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Transfer Function Compensation in Gyroscope-free Inertial Measurement Units for Accurate Angular Motion Sensing

W. T. Latt,

Department of Computing, and the Hamlyn Center for Robotic Surgery, Imperial College London
(phone:+44 0 7542163363)

U-X. Tan,

School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore

C. N. Riviere [Member IEEE], and

Robotics Institute, Carnegie Mellon University, USA

W. T. Ang [Member, IEEE]

School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore

W. T. Latt: w.tun-latt@imperial.ac.uk

Abstract

Gyroscope-free inertial measurement units have been gaining popularity in applications such as motion sensing of hand-held microsurgical instruments. Various accelerometer placement configurations have been proposed in the past. However, the effect of non-identical transfer functions of accelerometers on the accuracy of angular motion sensing has been underestimated and never been taken into consideration. In this paper, significant effect of different phases at different accelerometer outputs due to non-identical transfer functions on the accuracy is highlighted, and a method is proposed to make the transfer functions of all the accelerometers identical. Effectiveness of the method is confirmed with an experiment using ADXL-203 accelerometers.

Index Terms

Phase different; accelerometer transfer function; angular motion

I. Introduction

Gyroscope-free inertial measurement units have been gaining popularity in applications such as sensing of microsurgical instrument motion [1][2]. Various accelerometer placement configurations have been proposed in the past. Two or more accelerometers are used to calculate a particular acceleration component. The accelerometers are assumed to have identical transfer functions. In practice, it is very rare to have identical transfer functions. Even if the same model of accelerometers is employed, there is slight variation in transfer function parameters mainly due to inaccuracy in manufacturing process [3]. Non-identical transfer functions of the accelerometers create phase differences among their outputs, causing error in calculated angular acceleration. The amount of phase difference can be more significant if different model accelerometers are employed [3][5][6].

Research activities have been focusing on placement configurations of accelerometers for achieving compactness, robustness, high accuracy and resolution, etc [1][2][4]. However, the effect of non-identical transfer functions on the accuracy of angular acceleration has never been discussed. In this paper, the effect on the accuracy is described. A method is

proposed to compensate for different transfer functions in order to achieve identical transfer functions, and hence obtaining high accuracy in angular motion sensing.

II. Methodology

A. General Transfer Function

The transfer function of i^{th} accelerometer in a group of accelerometers can be written in general form as follows:

$$G_{p_i}(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n} = \frac{B_{p_i}(s)}{A_{p_i}(s)} \quad (1)$$

where parameters a 's and b 's are constant and $m \leq n$. The parameters a 's and b 's, and m and n are different from one transducer to another. However, they can be known from experiments.

B. Compensation

To have the same parameters for different accelerometer transfer functions, a compensator transfer function $G_{c_i}(s)$ is proposed and cascaded to the transfer function of the i^{th} accelerometer as follows:

$$G_{T_i}(s) = G_{p_i}(s) G_{c_i}(s) \quad (2)$$

where $G_{T_i}(s)$ is the resultant transfer function of i^{th} accelerometer. The $G_{c_i}(s)$ is designed so that the $G_{T_i}(s)$ is independent of a 's and b 's, and is written in general form as:

$$G_{c_i}(s) = \frac{A_{p_i}(s)}{B_{p_i}(s)C(s)} \quad (3)$$

where $C(s) = c_0 s^k + c_1 s^{k-1} + \dots + c_{k-1} s + c_k$

By substituting (1) and (3) in (2), $G_{T_i}(s)$ becomes

$$G_{T_i}(s) = \frac{1}{C(s)} \quad (4)$$

As can be seen from (4), the resultant transfer function becomes the same for all the accelerometers. The parameters c 's, and the order k can be chosen to meet a particular application requirement, while at the same time fulfilling stability requirement [7].

III. Experiment and Results

To evaluate effectiveness of the compensation method, an experiment is conducted. A hand-held instrument consisting of ADXL-203 accelerometers is mounted on a device (nanopositioning system, PI, Germany) that can produce precise translational motion. Transfer functions of two accelerometers (a_1 and a_2) whose outputs are used to calculate an angular acceleration component are measured with experiments. The accelerometers have first-order transfer functions and they can be expressed as

$$G_{p_i}(s) = \frac{1}{1 + \tau_i s} \quad (5)$$

where constant parameter τ_i is the time-constant.

Using (5), $G_{c_i}(s)$ in (3) is designed with $C(s) = 1 + \tau s$ as follows

$$G_{c_i}(s) = \frac{1 + \tau_i s}{1 + \tau s} \quad (6)$$

Therefore, $G_{T_i}(s)$ becomes

$$G_{T_i}(s) = \frac{1}{1 + \tau s} \quad (7)$$

Time constant τ_i of each accelerometer transfer function is measured from their step response. The time-constants of transfer functions of a_1 and a_2 are found to be 2.5 ms and 4 ms, respectively. The experiment setup is shown in Fig. 1. The translational sinusoidal motion (50 μ m amplitude, 12 Hz) generated by device is experienced by the accelerometers. Acceleration outputs from the accelerometers and their difference are shown in Fig. 2. As can be seen from the figure, magnitude of the difference is significant (~15% of the signal) although the phase difference between the two outputs is not so obvious.

The angular acceleration α is calculated using the following formula.

$$\alpha = \frac{a_1 - a_2}{d} \quad (8)$$

where d is the separation distance between the accelerometers.

IV. Discussion

Since true angular motion is zero, angular acceleration calculated with the accelerometer outputs should be zero. However, there exists some error acceleration without compensation (Fig. 3(a)). The error is reduced with compensation (Fig. 3(b)). Without compensation, the higher the motion frequency, the larger the phase-shift, and the more inaccurate the calculated angular acceleration becomes. Therefore, transfer function compensation is necessary for application involving high-frequency motion such as motion due to physiological tremor [8]. Time-constant (τ) has to be chosen based on the application requirement. If high signal-to-noise ratio is desired, the time-constant has to be big enough to eliminate the noise adequately. If fast response is desired, the time-constant has to be very small so that the phase-lag is negligible.

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Fig. 1. The experiment setup showing (1) the motion generation device (Nanopositioning system, P561.3CD), (2) the hand-held microsurgical instrument consisting of ADXL-203 accelerometers, (3) direction of motion generated by the device, and (4) accelerometer locations in the instrument and their sensing directions

Fig. 2.
Acceleration outputs from the accelerometers, $a1$ and $a2$, and their difference.

Fig. 3. Angular acceleration error at 12 Hz (a) without compensation, and (b) with compensation. Time-constant used is 8 ms.