Size Effects on Correlation Measures

ANA V. CORONADO and PEDRO CARPENA*

*Departamento de Física Aplicada II, E.T.S.I. de Telecomunicación, Universidad de Málaga,
29071 Málaga, Spain*

*(*Author for correspondence, e-mail: pcarpena@ctima.uma.es)*

Abstract. The detection and quantification of long-range correlations in time series is a fundamental tool to characterize the properties of different dynamical systems, and is applied in many different fields, including physics, biology or engineering. Due to the diversity of applications, many techniques for measuring correlations have been designed. Here, we study systematically the influence of the length of a time series on the results obtained from several techniques commonly used to detect and quantify long-range correlations: the autocorrelation analysis, Hurst's analysis, and detrended fluctuation analysis (DFA). Using the Fourier filtering method, we generate artificial time series with known and controlled long-range correlations and with a broad range of lengths, and apply on them the different correlation measures we have studied. Our results indicate that while the DFA method is practically unaffected by the length of the time series, and almost always provides accurate results, the results from Hurst's analysis and the autocorrelation analysis strongly depend on the length of the time series.

Key words: long-range correlations, time series analysis, brain dynamics, heartbeat correlations, DNA correlations

1. Introduction

Many physical and biological systems in which there exist a multiplicity of control mechanisms exhibit scale-invariant properties characterized by long-range power-law correlations in their outputs. Many examples can be mentioned: long-range correlations have been observed in DNA sequences [1, 2], in cardiac dynamics (measuring interbeat time intervals) [3–7], in human electroencephalographic fluctuations [8, 9], in human motor activity [10] and gait [11–13], in sensory receptors in neural systems [14], etc. Outside biophysical examples, long-range correlations are also present in meteorology [15–19], seismic signals [20], economics [21, 22] and even in the properties of music [23].

It is clearly important to properly characterize and quantify the strength of the long-range correlations, because they can unveil or explain the behavior found in such a gigantic diversity of data, or the hidden dynamics responsible for the observations. However, due to the diversity of fields in which long-range correlations appear, and also maybe due to the different academic background of the

researchers, it is not surprising that different methods have been designed and applied to quantify the strength of the correlations. In general, all the methods characterize the degree of correlations by a single parameter, called *correlation exponent*. In many cases, these different methods are considered as equivalent, and equations relating the different types of correlations exponents have been obtained. Thus, it is not unusual to find in the bibliography that, provided a correlation exponent is obtained from a particular method, the values of other correlation exponents are calculated and used by means of these equations.

In this paper, we study the effects of the length of the time series analyzed on the correlation exponents obtained from three different methods commonly used to measure correlations: the autocorrelation analysis, rescaled range analysis (or Hurst's analysis) and detrended fluctuation analysis (DFA). We show how size effects are very important in the first two methods, producing errors of up to 20% in the estimation of the corresponding correlation exponent. Thus, two straightforward conclusions can be derived: (i) DFA is the optimal method to measure long-range correlations, unless very large time series are available. In this case, size effects can be acceptably small. (ii) It is not appropriate to use equations relating different correlation exponents, because they can produce spurious results.

The paper is organized as follows: in Section 2, we explain how artificial long-range correlated time series are generated, which are used to test the different correlation measures. These measures are briefly described in Section 3, as well as the equations relating the different correlation exponents obtained from them. In Section 4, we discuss our results. Finally, we present our conclusions.

2. Generating Long-Range Correlated Time Series

In order to generate efficiently in a controlled way long-range correlated time series, we use the Makse algorithm [24], which in essence is an improved version of the Fourier filtering method. Briefly, the algorithm does the following: (i) generates a white noise in the frequency (f) domain; (ii) multiplies the white noise by a power law with the desired exponent β ; (iii) Fourier-transforms back into the time domain to obtain the correlated time series. In this way, by construction, the power spectrum (the squared modulus of the Fourier transform) of the time series is of the type

$$S(f) = \frac{1}{f^\beta}. \quad (1)$$

By choosing the exponent β in step (ii) of the algorithm, we control the strength of the long-range correlations of the time series, taking into account that $\beta = 0$ corresponds to the absence of correlations (white noise) and $\beta > 0$ corresponds to positive correlations. In Figure 1, we show three examples of time series $\{y_t\}$ generated with three different values of the correlation exponent β .

Figure 1. Examples of time series $\{y_t\}$ with different degrees of long-range correlations for a series length of $N = 2^{10}$.

In our numerical simulations (see Section 4), we consider a broad range of lengths (N) for the artificially generated time series (from $N = 2^8$ to $N = 2^{20}$) as well as a broad range of values of β (from $\beta = 0$, corresponding to uncorrelated noise, to $\beta = 2$, corresponding to the classical random walk). For any value of β and N , we generate a total number of $2^{25}/N$ different time series, in order to have a large enough sample to produce consistent average values of the correlation exponents provided by all the correlation measures. In addition, when we generate $2^{25}/N$ time series, on the one hand we consider the same number of data points for any N , and, on the other hand, we generate a larger number of time series for small N , where the dispersion of the measured correlations are expected to be larger.

3. Correlation Measures

We consider three of the most used methods to quantify correlations in time series, in order to compare the accuracy of the results obtained from them. These methods are briefly described below.

3.1. AUTOCORRELATION ANALYSIS

Given a discrete time series $\{y_t\}$ ($t = 1, \dots, N$), the autocorrelation function at distance τ , $C(\tau)$, is defined as:

$$C(\tau) = \frac{\langle y_t y_{t+\tau} \rangle - \langle y_t \rangle \langle y_{t+\tau} \rangle}{\langle (y_t - \langle y_t \rangle)^2 \rangle} \quad (2)$$

where $\langle \rangle$ denotes average values. This function characterizes the persistences or correlations in the time series, because it provides information of the time series at a time $t + \tau$, provided the time series at a time t . Random time series have a null correlation function, because the time series is not predictable at all. Time series with short-range correlations, like the ones derived from Markov processes, present an autocorrelation function which is basically a decaying exponential with a time constant, which defines the characteristic time scale of the process. In contrast, in time series with long-range correlations, the autocorrelation function behaves as a power law:

$$C(\tau) \propto \frac{1}{\tau^\gamma} \quad (3)$$

This power-law behavior indicates the lack of any characteristic scale (at least in the range where Eq. (3) is valid), and thus the scale invariance, or fractal behavior, of the time series. In addition, as a power law decreases slower than an exponential, these correlations are also known as long-range correlations. These correlations come from clear persistences in the time series, which makes it more ‘predictable’ than in the case of random or short-range correlations. A recent use of the autocorrelation function for DNA sequences can be seen in [25].

3.2. RESCALED RANGE ANALYSIS (HURST’S ANALYSIS)

Rescaled range analysis (or Hurst’s analysis) is another classical tool for exploring the existence of long-range correlations in a time series. It was firstly used by Hurst when he wanted to study the properties of the river Nile floods and the design of reservoirs [26, 27]. To apply Hurst’s analysis, we proceed as follows: giving a discrete time series $\{y_t\}$ ($t = 1, \dots, N$), we partition it into windows of size τ (the number of which depends on whether overlapping or not-overlapping windows are considered). The rescale range (R/S) at a time scale τ is calculated as follows: let us define the function $X(t, \tau)$ as

$$X(t, \tau) = \sum_{u=1}^t (y_u - \langle y \rangle_\tau) \quad (4)$$

where

$$\langle y \rangle_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} y_t \quad (5)$$

i.e., $X(t, \tau)$ represents the sum of the deviations of the time series with respect to the mean in the time period τ . Let us define the function $R(\tau)$ as

$$R(\tau) = \max_{1 \leq t \leq \tau} X(t, \tau) - \min_{1 \leq t \leq \tau} X(t, \tau) \quad (6)$$

i.e., $R(\tau)$ represents the range of variation of the sum of the deviations with respect to the mean in the time period τ . Let us call $S(\tau)$ to the standard deviation of the time series in the time period τ :

$$S(\tau) = \left(\frac{1}{\tau} \sum_{t=1}^{\tau} (y_t - \langle y \rangle_{\tau})^2 \right)^{1/2}. \quad (7)$$

Thus, we can study the behavior of the average value of $R(\tau)/S(\tau)$ (i.e., averaging over all the windows of size τ into which we divided the original time series) as a function of τ . Scale invariance appears if:

$$\left\langle \frac{R}{S}(\tau) \right\rangle \propto \tau^H. \quad (8)$$

The exponent H characterizes the long-range correlations of the time series. For uncorrelated (white noise) time series, $H = 1/2$, while $H > 1/2$ indicates positive long-range correlations (persistences) and $H < 1/2$ indicates anti-correlations (anti-persistences). Recent uses of Hurst's Analysis can be seen in [20, 28].

3.3. DETRENDED FLUCTUATION ANALYSIS

Detrended fluctuation analysis (DFA) is a scaling analysis method providing a simple quantitative parameter – the scaling exponent α – to represent the correlation properties of a signal. It was firstly introduced in [29]. To illustrate the DFA method, we consider a noisy time series, $\{y_t\}$ ($t = 1, \dots, N$). First, the time series $\{y_t\}$ is integrated:

$$Y(t) = \sum_{u=1}^t (y_u - \langle y \rangle) \quad (9)$$

where $\langle y \rangle$ is the global mean:

$$\langle y \rangle = \frac{1}{N} \sum_{t=1}^N y_t. \quad (10)$$

Second, the integrated time series is divided into boxes of equal length, τ . In each box, we fit the integrated time series by using a polynomial function (in our case, we consider a straight line) $Y_{fit}(t)$, which is called the local trend. The integrated time series, $Y(t)$, is detrended by subtracting the local trend $Y_{fit}(i)$ in each box and the detrended fluctuation function is obtained

$$\psi(t) = Y(t) - Y_{fit}(t). \quad (11)$$

For a given box size τ , the root mean square fluctuation is calculated

$$F(\tau) = \left(\frac{1}{N} \sum_{t=1}^N [\psi(t)]^2 \right)^{1/2}. \quad (12)$$

The above computation is repeated over different time scales (box size τ) to provide a relationship between $F(\tau)$ and τ . The power-law relation between $F(\tau)$ and the box size τ indicates the presence of scaling:

$$F(\tau) \sim \tau^\alpha. \quad (13)$$

The parameter α , called the scaling exponent, represents the correlation properties of the signal: if $\alpha = 0.5$, there is no correlation and the signal is an uncorrelated random series (white noise); if $\alpha < 0.5$, the signal presents anti-correlation, and if $\alpha > 0.5$, there are positive correlations.

The advantages of DFA over conventional methods (for example, Hurst's analysis) are that it permits the detection of long-range correlations embedded in seemingly non-stationary time series, and also avoids the spurious detection of apparent long-range correlations that are artifacts of non-stationarity. DFA has been used and studied in more than one hundred papers in recent years (see, for example, [30–32] and references therein).

3.4. RELATIONSHIPS BETWEEN THE DIFFERENT CORRELATION EXPONENTS

As we have shown above, the three tools described provide as a result an exponent (or correlation exponent) to quantify the strength of the correlations in the analyzed time series. Also, the method we use to generate the time series is controlled by another exponent (β), accounting for the correlations we impose on the time series. There exist theoretical calculations which relate all these exponents. In our case, we are interested in the relationships of all the exponents (γ , H and α) with β , because the latter is the one we control and use in the time series generation, and is our reference for comparison.

Concerning Hurst's exponent H , we have

$$H = \begin{cases} \frac{\beta + 1}{2} & \text{if } \beta < 1 \\ 1 & \text{if } \beta \geq 1 \end{cases} \quad (14)$$

where it can be seen how H saturates at $H = 1$.

Correspondingly, for the exponent γ of the autocorrelation function, we have

$$\gamma = \begin{cases} 1 - \beta & \text{if } \beta < 1 \\ 0 & \text{if } \beta \geq 1 \end{cases} \quad (15)$$

where again, we see how γ saturates at $\gamma = 0$.

With respect to the DFA exponent α , the relationship with β is the following:

$$\alpha = \frac{\beta + 1}{2}. \quad (16)$$

It can be seen how the relationships of H and α with β are identical (in the linear regime of H), the difference being that α does not saturate, at least in the range of β values we have considered ($0 < \beta < 2$), as we explain in the next section.

4. Results

4.1. SIZE EFFECTS ON THE AUTOCORRELATION FUNCTION

The autocorrelation function is known to be a not sufficiently accurate estimator of the correlations present in a time series, because it provides very noisy results. This is one of the reasons why, for example, for estimating correlations in DNA sequences, alternative techniques were developed when only short sequences were available in the public databases (see the discussion in ref. [25]). Another problem with autocorrelation analysis is that it can be only applied to stationary time series, while many signals in physics and biology exhibits non-stationarity, which can strongly affect the results.

We observe this inaccurate behavior of the autocorrelation function when analyzing our artificial time series. In particular, while the power spectrum of the artificially generated time series is a power law characterized by a correlation exponent β , the autocorrelation function of the time series we generate does not behave properly as a power law (and then there is no clear value of the exponent γ) for small lengths of the time series ($N < 2^{12}$) and for low values of the correlation exponent β ($\beta < 1/2$). Thus, in our numerical simulations, we have considered lengths in the range $2^{12} \leq N \leq 2^{20}$ and values of β in the range $1/2 < \beta < 2$. For any value of N and β , we have generated $2^{25}/N$ correlated time series. Once the series are generated, we calculate the autocorrelation function for any individual time series following Eq. (2), and fit it to a power law in the range $1 < \tau < 100$. Finally, we average to obtain the γ value corresponding to the values of β and N considered.

Our results are shown in Figure 2. In general, for any N , the behavior of γ as a function of β is monotonically decreasing, as expected. Nevertheless, we find strong deviations in the estimated values of γ compared to the theoretically expected values, calculated using Eq. (15) (solid line in Figure 2). These deviations lead to a systematic overestimation of γ . The behavior of γ as a function of β tends asymptotically from above to the expected theoretical value for increasing N . The deviations decrease in general for increasing time series length, showing clear size effects, and also depend on the specific values of β considered. While the deviations are small at both extrema of the correlation values (although still significant), in

Figure 2. Behavior of the correlation exponent γ of the autocorrelation function as a function of the exponent β for several time series lengths. The solid line represents the expected theoretical value of γ according to Eq. (15).

the range around $\beta = 1$ these deviations are very large indeed, even for large N . These findings suggest that one should avoid the use of the autocorrelation function exponent γ to characterize correlations in time series, unless very long time series with extreme long-range correlations (low or high) are available.

4.2. SIZE EFFECTS ON HURST'S ANALYSIS

To study possible size effects on the results (the exponent H) provided by Hurst's analysis, we follow a similar strategy to the previous case: we generate long-range correlated time series with lengths in the range $2^8 \leq N \leq 2^{20}$, and with values of β in the range $0 \leq \beta \leq 2$. For any value of N and β , we generate as before $2^{25}/N$ correlated time series. Once the series are generated, we perform Hurst's analysis in any individual time series, as explained in Section 3.2, to obtain $R/S(\tau)$. Then, we fit $R/S(\tau)$ vs. τ using a power law according to Eq. (8) to obtain the exponent H corresponding to the analyzed individual time series. The power-law fitting is performed in the range $4 < \tau < N/20$, where we check that there is good power-law behavior. Finally, we average to obtain the H value corresponding to the values of β and N considered.

The dependence of H as a function of β for different time series lengths N is shown in Figure 3. These results demonstrate also clear size effects in the determination of H , although of different character compared to the ones shown in the preceding section corresponding to the autocorrelation function exponent γ . In the present case, we observe that in the low range of β ($\beta < 1/2$) the exponent H is systematically overestimated, while in the range $\beta > 1/2$ we find that H is

Figure 3. Behavior of the correlation exponent H of Hurst's analysis as a function of the correlation exponent β for several time series lengths N . The solid line corresponds to the theoretical expectation according to Eq. (14).

always underestimated, compared to the theoretical expectation shown with a solid line in Figure 3 and obtained from Eq. (14). The deviations are larger for small (β around 0) and moderately large (β around 1) degree of correlations than for intermediate β values. Nevertheless, in general the values of Hurst's exponent H deviates substantially from the theoretically expected values, and could produce spurious results when analyzing time series: for example, for $\beta = 0$ (corresponding to purely random time series) Hurst's analysis provides, on average, H values of up to $H = 0.63$ for $N = 2^8$ or $H = 0.58$ for $N = 2^{12}$. These results would lead to the conclusion that long-range correlations exist in the time series, when actually the time series is randomly generated. In the case of larger correlations in the time series (around $\beta = 1$, corresponding to pure fractal behavior), the use of Hurst's analysis provides clear underestimations of the real correlations, which again may be a source of errors when characterizing the correlation properties of generic signals.

When increasing time series length N , the behavior of H as a function of β tends asymptotically to the theoretical expectation. This asymptotic approach takes place from above in the case of small β values, and from below in the case of large β values. Strikingly, there is a fixed point (marked with an arrow in Figure 3) for which the obtained value of H is the same *independently on the time series length* N . In addition, this value of H coincides with the correct one obtained from Eq. (14) using the corresponding value of β . The fixed point takes place at a correlation value very close to $\beta = 1/2$ (approximately, at $\beta \simeq 0.49$), where $H = 0.745$. Thus, for correlations close to this β value, Hurst's analysis provides practically correct results, because size effects disappear. This is not true in other correlation ranges. It is worth mentioning here that in Hurst's work [26, 27], when studying

Figure 4. Hurst's exponent H as a function of the time series length N for three different values of the correlation exponent β . The solid lines show the expected values of H according to Eq. (14). The case of $\beta = 0.49$ corresponds to the point marked with an arrow in Figure 3.

water level fluctuations in the Nile, he obtained $H = 0.77$. This value is practically correct, because it is very close to the fixed point, independently on the length of the time series analyzed. A similar statement can be formulated concerning the results presented in ref. [28], where $H = 0.76$ is obtained. We want also to point out that in [33] it is suggested that Hurst's analysis only provides correct results when $0.7 < H < 0.8$. This is in agreement with our results, because the fixed point is precisely at the center of this interval, and then size effects are small.

To show clearly the fixed point, as well as size effects in other correlation values, in Figure 4 we plot the behavior of H as a function of N for three values of β (the fixed point $\beta = 0.49$, $\beta = 0$ and $\beta = 1$). The solid lines correspond to the expected values of H according to Eq. (14).

4.3. SIZE EFFECTS ON DFA

Detrended fluctuations analysis quantifies correlations in time series providing the exponent α to characterize the correlations present in the analyzed signal. To study possible size effects on the exponent α , we follow a similar strategy as in the two previous cases: we generate long-range correlated time series with lengths in the range $2^8 \leq N \leq 2^{20}$, and with values of β in the range $0 \leq \beta \leq 2$. For any value of N and β , $2^{25}/N$ correlated time series are generated. Once the series are generated, we perform DFA in any individual time series as explained in Section 3.3 to obtain $F(\tau)$. Then, we fit $F(\tau)$ vs. τ using a power law according to Eq. (13) to obtain the exponent α corresponding to the analyzed individual time series. The power-law fitting is performed in the range $4 < \tau < N/10$, where we test the correct power-law behavior. Note that this range of scaling is larger than in the case

Figure 5. Behavior of DFA exponent α as a function of the correlation exponent β for varied lengths N of the time series. The solid line shows the theoretical expectation according to Eq. (16).

of Hurst's analysis, and also much larger than in the case of the autocorrelation function. Finally, we average to obtain the α value corresponding to the values of β and N considered.

We show our results in Figure 5. We find that size effects are practically negligible on average when using DFA to determine the correlations of a time series, because the results for all the time series lengths collapse on top of each other as far as $N \geq 2^{10}$. For smaller lengths (see the case $N = 2^8$ in Figure 5), we detect a small deviation of the estimated value of α (which is overestimated) with respect to all other N values. This happens only for low correlations (for $\beta < 0.5$), while this deviation disappears for higher values of β .

Not only size effects are practically absent, but also the results coincide with the theoretical expectation, obtained from Eq. (16), and shown with a solid line in Figure 5. These results indicate that DFA is the best method (compared to Hurst's analysis and the autocorrelation analysis) to estimate long-range correlations, because it provides accurate results almost independently on the length of the time series and on the strength of correlations present in the signal. The only size effect we are able to detect appears for extremely short time series ($N < 1000$), and only occurs for low values of β .

5. Conclusions

In this paper, we have studied how different correlation measures are affected by the length of the time series analyzed by generating artificial long-range correlated time

series of a broad range of lengths and a broad range of correlations. We find that the autocorrelation function is strongly affected by the length of the time series, which is reflected in a systematic overestimation of the correlation exponent γ . Although this effect decreases for increasing time series length, it still persists even for time series of length 2^{20} , being more critical at intermediate values of the correlations in the time series. Hurst's analysis also presents clear size effects, although of different nature: while the exponent H is systematically overestimated for a low degree of correlations, H is underestimated for a high degree of correlations in the time series. These deviations decrease for increasing time series length, but are still relevant for lengths of up to 2^{20} , especially for low and high degrees of correlations in the time series. Strikingly, we find a fixed point for the correlations in the time series for which size effects do not exist, and the H value obtained is always correct. Finally, we study DFA, and we find that this method is practically free of size effects, providing consistently correct results for all the range of time series lengths and correlations considered.

Acknowledgements

We acknowledge the Spanish Ministerio de Ciencia y Tecnología for financial support (Grant No. BFM2002-00183).

References

1. Peng, C.K. *et al.*: Long Range Correlations in Nucleotide Sequences, *Nature* **356** (1992), 168–171.
2. Carpena, P. *et al.*: Metal-Insulator Transition in Chains with Correlated Disorder, *Nature* **418** (2002), 955–959.
3. Ivanov, P.Ch. *et al.*: Scaling behaviour of Heartbeat Intervals Obtained by Wavelet-Based Time-Series Analysis, *Nature* **383** (1996), 323–327.
4. Ivanov, P.Ch. *et al.*: Sleep-Wake Differences in Scaling Behavior of the Human Heartbeat: Analysis of terrestrial and long-term space flight data, *Europhys. Lett.* **48** (1999), 594–600.
5. Ivanov, P.Ch. *et al.*: Multifractality in Human Heartbeat Dynamics, *Nature* **399** (1999), 461–465.
6. Kantelhardt, J.W., Havlin, S. and Ivanov, P.Ch.: Modeling Transient Correlations in Heartbeat Dynamics During Sleep, *Europhys. Lett.* **62** (2003), 147–153.
7. Bernaola-Galván, P. *et al.*: Scale Invariance in the Nonstationarity of Human Heart Rate, *Phys. Rev. Lett.* **87**, (2001), art. no. 168105.
8. Robinson, H.P.C. and Harsch, A.: Stages of Spike Time Variability During Neuronal Responses to Transient Inputs, *Phys. Rev. E* **66** (2002), art. no. 061902.
9. Sapir, N. *et al.*: Detecting Scaling in the Period Dynamics of Multimodal Signals: Application to Parkinsonian tremor, *Phys. Rev. E* **67** (2003), art. no. 031903.
10. Hu, K. *et al.*: Non-Random Fluctuations and Multi-Scale Dynamics Regulation of Human Activity, *Physica A* **337** (2004), 307–318.
11. Hausdorff, J.M. *et al.*: When Human Walking Becomes Random Walking: Fractal Analysis and Modeling of Gait Rhythm Fluctuations, *Physica A* **302** (2001), 138–147.
12. West, B.J. and Scafetta, N.: Nonlinear Dynamical Model of Human Gait, *Phys. Rev. E* **67** (2003), art. no. 051917.

13. Ashkenazy, Y. *et al.*: A Stochastic Model of Human Gait Dynamics, *Physica A* **316** (2002), 672–670.
14. Bahar, S. *et al.*: Long-Range Temporal Anti-Correlations in Paddlefish Electoreceptors, *Europhys. Lett.* **56** (2001), 454–460.
15. Koscielny-Bunde, E. *et al.*: Indication of a Universal Persistence Law Governing Atmospheric Variability, *Phys. Rev. Lett.* **81** (1998), 729–732.
16. Talkner, P. and Weber, R.O.: Power Spectrum and Detrended Fluctuation Analysis: Application to Daily Temperatures, *Phys. Rev. E* **62** (2000), 150–160.
17. Ausloos, M. and Ivanova, K.: Power-Law Correlations in the Southern-Oscillation-Index Fluctuations Characterizing El Nino, *Phys. Rev. E* **63** (2001), art. no. 047201.
18. Govindan, R.B. *et al.*, Long-Range Correlations and Trends in Global Climate Models: Comparison with Real Data, *Physica A* **294** (2001), 239–248.
19. Kiraly, A. and Janosi, I.M.: Stochastic Modeling of Daily Temperature Fluctuations, *Phys. Rev. E* **65** (2002), art. no. 051102.
20. Varotsos, P.A., Sarlis, V.M. and Skordas, E.S.: Long-Range Correlations in the Electric Signals that Precede Rupture: Further Investigations, *Phys. Rev. E* **67** (2003), art. no. 021109.
21. Vandewalle, N. and Ausloos, M.: Crossing of two Mobile Averages: A Method for Measuring the Roughness Exponent, *Phys. Rev. E* **58** (1998), 6832–6834.
22. Ivanov, P.Ch. *et al.*: Common Scaling Patterns in Intertrade Times of U.S. Stocks, *Phys. Rev. E* **69** (2004), art. no. 056107.
23. Jennings, H.D. *et al.*: Variance Fluctuations in Nonstationary Time Series: A Comparative Study of Music Genres, *Physica A* **336** (2004), 585–594.
24. Makse, H.A. *et al.*: Method for Generating Long-Range Correlations for Large Systems, *Phys. Rev. E* **53** (1996), 5445–5449.
25. Bernaola-Galván, P. *et al.*: Study of Statistical Correlations in DNA Sequences, *Gene* **300** (2002), 105–115.
26. Hurst, H.E.: Suggested Statistical Model of Some Time Series which Occur in Nature, *Nature* **180** (1957), 494.
27. H.E. Hurst, *Long-Term Storage: An Experimental Study* (Constable & Co. Ltd., London, 1965).
28. Peters, O., Hertlein, C. and Christensen, K.: A Complexity View of Rainfall, *Phys. Rev. Lett.* **88** (2002), art. no. 018701.
29. Peng, C.K. *et al.*: Mosaic Organization of DNA Nucleotides, *Phys. Rev. E* **49** (1994), 1685–1689.
30. Hu, K. *et al.*: Effect of Trends on Detrended Fluctuation Analysis, *Phys. Rev. E* **64** (2001), art. no. 011114.
31. Chen, Z. *et al.*: Effect of Nonstationarities on Detrended Fluctuation Analysis, *Phys. Rev. E* **65** (2002), art. no. 041107.
32. Chen, Z. *et al.*: Effect of Nonlinear Filters on Detrended Fluctuation Analysis, *Phys. Rev. E* **71** (2005), art. no. 011104.
33. Mandelbrot, B.B. and Wallis, J.R.: Some Long-Run Properties of Geophysical Records, *Water Resources Res.* **5** (1969), 321.