Estimation of the two-dimensional presampled modulation transfer function of digital radiography devices using one-dimensional test objects

Jered R. Wells^{a)}

Carl E. Ravin Advanced Imaging Laboratories, Duke University Medical Center, Durham, North Carolina 27705 and Medical Physics Graduate Program, Duke University, Durham, North Carolina 27705

James T. Dobbins III

Carl E. Ravin Advanced Imaging Laboratories, Duke University Medical Center, Durham, North Carolina 27705; Department of Radiology, Duke University Medical Center, Durham, North Carolina 27705; Department of Biomedical Engineering, Duke University, Durham, North Carolina 27705; Department of Physics, Duke University, Durham, North Carolina 27705; and Medical Physics Graduate Program, Duke University, Durham, North Carolina 27705

(Received 23 April 2012; revised 27 August 2012; accepted for publication 28 August 2012; published 21 September 2012)

Purpose: The modulation transfer function (MTF) of medical imaging devices is commonly reported in the form of orthogonal one-dimensional (1D) measurements made near the vertical and horizontal axes with a slit or edge test device. A more complete description is found by measuring the two-dimensional (2D) MTF. Some 2D test devices have been proposed, but there are some issues associated with their use: (1) they are not generally available; (2) they may require many images; (3) the results may have diminished accuracy; and (4) their implementation may be particularly cumbersome. This current work proposes the application of commonly available 1D test devices for practical and accurate estimation of the 2D presampled MTF of digital imaging systems.

Methods: Theory was developed and applied to ensure adequate fine sampling of the system line spread function for 1D test devices at orientations other than approximately vertical and horizontal. Methods were also derived and tested for slit nonuniformity correction at arbitrary angle. Techniques were validated with experimental measurements at ten angles using an edge test object and three angles using a slit test device on an indirect-detection flat-panel system [GE Revolution XQ/i (GE Healthcare, Waukesha, WI)]. The 2D MTF was estimated through a simple surface fit with interpolation based on Delaunay triangulation of the 1D edge-based MTF measurements. Validation by synthesis was also performed with simulated images from a hypothetical direct-detection flat-panel device.

Results: The 2D MTF derived from physical measurements yielded an average relative precision error of 0.26% for frequencies below the cutoff (2.5 mm^{-1}) and approximate circular symmetry at frequencies below 4 mm^{-1} . While slit analysis generally agreed with the results of edge analysis, the two showed subtle differences at frequencies above 4 mm^{-1} . Slit measurement near 45° revealed radial asymmetry in the MTF resulting from the square pixel aperture ($0.2 \text{ mm} \times 0.2 \text{ mm}$), a characteristic which was not necessarily appreciated with the orthogonal 1D MTF measurements. In simulation experiments, both slit- and edge-based measurements resolved the radial asymmetries in the 2D MTF. The average absolute relative accuracy error in the 2D MTF between the DC and cutoff (2.5 mm^{-1}) frequencies was 0.13% with average relative precision error of 0.11%. Other simulation results were similar to those derived from physical data.

Conclusions: Overall, the general availability, acceptance, accuracy, and ease of implementation of 1D test devices for MTF assessment make this a valuable technique for 2D MTF estimation. © 2012 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4752442>]

Key words: modulation transfer function (MTF), digital radiography, resolution, edge spread function, line spread function

I. INTRODUCTION

The modulation transfer function (MTF) is widely accepted as the standard measure of linear imaging system resolution performance. It is commonly reported in one dimension as the normalized magnitude of the Fourier transform (FT) of the line spread function (LSF) that is readily obtained by imaging a slit or edge test device.^{1–3} These test devices are typi-

cally used to report orthogonal measures of the MTF. While orthogonal measures classify the resolution performance of the system near the two major axes, it has generally been impractical to reliably estimate the off-axis portions of the two-dimensional (2D) MTF.

It would be useful to assess the full 2D resolution properties of medical imaging systems. For instance, previous reports on the MTF of computed radiography systems have

shown distinctive asymmetries in the scan and subscan directions which can make the values at intermediate angles difficult to estimate.^{4,5} Examples from digital radiography have also shown that 2D undersampling and aliasing effects are important to consider, especially in calculations of noise equivalent quanta (NEQ) and detective quantum efficiency (DQE).^{6–8} Complete assessments of the 2D NEQ and DQE are valuable in assessments of observer task performance since image interpretation is an inherently 2D task.⁵ And although the 2D noise power spectrum (NPS) is easily measured, regular evaluations of the full 2D NEQ and DQE have been difficult due to various challenges in obtaining accurate estimates of the 2D MTF.

Direct measurement of the 2D MTF has been difficult due to the lack of a suitable 2D test device and a practical measurement procedure. In a recent effort to measure the 2D MTF without the use of physical test devices, Kuhls-Gilcrist *et al.* derived an exact relationship between the 2D NPS and presampled MTF.⁹ Although accurate, the technique requires in-depth cascaded systems analysis to produce a functional form of the system MTF for fitting purposes. Still, several authors have reported on efforts to develop 2D resolution test objects. Fetterly *et al.* and Báth *et al.* both demonstrated the use of pinhole test devices for the measurement of 2D detector point spread function (PSF).^{5,10,11} Their techniques utilize precision-machined arrays of pinholes to produce “point” sources of x-rays, and such devices may not be generally available. Also, to measure the (5×) oversampled and tail-smoothed PSF of a computed radiography system, Fetterly *et al.* required in excess of 6000 individual point-source measurements (obtained from 25 images of a 16 × 16 matrix of pinholes) for precise computation of the 2D MTF.⁵ Furthermore, opaque pinhole devices may require cumbersome alignment procedures (due to the thickness of the stock material), while nonopaque devices may require background detrending which can affect low-frequency MTF measurements. Other authors have proposed the use of a disk test device and Wiener filtering for the measurement of 2D MTF.^{12,13} Although this technique requires only a single measurement, reports indicate inaccuracies in 2D MTF assessment due to aliasing at high frequencies resulting from failure to acquire the presampled MTF.¹¹ Thus, while these efforts at developing 2D test objects for MTF are laudable, there still remain issues of practicality and precision in such direct 2D measurements.

In this paper, we describe a simple and accurate method for estimating the 2D MTF using edge and slit test devices that have been routinely used for one-dimensional (1D) MTF measurement. By making measurements with these devices at several angles, we assemble an estimate of the 2D MTF from an ensemble of 1D MTF measurements. We also describe methods to ensure adequate fine sampling of edge and slit devices cast at arbitrary angle. The proposed methods are evaluated with experimental measurements on an indirect-detection flat-panel detector (FPD) device and with simulation studies of a direct-detection FPD. We also compare the results from edge and slit devices and find subtle differences at low and high frequencies.

II. MATERIALS AND METHODS

It is well known that a 2D system PSF can be formed if the system LSF at all possible orientations θ is measured.¹⁴ Although it is impractical to measure the LSF at *all* angles, we propose that a good estimate of the PSF can be achieved through the appropriate combination of a reasonable number of finely sampled LSF measurements and 2D interpolation.

In Subsections II.A–II.C, we describe experimental and mathematical methods for the estimation of 2D MTF from several 1D MTF measurements at various angles relative to the sampling lattice. We derive a method to ensure adequate fine-sampling of the system LSF in addition to correction factors applied to the edge and slit methods. We demonstrate experimental methods used to acquire and process edge and slit images from an indirect-detection device and from simulations of a hypothetical direct-detection imaging device. All data processing was performed using a commercially available software package [MATLAB 7.7.0 (R2008b), The MathWorks Inc., Natick, MA, 2008].

II.A. Methodology

Methods are described here to ensure proper sampling of the system LSF at arbitrary angle. For slit-based assessments of MTF, a technique is also proposed for slit-uniformity correction at arbitrary angle.

II.A.1. Fine-sampling at arbitrary angle

The system LSF can be measured using the techniques of Fujita *et al.*² who provided a simple method for determining the presampled 1D MTF using a slit test device. Their methods have been used extensively by many investigators.^{1,3,4,11,15–17} However, the slit method has not been sufficiently generalized to afford presampled MTF estimation at angles which do not approximate the vertical or horizontal. Therefore, we propose an approach towards the generalization of the original MTF estimation method of Fujita *et al.*, which ensures fine-sampling of the LSF at arbitrary angle.

Fine-sampling of the system LSF is required to reduce the effects of aliasing on the measured signal and to recover the presampled MTF. Therefore, a slit-test object must be measured at angles, which afford sufficient fine-sampling of the LSF. For a slit at arbitrary angle θ , the slope of the projected slit, m , can be represented using a rational approximation (in simplest form)

$$m = \frac{a}{b} \approx \tan(\theta). \quad (1)$$

In Eq. (1), a and b are integers used to approximate $\tan(\theta)$. For the special cases of $0^\circ = \theta \bmod 180^\circ$ (horizontal) and $90^\circ = \theta \bmod 180^\circ$ (vertical), $m = 0/1$ and $m = 1/0$, respectively. The simplest-form rational approximation of m is required in order to accurately determine the subpixel sampling resolution achievable at a particular angle. The effective sampling aperture is defined as

$$\Delta s = p/n, \quad (2)$$

where pixel dimension p in the two orthogonal directions is assumed to be equal. Parameter n is computed from the simplest-form rational approximation in Eq. (1) as

$$n = \sqrt{a^2 + b^2}. \quad (3)$$

It is evident from Eqs. (1)–(3) that large-integer rational approximations of m result in large values of n which in turn provide small effective sampling apertures. In other words, $1/n$ represents the minimum fractional pixel width achievable through subpixel sampling of a 1D resolution test signal at angle $\theta \approx \tan^{-1}(m)$. This distinction is an important generalization of statements made by both Fujita *et al.*² and Samei *et al.*³ Given our definitions in the equations above, we conclude that a slit or edge at an angle with $1/n$ greater than some predefined value for $\Delta s/p$ will provide inadequate subpixel resolution. At these angles, the coarseness of subpixel sample spacing may lead to aliasing in the response function. Therefore, a 1D resolution test device imaged at any other angle with sufficiently small $1/n$ will yield adequate results.

Finally, once a satisfactory angle is found, the finely sampled LSF can be obtained as a function of pixel distance d from the slit center

$$d = \frac{y - (mx + y_0)}{\sqrt{m^2 + 1}}, \quad (4)$$

where d is the exact perpendicular distance from pixel center coordinate (x, y) to the center of the imaged slit defined by slope m and y -intercept y_0 . Simple arrangement of pixel values as a function of increasing d between $\pm d_0$ (Fig. 1) forms a precise estimate of the finely sampled LSF.

FIG. 1. Perpendicular slit profiles (thick dashed rectangular enclosures) shown for a slit at or near 45° . For $\theta \approx 45^\circ$, $\tan(\theta) \approx 1/1 = a/b = m$. Variables n and d_0 are displayed for convenience. Alternating black and gray pixel centers indicate a particular pixel's profile membership. Pixels with white centers are ignored.

II.A.2. Slit nonuniformity

One hurdle to physical implementation of slit-based measurements of MTF at arbitrary angle is the absence of a generalized uniformity correction for imperfections in the slit. Slight inaccuracies in slit fabrication due to imprecision in machining may result in slit width variation leading to nonuniformities in x-ray transmission along the length of the test device. In a previous report,⁴ slit width imperfection was corrected by normalizing the intensity of pixel values by the integral intensity across the slit in a direction perpendicular to the slit for slits positioned near the image axes. We have extended this approach for application at arbitrary angle.

The new approach to slit nonuniformity correction involves rounding the slit angle to one that permits estimation of m using small integers a and b [Eq. (1)]. The small-integer approximation of m results in a relatively small value for n [Eq. (3)] which can be interpreted as the regular interval (in pixels) approximately parallel to the slit at which consecutive perpendicular profiles should be formed. Note that the small-integer approximation of m is recommended exclusively for the purposes of slit nonuniformity correction and not for the construction of the finely sampled LSF due to the potential for aliasing in the response function. For example, nonuniformity correction of an approximately horizontal slit involves normalization of pixel values to the integral of vertical profiles at regular increments along the 0° direction ($m = 0/1$). At 0° , $n = 1$ implying that profiles should be generated at 1-pixel increments along the slope $m = 0$. In another example, a slit at 44° ($m \approx 28/29$) requires nonuniformity correction using profiles perpendicular to the 45° direction ($m = 1/1$). At 45° , $n = 1.414$ indicating that nonuniformity correction can be achieved through pixel value normalization by the perpendicular profile integral obtained at increments of 1.414 pixels along the 45° angle (Fig. 1). This same procedure can be implemented at any other angle thus generalizing the slit nonuniformity correction technique.

II.B. Experimental implementation with an indirect detection device

The measurement of MTF at angles between horizontal and vertical was demonstrated on an indirect-detection FPD using both edge and slit devices.

II.B.1. Imaging system

A prototype indirect-detection FPD (Revolution XQ/i, GE Healthcare, Waukesha, WI) was investigated. The Bucky-mounted detector has a $41 \times 41 \text{ cm}^2$ field of view (FOV) with 0.2 mm pixel pitch. Other detector properties have been described extensively in the literature.^{6,16} A standard MX 100 x-ray tube insert and housing (GE Healthcare, Waukesha, WI) were used with an ULTRANET SA beam collimator (Medys, Monza, Italy). The system source to image distance (SID) was 186 cm.

Prior to system calibration, several pieces of hardware were removed from the x-ray beam: beam conditioning

filters; collimator-mounted crosshairs and exposure meter; detector cover plate; antiscatter grid; and the automatic exposure control. In accordance with manufacturer specifications, a 20 mm aluminum block was placed in the beam (for calibration purposes only). No other hardware was added. The system was then calibrated for gain nonuniformities, offset correction, and defective pixels using the “no grid” protocol as specified by the manufacturer. Previous studies showed that the system response function demonstrated excellent linearity within the relevant exposure range,¹ hence raw images required no linearization correction. Gain map, offset, and bad pixel corrections constituted the full extent of image preprocessing by the imaging system.

II.B.2. Beam conditions

The x-ray beam was conditioned in accordance with the IEC RQA5 standard which requires a 21 mm aluminum block in the beam.^{18,19} Standard shop-grade aluminum (type-1100) was used following the findings in a previous publication.²⁰ To approximate narrow beam geometry, the beam was first tightly windowed with the system’s internal collimator followed by a second lead collimator attached to the detector side of the aluminum block. With phantom and collimators in place, a half-value layer (HVL) of 6.8 mm Al was achieved at 71.5 kV_p. Exposure measurements for HVL calculation were made using a calibrated ionization chamber and kV meter (Accu-Pro, MDH Model 1015, 10 × 5-6 ionization chamber, Radcal, Moravia, CA). The chamber was positioned 93 cm from the source for all exposure measurements made in this study.

All beam conditions associated with MTF measurements closely followed the IEC standard with one exception: The x-ray exposure was set to the maximum value allowed by the system. The technique was fixed at 71.5 kV_p, 64 mAs, 250 mA, 250 ms, and 0.6 mm nominal focal spot for all MTF test images in this study. It provided an exposure of 4.0 mR at the detector (higher than the IEC recommendation). Other investigations have shown that this deviation from the prescribed protocol is justified in order to reduce the influence of image noise since the MTF of digital FPDs rarely exhibit substantial exposure dependency.^{3,21}

II.B.3. Edge image acquisition

An opaque edge was used in the first set of physical measurements of MTF. It consisted of a $0.2 \times 5 \times 10$ cm³ tungsten

slab with one polished edge (TX5 W Edge Test Device, Scanditronix Wellhöfer, Schwarzenbruck, Germany). The other three edges were surrounded by a 3 mm thick lead frame. The edge test device was placed in contact with the detector surface and visually aligned with the central axis of the x-ray beam. To ensure proper alignment of the beam axis, detector center, and edge, a bubble level was used to measure and correct for any sagging of the x-ray tube housing and detector Bucky. After verification of system alignment, the x-ray beam was collimated to 8×8 cm² at the detector. This ensured proper exposure of the full length of the available edge without exposing the lead frame. The edge was rotated and imaged through a total of ten angles between 0° (horizontal) and 90° (vertical). It was supported at each angle by wooden wedges and fixed in place with common tape. At each angle, the entire collimator assembly was rotated to align the square x-ray field with the edge. Five images were acquired at every edge orientation with at least 2 min between each acquisition to minimize the influence of detector lag. The repeated measurements at each angle were taken to compute the uncertainty in 2D edge-based MTF assessments.

II.B.4. Edge image processing

The method of Samei *et al.*³ was used to process the individual edge images at each angle. Edges were extracted using a 256×256 pixel² ROI. A binary threshold operation was applied to the edge data in the ROI,²² and the edge was extracted using Canny edge detection.²³ The edge angle was determined using a double Hough transform with 0.1° precision. Each of the five edge images at each angle were processed independently, and key measurements are summarized in Table I. The data were projected parallel to each edge and rebinned with 0.02 mm (0.1 pixel) spacing to form the finely sampled edge spread function (ESF).

In order to reduce the impact of noise in the finely sampled ESF measurement, two denoising methods were compared. One method employed modest smoothing with a Gaussian-weighted polynomial kernel (0.34 mm width, fourth-order polynomial fitting) according to the recommendations of Samei *et al.*³ Alternatively, the ESF was conditioned using the algorithm of Maidment and Albert.²⁴ Whereas smoothing of the ESF has been shown to introduce systematic errors in the estimation of the MTF (namely, amplitude suppression at high frequencies), the ESF conditioning algorithm has been shown to preserve fine features of the MTF.²⁴ This is

TABLE I. Summary of values obtained from physical edge measurement. Using the estimated edge angles, a/b ratios were obtained using the MATLAB function *rat* with tolerance factor of 0.001. Parameter n indicates that all edge angles can accommodate fine-sampling of the edge spread function with subpixel sampling bins which are at least 10× smaller than the physical pixel dimension.

Parameter	Acquisition									
	1	2	3	4	5	6	7	8	9	10
θ	1.33°	7.59°	20.22°	30.29°	37.90°	48.07°	58.85°	69.78°	78.13°	88.02°
a/b	1/43	2/15	7/19	7/12	7/9	59/53	43/26	19/7	157/33	781/27
n	43.01	15.13	20.25	13.89	11.40	79.31	50.25	20.25	160.43	781.47

FIG. 2. LSF following numerical differentiation of a finely sampled edge at 1.33° (a) without denoising, (b) with smoothing, and (c) with conditioning. The effects of denoising are most evident on the positive side of the LSF which corresponds to the unattenuated side of the edge.

accomplished by constraining the ESF data through quadratic optimization according to two requirements: monotonicity of the ESF and squared error minimization. We compare the results of these two denoising processes in Sec. III.

After denoising the finely sampled ESF, differentiation using the standard central-difference algorithm produced the LSF. A sample LSF from an approximately horizontal edge is shown in Fig. 2 without denoising, with smoothing, and with conditioning. To remove low-frequency nonuniformities, the baseline of the LSF was subtracted using a linear fit to 1 cm portions of the LSF tails. This was followed by windowing with a Hanning window of 2 cm width. The 1D FT of the windowed LSF produced the optical transfer function (OTF). The magnitude of the complex OTF normalized to unity at zero frequency formed the MTF with bin spacing of 0.05 mm^{-1} . An iterative process refined the edge angle estimate down to 0.01° precision through maximization of the integral of the MTF between the zero (dc) and cutoff (f_c) frequencies.

We digress briefly to describe two correction factors which improve the edge method of Samei *et al.*³ Their subpixel binning procedure is in effect the convolution of a rectangular function of subpixel width with the finely sampled 1D ESF data. Therefore, the resultant MTF must be corrected through division by $|\text{sinc}(\Delta s \sqrt{u^2 + v^2})|$, where variables u and v are spatial frequencies in the horizontal and vertical directions, respectively. Furthermore, differentiation using the standard central-difference algorithm introduces additional blurring effects which must be modeled to correct the MTF. For evenly spaced data, the central-difference algorithm is simply the average of the forward and backward (two-point) differences. The averaging process may be modeled as a convolution of the LSF by a rectangular function of width equal to two subpixel bins, which is corrected through division of the MTF by $|\text{sinc}(2\Delta s \sqrt{u^2 + v^2})|$. This correction, which is equivalent to the transfer function of central differences algorithm, is consistent with the findings of Buhr *et al.*²⁵

Once the corrected 1D MTF estimates at each of ten angles was obtained, the 2D MTF was estimated through a simple surface fit to the ensemble of 1D data. Due to the symmetry of the detector elements in our studies, we assumed reflective symmetry of the 2D PSF across both the x and y axes. Therefore, the 1D MTF measurements generated from edge images between 0° and 90° were reflected across the u and v axes to fill the remainder of 2D frequency space in advance of data

interpolation onto a 192×192 Cartesian lattice with evenly spaced samples between $\pm 1.5f_c = \pm 3.75 \text{ mm}^{-1}$. This grid size was chosen based on the recommended size for 2D NPS measurements from a previous study.²⁶ Consistency between the MTF and NPS grid sizes is important for the purposes of 2D DQE and NEQ computations.^{4,6-8,11} Finally, the ensemble of 1D MTF data were interpolated onto the 2D grid using three interpolation methods: nearest neighbor, linear, and cubic interpolation. All interpolation methods were based on Delaunay triangulation due to the scattered (unevenly sampled) nature of the source data. Comparisons of the accuracy and precision of these interpolation methods are reported in Sec. III.

II.B.5. Slit image acquisition

A narrow slit test device was imaged to compare against results from the edge method. A sturdy aluminum jig held two 2 mm thick lead blocks with highly polished edges spaced $12 \mu\text{m}$ apart. The slit was positioned in the center of a $35 \times 35 \text{ mm}^2$ aluminum window. Following the same tube and Bucky alignment procedure as with the edge, the slit was positioned 1 cm from the detector surface (approximately 2 cm from the actual detection plane) near the center of the imaging plate. The beam apertures were set to deliver a $3.5 \times 3.5 \text{ cm}^2$ field at the detector to avoid excessive exposure of the aluminum frame. The slit was imaged at three orientations: vertical, horizontal, and 45° . At each position, the slit was adjusted so that its angle was 1° – 4° off of true vertical, horizontal, and 45° alignment. An iterative alignment procedure was implemented to maximize the x-ray flux through the slit indicating optimal slit positioning.⁴ Once aligned, the slit was imaged 20 times with at least 2 min between each acquisition.

II.B.6. Slit image processing

We applied our generalizations to the methods of Fujita *et al.* and Dobbins *et al.* which both served as guides for slit processing.^{2,4} At each angle, the 20 slit images were averaged to form a single composite image which reduced the influence of noise. Slits were extracted using a $160 \times 160 \text{ pixel}^2$ ROI. In a process similar to that implemented for the edge, slit angle was determined using a double Hough transform following thresholding and morphological skeletonization of the slit image. This yielded slit angle measurements of 4.15° , 44.07° ,

FIG. 3. Normalized LSF from a slit at 4.15° (a) before and (b) after exponential extrapolation below the 0.01 level.

and 87.03° . Irregularities in pixel intensity due to nonuniformities in slit width were corrected using our proposed technique. Finally, the finely sampled LSF was found using the same binning operation described for the edge (0.02 mm subpixel bins).

To promote consistency between LSF measurements, the LSF tails were extrapolated exponentially beyond 1% of the peak value following the work of other authors.^{2,4,27} A sample LSF is shown in Fig. 3 before and after extrapolation. Following extrapolation and 1D FT of the LSF, the magnitude of the complex OTF was normalized to unity at zero frequency to form the MTF. Finally, two correction factors were applied to the MTF. It was first corrected through division by $\text{sinc}[(14 \mu\text{m})\sqrt{u^2 + v^2}]$ to account for the finite width of the slit, its magnification, and focal spot blur. The measure of $14 \mu\text{m}$ is a value derived from past work with the same slit test device.⁴ Then, the MTF was corrected for the blur due to subpixel binning through division by $|\text{sinc}(\Delta s\sqrt{u^2 + v^2})|$.

II.C. Validation by synthesis with a hypothetical direct detection device

II.C.1. 2D MTF simulation

To better understand the utility of 2D MTF estimation techniques, test images were synthesized assuming a hypothetical direct-detection FPD device. The direct-detection device was chosen in order to estimate the 2D MTF in a case where a substantial amount of aliasing is likely to be present. The blurring function of the hypothetical detector was approximated by the square pixel aperture function modeled as a rectangular window of height h_a and width w_a .^{25,28,29} This blurring function is associated with the following presampled MTF:

$$\text{MTF}_{\text{aperture}}(\rho, \theta) = |\text{sinc}(\rho w_a \cos \theta) \text{sinc}(\rho h_a \sin \theta)|. \quad (5)$$

TABLE II. Summary of values obtained from simulated edge measurement. Using the estimated edge angles, a/b ratios were obtained using the MATLAB function *rat* with tolerance factor of 0.001. Parameter n indicates that all edge angles can accommodate fine-sampling of the edge spread function with subpixel sampling bins which are at least $10\times$ smaller than the physical pixel dimension.

Parameter	Acquisition									
	1	2	3	4	5	6	7	8	9	10
θ	4.50°	13.50°	22.50°	31.50°	40.50°	49.50°	58.50°	67.50°	76.50°	85.50°
a/b	3/38	6/25	12/29	19/31	41/48	48/41	31/19	70/29	504/121	216/17
n	38.12	25.71	31.38	36.36	63.13	63.13	36.36	75.77	518.32	216.67

In Eq. (5), $\text{MTF}_{\text{aperture}}(\rho, \theta)$ is described by polar coordinates ρ and θ . In terms of the more traditional Cartesian coordinates, radial spatial frequency variable $\rho = \sqrt{u^2 + v^2}$ and $\theta = \tan^{-1}(v/u)$ is the polar angle. The effects of detector MTF were incorporated into the simulation model which is described in full mathematical detail in the Appendix. A graphical description of model variables is provided in Fig. 9.

To test the precision and accuracy of our methods, the 2D MTF of the hypothetical direct-detection device was estimated from ten simulated edge images. The angles of the edges in each image were evenly spaced between 0° and 90° at the angles shown in Table II. Images were synthesized by executing Eq. (A8) using Simpson's trapezoid rule for numerical integration with 200 001 evenly spaced spatial frequency locations between -50 and 50 mm^{-1} with parameters $\omega = 125 \text{ mm}$ and $\ell = \omega/2$ on a 384×384 pixel² lattice. Pixels were square with pitch of 0.2 mm , and simulated pixel intensities in all images ranged from 10 to 16 383. Poisson random noise was added to the simulated edge images. Finally, the presampled MTF was estimated at each angle and the 2D MTF estimate constructed in accordance with the procedures outlined in edge image processing. Results were plotted in the range $\pm 4f_c = \pm 10 \text{ mm}^{-1}$ in the u and v directions on an evenly sampled 512×512 Cartesian lattice.

II.C.2. Nonuniform slit simulation

Simulation of nonuniform slits permitted testing of the proposed slit nonuniformity correction at arbitrary angle. Wedge-shaped slits were produced by subtracting two offset edges with slightly different angles. We simulated slits of average width $s = 12.5 \mu\text{m}$ at two representative angles (1.5° and 43.5°) on a 384×384 pixel² lattice with 0.2 mm pixel pitch in the horizontal and vertical directions. Slit width varied linearly by $\pm 6.25 \mu\text{m}$ ($\pm 50\%$) over 160 pixels along the length of the slit at either angle. This type of slit-width variation was similar to that observed on the physical slit test device.

Using methods from the Appendix, the slit at 1.5° was produced by subtracting an edge image with parameters $\omega = 125 \text{ mm}$, $\ell = (\omega + s)/2$, and $\theta = 90^\circ - 1.5^\circ + \text{atan}[6.25 \mu\text{m}/(160 \times 200 \mu\text{m})]$ from another edge image with parameters $\omega = 125 \text{ mm}$, $\ell = (\omega - s)/2$, and $\theta = 90^\circ - 1.5^\circ - \text{atan}[6.25 \mu\text{m}/(160 \times 200 \mu\text{m})]$. The slit at 43.5° was produced by subtracting an edge image with parameters $\omega = 125 \text{ mm}$, $\ell = (\omega + s)/2$, and $\theta = 90^\circ - 43.5^\circ + \text{atan}[6.25 \mu\text{m}/(160 \times 200 \mu\text{m})]$ from another edge image with parameters $\omega = 125 \text{ mm}$, $\ell = (\omega - s)/2$, and $\theta = 90^\circ - 43.5^\circ$.

— $\tan[6.25 \mu\text{m}/(160 \times 200 \mu\text{m})]$. Simulated pixel intensities in both images ranged from 10 to 16 383, and Poisson random noise was added. The presampled MTF was estimated at each angle in accordance with the procedures outlined in slit image processing. Simulation results were compared for presampled MTFs produced with and without the slit nonuniformity correction.

III. RESULTS

In this section, we compare 1D and 2D estimates of the MTF from slit- and edge-based data.

III.A. Results from experimental implementation

III.A.1. 1D MTF measurements

Figure 4 shows the comparison of edge- and slit-based MTF measurements for the indirect-detection FPD. Averaging between the DC and cutoff frequencies, we find $0.78\% \pm 0.31\%$ ($\mu \pm \sigma$) difference between the orthogonal MTF measurements for the slit method, $0.31\% \pm 0.38\%$ difference between the orthogonal MTF measurements for the smoothed edge method, and $0.12\% \pm 0.40\%$ difference between the orthogonal MTF measurements for the conditioned edge method. For a given MTF response, the slit-based assessment is slightly higher than the edge-based assessment from the dc-component up to about 3.5 mm^{-1} . Averaged between zero and the cutoff frequency, the slit method returns MTF values $1.21\% \pm 0.67\%$ higher than those of the conditioned edge. These observations are consistent with other comparisons of slit- and edge-based measurements.^{1,3,30}

III.A.2. 2D MTF measurements

Figure 5(a) shows an estimate of the 2D MTF of the GE Revolution XQ/i based on an ensemble of ten 1D MTF measurements computed from conditioned data using a single edge-test device imaged once at each of ten angles. Based on five independent estimates of the 2D MTF generated from five different ensembles of edge data, the relative precision error

FIG. 4. MTF estimates from approximately horizontal and vertical slit and edge test devices. The edge-based MTF shown was computed using conditioned data. Symbols are plotted at every third data point.

TABLE III. Physical measurement results comparing different ESF denoising techniques and 2D interpolation methods in terms of relative precision error averaged over $u = v = [0, f_c]$.

Technique	Interpolation method		
	Nearest	Linear	Cubic
Nondenoised	0.42%	0.32%	0.37%
Smoothed	0.41%	0.32%	0.36%
Conditioned	0.34%	0.27%	0.30%

(standard deviation divided by the mean) for the non-negative frequencies only is reported in Fig. 5(b). From Table III, conditioned edge data with linear interpolation yield results with lowest average relative precision error which, averaged over $[0, f_c]$ in the u and v directions, is 0.26%. No perceivable patterns or differences could be detected in edge-based MTF measurements as a function of measurement angle below 4 mm^{-1} . This implies approximate radial symmetry which is emphasized by the contour plot in Fig. 5(c). Finally, in Fig. 5(d), orthogonal profiles through the 2D MTF are compared to the 1D MTF obtained from slit-based measurements.

III.A.3. Slit-based measurements and nonuniformity correction

Results from the slit-based measurements demonstrate features of the radial asymmetry of the 2D MTF. Figure 6 shows comparisons of 1D MTF measurements using a slit test device at 4.15° and 44.07° . At frequencies below 4 mm^{-1} , 1D MTF plots at the two angles are nearly identical with maximum absolute difference of 0.0076. However, above 4 mm^{-1} , the first zeros of the plots differ markedly. This observation agrees with theory which, under the assumption of 100% pixel fill factor, predicts zero-values at 5.0 mm^{-1} and 7.1 mm^{-1} for the 1D MTF measurements near 0° and 45° , respectively. And although the magnitude of the MTF for this indirect detector is quite low near the described frequencies, it clearly indicates that the approximation of circular symmetry does not apply in the high spatial frequency range. Rather, the MTF in the high-frequency range lends itself more towards the 2D separable behavior reminiscent of the FT of the rectangular pixel aperture function, an observation consistent with expectations based on detector geometry.

Also shown in Fig. 6 are comparisons of the 1D MTF produced with and without slit nonuniformity correction. Whereas the corrected plots approach zero near $2f_c$, the uncorrected plots deviate in the positive direction. To compare corrected and uncorrected MTFs, we examine the relative difference (absolute difference divided by the average) averaged between the dc and cutoff components. At 4.15° , the average relative difference between corrected and uncorrected MTFs is $0.10\% \pm 0.09\%$ with maximum absolute difference of 0.053 (near $2f_c$). At 44.07° , the average relative difference between corrected and uncorrected MTFs is $0.08\% \pm 0.07\%$ with maximum absolute difference of 0.022 (near $2f_c$). Similar results are observed in the simulated data (Fig. 8).

FIG. 5. 2D MTF estimate based on ten edge measurements between 0° and 90° . Results using conditioned edge data are shown for $\pm 1.5f_C$ in the u and v directions using 192×192 linearly interpolated points. (a) 2D rendition constructed under the assumption of reflective symmetry across the u and v axes. (b) Plot of the relative precision error for non-negative frequencies. (c) Contour plot of (a) with contours drawn at increments of 0.1 from 0.1 to 1. (d) Comparison of the MTF from approximately horizontal and vertical slits with vertical and horizontal profiles through the 2D MTF. Symbols are plotted every fourth data point.

III.B. Results from validation by synthesis

III.B.1. 2D MTF estimation

Figure 7(a) shows an estimate of the 2D MTF of the hypothetical direct-detection FPD based on an ensemble of ten 1D MTF measurements computed from conditioned data. Data

are plotted from $-4f_C$ to $+4f_C$ in both the u and v directions in order to display the first side-lobes of the 2D MTF. Figure 7(b) compares horizontal profiles through different estimates of the 2D MTF based on results using different ESF denoising methods. Figure 7(c) shows the error associated with the plots in Fig. 7(b) and reveals that the conditioned data yield

FIG. 6. Comparison of slit-based MTF measurements from physical slits at two angles with and without slit nonuniformity correction. (a) Comparison of the MTF at angles approximating 0° and 45° . (b) Detail of the MTF plot in (a) showing deviation between the MTF near 0° and 45° beyond 4 mm^{-1} . The difference between corrected and uncorrected plots can also be appreciated.

FIG. 7. 2D MTF measured from a hypothetical direct-detection device using the proposed edge method. (a) The 2D MTF was estimated from ten equally spaced edge measurements under the assumption of reflective symmetry across the u and v axes. Linear interpolation was used to estimate MTF values on a 512×512 pixel² Cartesian lattice in the range $\pm 4f_C$. (b) Using different ESF denoising methods, horizontal profiles through the 2D MTF are compared to the true MTF. (c) Error associated with (b).

superior results. This is confirmed numerically in Tables IV and V where different combinations of ESF denoising techniques and interpolation methods are compared. Conditioned data in combination with linear interpolation deliver the most accurate results. Conditioned data in combination with either linear or cubic interpolation deliver the most precise results.

III.B.2. Slit nonuniformity correction

Figure 8 demonstrates the relative impact of the slit nonuniformity correction on estimation of the MTF when applied to simulated nonuniform slits at 1.5° and 43.5° . Figure 8(a) demonstrates the improvement in MTF accuracy

when the slit nonuniformity correction is applied to slit data at 1.5° . Figure 8(b) plots the associated error. Averaged between dc and f_C , the relative accuracy error of the corrected MTF is $-0.06\% \pm 0.05\%$ while that of the uncorrected MTF is $-0.09\% \pm 0.08\%$. At 1.5° , the maximum absolute error in the corrected plot is 0.002 while that of the uncorrected plot is 0.044 (both near $2f_C$). Figures 8(c) and 8(d) show the same comparisons at 43.5° . Averaged between dc and f_C , the relative accuracy error of the corrected MTF is $-0.19\% \pm 0.16\%$ while that of the uncorrected MTF was $-0.27\% \pm 0.24\%$. At 43.5° , the maximum absolute error in the corrected plot is 0.017 while that of the uncorrected plot is 0.031 (both near $2f_C$).

TABLE IV. Simulation results comparing different ESF denoising techniques and 2D interpolation methods in terms of absolute relative accuracy error averaged over $u = v = [0, f_C]$.

Technique	Interpolation method		
	Nearest	Linear	Cubic
Nondenoised	0.59%	0.35%	0.42%
Smoothed	0.57%	0.34%	0.40%
Conditioned	0.42%	0.13%	0.20%

TABLE V. Simulation results comparing different ESF denoising techniques and 2D interpolation methods in terms of relative precision error averaged over $u = v = [0, f_C]$.

Technique	Interpolation method		
	Nearest	Linear	Cubic
Nondenoised	0.51%	0.39%	0.45%
Smoothed	0.51%	0.39%	0.44%
Conditioned	0.12%	0.11%	0.11%

FIG. 8. Slit-based MTF measurements using synthesized images from a hypothetical direct-detection device showing the impact of slit nonuniformity correction on a nonuniform slit. (a) MTF estimates at 1.5°. (b) Error in MTF estimates at 1.5°. (c) MTF estimates at 43.5°. (d) Error in MTF estimates at 43.5°.

IV. DISCUSSION

Although there is appeal for the use of 2D MTF test objects, measurement of the 2D MTF of digital medical imaging devices has traditionally been very difficult to achieve. Point-like test objects (e.g., pinholes) have the challenges of being difficult to machine and having very little transmitted flux. For opaque pinhole test devices, proper alignment may be problematic due to the thickness of the stock material. On the contrary, nonopaque devices may require background detrending which can affect low-frequency MTF measurements. More complicated point-like test objects (pinhole arrays) have been described in the literature to address some of these challenges, but they are still difficult to manufacture.^{5,10} Disk-test devices have also been described for use in the assessment of 2D MTF, but reports indicate that they lack accuracy due to signal aliasing.^{11–13} Recent advances have yielded promising results for extraction of the 2D MTF from NPS data, but the precision and accuracy of these results come at the cost of intensive cascaded systems analysis and fitting of measured data to an assumed functional form of the MTF.⁹

By comparison with existing 2D MTF test objects, namely, 2D arrays of pinholes,^{5,11} our techniques that estimate the 2D MTF from multiple measurements with 1D devices have certain benefits. First, 1D test devices have been used extensively in laboratory and clinical settings, and their use is well un-

derstood and widely accepted. Second, 1D test devices are generally available, precluding the need for new equipment. Third, the use of an edge-test device in the estimation of 2D MTF affords a reasonable number of measurements for high precision and practical application. And fourth, our evidence suggests that the use of multiple 1D assessments of MTF at a wide range of angles is a robust, precise, and accurate method for 2D interpretation of detector resolution properties. In particular, the edge method is quite tolerant of modest misalignment errors, and we have shown that it provides results similar to those derived using the slit method.^{1,3}

While the edge method for 2D MTF estimation has several strengths, there are also some limitations to its implementation. First, the number of angles required for accurate representation of the 2D MTF is highly dependent upon its asymmetry. If the spacing of the radial (1D) MTF samples is too coarse, then fine details may be lost during the 2D interpolation process. Furthermore, if certain assumptions regarding symmetry of the MTF are not valid (e.g., reflective symmetry across the axes), then data collection at angles between 0° and 90° may need to be extended to 180°. Second, if the 2D MTF at higher frequencies is to be accurately estimated, then finer radial sampling may be required to account for the reduction in radial sample density with increasing frequency. And third, it is well known that the edge method for MTF determination at very high frequencies is limited by lower

signal-to-noise ratio (SNR) compared to other methods (e.g., the slit method).^{30–33} If these high frequencies are of particular interest, then special care must be taken to ensure that adequate SNR is achieved.

In our work, we note upon qualitative comparisons of our edge method of 2D MTF assessment to the results of Fetterly *et al.*⁵ who measured the 2D MTF of the GE Revolution XQ/i using an array of pinholes. They concluded that the 2D MTF of this indirect-detection FPD is circularly symmetric (out to $\pm 3.75 \text{ mm}^{-1}$ in the u and v directions). We draw similar conclusions for this same detector in the same frequency range. However, it should be recognized that the pinhole-based 2D MTF results of Fetterly *et al.* differ from their edge-based measurements near the vertical and horizontal axes by up to 8% at frequencies between the dc and cutoff components. Fetterly *et al.* speculate that this deviation may be the result of low-frequency glare from neighboring pinholes and from detector exposure near the edges of the test device.⁵ Regardless of the source, any error in MTF estimation will be amplified in calculations of the 2D NEQ and DQE since these quantities are proportional to the square of the MTF. Analyses of the digital NEQ and DQE are limited to $\pm f_c$ in the Cartesian coordinate plane,³³ and in this range, our edge-based methods provide average relative uncertainty in the presampled MTF of 0.26% for physically obtained data. According to the accuracy achieved in our simulation studies, it is likely that the average absolute relative accuracy error is of similar magnitude.

The high accuracy of our methods has demonstrated the importance of fully 2D MTF assessment. Whereas orthogonal 1D measurements of MTF have traditionally been used to classify system resolution properties, accurate assessment of the 2D MTF can provide more highly detailed measures of system performance. In particular, the MTF measurements of the indirect-detection FPD in our studies showed differences in the observed symmetry at low- and high-frequencies. Both slit- and edge-based measurements of the GE Revolution XQ/i suggest approximate circular symmetry of the MTF at frequencies below 4 mm^{-1} . However, at frequencies above 4 mm^{-1} , slit-based measurements of the GE Revolution XQ/i near 45° suggest that the 2D separable pixel aperture function dominates the behavior of the MTF. The difference in symmetry at low- and high-frequencies was only observed when using the slit method, likely due to its superior SNR at higher frequencies.³⁰ While the edge method exhibits higher SNR at lower frequencies,³⁰ it is subjected to positive bias at higher frequencies due to quantum,³ electronic, and mechanical noise.^{2,31} In addition, high-frequency noise in the ESF is amplified by differentiation (required to compute the LSF).^{32,34} Although the amplitude of the MTF at high frequencies is quite low for most indirect-detection FPD devices, high accuracy and precision at high frequency has more profound implications when direct-detection devices are considered.

Images from a hypothetical direct-detection FPD were simulated in this work, and they served several purposes. First, the simulations provided introspection into the anticipated performance of our 2D MTF estimation methods for a

detector with appreciable MTF amplitude beyond $2f_c$. This can lead to appreciable aliasing in the response function, and we showed that our methods are likely to perform well, even out to $\pm 4f_c$ for this type of detector. Second, the simulated device permitted the computation of accuracy and precision figures given that the true 2D MTF was known exactly (Tables IV and V). And third, simulations of a nonuniform slit were conducted to investigate the impact of the nonuniformity and corresponding nonuniformity correction in terms of MTF estimation error. We showed that slit nonuniformity has the greatest effect on the MTF near $2f_c$ where the amplitude is quite low. At 1.5° and 43.5° , slit nonuniformity correction improved the error in MTF estimation, but we must also remark on some of its limitations.

The proposed slit nonuniformity correction corrects for some, but not all, of the effects encountered in imperfect slits. The nonuniformity correction improved measurements of the MTF near $2f_c$ for both real and simulated slits; however, the correction was less impactful at lower frequencies. Furthermore, slits with variable width produce perpendicular intensity profiles which are not only variable in terms of transmitted intensity but also in terms of profile width. Although changes in the profile width are not corrected by the intensity normalization technique, such fluctuations may be of little consequence as the sinc correction used to correct for the width of very narrow slits only adjusts the amplitude of the MTF by a few percent even at high frequencies. Of perhaps greater interest would be an investigation into the effects of high-frequency fluctuations in slit width. These types of variations could introduce large errors in the finely sampled LSF, especially in regions where the magnitude of its derivative is greatest. Such investigations are deferred to future work.

Given all of these considerations, we conclude that the edge- and slit-based assessments of 2D MTF presented here provide very favorable trade-offs between ease-of-implementation and achievable accuracy. Edge-based assessments of MTF are relatively simple and well established, and many facilities already possess the proper equipment for this type of analysis, while slit-based methods offer better performance beyond the cutoff frequency. Results from imaging studies and simulations show that the proposed methods are not only feasible, but they may provide higher accuracy than some 2D measurements produced using pinhole- and disk-test devices. Measuring the full 2D MTF provides details of resolution performance not seen with traditional 1D methods along only the vertical and horizontal axes thus providing better data for the assessment of 2D NEQ and DQE.

ACKNOWLEDGMENTS

This work was supported in part by a training grant from the National Institutes of Health (TG32 EB007185). The indirect-detection flat-panel detector used in this study was provided through a research agreement with GE Healthcare. The authors wish to thank Dr. Ehsan Samei and the members of the Duke University Clinical Imaging Physics Group (CIPG) for their helpful conversations and provision of equipment for experiments.

APPENDIX: SYNTHESIS OF DIGITIZED SLIT AND EDGE OBJECTS WITH SYSTEM BLURRING EFFECTS

To predict the accuracy of MTF estimates generated from arbitrarily angled resolution test devices, a generalized model was developed for computer simulation purposes. Ideally, 1D MTF analysis would be conducted using a Dirac delta line impulse object of the form

$$\delta_\ell(x, y) = \delta(x \cos \theta_0 + y \sin \theta_0 - \ell). \quad (\text{A1})$$

The Dirac delta line impulse is alternatively defined in the following limit derived from the definition of an angled and shifted rectangular function:

$$\delta_\ell(x, y) = \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{rect} \left(\frac{x \cos \theta_0 + y \sin \theta_0 - \ell}{\omega} \right). \quad (\text{A2})$$

The test signal is parameterized by width ω , unit normal at angle θ_0 , and distance ℓ from the origin in the direction of the unit normal. For small ω , Eq. (A2) approximates an ideal physical slit imaged using ideal parallel beam geometry (illustrated in Fig. 9). For sufficiently large ω , the same model may be used to approximate an edge test function assuming proper assignment of ℓ such that only a single edge is visible in the relevant field of view.

The rectangular function of infinite extent $f(x, y)$ is defined such that the volume per unit length in the “long” dimension is unity in order to ensure that, in the limit as $\omega \rightarrow 0$, the integral of the Dirac delta function [Eq. (A2)] is also unity. Therefore, the unsampled, unblurred signal in the spatial domain is defined as

$$f(x, y) = \frac{1}{\omega} \text{rect} \left(\frac{x \cos \theta_0 + y \sin \theta_0 - \ell}{\omega} \right). \quad (\text{A3})$$

Modification of $f(x, y)$ by some system PSF is modeled as the convolution of $f(x, y)$ with the PSF to produce $g(x, y)$. To avoid the computational complexities associated with convolution, the signal is converted to its frequency space representation

FIG. 9. Depiction of variables used to define a resolution test object of finite width and infinite length cast onto a Cartesian coordinate system.

tation by the Fourier transform as outlined by the convolution theorem (* denotes convolution)

$$\begin{aligned} \mathcal{F}_{2D}\{f * \text{PSF}\}(x, y)(u, v) \\ = \mathcal{F}_{2D}\{f\}(x, y)(u, v) = G(u, v) = F(u, v)\text{MTF}(u, v). \end{aligned} \quad (\text{A4})$$

Expanding $F(u, v)$ yields

$$\begin{aligned} F(u, v) = \mathcal{F}_{2D}\{f\}(x, y)(u, v) \\ = \text{sinc} \left(\frac{u\omega}{\cos \theta_0} \right) e^{-2\pi i u \ell / \cos \theta_0} \frac{\delta(v - u \tan \theta_0)}{\cos \theta_0}. \end{aligned} \quad (\text{A5})$$

The Dirac delta line impulse in Eq. (A5) may be modified for computational convenience³⁵

$$F(u, v) = \text{sinc} \left(\frac{u\omega}{\cos \theta_0} \right) e^{-2\pi i u \ell / \cos \theta_0} \delta(v \cos \theta_0 - u \sin \theta_0). \quad (\text{A6})$$

The sifting property of the Dirac delta line impulse greatly simplifies the equation, especially when converted to representation in polar coordinates

$$F(\rho, \theta_0) = \text{sinc}(\rho\omega) e^{-2\pi i \rho \ell}. \quad (\text{A7})$$

To incorporate the effects of system blur, $F(\rho, \theta_0)$ must be modified by the MTF to produce $G(\rho, \theta_0)$. In turn, $G(\rho, \theta_0)$ must undergo a 1D inverse Fourier transform to return the problem to the spatial domain using

$$\begin{aligned} g(x, y) = \mathcal{F}_{1D}^{-1}\{G\}(\rho, \theta_0)(x, y) \\ = \int_{-\infty}^{\infty} \text{sinc}(\rho\omega) \text{MTF}(\rho, \theta_0) e^{2\pi i \rho (x \cos \theta_0 + y \sin \theta_0 - \ell)} d\rho. \end{aligned} \quad (\text{A8})$$

It should be noted that the MTF in Eq. (A8) involves all components of detector blur (including signal integration across finite pixel apertures). If the MTF of the system is expressed in analytical form, Eq. (A8) lends itself readily to straightforward calculation. However, the MTF of real systems are typically computed from measured data which may lack a convenient analytical form. In either case, Simpson's rule for numerical integration can be used to conduct the 2D inverse Fourier transform of $G(\rho, \theta_0)$. To verify the accuracy of this derivation, the IFT is performed without loss of generality by assuming unit MTF,

$$\begin{aligned} g(x, y) &= \int_{-\infty}^{\infty} \text{sinc}(\rho\omega) e^{2\pi i \rho (x \cos \theta_0 + y \sin \theta_0 - \ell)} d\rho \\ &= \frac{1}{\omega} \text{rect} \left(\frac{x \cos \theta_0 + y \sin \theta_0 - \ell}{\omega} \right) = f(x, y) \quad \blacksquare \end{aligned} \quad (\text{A9})$$

^{a)} Author to whom correspondence should be addressed. Electronic mail: jered.wells@duke.edu

¹E. Samei, N. T. Ranger, J. T. Dobbins III, and Y. Chen, “Intercomparison of methods for image quality characterization. I. Modulation transfer function,” *Med. Phys.* **33**, 1454–1465 (2006).

- ²H. Fujita, D. Y. Tsai, T. Itoh, K. Doi, J. Morishita, K. Ueda, and A. Ohtsuka, "A simple method for determining the modulation transfer function in digital radiography," *IEEE Trans. Med. Imaging* **11**, 34–39 (1992).
- ³E. Samei, M. J. Flynn, and D. A. Reimann, "A method for measuring the presampled MTF of digital radiographic systems using an edge test device," *Med. Phys.* **25**, 102–113 (1998).
- ⁴J. T. Dobbins III, D. L. Ergun, L. Rutz, D. A. Hinshaw, H. Blume, and D. C. Clark, "DQE(f) of four generations of computed radiography acquisition devices," *Med. Phys.* **22**, 1581–1593 (1995).
- ⁵K. A. Fetterly, N. J. Hangiandreou, B. A. Schueler, and E. R. Ritenour, "Measurement of the presampled two-dimensional modulation transfer function of digital imaging systems," *Med. Phys.* **29**, 913–921 (2002).
- ⁶P. R. Granfors and R. Aufrichtig, "Performance of a 41×41-cm² amorphous silicon flat panel x-ray detector for radiographic imaging applications," *Med. Phys.* **27**, 1324–1331 (2000).
- ⁷W. Zhao and J. A. Rowlands, "Digital radiology using active matrix readout of amorphous selenium: Theoretical analysis of detective quantum efficiency," *Med. Phys.* **24**, 1819–1833 (1997).
- ⁸J. T. Dobbins III, "Effects of undersampling on the proper interpretation of modulation transfer function, noise power spectra, and noise equivalent quanta of digital imaging systems," *Med. Phys.* **22**, 171–181 (1995).
- ⁹A. Kuhls-Gilcrist, A. Jain, D. R. Bednarek, K. R. Hoffmann, and S. Rudin, "Accurate MTF measurement in digital radiography using noise response," *Med. Phys.* **37**, 724–735 (2010).
- ¹⁰M. Báth, P. Sund, and L. G. Mansson, "Method for determining the two-dimensional presampling modulation transfer function in digital radiography," *Proc. SPIE* **4320**, 268–279 (2001).
- ¹¹M. Báth, M. Hakansson, and L. G. Mansson, "Determination of the two-dimensional detective quantum efficiency of a computed radiography system," *Med. Phys.* **30**, 3172–3182 (2003).
- ¹²D. A. Reimann, H. A. Jacobs, and E. Samei, "Use of Wiener filtering in the measurement of the two-dimensional modulation transfer function," *Proc. SPIE* **3977**, 670–680 (2000).
- ¹³D. A. Reimann, E. Samei, and H. A. Jacobs, "Evaluation of a new technique for the measurement of the two dimensional MTF of digital radiographic systems," *Med. Phys.* **28**, 1187 (2001).
- ¹⁴E. W. Marchand, "From line to point spread function: The general case," *J. Opt. Soc. Am.* **55**, 352–354 (1965).
- ¹⁵C. D. Bradford, W. W. Peppler, and J. T. Dobbins III, "Performance characteristics of a Kodak computed radiography system," *Med. Phys.* **26**, 27–37 (1999).
- ¹⁶C. E. Floyd, Jr., R. J. Warp, J. T. Dobbins III, H. G. Chotas, A. H. Baydush, R. Vargas-Voracek, and C. E. Ravin, "Imaging characteristics of an amorphous silicon flat-panel detector for digital chest radiography," *Radiology* **218**, 683–688 (2001).
- ¹⁷J. M. Boone, "Determination of the presampled MTF in computed tomography," *Med. Phys.* **28**, 356–360 (2001).
- ¹⁸International Electrotechnical Commission, "Medical diagnostic x-ray equipment—Radiation conditions for use in the determination of characteristics," (International Electrotechnical Commission, Geneva, Switzerland, 2005), Vol. IEC 61267.
- ¹⁹International Electrotechnical Commission, "Medical electrical equipment—Characteristics of digital x-ray imaging devices—Part 1: Determination of the detective quantum efficiency," (International Electrotechnical Commission, Geneva, Switzerland, 2003), Vol. IEC 62220-1.
- ²⁰N. T. Ranger, E. Samei, J. T. Dobbins III, and C. E. Ravin, "Measurement of the detective quantum efficiency in digital detectors consistent with the IEC 62220-1 standard: Practical considerations regarding the choice of filter material," *Med. Phys.* **32**, 2305–2311 (2005).
- ²¹E. Samei, N. T. Ranger, J. T. Dobbins III, and Y. Chen, "Intercomparison of methods for image quality characterization. I. Modulation transfer function," *Med. Phys.* **33**, 1454–1465 (2006).
- ²²T. W. Ridler and S. Calvard, "Picture thresholding using an iterative selection method," *IEEE Trans. Syst. Man Cybern.* **8**, 630–632 (1978).
- ²³J. Canny, "A computational approach to edge detection," *IEEE Trans. Pattern Anal. Mach. Intell.* **8**, 679–698 (1986).
- ²⁴A. D. Maidment and M. Albert, "Conditioning data for calculation of the modulation transfer function," *Med. Phys.* **30**, 248–253 (2003).
- ²⁵E. Buhr, S. Gunther-Kohfahl, and U. Neitzel, "Accuracy of a simple method for deriving the presampled modulation transfer function of a digital radiographic system from an edge image," *Med. Phys.* **30**, 2323–2331 (2003).
- ²⁶J. T. Dobbins III, E. Samei, N. T. Ranger, and Y. Chen, "Intercomparison of methods for image quality characterization. II. Noise power spectrum," *Med. Phys.* **33**, 1466–1475 (2006).
- ²⁷K. Doi, K. Strubler, and K. Rossmann, "Truncation errors in calculating the MTF of radiographic screen-film systems from the line spread function," *Phys. Med. Biol.* **17**, 241–250 (1972).
- ²⁸E. Samei and M. J. Flynn, "An experimental comparison of detector performance for direct and indirect digital radiography systems," *Med. Phys.* **30**, 608–622 (2003).
- ²⁹J. Yorkston, L. E. Antonuk, N. Seraji, W. Huang, J. H. Siewerdsen, and Y. El-Mohri, "Evaluation of the MTF for a-Si:H imaging arrays," *Proc. SPIE* **2163**, 141–149 (1994).
- ³⁰I. A. Cunningham and B. K. Reid, "Signal and noise in modulation transfer function determinations using the slit, wire, and edge techniques," *Med. Phys.* **19**, 1037–1044 (1992).
- ³¹J. C. Dainty and R. Shaw, *Image Science: Principles, Analysis and Evaluation of Photographic-Type Imaging Processes* (Academic, San Diego, CA, 1992).
- ³²I. A. Cunningham and A. Fenster, "A method for modulation transfer function determination from edge profiles with correction for finite-element differentiation," *Med. Phys.* **14**, 533–537 (1987).
- ³³I. A. Cunningham, "Applied linear-systems theory," in *Handbook of medical imaging: Physics and psychophysics*, edited by J. Beutel, H. L. Kundel and R. L. Van Metter (SPIE, Bellingham, WA, 2000), Vol. 1, pp. 79–159.
- ³⁴S. E. Reichenbach, S. K. Park, and R. Narayanswamy, "Characterizing digital image acquisition devices," *Opt. Eng.* **30**, 170–177 (1991).
- ³⁵R. L. Easton, *Fourier Methods in Imaging* (Wiley, Chichester, West Sussex, 2010).