Two methods for dealing with nuisance parameters are: A) to estimate them along with the desired parameters and then discard their result or B) to marginalize their effect on the probability model. Further information on nuisance parameters is found in [3]. Either method, A or B, can be accomplished with or without prior information for the distribution of estimate parameters.

In each case above, we begin by defining the signal-probability model for a coincidence pair of multiple-hit events, $\text{pr}(\mathbf{G}|\boldsymbol{\theta})$, where $\mathbf{G} = \{\mathbf{g}_1, \mathbf{g}_2\}$ are the observed signals and $\boldsymbol{\theta} = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2\}$ are the dependent parameters; we use bold-faced characters here and throughout to denote vectors. The vector $\mathbf{g}_j = \{g_{j1}, g_{jM_j}\}$ is the set of the M_j signals for photon j . The vector $\boldsymbol{\theta}_j = \{\varepsilon_n, K_j, R_{j1}, \dots, R_{jK_j}, E_{jK_j}\}$ is the set of dependent parameters associated with photon j . Here, ε_n is incident-photon energy, K_j is the number of hits, $\{R_{j1}, \dots, R_{jK_j}\}$ are the 3D hit positions, and E_{jK_j} is the deposited energy of the final hit. Energies deposited at $k < K_j$ are constrained by the Compton Equation given $\boldsymbol{\theta}_j$ and the incident direction $\Delta\mathbf{R}_j \equiv (\mathbf{R}_{j1} - \mathbf{R}_{j'1})$, where both $\{j, j'\} \in \{1, 2\}$ and $j \neq j'$. The energy incident upon the k^{th} -hit location, $\varepsilon_{j(k-1)}$, is thus a function of $\varepsilon_n > \Delta\mathbf{R}_j$ and the preceding hit positions, $\{\mathbf{R}_{j1}, \dots, \mathbf{R}_{j(k-1)}\}$.

The probability model for a pair of coincidence photons, $\text{pr}(\mathbf{G}|\boldsymbol{\theta})$, can be factored as $\text{pr}(\mathbf{g}_1|\boldsymbol{\theta}_1, \Delta\mathbf{R}_1)\text{pr}(\mathbf{g}_2|\boldsymbol{\theta}_2, \Delta\mathbf{R}_2)$, where:

$$\text{pr}(\mathbf{g}_j|\boldsymbol{\theta}_j, \Delta\mathbf{R}_j) = \int_0^{g_j} d^{M_j} g_{jK_j} \dots \int_0^{g_j} d^{M_j} g_{j2} \delta(g_j - \sum_{k=1}^{K_j} g_{jk}) \prod_{k=1}^{K_j} \text{pr}(g_{jk}|\mathbf{R}_{jk}, E_{jk}). \quad (\text{Eq.1})$$

We assume here that light from each hit independently contributes to the resultant signals:

$g_j = \sum_{k=1}^{K_j} g_{jk}$ the Dirac delta, $\delta(g_j - \sum_{k=1}^{K_j} g_{jk})$, in Eq. 1 enforces this constraint on the integral limits. Note that the term $\text{pr}(g_{jk}|\mathbf{R}_{jk}, E_{jk})$ here is just the one-hit signal-probability model, which can be experimentally calibrated [4].

The ML estimate of $\boldsymbol{\theta}$ given $\mathbf{G}$ is found by maximizing $\mathcal{L}(\boldsymbol{\theta}) = \text{pr}(\mathbf{G}|\boldsymbol{\theta})$. A MAP estimate of $\boldsymbol{\theta}$ is found by maximizing $\mathcal{L}(\boldsymbol{\theta})\text{pr}(\boldsymbol{\theta})$, where $\text{pr}(\boldsymbol{\theta})$ is our *prior* (prior knowledge for the distribution) of $\boldsymbol{\theta}$. Our priors include the Klein-Nishina distribution, the Beer-Lambert attenuation equation, and energy-dependent branching ratios for Compton and photoelectric interactions (section 2D). To estimate just a subset of parameters, $\boldsymbol{\theta}_{\text{keep}}$ we can maximize the integral of either $\mathcal{L}(\boldsymbol{\theta})$ or $\mathcal{L}(\boldsymbol{\theta})\text{pr}(\boldsymbol{\theta})$ over $\boldsymbol{\theta}_{\text{nuis}}$, where $\boldsymbol{\theta} = \{\boldsymbol{\theta}_{\text{nuis}}, \boldsymbol{\theta}_{\text{keep}}\}$

Again, using a likelihood threshold, we can filter events such as escaped secondaries that are not likely to have been produced under the assumed probability model (section 2F).

C. Practical application

As a practical example of multi-hit estimation for monolithic detectors, we consider an estimator of up to two (one or two) hits per photon with no escaped secondaries (i.e. photopeak events). This choice is made as a trade-off between computational complexity (3 additional search parameters per hit per photon) and the fraction of accurately modeled photopeak events (i.e. Fig. 2).

To further reduce the volume of parameter space that we search in PET, we can separate the search for the parameters of the two coincident photons; to do so, we use an initial estimate of the incident direction generated from a ML 1-hit position estimate. As is often the case, if the detector-block separation is large compared to the mean-free pathlength of photon scatter within a detector, the error in this initial estimate of the incidence angle will tend to be small. The incident direction and the final line-of-response estimate can then be iteratively improved after the multiple-hit estimation procedure. Subsequently, a localized simultaneous search for both coincident-photon parameters can be performed. In this work, we focus on multiple-hit parameter estimation given the incident direction.

As described in [7] and in section II of [8], the input to photodetectors of a monolithic detector is a low-efficiency binomial-selection process. Assuming low-noise amplification

and electronic readout (as we do in this simulation study), the joint probability density of the resulting output signals are well described by a multivariate scaled Poisson:

$$\text{pr}(\mathbf{g}|\mathbf{R}, E) = \prod_{m=1}^M \mathcal{P}(N_m|\bar{N}_m) \equiv \prod_{m=1}^M \frac{\bar{N}_m^{N_m} e^{-\bar{N}_m}}{N_m!}, \quad (\text{Eq.2})$$

where $N_m \equiv \text{round}(g_m/A_m)$, $\bar{N}_m \equiv \bar{g}_m(\mathbf{R}, E)/A_m$, and A_m is the m^{th} -channel gain. With this one-hit signal-probability model, the signal probability model for a superposition of hits (Eq. 1) then becomes:

$$\text{pr}(\mathbf{g}_j|\theta_j, \Delta\mathbf{R}_j) = \prod_{m=1}^{M_j} \mathcal{P}(N_{jm}|\sum_{k=1}^{K_j} \bar{N}_{jm}(\mathbf{R}_{jk}, E_{jk})). \quad (\text{Eq.3})$$

D. Prior density: $\text{pr}(\Theta)$

We generate a maximum-a-priori estimate using knowledge of the underlying physical processes (Fig. 3). In this work, we will discretize the parameter space. So, we express the prior as a probability of the parameter falling within a small volume about a discrete coordinate (i.e. $\pm \delta\mathbf{V}_k$ about a discrete coordinate, $\mathbf{R}_k$, for the k^{th} hit).

For a one-hit estimator, we weight the estimate likelihood by the probability of the primary interaction voxel given the location of a coincidence event and given that an interaction has occurred (i.e. the prior is normalized over the volume of the detector). The one-hit prior distribution is thus:

$$\text{Pr}_{\text{PE}}(\mathbf{R}_1 \pm \delta\mathbf{V}_1|\mathbf{R}_X, E, \text{hit}) = \frac{f_{\text{PE}}(E) \text{Pr}_{\mu}(\Delta R_{10} \pm \delta R_1|E)}{\text{Pr}(\text{hit}|\mathbf{R}_X, E)}, \quad (\text{Eq.4})$$

where $f_{\text{PE}}(E)$ is the energy-dependent photoelectric branching ratio, $\text{Pr}_{\mu}(\Delta R_{10} \pm \delta R_1|E)$ is the integral of total attenuation at an energy E over the interval $[\Delta R_{10} - \delta R_1, \Delta R_{10} + \delta R_1]$, $\text{Pr}(\text{hit}|\mathbf{R}_X, E)$ is the total probability of a hit given the incident energy, E , and the coincident photon coordinate, $\mathbf{R}_X$. For this purpose, we used attenuation-coefficient data for LSO from NIST [9].

The prior distribution for the two-hit MAP estimate has a branching ratio of $f_{\text{C}}(E) = 1 - f_{\text{PE}}(E)$ and has additional factors associated with the scatter distribution and attenuation of the second interaction. We assume here that the second interaction is photoelectric, such that the total incident energy is deposited. The two-hit prior distribution can be written as:

$$\text{Pr}_{\text{C} \rightarrow \text{PE}}(\mathbf{R}_1 \pm \delta\mathbf{V}_1, \mathbf{R}_2 \pm \delta\mathbf{V}_2|\mathbf{R}_X, E, \text{hit}) = \text{Pr}_{\text{C}}(\mathbf{R}_1 \pm \delta\mathbf{V}_1|\mathbf{R}_X, E, \text{hit}) \text{Pr}_{\text{KN}\mu\text{PE}}(\Theta, \Delta R_{21}|E), \quad (\text{Eq. 5})$$

where

$$\text{Pr}_{\text{C}}(\mathbf{R}_1 \pm \delta\mathbf{V}_1|\mathbf{R}_X, E, \text{hit}) = \frac{f_{\text{C}}(E) \text{Pr}_{\mu}(\Delta R_{10} \pm \delta R_1|E)}{\text{Pr}(\text{hit}|\mathbf{R}_X, E)}, \quad (\text{Eq.6})$$

and

$$\Pr_{\text{KN}\mu\text{PE}}(\boldsymbol{\theta}, \Delta R_{21}|E) = \iiint_{R_2 + \delta V_2} r^2 dr d\omega \Pr_{\text{KN}}(\vartheta|E) \Pr_{\mu}(\Delta r_{21}|E - E_1(\vartheta)) f_{\text{PE}}(E - E_1(\vartheta)). \quad (\text{Eq. 7})$$

The volume integral about the secondary position (Eq. 7) is over a voxel ($R_2 + \delta V_2$) in spherical-polar coordinates (r , ϑ , and ϕ , where $d\omega = \sin(\vartheta) d\vartheta d\phi$); (Eq. 7) accounts for the scatter-angle, ϑ , probability density (Klein-Nishina equation), for the total-attenuation probability density, and for the photoelectric cross-section of the secondary. Energy of the secondary, $E - E_1(\vartheta)$, is constrained by the Compton equation.

E. One-hit or two-hit classification

To accurately evaluate the signal likelihood, we attempt to classify events by their number of hits (one or two). We do so, by integrating the two-hit a-priori-weighted likelihood over the secondary parameters about the two-hit MAP estimate of the primary parameters. We then determine the a-priori-weighted likelihood of one-hit MAP estimate and choose the interaction sequence that is most likely.

F. Event filtering

Events that do not conform to the assumed signal-probability model may be subject to significant estimate error. We therefore conduct event filtering by applying a threshold to the prior-weighted likelihood of estimates. A common threshold is applied to one-hit and two-hit estimates. In practice, this threshold can be selected to optimize a given imaging task. For this study, we select this threshold to minimize primary-position estimate variance for the 1-hit estimator.

G. Simulation tool and detector model

We use an in-house Monte-Carlo tool for modeling Scintillation Camera OUTPUT named SCOUT [10], which includes high-energy-photon and optical-photon tracking. For comparing multiple-hit and one-hit estimation methods, we consider a $50 \times 50 \text{ mm}^2$ monolithic scintillation detector with single-ended or double-ended readout that are similar to Hamamatsu H8500 and H9500 multi-anode PMTs. Table 1 is a summary the detector configurations used in this work; we examine excursions of a baseline configuration to see how design parameters would affect the comparison of single-hit and multi-hit estimators.

To calibrate the likelihood model, we measure the single-hit mean detector response as a function of the 3D hit position. To do so, we use SCOUT to simulate 2,000 photoelectric interactions at each location on a 3D grid over the detector volume with 1-mm spacing. This calibration was repeated for each detector configuration. The resulting mean detector response function (MDRF) was smoothed (separately for each channel) using a locally fitted 2nd-order 3D polynomial with adaptive window size. The window width was increased from 3 pixels in each dimension near the peak mean response to a maximum of 5 pixels at an eighth of the MDRF peak. The MDRF was then interpolated at 0.2-mm-spaced grid points using a tri-cubic spline. Further interpolation is used to determine the MDRF for secondary-hit positions, which are discretized on a polar grid about the primary position.

To examine ensemble statistics (variance, bias, etc.) of the estimation methods considered, we consider primary 511-keV interactions forced to be at the detector center with a Compton-to-photoelectric branching ratio appropriate for the scintillator considered [9]. We compare estimator performance using an ensemble of 40,000 511-keV photons for detector configuration at normal angle of incidence.

H. MAP estimate search strategy

To find parameters for a MAP estimate, we use a *contracting grid* search algorithm devised by [13]. A 3D search is performed for the one-hit estimator. For the two-hit estimator, we also perform a 3D contracting-grid of the primary hit position. However, at each primary test position, we determine the MAP likelihood of the secondary position for the given primary position using a nested 3D contracting grid search (Fig. 4). After finding the best secondary position for each primary position at one scale, we then contract the primary test grid about this current best point.

Prior-weighted likelihood functions we have examined appear to be smoothly varying with a well-defined global maximum (Fig. 5). Occasionally, for low-probability events, small local maxima can appear at the boundary of the explored parameter space due to larger signal variance in this region. However these regions do not appear to attract significant attention by this multi-scaled search method. Of 10K randomly sampled contracting-grid searches performed, an exhaustive search yielded nearly the same solution in every case with a mean error of 0.05 pixels (Fig. 6).

III. Results

The process of 1-hit vs. 2-hit event classification for the multi-hit estimator is depicted in Fig. 7 for one of the detector configurations considered. Accuracy of this event classification is summarized in Table 2. Use of a likelihood threshold and its relation to the deposited energy and interaction type are given in Fig. 8 and Fig. 9. Cross-sectional views of the 3D position-estimate error for the primary interaction are given for two detector configurations in Fig. 10 and Fig. 11. Finally, a summary of the estimate resolution (FWHM and FWTM) and estimate bias for the first-hit 3D- position estimate for each of the detector configurations considered are given in Table 3.

IV. Discussion

We have demonstrated a method of multiple-hit estimation in a monolithic detector. We estimated 1-hit parameters and also jointly estimated parameters for 2-hit parameters by maximizing the prior-weighted likelihood. We then classified events as one-hit or two-hit events by likelihood comparison. We used likelihood windowing to filter unaccounted event sequences (i.e. escaped secondaries).

Slight improvement to resolution (detector FWHM) is observed. More modest improvement is observed in contrast (detector FWTM). The relative error of 2-hit vs. 1-hit estimates was most affected by improving signal-to-noise (e.g. PDE) and less so by photodetector spacing, dual-sided readout, and decreasing detector thickness (in that order).

Little bias is observed perpendicular to the incident direction either for 1 or 2 hit estimates. There is significant bias in the direction parallel to the incident direction, which is reduced by the 1-or-2-hit estimator relative to the 1-hit estimator. However, this type of bias does not cause parallax, but may slightly skew timing estimates.

The likelihood of 1-or-2-hit estimates is well separated from escape events, in contrast to the 1-hit estimates. About 6–7% of photopeak events were rejected for the 1-hit estimator.

This modest improvement comes at the expense of increased processing needs. Computational requirements for the 1-or-2-hit estimator are nearly 500-fold greater than the 1-hit estimator. However, recent improvements in processor technologies (e.g. GPU) make this level of processing feasible for real-time applications.

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Fig. 1.

Significant bias can result in a line-of-response (LOR) estimate if we falsely assume a one-hit interaction. The energy-weighted centroid (R_{COM}) is confused for the primary interaction position (R_1) in this case. Detector resolution is blurred for an ensemble of such event. If we filter events based on likelihood of the observed signals under an assumed one-hit signal-probability model, then scatter might be filtered, effectively reducing gamma detection efficiency.

Fig. 2.

Branching ratios for 511-keV photons at normal incidence to various thicknesses of a 50×50 mm² Lu₂SiO₅ (LSO). The fraction of undetected photons is computed from NIST attenuation data. Other branching ratios were determined by Monte-Carlo simulation using 10⁵ simulated interactions for each detector thickness. For compactness, Compton is abbreviated as *Comp*, escaped secondary as *Esc*, and photoelectric as *PE*.

Fig. 3.
Illustration of a-priori densities for 1-hit and 2-hit interaction events.

Fig. 4.

Contracting grid search of the secondary position for a given primary position. We use a Cartesian search pattern to begin and search over a finer polar grid about the finest Cartesian estimate. Cartesian test points are rounded to the nearest polar grid point and prior-weighted likelihood is integrated over a polar voxel.

Fig. 5. Polar cross section of likelihood (left) and log-likelihood (right) at a radius going through MAP estimate.

Fig. 6.

Distribution of contracting-grid estimate error compared with exhaustive search. This error is mitigated by expanding the number of search points in the last iteration of the contracting-grid search from 4 to 5 pixels wide in each dimension.

Fig. 7.

Scatter plot of 1-hit vs. 2-hit MAP log-likelihood for an ensemble of 40,000 normally incident 511-keV gammas with primary interaction at the middle of a 10-mm-thick LSO scintillator with back-face readout using 8×8 photodetectors with 35% photodetection efficiency.

Fig. 8.

Log-likelihood-threshold filtering for 1-hit estimator: (left) scatter plot of event likelihood vs. deposited energy, and (right) histogram of log-likelihood values MAP estimates for all 40K events. Results here are shown for the detector configuration with 8×8 photodetectors on the back face, 35% POE, and 10-mm LSO thickness. We constrain this threshold to be below the peak-likelihood distribution and select a value that minimizes primary-position estimate variance for the 1-hit estimator.

Fig. 9.

Log-likelihood-threshold filtering for 1-or-2-hit estimator. The same threshold is applied as for the 1-hit distribution. Better separability of multiple-hit events and escape events is observed than for the 1-hit estimator.

Fig. 10.

(Top) lateral cross-section and (bottom) transverse cross-sections of 3D-histograms for the first-hit position estimates of 40,000 events for a detector with 8×8 photodetectors on the back face, 35% POE, and 20-mm LSO thickness. Two-dimensional cross sections of estimate-error distribution are shown for (left) 1-hit estimator, and (middle) 1-or-2-hit estimator. A one-dimensional profile of each estimator is shown together at the (right). X and Y directions of the lateral distribution (termed $\Delta R_{\perp}$) were averaged. By contrast, the depth or Z direction is termed $\Delta R_{\parallel}$.

Fig. 11.

Comparable results to Fig. 10 for a detector configuration with 8x8 photodetectors on the back face, 35% POE, and 10-mm LSO thickness.

Table 1

Detector configuration parameters used for this simulation study. We used scintillator light-yield statistics from [11,12].

Scintillator: LSO	Readout side: <u>Back</u> , Front&Back
Area: 50×50 mm ²	Channels/face: <u>8</u> ×8, 16×16
Thickness: 10, <u>20</u> mm	Fill factor: 90%
Side treatment: Lambertian	PDE: <u>35</u> %, 70%
Side absorption: 95%	Gain: 1
Interface treat.: Polished	Gain var.: 0
1-sided reflector: Lambertian	Dark current: 0

For parameters with more than one value, the underlined quantities are the baseline configuration and the second value represents an excursion that we have examined.

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Table 2

Classification accuracy.

PD array type	B8×8	B8×8	B16×16	F8×8+B8×8	B8×8
PD Efficiency [%]	35	35	35	35	70
Thickness [mm]	20	10	20	20	20
Pr ($\Lambda_1 > \Lambda_2$ 0C+PE)	74%	84%	73%	77%	81%
Pr ($\Lambda_2 > \Lambda_1$ 1C+PE)	55%	54%	56%	64%	70%
Pr ($\Lambda_2 > \Lambda_1$ > 1C+PE)	72%	75%	75%	85%	92%

The following definitions apply here:

$\Lambda_1 \equiv \log(1\text{-hit MAP likelihood});$	$\Lambda_2 \equiv \log(1\text{-or-2-hit MAP likelihood});$
#C $\equiv$ Number of Compton scatters;	PE $\equiv$ final photoelectric interaction

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Table 3

Estimate resolution and bias for two estimators.

Array Type	PDE %	Thick [mm]	$FWHM_{\perp}$ [mm]	$FWHM_{\parallel}$ [mm]	$FWTM_{\perp}$ [mm]	$FWTM_{\parallel}$ [mm]	Bias $_{\perp}$ [mm]	Bias $_{\parallel}$ [mm]	Filter loss of Photopeak events [%]
B8×8	35	20	0.78 [0.79]	0.76 [0.75]	1.52 [1.50]	1.62 [1.59]	-0.01 [-0.00]	+0.64 [+0.43]	7.1 [0.0]
B8×8	35	10	0.52 [0.51]	0.69 [0.63]	1.02 [0.98]	1.36 [1.22]	+0.00 [-0.01]	+0.19 [+0.17]	6.3 [0.0]
B16×16	35	20	0.70 [0.65]	0.73 [0.67]	1.49 [1.43]	1.60 [1.57]	+0.01 [+0.00]	+0.63 [+0.44]	6.8 [0.0]
F8×8 & B8×8	35	20	0.54 [0.52]	0.53 [0.51]	1.06 [1.02]	1.02 [0.99]	-0.01 [-0.01]	+0.64 [+0.34]	6.5 [0.0]
B8×8	70	20	0.39 [0.35]	0.59 [0.52]	0.70 [0.68]	0.94 [0.88]	+0.01 [-0.01]	+0.60 [+0.39]	6.2 [0.0]

Metrics are given for two directions: parallel ($\parallel$) and perpendicular ($\perp$) to the incident direction. Results for the 1-or-2-hit estimator are bracketed and those for the 1-hit estimator are the non-bracketed values. In both cases, we filter unlikely events using a threshold on estimate prior-weighted likelihood, which is set as described in subsection F of the methods section and in Figs. 8 and 9. Results shown are for normal (0°) incidence 511-keV photons with primary interaction at the 3D-center of the detector block for each detector configuration. Preliminary results for 45° -incidence angle (not shown here) indicate FWHM and FWTM values that roughly equal in the $\parallel$ and $\perp$ directions, which are also equal to the geometric average of the $\parallel$ and $\perp$ values for the normal-incidence case. Estimate bias for 45° -incidence angle is roughly the same as for normal incidence in both $\parallel$ and $\perp$ directions. We also conducted ten trials for the 20-mm thick, 35% PDE, and B8×8 case in order to quantify the uncertainty of our results; in this case, we find a 1-sigma uncertainty of 0.004 mm for FWHM, 0.009 mm for FWTM and 0.003 mm for bias.