Statistical hydromechanics of the solar differential rotation

(astrophysics/turbulence theory)

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ABSTRACT An axisymmetric hydrodynamic model of the solar differential rotation is suggested that differs from the previous axisymmetric models (with arbitrarily given anisotropic viscosity) in the use of the hydrodynamic equations (the Friedmann–Keller equations) for the statistical second moments of the fluctuations of velocity and entropy—i.e., turbulent fluxes of momentum and entropy. The model gives a correct reproduction of the differential rotation (without unreal temperature differences on the Sun's surface peculiar to the anisotropic viscosity models) and makes possible calculations of the second moments.

Among the problems of the postwar developed geophysical hydrodynamics (i.e., hydrodynamics of natural flows of rotating baroclinic stratified fluids), particularly interesting are global problems devoted to the analysis of hydrodynamical processes on the scales of a whole given planet or a star. The first global problem of solar hydrodynamics [see, for instance, the reviews by Gilman (1), Durney (2), Howard (3), and Monin (4)] is the solar differential rotation—i.e., the observed increase of the angular velocity of the Sun's surface rotation from its poles toward its equator, amounting to nearly 40%. Such a behavior of a rotating gaseous ball left to itself requires, certainly, a hydro-mechanical explanation.

The differential rotation is accompanied, of course, by energy losses for overcoming turbulent viscosity resulting from small-scale convective motions in granules and supergranules. Meridional and radial angular momentum transport in the solar convection zone should be such a maintenance mechanism. It may be created by both (i) regular "circulation wheels" in the meridian plane (however, calculations show that these "wheels" are very weak, and observations within the available limits of error practically do not reveal them) and (ii) irregular large-scale motions—apparently, first of all, by the so-called giant convection cells whirled by solar rotation, forming the so-called helical macroturbulence (in which vorticity is not orthogonal to velocity).

Thus, calculations of the solar differential rotation require a description of macroturbulence in the convection layer formed by giant convection cells. For such a description, use can be made of the turbulence theory in which statistical parameters of turbulence are immediate objects of study. Dynamic equations are used for these statistical parameters (which appear to be not closed, so that they should be closed with the aid of some approximate methods).

HYDROMECHANICS

We shall use a spherical system of coordinates r, θ, λ (where r is the radial coordinate, θ is colatitude, and λ is longitude) rotating at an average angular velocity $\Omega = \text{constant}$ of the Sun's rotation. The continuity equation is written for the time being

in the full form

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \rho v_r + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \rho v_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \lambda} \rho v_\lambda = 0, \quad [1]$$

where ρ is density, and v_r, v_θ, v_λ are the motion velocity components. The equations of motion are written with the linearized buoyancy force (this simplification with an additional requirement of replacing the continuity equation by the velocity nondivergence condition is called a Boussinesq approximation); in the tensor form they look like

$$\frac{\partial v_i}{\partial t} + v_\alpha \nabla^\alpha v_i + f_i = -\frac{1}{\rho_*} \frac{\partial P}{\partial x^i} + g \zeta \ell_i + \frac{1}{\rho_0} \nabla^\alpha \mathcal{T}_{i\alpha}, \quad [2]$$

where f_i is the Coriolis acceleration; ℓ_i is the unit vertical vector; $\mathcal{T}_{i\alpha}$ is the viscous stress tensor; $\rho_* = \rho_*(r)$ is the hydrostatic potential density [defined by expression

$$\frac{1}{\rho_*} \frac{d\rho_*}{dr} = \frac{1}{\rho_0} \frac{d\rho_0}{dr} + \frac{g}{c_0^2},$$

where $\rho_0 = \rho_0(r)$ and $c_0 = c_0(r)$ are the hydrostatic density and sound velocity, and $g = g_0 \left(\frac{r_0}{r} \right)^2$ is the acceleration of gravity with r_0 , the solar radius, and g_0 , the acceleration of gravity on the Sun's surface]; $P = \frac{\rho_*}{\rho_0} (p - p_0)$ is the normalized deviation of pressure p from its hydrostatic value $p_0 = p_0(r)$; finally ζ is the deviation of entropy η from its hydrostatic value η_0 , divided by specific heat capacity at constant pressure C_p , and linearized with respect to $p - p_0$ and $\rho - \rho_0$ —i.e., $\zeta = \frac{1}{\rho_0 c_0^2} [(p - p_0) - c_0^2(\rho - \rho_0)]$. The replacement of terms $-\frac{1}{\rho} \frac{\partial P}{\partial x^i} - g \ell_i$ by terms

$-\frac{1}{\rho_*} \frac{\partial P}{\partial x^i} + g \zeta \ell_i$ in Eq. 2 is advantageous in that instead of the unpleasant factor $\frac{1}{\rho_0}$ at a pressure gradient changing within the solar convection layer by many orders now at $\frac{\partial P}{\partial x^i}$, we have factor

$\frac{1}{\rho_*}$ approximately constant in the convection layer (because in the latter layer, due to convection, stratification close to adiabatic sets in). Therefore, term $-\frac{1}{\rho_*} \frac{\partial P}{\partial x^i}$ can be regarded as approximately irrotational.

† Deceased.

Eqs. 1 and 2 should be supplemented with the entropy evolution equation in which, in the solar convection layer, we shall neglect a radiation heat flux compared to convective transport, and, of the nonadiabatic factors, we shall take into account only heat influx due to molecular heat conductivity $C_{p\rho_0}\chi$. Such an equation is written in the form similar to Eq. 1

$$\frac{\partial \zeta}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \zeta v_r + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \zeta v_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \lambda} \zeta v_\lambda = \frac{N^2}{g} v_r + \frac{1}{\rho_0 T_0} \operatorname{div} \rho_0 \chi \nabla T, \quad [3]$$

where $N^2 = \frac{g}{\rho_*} \frac{d\rho_*}{dr}$, and T and T_0 are temperature and its hydrostatic value.

STATISTICAL HYDROMECHANICS

From Eqs. 1–4, let us derive equations for the averaged parameters of the hydrodynamic fields of the solar convection layer by using the averaging in longitude λ (denoted with a bar over letters), so that the averaged quantities will not depend on λ ; moreover, we shall consider them independent of time t (i.e., the averaged zonal circulation will be considered steady). Thus, each hydrodynamic field $a(r, \theta, \lambda, t)$ will be represented as a sum of its averaged value $\bar{a}(r, \theta)$ and a deviation from the latter – the fluctuation $a'(r, \theta, \lambda, t) = a - \bar{a}$. The zonal velocity component of the averaged circulation will be represented in the form $\bar{V}_\lambda(r, \theta) = \omega(r, \theta) r \sin \theta$, so that $\omega(r, \theta)$ will denote the angular velocity of the solar differential rotation, and the full angular velocity of the solar rotation will be $\omega_0 = \Omega + \omega(r, \theta)$. Let us introduce another designation $\zeta_1 = \bar{\zeta} - \zeta_0$, where $\zeta_0 = \zeta_0(r)$ is

determined from the expression $\frac{d\zeta_0}{dr} = \frac{N^2}{g}$, and denote one-point second moments of the fluctuations $\overline{a'(r, \theta, \lambda, t) b'(r, \theta, \lambda, t)}$ by the symbols $B_{ab}(r, \theta)$. Taking into account that $\overline{ab} = \bar{a}\bar{b} + B_{ab}$, the averaging of continuity Eq. 1 results in

$$\frac{1}{r^2} \frac{\partial}{\partial r} r^2 (\bar{\rho} \bar{v}_r + B_{r\rho}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta (\bar{\rho} \bar{v}_\theta + B_{\theta\rho}) = 0. \quad [4]$$

With the same precision as above, we can approximately assume $\bar{\rho} \approx \rho_0$ and $\rho' \approx -\rho_0 \zeta'$. In this case Eq. 4 can be satisfied by introducing a stream function ψ with the aid of equations

$$\begin{aligned} \bar{v}_r &= B_{r\zeta} - \frac{1}{\rho_0 r^2 \sin \theta} \frac{\partial \rho_0 \psi}{\partial \theta} = B_{r\zeta} - \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}; \\ \bar{v}_\theta &= B_{\theta\zeta} + \frac{1}{\rho_0 r \sin \theta} \frac{\partial \rho_0 \psi}{\partial r} \\ &= B_{\theta\zeta} + \frac{1}{r \sin \theta} \left(\frac{\partial \psi}{\partial r} + \frac{\psi}{\rho_0} \frac{d\rho_0}{dr} \right). \end{aligned} \quad [5]$$

With the use of these equations, the averaged meridional circulation vorticity is simply expressed in terms of the stream function ψ :

$$\begin{aligned} \Omega_\lambda &= \frac{1}{r} \left(\frac{\partial r \bar{v}_\theta}{\partial r} - \frac{\partial \bar{v}_r}{\partial \theta} \right) - \frac{1}{r \sin \theta} \left(\Delta \psi - \frac{2}{r} \frac{\partial \psi}{\partial r} - \frac{2 \cot \theta}{r^2} \frac{\partial \psi}{\partial \theta} \right) \\ &\quad + \frac{1}{r} \left[\left(\frac{\partial r B_{\theta\zeta}}{\partial r} - \frac{\partial B_{r\zeta}}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial r} \frac{\psi}{\rho_0} \frac{d\rho_0}{dr} \right], \end{aligned} \quad [6]$$

where

$$\Delta \psi = \frac{1}{r^2} \left(\frac{\partial}{\partial r} r^2 \frac{\partial \psi}{\partial r} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial \psi}{\partial \theta} \right)$$

is the two-dimensional Laplacian operator in the meridian plane. In our subsequent calculations in the present paper, we shall use Eqs. 5 in the simplified form, neglecting terms $B_{r\zeta}$ and $B_{\theta\zeta}$ in their right-hand parts (i.e., neglecting the average convective mass transport compared to its transport by average

meridional circulation) and term $\frac{\psi}{\rho_0} \frac{d\rho_0}{dr}$; in sum, these simplifications are equivalent to the replacement of the equation of continuity by the nondivergence condition of the velocity field. Then Eqs. 5 will be substituted for by simpler equations

$$\bar{v}_r \approx -\frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}; \quad \bar{v}_\theta \approx \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}. \quad [5']$$

With the use of these equations, the term in brackets in the right-hand part of Eq. 6 falls out.

Returning to the use of the more precise Eqs. 5 (in which the quantity $\frac{1}{\rho_0} \frac{d\rho_0}{dr}$ should then be given with the aid of some solar interior model) will serve in the future as one of the ways for the improvement of the calculations discussed in the present paper.

Averaging of entropy evolution Eq. 3 and neglecting in it a molecular heat flux compared to a convective flux results in an equation similar to Eq. 4:

$$\begin{aligned} \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \left(-\frac{\zeta_1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} + B_{r\zeta} \right) \\ + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \left(\frac{\zeta_1}{r \sin \theta} \frac{\partial \psi}{\partial r} + B_{\theta\zeta} \right) = 0. \end{aligned} \quad [7]$$

Finally, we write equations of motion (Eq. 2) in projections on the spherical coordinate axes and then average them, neglecting a molecular momentum flux compared to a convective flux. The averaged equation for a zonal motion acquires the form similar to Eq. 7:

$$\begin{aligned} \frac{1}{r^3} \frac{\partial}{\partial r} r^3 \left[-\frac{\omega + \Omega}{r} \frac{\partial \psi}{\partial \theta} + B_{r\lambda} \right] \\ + \frac{1}{r \sin^2 \theta} \frac{\partial}{\partial \theta} \sin^2 \theta \left[(\omega + \Omega) \frac{\partial \psi}{\partial r} + B_{\theta\lambda} \right] = 0. \end{aligned} \quad [8]$$

FIG. 1. Meridional section of the convection layer isolines $\omega/\Omega = \text{constant}$ (—) and stream lines $\psi = \text{constant}$ (----).

Elimination of the pressure gradient from the averaged equations of motion in the meridian plane by cross-differentiation gives an equation for vorticity:

$$\begin{aligned} & \frac{1}{r^2 \sin \theta} \left[\frac{\partial(\psi, \Omega_\lambda)}{\partial(r, \theta)} + \Omega_\lambda \left(\frac{1}{r} \frac{\partial \psi}{\partial \theta} - \cotan \theta \frac{\partial \psi}{\partial r} \right) \right] \\ & + r \sin^2 \theta \left[\frac{1}{r} \frac{\partial(\omega + \Omega)^2}{\partial \theta} - \cotan \theta \frac{\partial(\omega + \Omega)^2}{\partial r} \right] \\ & = - \frac{g}{r} \frac{\partial \zeta_1}{\partial \theta} + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} r^2 B_{rr} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta B_{r\theta} \right. \\ & \left. - \frac{B_{\theta\theta} - B_{\lambda\lambda}}{r} \right) - \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} r^3 B_{r\theta} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta B_{\theta\theta} \right. \\ & \left. - \cotan \theta B_{\lambda\lambda} \right). \end{aligned} \quad [9]$$

The three Eqs. 7-9 contain three unknown functions ψ , ω , and ζ_1 and nine additional unknown quantities—Reynolds stresses B_{ij} and components of a convective entropy flux $B_{i\epsilon}$ —so that these equations are not closed, and their closure requires that some additional relationships be used.

SECOND MOMENT EQUATIONS

Instead of the attempts to somehow express the additional unknown quantities B_{ij} and $B_{i\epsilon}$ in terms of the basic unknown quantities ψ , ω , and ζ_1 , we add dynamic equations for one-point second moments of the fluctuations B_{ij} , $B_{i\epsilon}$, and $B_{\epsilon\epsilon}$ deduced from the equations of hydrodynamics (the so-called Friedmann-Keller equations) to Reynolds equations. Such a system of equations will be nonclosed all the same: in addition to the basic unknown

quantities ψ , ω , ζ_1 and B_{ij} , $B_{i\epsilon}$, $B_{\epsilon\epsilon}$, it will contain some new unknown quantities—one-point third moments of fluctuations and second moments relating velocity fluctuations to the fluctuations of stress gradients—and we shall have to accept some hypotheses with a certain degree of arbitrariness for these new additional unknown quantities. As one of the simplifications of the Friedmann-Keller equations for the one-point second moments, we shall neglect the third moments in them. Such simplified equations for the second moments are derived from the equations of hydrodynamics linearized with respect to fluctuations and in the tensor designations having the form

$$\begin{aligned} \frac{\partial v'_i}{\partial t} + \bar{v}_\alpha \nabla^\alpha v'_i + v'_\alpha \nabla^\alpha \bar{v}_i + f'_i - g \zeta'_i \ell_i \\ = - \frac{1}{\rho_*} \nabla_i P' + \frac{1}{\rho_0} \nabla^\alpha \mathcal{T}'_{ia}; \end{aligned} \quad [2']$$

$$\frac{\partial \zeta'}{\partial t} + \bar{v}_\alpha \nabla^\alpha \zeta' + v'_\alpha \nabla^\alpha \zeta_1 = \frac{1}{\rho_0 T_0} \nabla^\alpha \rho_0 \chi \nabla_\alpha T'. \quad [3']$$

From Eq. 2' for the moments B_{ij} , we obtain equations

$$\begin{aligned} \bar{v}_\alpha \nabla^\alpha B_{ij} + (B_{i\alpha} \nabla^\alpha \bar{v}_j + B_{j\alpha} \nabla^\alpha \bar{v}_i) + (\overline{f'_i v'_j} + \overline{f'_j v'_i}) - g(B_{i\epsilon} \ell_j + B_{j\epsilon} \ell_i) \\ \approx - C_1 \frac{B}{L} \left(B_{ij} - \frac{1}{3} B^2 g_{ij} \right) - C_2 \frac{B^3}{L} \left(\ell_i \ell_j - \frac{1}{3} g_{ij} \right) \\ - 2 \frac{B^3}{L} [(C_3 - 3C_4) \ell_i \ell_j + C_4 g_{ij}]. \end{aligned} \quad [10]$$

FIG. 2. Angular velocity of the differential rotation on the Sun's surface $10^7 \omega$ (sec^{-1}) from the data of F. Ward, V. Starr, and C. Allen (—) and from calculation (----).

FIG. 3. Distribution by latitudes of an averaged meridional velocity $\bar{v}_\theta$ at some depths in the northern hemisphere. Curves: 1, $r_0 - \frac{1}{6}(r_0 - r_1)$; 2, $r_0 - \frac{1}{3}(r_0 - r_1)$; 3, $r_1 + \frac{1}{3}(r_0 - r_1)$; 4, $r_1 + \frac{1}{6}(r_0 - r_1)$.

Here the right-hand part contains one-point second moments relating velocity fluctuations to the fluctuations of stress gradients. We express these moments in terms of B_{ij} by semiempirical equations used earlier by Monin (5–7) in the theory of the turbulent atmospheric boundary layer. Here $B^2 = B_{rr} + B_{\theta\theta} + B_{\lambda\lambda}$ is the mean square of fluctuation velocity; L is the vertical scale of turbulence; g_{ij} is the metric tensor; and C_1 , C_2 , C_3 , and C_4 are numerical constants. The first term in the right-hand part describes an isotropifying action of pressure fluctuations; the second one, on the contrary, described a tendency toward anisotropy near the upper and the lower boundaries of the convection layer; and the third one describes an anisotropic viscous dissipation of the turbulent energy.

Equations for moments B_{ij} similar to Eqs. 10 are obtained from Eqs. 2' and 3' in the form

$$\bar{v}_a \nabla^a B_{ij} + B_{\alpha i} \nabla^a \bar{v}_j + B_{ja} \nabla^a \bar{v}_i + \bar{f}_{ij} - g B_{ij} \ell_i \approx -C_5 \frac{B}{L} B_{ij} - C_6 \frac{B^2}{L} B_{ij}^{1/2} \ell_i, \quad [11]$$

where C_5 and C_6 are new numerical constants.

Finally, an equation for entropy variance is derived from Eq. 3' in the form

$$\frac{1}{2r^2 \sin \theta} \frac{\partial(\psi, B_{ij})}{\partial(r, \theta)} + B_{rj} \frac{\partial \zeta_1}{\partial r} + \frac{B_{\theta j}}{r} \frac{\partial \zeta_1}{\partial \theta} \approx -C_7 \frac{B}{L} B_{ij}, \quad [12]$$

where C_7 is another numerical constant.

According to the papers by Monin (5–7), the numerical constants in Eqs. 10–12 are expressed in terms of the limiting values under neutral stratification of some turbulence parameters in the atmospheric surface layer. Hence, in the limit of neutral stratification, we have at $q > 0$ $C_1 \approx 0.21$, $C_3 \approx 0.035$, $C_4 - \frac{C_2}{C_6} \approx 0.0025$, $C_5 \approx 0.204$, $C_6 \approx -0.00226$, and $C_7 \approx 0.0292$.

If the turbulence scale L is given somehow, Eqs. 7–9 and 10–12 form a closed system relative to 13 unknown quantities ψ , ω , ζ , B_{ij} , B_{ir} , $B_{i\theta}$, which represents a model of the steady zonal circulation in the global gaseous layer of a celestial body.

FIG. 4. Distribution by latitudes of an averaged vertical velocity $\bar{v}_r$ at some depths. Curves (radial velocity): 1, $r = r_0 - \frac{1}{6}(r_0 - r_1)$; 2, $r = r_0 - \frac{1}{2}(r_0 - r_1)$; 3, $r = r_1 + \frac{1}{6}(r_0 - r_1)$.

For the solar convection zone, we took $r_1 = 0.7$ and $L = 0.4$ ($r_0 - r_1$).

The system of equations should be integrated in the meridian plane (r, θ) , under conditions of symmetry with respect to the equator, by assuming $\psi = 0$ at all boundaries of the integration region and $B_{r\theta} = B_{r\lambda} = 0$ at $r = r_0$ and $r = r_1$, by prescribing at the lower boundary $r = r_1$ a constant radial heat flux $q_1 = q_0 \left(\frac{r_0}{r_1}\right)^2$ (where $q_0 = 6.27 \times 10^{10}$ erg/cm²·sec is the solar radiance), and by requiring there the equality to zero of radial gradients (fluxes) of the rest of the unknown quantities (no boundary conditions are stipulated for ω and ζ_1). In this case, $B_{r\theta} = \frac{1}{T} \frac{q}{v_r T} = \frac{q}{C_P \rho T} = \frac{\mathcal{H} - 1}{\mathcal{H}} \frac{q}{p}$ and, at $p_1 = 3.3 \times 10^{13}$ dyne/cm², $\mathcal{H} = 5/3$ and $r_1 = \frac{2}{3} r_0$; the boundary condition for $B_{r\theta}$ at the inner boundary of the convection layer has the form $B_{r\theta} = 1.7 \times 10^{-3}$ cm/sec.

NUMERICAL INTEGRATION

The equations were reduced to the nondimensional form by measuring the variables r , ψ , ω , ζ_1 , B_{ij} , B_{ir} , and $B_{i\theta}$ in units r_0 , Ωr_0^3 , Ω , $\frac{\Omega^2 r_0}{g_0}$, $\Omega^2 r_0^2$, $\frac{\Omega^3 r_0^2}{g_0}$, and $\frac{\Omega^4 r_0^2}{g_0^2}$. The single parameter of the problem is the boundary value of the quantity $B_{r\theta}$.

FIG. 5. Distribution by latitudes of root mean square fluctuations of vertical velocity $B_{r\theta}^{1/2}$ (—) and meridional velocity $B_{\theta\theta}^{1/2}$ (----) at some depths. Curves: 1, $r_0 - \frac{1}{6}(r_0 - r_1)$; 2, $r_1 + \frac{1}{2}(r_0 - r_1)$; 3, $r_1 + \frac{1}{6}(r_0 - r_1)$.

FIG. 6. Distribution by latitudes of turbulent friction stresses $B_{r\theta}$ and $B_{r\lambda}$ on the Sun's surface.

The numerical solution of the equations was made by the finite difference method. In the motion region, a rectangular grid was introduced for which $\theta_i = i\Delta\theta$; $r_K = K\Delta r$; $i = 0, 1, 2, \dots, \mathcal{J}$; $K = 0, 1, 2, \dots, \mathcal{K}$. In all of the equations, the derivatives not entering into Jacobians were replaced by symmetric difference ratios. Advective derivatives were replaced by difference ratios with account for advection direction. In Eqs. 7 and 8, the derivatives were replaced by symmetric difference ratios of the expressions whose derivative was taken so that, for these equations, no special difference form of a Jacobian was used. In this case, no boundary conditions for ω and ζ_1 are required because the factors standing with them in Eqs. 7 and 8 are equal to zero at the boundary.

The iteration solution procedure was used. Solution was found in inner points of the region. Values of the desired quantities not given at the boundaries were found in points of boundaries by extrapolation. The convection zone in the calculations was divided into 10 nine-degree zones between the equator and the pole and into six equally thick radial layers. Check calculation was made for 10 radial layers. The results turned out to be close. For convergence of stream function values with about 1% accuracy, nearly 10 iterations were required; the remaining quantities converged slower, and their determination with the same accuracy required nearly 1,000 iterations.

RESULTS

Circulation in the meridian plane appeared to be unicellular with rises at the poles, a surface flow from the poles toward the equator, and sinking at the equator. Differential rotation ω/Ω at the surface turned out to be growing from -0.28 near the pole to $+0.07$ at the equator, and under the surface to be growing with depth, the value ω/Ω changing near the lower boundary of the convection layer from -0.14 at the pole to $+0.28$ at the equator. Fig. 1 represents a meridional section of the convec-

tion layer with stream lines ψ and isolines ω/Ω . Fig. 2 shows a comparison of the angular velocities on the Sun's surface yielded by calculations with the angular velocities obtained from observations and given by Starr (8) (angular velocity distribution in the near-equatorial zone was obtained by F. Ward; in the high latitudes, Ward's results were supplemented with C. Allen's 1973 results based on Doppler-shift measurements). Fig. 3 gives the distribution of a meridional velocity $\bar{v}_\theta$ in some radii in the convection layer. It is well seen that the maximum velocities at the layer boundaries reach tens of meters per second. Fig. 4 presents radial velocities $\bar{v}_r$ in the convection zone. It is seen that the maximum absolute values of the radial velocity are in the middle part of the convection layer and the radial motion near the pole is much more intensive than the radial flow near the equator.

Nondimensional entropy deviation $\frac{g_0\zeta_1}{\Omega^2 r_0}$ appeared to be practically not changing with latitude (this refers to temperature as well, which is an undoubted advantage of our model: in less detailed models, either temperature or radial heat flux were essentially changing with latitude in contradiction to the observational data) and to be negative and decreasing with depth from -0.13×10^{-5} at the upper boundary to -0.19×10^{-5} at the lower boundary.

Root mean square horizontal velocity fluctuations $B_{\theta\theta}^{1/2}$ and $B_{\lambda\lambda}^{1/2}$ were of the order of 20–30 cm/sec, close to each other, and several times smaller than vertical fluctuations $B_{\lambda\lambda}^{1/2}$, whose order of magnitude was 125–175 cm/sec. The vertical fluctuations grow with depth and vary only slightly with latitude, especially in the near-equatorial region. Fig. 5 shows vertical fluctuations $B_{rr}^{1/2}$ and horizontal fluctuations $B_{\theta\theta}^{1/2}$ at some depths of the convection layer.

Fig. 6 represents changes of $B_{r\theta}$ and $B_{r\lambda}$ at the surface of the convection layer $r = r_0$. It is seen that on the surface these quantities are negative, and $B_{r\theta}$ by its absolute value is smaller than $B_{r\lambda}$. The quantities $B_{\theta\lambda}$ are considerably (by 2–3 orders of magnitude) less than the quantities $B_{r\theta}$ and $B_{r\lambda}$. The quantities $B_{r\lambda}$ appeared to be only slightly changing with depth and very close to the limiting value $B_{r\lambda} = 1.7 \times 10^{-3}$ cm/sec. The quantities $B_{\theta\lambda}$ and $B_{\lambda\lambda}$ are considerably (2–3 orders of magnitude) less than the quantities $B_{r\lambda}$. In the nondimensional form, the quantity $B_{r\lambda}$ is several orders less than $B_{\theta\lambda}$ and $B_{\lambda\lambda}$.

Useful further modifications of the model may be a substitution of the nondivergence of an averaged mass flow for velocity nondivergence, the employment of other models for $L(r)$, and other variants of boundary conditions.

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