A nonlinear lag correction algorithm for a-Si flat-panel x-ray detectors

Jared Starman^{a)}

Department of Electrical Engineering, Stanford University, Stanford, California 94305
and Department of Radiology, Stanford University, Stanford, California 94305

Josh Star-Lack, Gary Virshup, and Edward Shapiro

Varian Medical Systems, Palo Alto, California 94304

Rebecca Fahrig

Department of Radiology, Stanford University, Stanford, California 94305

(Received 7 March 2012; revised 17 July 2012; accepted for publication 26 August 2012;
published 18 September 2012)

Purpose: Detector lag, or residual signal, in a-Si flat-panel (FP) detectors can cause significant shading artifacts in cone-beam computed tomography reconstructions. To date, most correction models have assumed a linear, time-invariant (LTI) model and correct lag by deconvolution with an impulse response function (IRF). However, the lag correction is sensitive to both the exposure intensity and the technique used for determining the IRF. Even when the LTI correction that produces the minimum error is found, residual artifact remains. A new non-LTI method was developed to take into account the IRF measurement technique and exposure dependencies.

Methods: First, a multiexponential ($N = 4$) LTI model was implemented for lag correction. Next, a non-LTI lag correction, known as the nonlinear consistent stored charge (NLCSC) method, was developed based on the LTI multiexponential method. It differs from other nonlinear lag correction algorithms in that it maintains a consistent estimate of the amount of charge stored in the FP and it does not require intimate knowledge of the semiconductor parameters specific to the FP. For the NLCSC method, all coefficients of the IRF are functions of exposure intensity. Another nonlinear lag correction method that only used an intensity weighting of the IRF was also compared. The correction algorithms were applied to step-response projection data and CT acquisitions of a large pelvic phantom and an acrylic head phantom. The authors collected rising and falling edge step-response data on a Varian 4030CB a-Si FP detector operating in dynamic gain mode at 15 fps at nine incident exposures (2.0%–92% of the detector saturation exposure). For projection data, 1st and 50th frame lag were measured before and after correction. For the CT reconstructions, five pairs of ROIs were defined and the maximum and mean signal differences within a pair were calculated for the different exposures and step-response edge techniques.

Results: The LTI corrections left residual 1st and 50th frame lag up to 1.4% and 0.48%, while the NLCSC lag correction reduced 1st and 50th frame residual lags to less than 0.29% and 0.0052%. For CT reconstructions, the NLCSC lag correction gave an average error of 11 HU for the pelvic phantom and 3 HU for the head phantom, compared to 14–19 HU and 2–11 HU for the LTI corrections and 15 HU and 9 HU for the intensity weighted non-LTI algorithm. The maximum ROI error was always smallest for the NLCSC correction. The NLCSC correction was also superior to the intensity weighting algorithm.

Conclusions: The NLCSC lag algorithm corrected for the exposure dependence of lag, provided superior image improvement for the pelvic phantom reconstruction, and gave similar results to the best case LTI results for the head phantom. The blurred ring artifact that is left over in the LTI corrections was better removed by the NLCSC correction in all cases. © 2012 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4752087>]

Key words: x-ray detector, flat-panel, lag, a-Si, radar artifact

I. INTRODUCTION

Amorphous silicon (a-Si) flat panel (FP) detectors can exhibit detector lag that affects projection images in fluoroscopy and cone-beam computed tomography (CBCT) reconstructions.^{1–6} Detector lag is defined as signal present in frames following the frame in which it was generated. In fluoroscopy, lag causes temporal blurring and reduces temporal resolution. For a-Si FP detectors designed to be used

for mixed radiography and fluoroscopy applications, lag can cause severe ghosting of the high-dose radiography image during low-dose fluoroscopy.^{1,2,7} In CBCT reconstructions, detector lag can lead to image artifacts such as streaking, blurring, and severe shading artifacts referred to here as the radar artifact.^{2–6}

The main source of detector lag in a-Si FP x-ray detectors is charge trapping in the a-Si photodiode.^{2,7,8} Defect states, known as charge traps, fill with charge during photodiode

illumination and then release the charge during later detector frames. Contrast in projection images, or ghosts, arises from differences in the trapping history of different pixels. For CBCT reconstructions, the different trapping histories cause shading artifacts for parts of the object that correspond to detector pixels that experience large dynamic range changes.⁶

Several hardware approaches exist to remove lag from an a-Si FP detector. Typically, the hardware methods involve waiting an extra amount of time for the lag signal to decay,³ which slows data acquisition, or involve making the lag signal into a completely uniform offset signal that can then be subtracted out. The second method can be achieved using either LEDs⁷ or the pixel photodiode operated in a forward bias mode.^{9–11} Drawbacks of these two methods are that they cannot correct the temporal behavior of the scintillator, require hardware redesigns, limit frame rates, and increase dark currents.

Software lag compensation methods also exist for reducing lag in a-Si FP detectors. For these methods, a suitable model of the lag decay is fit to the a-Si FP detector data with a multiexponential¹² signal or a power function.¹ Typically, the a-Si FP is modeled as a linear time-invariant (LTI) system, which is described by an impulse response function (IRF). The correction is a temporal deconvolution of the detector output with the modeled IRF.¹³

Problems with temporal IRF deconvolution have been identified.⁶ First, there are many ways to measure the IRF, such as measured directly from the response to a single frame of radiation² or derived from the response to a much longer period of irradiation (e.g., a step function). The rising step-response function (RSRF) looks at the a-Si FP response to the onset of x-ray exposure, while the falling step-response function (FSRF) is the response after a long irradiation has stopped. Our work focuses solely on the step-response techniques because we are most interested in characterizing the long-time-constant charge traps which are responsible for the radar artifact. For a LTI system, the IRF should be independent of the technique used, but large differences between the responses determined from the RSRF and FSRF have been shown for both a-Si FPs and image intensifiers using cesium iodide (CsI) scintillators.^{6,14,15} It has also been shown that the IRF is dependent on the exposure level of the step function used. Taken together, there is uncertainty in what is the best technique to use for lag correction using a LTI model. Furthermore, the charge trapping mechanism is not adequately described by a LTI model since residual image artifact in the CBCT reconstructions can remain after correction. An algorithm based on the semiconductor physics has been developed and is discussed further in Appendix A.¹⁶ It assumes that the input signal is a scaled step-response and requires intimate knowledge of the semiconductor parameters. This work explores the effectiveness of a novel nonlinear time-variant (non-LTI) algorithm compared to standard LTI algorithms.

The paper is organized as follows. Section II describes the non-LTI algorithm known as the nonlinear consistent stored charge (NLCSC) method that has been developed and how to calibrate the parameters for the algorithm. Next, step-response and CBCT experiments are described that are used

to compare the NLCSC to other LTI and non-LTI algorithms. Section III presents the results of the NLCSC calibration, the step-response experiments, and the CBCT experiments. Finally, Sec. IV summarizes and discusses the findings of this work.

II. METHODS AND MATERIALS

In this section, an existing LTI lag correction is introduced and then extended to a novel, non-LTI correction that takes into account the nonlinearity and time variance of the charge trapping mechanism. Step-response and CBCT experiments are then described to compare the algorithms. All experiments were carried out on a table-top x-ray system using a Varian 4030CB detector running at 15 fps. For the purposes of deriving the coefficients for all the corrections, the central 100×1024 pixels of the detector were averaged together. Previous work on a linear lag correction found that using a large ROI produced nearly the same results as many small ROIs for radar artifact removal in CBCT reconstructions.⁶

II.A. LTI lag theory

In his work, Hsieh¹³ represents the temporal impulse response for afterglow as a multiexponential signal, where the time constants and coefficients are known *a priori* for the underlying continuous process. For operating on frame-integrated data that come from an a-Si FP detector, it is useful to have a discrete-time version, since the detector output is inherently discrete in time. A discrete-time version can be easily derived, starting from the multiexponential impulse response

$$h(k) = b_0\delta(k) + \sum_{n=1}^N b_n e^{-a_n k}, \quad (1)$$

where k is the discrete-time variable for x-ray frames, and N is the number of exponential terms in the impulse response, typically between two and four.^{4,6,13} $\delta(k)$ is the impulse function of magnitude 1.0, and $b_0\delta(k)$ represents the portion of the input signal that is unaffected by lag. The coefficients b_n will be referred to as the lag coefficients and the exponential rates a_n will be referred to as the lag rates. The sum of all b_n will be forced to equal 1.0 so that $h(0)$ equals 1.0. The sum could be scaled to other values, but that will only result in an overall scaling of the corrected output. Given $h(k)$, the actual FP output is

$$y(k) = x(k) * h(k). \quad (2)$$

The ideal output of the FP with no charge trapping is $x(k)$, what we wish to determine. A direct measure of the incident exposure on the FP detector, $x(k)$ is measured in units of detector counts.

The recursive deconvolution algorithm for this IRF model is modified version of Hsieh's afterglow correction¹³

$$x(k) = x_k = \frac{y(k) - \sum_{n=1}^N b_n S_{n,k} e^{-a_n}}{\sum_{n=0}^N b_n}, \quad (3)$$

$$S_{n,k+1} = x_k + S_{n,k} e^{-a_n}. \quad (4)$$

FIG. 1. (a) Flow chart for the LTI deconvolution algorithm and (b) flow chart for the NLCSC algorithm. An important modification made for the NLCSC algorithm is changing the state variable to the estimated stored charge.

This version is easily derived by indexing the data by frame number. Determination of the lag coefficients and lag rates from a step response is discussed further in Ref. 6.

$S_{n,k}$ is a state variable that contains the contribution from previous inputs, where each input is recursively weighted by e^{-a_n} . The state variable transmits the weighted history of the FP from one frame of the deconvolution algorithm to the next, and is graphically depicted in Fig. 1(a).

II.A.1. Estimated stored charge

When the deconvolution algorithm is applied, an estimate of the charge stored in the FP detector, q_k , can be calculated at every frame. This calculation is an essential part of the NLCSC algorithm, but is also applicable to the LTI algorithm. The estimate q_k is equivalent to the summation of all future corrections the deconvolution algorithm would apply if all future inputs equal 0 (i.e., $x(i) = 0$ for all $i > k$). In this case, the algorithm should exactly remove the lag signal and return 0. Another way to think about q_k is as the summation of all future lag signals, which is an estimate of the current charge stored in the FP. Future lag signals can be calculated from the LTI model and state variable by fixing $x_k = 0$ in Eqs. (3) and (4) and calculating the predicted $y(k)$. From Eq. (3) and defining L_j as the lag in the j th frame beyond the current frame k ,

$$y(k+1) = L_1 = \sum_{n=1}^N b_n e^{-a_n} S_{n,k+1}. \quad (5)$$

In the next frame, again with no input,

$$y(k+2) = L_2 = \sum_{n=1}^N b_n e^{-a_n} S_{n,k+2}, \quad (6)$$

where, from Eq. (4) with no input ($x_{k+1} = 0$),

$$S_{n,k+2} = S_{n,k+1} e^{-a_n}. \quad (7)$$

The sum of all residual signals in the predicted lag frames is defined as q_{k+1} ,

$$\begin{aligned} q_{k+1} &= \sum_{j=1}^{\infty} y(k+j) = \sum_{j=1}^{\infty} L_j \\ &= \sum_{j=1}^{\infty} \sum_{n=1}^N b_n S_{n,k+1} (e^{-a_n})^j \end{aligned}$$

$$\begin{aligned} &= \sum_{n=1}^N b_n e^{-a_n} S_{n,k+1} \sum_{m=0}^{\infty} (e^{-a_n})^m \\ &= \sum_{n=1}^N S_{n,k+1} \left(\frac{b_n e^{-a_n}}{1 - e^{-a_n}} \right). \end{aligned} \quad (8)$$

Note, the subscript for q is $k+1$ and the notation is meant to mimic that for $S_{n,k}$ in Eq. (4). The stored charge immediately preceding irradiation in frame k is q_k , and q_{k+1} is the stored charge immediately after the irradiation of the k th frame.

We also define $q_{n,k}$ as the estimated stored charge in the FP associated with the n th exponential term in Eq. (8),

$$q_{n,k} = S_{n,k} \frac{b_n e^{-a_n}}{1 - e^{-a_n}}, \quad (9)$$

where

$$q_k = \sum_{n=1}^N q_{n,k}. \quad (10)$$

II.B. NLCSC lag algorithm

For the NLCSC algorithm, different exposure-dependent IRFs are used to deconvolve the detector output to account for the nonlinearity and time variance properties of the a-Si FP.⁶ The current estimate of signal without lag, x_k , will be used as the measure of the exposure level. The function that characterizes the change in $h(k)$ as a function of x_k will be referred to as the impulse response generator function

$$h(k, x_k) = b_0(x_k) \delta(k) + \sum_{n=1}^N b_n(x_k) e^{-a_n(x_k)k}. \quad (11)$$

For fixed values of x_k , $h(k, x_k)$ is a multiexponential IRF such as Eq. (1). When the exposure intensity on the FP changes, the lag rates and lag coefficients can change, creating a new IRF. This is in accordance with the model of charge trapping in a-Si by Wiczcerek,¹⁶ where the trap dynamics and the trap occupation function change as a function of illumination intensity on the photodiode.

The important requirement enforced by the NLCSC algorithm is that the estimate of stored charge associated with the n th exponential term, $q_{n,k}$, should not change when $h(k, x_k)$ changes to a new function. In other words, the effects of previous irradiation on the FP do not change when the current

exposure intensity on the FP changes. Only the present charge filling and emptying dynamics should change.

In Eq. (9), $q_{n,k}$ depends on the lag rates (a_n) and the lag coefficients (b_n). Now, if a_n and b_n change on a frame by frame basis, as the NLCSC algorithm allows, $q_{n,k}$ would also suddenly change if $S_{n,k}$ remains constant. This does not make physical sense and violates the requirement being used to construct the NLCSC algorithm. The conservation of estimated charge requirement in the model can be forced upon the NLCSC algorithm by propagating $q_{n,k}$ as a separate state variable for every frame. A new state variable, $S_{n,k}^*$, is defined such that it is consistent with the stored charge after the previous frames, $q_{n,k}$, and the IRF of the current frame, $h(k, x_k)$. Prior to the deconvolution step in Eq. (3), $S_{n,k}^*$ will be calculated from $q_{n,k}$, $a_n(x_k)$, and $b_n(x_k)$.

The deconvolution algorithm in Eqs. (3) and (4) becomes

$$S_{n,k}^* = F_1(q_{n,k}, h(k, x_k)), \quad (12)$$

$$x_k = \frac{y(k) - \sum_{n=1}^N b_n(x_k) S_{n,k}^* e^{-a_n(x_k)}}{\sum_{n=0}^N b_n(x_k)}, \quad (13)$$

$$S_{n,k+1} = x_k + S_{n,k}^* e^{-a_n(x_k)}, \quad (14)$$

$$q_{n,k+1} = F_2(S_{n,k+1}, h(k, x_k)). \quad (15)$$

The F_1 and F_2 functions are inverses of each other and are explicitly given in Sec. II.B.1. For a true LTI system where the lag rates and lag coefficients are not functions of x_k , Eqs. (12)–(15) are equivalent to Eqs. (3) and (4) and $S_{n,k}^*$ always equals $S_{n,k}$. Also, $S_{n,k}^*$ is equal to $S_{n,k}$ if $x_{k-1} = x_k$ in Eq. (12). Otherwise, $S_{n,k}^*$ is forced to be consistent with $q_{n,k}$, $a_n(x_k)$, and $b_n(x_k)$. The information that is transmitted to the next step in the deconvolution is now $q_{n,k}$, which is depicted graphically in Fig. 1(b). The abstract structure of the NLCSC algorithm mirrors that of the LTI algorithm, but with a different state variable and exposure-dependent equations.

In Eq. (12), $S_{n,k}^*$ depends on x_k through $h(k, x_k)$, which is the unknown value to be solved for. It is assumed that $h(k, x_k)$ is a slowly varying function of x_k and that $h(k, y(k)) \approx h(k, x_k)$ for purposes of the deconvolution. This can be improved upon by iterating between Eqs. (12) and (13).

After the update functions $F_1()$ and $F_2()$ are given in Sec. II.B.1, details on how $b_n(x_k)$ and $a_n(x_k)$ vary with exposure will be given.

II.B.1. Stored charge constraint

Physically, the amount of estimated charge stored in the FP should remain constant when switching from one set of linear system parameters to another. Because the system is causal, current inputs will not affect charge storage history. The state variable update functions, F_1 and F_2 in Eqs. (12) and (15) enforce this constraint by calculating a consistent $S_{n,k}^*$ with

$$S_{n,k}^* = F_1(q_{n,k}, h(k, x_k)) = q_{n,k} \frac{1 - e^{-a_n(x_k)}}{b_n(x_k) e^{-a_n(x_k)}}, \quad (16)$$

and conversely,

$$q_{n,k+1} = F_2(S_{n,k+1}, h(k, x_k)) = S_{n,k+1} \frac{b_n(x_k) e^{-a_n(x_k)}}{1 - e^{-a_n(x_k)}}. \quad (17)$$

Another way to look at F_1 and the calculation of $S_{n,k}^*$ is as a weighting or adjustment of the exposure-dependent lag coefficients. An equation for $S_{n,k}^*$ can be written out to give

$$S_{n,k}^* = \left(\frac{1 - e^{-a_n(x_k)}}{b_n(x_k) e^{-a_n(x_k)}} \right) \left(\frac{b_n(x_{k-1}) e^{-a_n(x_{k-1})}}{1 - e^{-a_n(x_{k-1})}} \right) S_{n,k}. \quad (18)$$

The $S_{n,k}^*$ term in the numerator of Eq. (13) can be replaced by Eq. (18). Thought of this way, it indicates that the lag coefficients and lag rates cannot change independently; they must change in such a way that $q_{n,k}$ is preserved.

II.B.2. Exposure-dependent lag coefficients: $b_n(x)$

For a LTI system with a multiexponential impulse response of the form in Eq. (1), the analytical, unnormalized FSRF is found by convolving a falling step function of magnitude x with the IRF. For frames after the source is turned off, the analytical FSRF is

$$\text{FSRF}_{\text{unnormalized}}(k) = \sum_{n=1}^N x \frac{b_n}{1 - e^{-a_n}} e^{-a_n k}. \quad (19)$$

To determine the amount of charge that the FP can store and release, define $Q(x)$ as the summation of the FSRF over all k ,

$$Q(x) = \sum_{k=1}^{\infty} \sum_{n=1}^N x \frac{b_n}{1 - e^{-a_n}} e^{-a_n k} = \sum_{n=1}^N \frac{x b_n e^{-a_n}}{(1 - e^{-a_n})^2}. \quad (20)$$

For an ideal falling step function all available charge traps should be filled. Thus, $Q(x)$ is both an estimate of the number of available charge traps at an exposure x and an estimate of the maximum amount of charge that the FP can store. An assumption here is that an equilibrium has been reached because the rates of trap filling and emptying have become equal.

$Q(x)$ can be experimentally derived from FSRF measurements of different magnitude step responses. From our experience the measured $Q(x)$ changes nonlinearly with step-response magnitude on the FP,⁶ which is consistent with the lag rates and coefficients changing with x .

$Q_n(x)$ is defined as the charge associated with a single exponential term,

$$Q(x) = \sum_{n=1}^N Q_n(x). \quad (21)$$

As shown in Appendix B, the FSRF for traps at a single energy level, which is similar to considering a single exponential term, can also be written out in terms of the parameters of the photodiode as

$$y(t) = \beta_2 e^{-a_{\text{thermal}} t}, \quad (22)$$

where traps are considered for a single energy level in the bandgap, E_{tr} , and

$$\beta_2 = (f(E_{tr}, 0) - f_{\text{Fermi}}(E_{tr}))N_t(E_{tr})a_{\text{thermal}}. \quad (23)$$

$f(E_{tr}, 0)$ is the initial value of the trap occupation function, $f_{\text{Fermi}}(E_{tr})$ is the occupation function after an equilibrium has been reached, $N_t(E_{tr})$ is the trap density inside the bandgap at the energy E_{tr} , and a_{thermal} is the rate at which charge is released from the traps.

Similar to Eq. (20), the amount of stored charge from the photodiode parameters can be calculated by integrating Eq. (22) over time to give

$$Q = \beta_2/a_{\text{thermal}} = (f(E_{tr}, 0) - f_{\text{Fermi}}(E_{tr}))N_t(E_{tr}). \quad (24)$$

While Eq. (24) does not explicitly depend on the exposure x , it does implicitly through f_{Fermi} . Equating the stored charge associated with a single exponential term in Eq. (20), $Q_n(x)$, and Eq. (24) gives

$$\frac{x b_n e^{-a_n}}{(1 - e^{-a_n})^2} = (f(E_{tr}, 0) - f_{\text{Fermi}}(E_{tr}))N_t(E_{tr}). \quad (25)$$

$f(E_{tr}, 0)N_t(E_{tr})$ is the initial charge stored in the traps and $f_{\text{Fermi}}(E_{tr})N_t(E_{tr})$ is the final, or equilibrium value, of charge stored for a given exposure value. For the FSRF experiment, the traps start full at x-ray turn-off, so a value of Q charge is stored, and the final value is taken as zero, since we are only interested in changes in the stored charge from the equilibrium value with no exposure. Rewriting Eq. (25) in terms of initial and final charge notation gives

$$\frac{x b_n e^{-a_n}}{(1 - e^{-a_n})^2} = Q_I - Q_F. \quad (26)$$

Rearranging Eq. (26) to solve for b_n gives

$$b_n(x) = \frac{|q_{n,k} - Q_n(x)| (1 - e^{-a_n(x)})^2}{x e^{-a_n(x)}}. \quad (27)$$

The absolute value has been introduced because the recursion formula will handle the sign. The initial charge is replaced with $q_{n,k}$ and the final charge is replaced with $Q_n(x)$ for a given frame.

For $x = 0$, Eq. (27) looks to be ill-defined, which would be problematic for the FSRF measurements. However, it does not matter what the value of b_n is when $x = 0$ in Eqs. (12)–(15), because they can be reduced to the following two equations with the single state variable $q_{n,k}$:

$$x_k = \frac{y(k) - \sum_{n=1}^N q_{n,k}(1 - e^{-a_n(x_k)})}{\sum_{n=0}^N b_n(x_k)}, \quad (28)$$

$$q_{n,k+1} = x_k b_n(x_k) \frac{e^{-a_n(x_k)}}{1 - e^{-a_n(x_k)}} + q_{n,k} e^{-a_n(x_k)}. \quad (29)$$

The contribution to $q_{n,k+1}$ from the current input x_k becomes the only term weighted by b_n , and will equal 0 when $x_k = 0$. Note, the denominator in Eq. (28) is still forced to equal 1. Even for small values of x_k , the contribution to $q_{n,k+1}$ will be small. Thus, a value of b_n can be defined for small values of x_k .

Finally, Eq. (27) can be further approximated for implementation as

$$b_n(x) = \frac{Q_n(x)}{x} \frac{(1 - e^{-a_n(x)})^2}{e^{-a_n(x)}}. \quad (30)$$

The value of $b_n(x)$ from Eq. (30) will be equal to or larger than that of Eq. (27). The approximation simply uses the b_n associated with the FSRF measurement of the same exposure intensity x .

Equation (30) gives an expression for the coefficient $b_n(x)$ in terms of the total charge that can be stored in the FP during the FSRF measurement, the current exposure level on the FP, and the lag rate.

II.B.3. Exposure-dependent lag rates: $a_n(x)$

In Eq. (11), the dynamics of detector lag are split into N groups, each with its own lag rate, $a_n(x)$. For a detailed semiconductor model, Wieczorek¹⁶ describes the dynamics of trap filling and emptying at a specific energy level, E_{tr} with a single, illumination dependent, exponential rate given by

$$a(E_{tr}) = \nu_0 [e^{-(E_c - E_{tr})/kT} + e^{-(E_c - E_{Fn})/kT}], \quad (31)$$

where ν_0 is the attempt-to-escape frequency for electrons, E_c is the energy level of the conduction band edge, E_{Fn} is the quasi-Fermi level, k is the Boltzmann constant, and T is temperature. In Eq. (31), $a(E_{tr})$ is continuously modeled for each energy level, instead of grouped into two to four energy ranges as in a multiexponential model.

Equation (31) can be broken into two parts, a_{thermal} and $a_{\text{exposure}}(n_e)$,

$$a_{\text{thermal}} = \nu_0 e^{-(E_c - E_{tr})/kT}, \quad (32)$$

$$a_{\text{exposure}}(n_e) = \nu_0 e^{-(E_c - E_{Fn})/kT} = \nu_0 n_e / N_c, \quad (33)$$

where n_e is the free carrier density, and N_c is the effective density of states at the conduction band edge. Note that $a_{\text{exposure}}(n_e)$ is directly proportional to the number of free carriers, which is directly proportional to the incident signal, x , on the FP. Equation (32) represents the minimum rate at which trap dynamics occur, which is only dependent on the thermodynamic level of the charge trap.

Similar to what can be done for Wieczorek's rate in Eq. (31), we break a_n in Eq. (11) into $a_{1,n}$ and $a_{2,n}$, where

$$a_n(x) = a_{1,n} + a_{2,n}(x). \quad (34)$$

The base lag rate, $a_{1,n}$, is a constant that corresponds to a_{thermal} and is independent of the exposure level. The exposure-dependent lag rate, $a_{2,n}(x)$, corresponds to a_{exposure} and depends on the input exposure intensity level. Now, Eq. (11) is written as

$$h(k, x_k) = b_0(x_k)\delta(k) + \sum_{n=1}^N b_n(x) e^{-(a_{1,n} + a_{2,n}(x))k}. \quad (35)$$

II.C. Weighting only non-LTI

A different non-LTI algorithm has been suggested that only provides an intensity-based weighting to previous projection

data.² Compared to the non-LTI algorithm just described, this intensity weighted lag correction is the same as setting $a_{2,n} = 0$ for all n and removing the stored charge constraint used in calculating $S_{n,k}^*$. The lag coefficients are still adjusted as in Eq. (30) as a function of exposure to provide the intensity weighting. From Eq. (9), changes in the lag coefficients result in a changing estimate of $q_{n,k}$, so the stored charge estimate will become inconsistent.

II.D. Calibration of lag correction algorithm

It was found that $N = 4$ exponentials works well for describing the FSRF and IRF for the Varian 4030CB FP,⁶ and so four constants and eight functions of x must be found in Eq. (35). However, the radar artifact presents in CBCT imaging with a-Si FPs is due to the longer time constant (smaller rate) exponentials of the impulse response. Therefore, only the two longest time constant exponentials ($n = 1, 2$) need to be treated as functions of exposure. To reduce complexity, the two shortest time constant terms ($n = 3, 4$) will be treated as a true LTI system with constant lag coefficients b_n and exposure-dependent lag rates $a_{2,n} = 0$. Therefore, the system will be described by six constants and four functions of exposure.

Figure 2 summarizes the calibration process, which has three main steps: determining the base lag rates $a_{1,n}$, measuring the maximum stored charge $Q(x)$ at different exposure levels, and determining the exposure-dependent rates $a_{2,n}(x)$.

The first calibration step is to determine the base lag rate $a_{1,n}$. The base lag rate determines the rate of charge release from the traps when there is no appreciable amount of charge in the conduction band to cause additional trapping to occur, i.e., the x-ray source is turned off. To estimate this parameter, FSRF data are fit with a multiexponential function with $N = 4$. The fit exponential rates are taken directly as the base lag rates. This can be done for any exposure level and should be independent of exposure. In general, choosing an exposure

in the middle of the dynamic range of the FP is best to avoid any potential saturation effects of the amplifier electronics and to maintain excellent SNR in the fitting process of the long time constant FSRFs.

The second calibration step is to measure $Q_n(x)$, which in turn is used along with the lag rates a_n to determine the lag coefficients $b_n(x)$. $Q_n(x)$ is measured from a series of FSRF experiments with different step-response magnitudes. At each exposure level, the FSRF data are fit to a multiexponential model, and then each exponential term in the model is integrated from the first lag frame to infinity to determine $Q_n(x)$.

The third calibration step uses a global search method to determine the best value for $a_{2,n}(x)$ at a number of exposure levels. For RSRF data, the exposure-dependent lag rates remain constant because the input to the FP is constant. The search method will apply the deconvolution described in Eqs. (12)–(15) for many values of $a_{2,1}$ and $a_{2,2}$ for each exposure level. For each point in the search space, the values of $a_{2,1}$ and $a_{2,2}$ will be used to calculate the lag coefficients. The optimal values of $a_{2,1}$ and $a_{2,2}$ for each exposure are those that produce a constant, lag-free RSRF after the correction. The metric used to determine the optimal exposure-dependent lag rates is the standard deviation of the RSRF, which is at a minimum for the ideal, flat response.

The exposure-dependent lag rates were found to be reasonably well fit by an exponential function of the form

$$a_{2,n}(x) = c_1(1 - e^{-c_2 x}). \quad (36)$$

This model forces the exposure-dependent lag rates equal to 0 at $x = 0$. If there were an offset, it should show up in the measurement of the base lag rate.

II.E. Experiments

Step-response and CBCT experiments were performed on a Varian 4030CB FP to evaluate the differences between using a LTI algorithm and the NLCSC algorithm for lag correction.

II.E.1. Step-response measurements

For the step-response measurements, the lengths of the rising and falling step functions were fixed. Traps with time constants ranging from a few milliseconds to several days exist. However, the time constants of concern for our investigations are those between the duration of a single frame and the entire scan length. Thus, similar to previous LTI lag corrections,⁶ our step-response measurements consisted of 600 RSRF frames followed by 600 FSRF frames, unless otherwise noted, since our CT acquisitions are on the order of 600 frames.

Step-response measurements were collected at nine different exposure levels ranging from 2% to 92% of the FP saturation level and used to calibrate the LTI and NLCSC algorithms. The exposure levels chosen correspond to available x-ray source settings that span the detector dynamic range and are listed in Table I. The LTI, NLCSC, and weighting only non-LTI algorithms were performed on select cases of step-response data to compare the lag correction

FIG. 2. Flowchart of the NLCSC calibration algorithm.

TABLE I. Percent of a-Si FP saturation signal for different exposure intensities at 125 kVp with 0.5 mm Ti.

Exposure level	1 ^a	2 ^b	3	4	5	6	7	8	9	10
mA	10	10	10	10	10	10	25	25	40	40
ms	2	2	2	8	14	22	14	19	15	19
% a-Si FP saturation	0.6	2.0	4.3	11	18	27	42	58	72	92

^aUsed 1.75 mm Cu.^bUsed 0.5 mm Cu.

effectiveness of each calibration. The first frame lag is often quoted when discussing lag measurements. Since the longer charge trapping time constants are of concern for the radar artifact, the 50th lag frame is also evaluated.

The RSRF measurements were normalized by the output of a separate silicon photodiode attached to a ceramic GOS scintillator (Hamamatsu S8193) to form a normalization chamber. The silicon photodiode has no lag and the ceramic scintillator is extremely fast. Using the output of the normalization chamber, any variation in the measured RSRF due to the x-ray tube or generator can be removed. The output of the photodiode was amplified and digitized using a National Instruments A/D converter (USB-6008).

II.E.2. CBCT measurements

CBCT data sets were acquired for two different phantoms: a large pelvic phantom (42 cm × 26 cm) and a uniform acrylic head phantom (20 cm × 16 cm). The large pelvic phantom is made up of a smaller pelvic phantom with an additional layer of attenuating material to simulate larger patients. A total of 625 projections were collected in a 2 × 2 binned pixel mode. The tube protocols were chosen to give high SNR. For the large pelvic phantom, a pulsed fluoroscopy protocol of 125 kVp, 80 mA, and 30 ms was used in an offset detector geometry (16 cm offset). For the uniform acrylic head phantom, a tube protocol of 125 kVp, 63 mA, and 18 ms was used. For all setups, a 10:1 antiscatter grid, an appropriately designed (i.e., centered or offset geometry) 8:1 Al bowtie filter, and 0.5 mm Ti filter were also used. The x-ray beam was collimated to approximately 2 cm at the detector for data acquisition to reduce scatter effects. To further reduce the effects of beam hardening and scatter, all data sets made use of appropriately sized normalization phantoms.¹⁷ The FDK algorithm¹⁸ was used to reconstruct the data sets at a reconstruction pixel size of 1 mm for the pelvic phantom and 0.5 mm for the head phantom. The slices were averaged to a thickness of 5.0 mm to reduce noise. As is standard in CT imaging, all reconstructions were corrected for ring artifact.¹⁹

The CBCT projection data were corrected for lag using the LTI and NLCSC algorithms. An intensity-weighted LTI algorithm was also used for comparison. Since an IRF derived from the detector FSRF was found to provide the best LTI correction of the radar artifact in CBCT images, only IRFs derived from the FSRFs at different exposures were used.⁶ The reconstructions using LTI corrections that give the minimum and maximum average error are compared to the reconstruction that made use of the NLCSC algorithm.

The error metric used in Mail *et al.*⁴ was a nonuniformity measure that examined the maximum difference in HU signal level among several ROIs. We define our error metric in a similar way. Five pairs of ROIs, 10 pixels on each side, that straddle the obvious lag artifact boundary in the uncorrected data sets are defined. The difference in the mean signals is calculated for each ROI pair and the average and maximum differences for each image are used as the error metrics. Thus, other factors potentially affecting image nonuniformity are minimized since local regions are being compared.

Furthermore, to gain a preliminary understanding of the noise performance of the lag correction algorithms, the standard deviation of centrally located ROIs (50 pixels on each side) were compared for the uncorrected, LTI corrected, and NLCSC corrected CBCT data.

III. RESULTS

III.A. Calibration of lag correction algorithm

Table II lists the results of performing a four-exponential model fit to the FSRF data at an exposure level of 27%. The listed values correspond to all of the necessary parameters for the LTI correction calibrated at that exposure. The units for the lag rates a_n are frames^{-1} and for the lag coefficients b_n are (detector counts with lag/detector counts without lag).

For the NLCSC algorithm, the a_n entries correspond to the base lag rates $a_{1,n}$. Furthermore for the NLCSC algorithm, b_1 and b_2 will become functions of exposure and $a_{2,1}(x)$ and $a_{2,2}(x)$ will be added to the calibration information (shown further down). For the shorter time constants ($n = 3, 4$), the lag coefficients will remain the constants listed and the exposure-dependent lag rates will be set to 0.

Figure 3 shows the estimates for the number of available charge trap states associated with the first two exponential terms, $Q_1(x)$ and $Q_2(x)$, from the step-response experiments at different exposure levels. The data points are interpolated using a fourth-order polynomial. The amount of stored

TABLE II. LTI IRF parameters from calibration FSRF data at an exposure level of 27%. The base lag rates $a_{1,n}$ have units of frames^{-1} and the lag coefficients b_n have units of (detector counts with lag/detector counts without lag).

a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
2.5×10^{-3}	2.1×10^{-2}	1.6×10^{-1}	7.6×10^{-1}	7.1×10^{-6}	1.1×10^{-4}	1.7×10^{-3}	1.8×10^{-2}

FIG. 3. Stored charge estimates Q_n for (a) the longest time constant and (b) the second longest time constant charge traps. Data points were fit with a fourth-order polynomial.

charge in the smallest time constant exponential ($n = 1$) nearly equals the incident exposure on the FP for a single frame of exposure [Fig. 3(a)], thus highlighting that charge traps have huge charge storage capabilities.

The optimal $a_{2,1}$ and $a_{2,2}$ found for each exposure level (black boxes in Fig. 4) using a global search optimization of Eqs. (12)–(15) are fit with an exponential model (curves and equations in Fig. 4) that guarantees $a_{2,n}(0) = 0$. When the RSRF data are corrected using the exposure-dependent lag rates corresponding to the correct exposure intensity, the normalized RSRF shows no appreciable rise time. This is shown in Fig. 5 for the 27% exposure.

For Figs. 3 and 4, the x-axis is exposure intensity measured in units of detector counts. For the Varian 4030CB detector used in these experiments, 10^5 detector counts is equal to 100% of detector saturation.

III.B. Experiments

III.B.1. Step-response measurements

Figure 6(a) shows uncorrected and corrected FSRF data at different exposure intensities (2.0%–92.0%) using an IRF that was derived for an exposure of 27% of FP saturation. The FSRFs are normalized to the average of the detector signal during irradiation, and it is evident from the uncorrected FSRFs that the lag signal for the higher exposure (92%) FSRF is a smaller fraction of the incident signal than the FSRF at 2%. Because of this intensity dependence, the LTI corrections show a range of responses. Since the IRF was calculated from a single exposure, the 27% corrected FSRF is well corrected, but the 2.0% FSRF is undercorrected and the 92% FSRF is overcorrected. Figure 6(b) shows a similar comparison, but for the NLCSC algorithm. Looking at the three corrected

FIG. 4. Exposure-dependent lag rates for (a) the longest time constant and (b) the second longest time constant charge traps. Data points were fit with an exponential function. The form of the function ensures that $a_{2,n}(0) = 0$.

FIG. 5. The uncorrected and corrected RSRF found during the global search for the optimal $a_{2,n}(x)$ at $x = 27\%$.

FSRFs at different exposures, the NLCSC calibration clearly accounts for the exposure dependence of the charge trapping.

The uncorrected and corrected FSRF data using the intensity weighted algorithm is shown in Fig. 6(c) and is nearly identical to the results for the NLCSC algorithm. The results should be similar for the FSRF data because all available charge traps should have been filled by the long irradiation time prior to the FSRF measurements.

Table III summarizes the 1st and 50th frame residual lags for step-response data that is: (1) uncorrected, (2) corrected with a LTI algorithm calibrated at 27% of the FP saturation exposure, (3) corrected with the intensity weighted non-LTI algorithm, and (4) corrected with the NLCSC algorithm. The listed values are the averages over the central 4 cm of the FP. The LTI algorithm performs slightly better at the 27% exposure since it was calibrated specifically for that data, although both algorithms do well. The first frame lag for the 92% exposure is also lower for the LTI algorithm, but it is a large

FIG. 6. Uncorrected and corrected FSRFs for step-response data at several different exposures, (a) LTI corrections derived from a 27% step-response, (b) NLCSC correction which takes into account the exposure dependence, and (c) the intensity weighted only non-LTI correction.

TABLE III. Summary of 1st and 50th frame residual lag for uncorrected, LTI, and NLCSC algorithms.

Exposure level	Lag correction algorithm	Residual 1st frame lag	Residual 50th frame lag
2%	Uncorrected	3.7%	0.96%
2%	LTI	1.4%	0.48%
2%	Intensity weighted	0.33%	0.10%
2%	NLCSC	0.25%	−0.0028%
27%	Uncorrected	2.8%	0.45%
27%	LTI	0.25%	0.0038%
27%	Intensity weighted	0.30%	0.051%
27%	NLCSC	0.29%	0.0052%
92%	Uncorrected	2.4%	0.28%
92%	LTI	0.0053%	−0.16%
92%	Intensity weighted	0.15%	0.0032%
92%	NLCSC	0.16%	0.0034%

overcorrection for all of the other lag frames. The NLCSC algorithm consistently has 1st frame lag that is less than 0.3% in magnitude and 50th frame lag that is less than 0.01% in magnitude.

III.B.2. CBCT measurements

Figure 7(a) shows the reconstruction from an uncorrected projection data set along with the ROIs used for error calculations. The average and maximum ROI errors are 35 HU and 52 HU, similar to measurements from a different Varian 4030CB FP.⁶ The other FP had errors of 35 HU and 51 HU for the same phantom and with ROIs defined in similar, but different locations to capture the contrast of the radar artifact. A larger study of several FPs is needed to determine whether detector-specific calibrations are needed or whether similar FPs can use the same set of coefficients.

Figure 7(b) is the reconstruction using the LTI correction with the smallest average ROI error (best case) and Fig. 7(c) is the reconstruction with the largest ROI error (worst case). The reconstruction using the best-case IRF reduced the average ROI error to 14 HU, although a blurred ring artifact remains.⁶

FIG. 7. $1 \times 1 \times 5 \text{ mm}^3$ reconstructions of a pelvic phantom ($42 \times 26 \text{ cm}^2$) with (a) no lag correction, (b) best LTI correction, (c) worst LTI correction, (d) intensity weighted lag correction, and (e) NLCSC correction. Window, level = 200, −25 HU.

FIG. 8. $0.5 \times 0.5 \times 5 \text{ mm}^3$ reconstructions of a pelvic phantom ($20 \times 16 \text{ cm}^2$) with (a) no lag correction, (b) best LTI correction, (c) worst LTI correction, and (d) NLCSC correction. Window, level = 100, −25 HU.

Figure 7(d) shows the pelvic reconstruction using the intensity weighted lag correction algorithm where $a_{2,n}(x)$ was assumed to be 0 for all n . The center shading has been slightly reduced by this algorithm compared to the other reconstructions, but the radar artifact is still clearly visible. The NLCSC reconstruction [Fig. 7(e)] of the pelvic phantom shows the least amount of error and no residual blurred ring artifact. Thus, the exposure-dependent rates in the NLCSC algorithm, $a_{2,n}(x)$, and the stored charge constraint are important aspects in the correction procedure.

Figure 8 shows reconstruction results for the uniform acrylic head phantom with a narrow window of 100 HU. The outline artifact that is especially prominent on the right side of each reconstruction is a result of the dynamic gain mode of the detector and none of the lag corrections should affect it. In spite of this artifact, the radar artifact is still quite evident in the uncorrected reconstruction [Fig. 8(a)], which also shows the five pairs of defined ROIs. The worst-case LTI correction [Fig. 8(c)] only partially corrects the radar artifact to an average ROI error of 11 HU. Both the best-case LTI [Fig. 8(b)] and NLCSC [Fig. 8(d)] corrections work well at reducing the average ROI error to 2 or 3 HU.

For the pelvic and head phantoms, Table IV summarizes the average and maximum errors that result from using the different lag correction techniques on the CBCT projection data. For the pelvic phantom, the NLCSC correction outperforms all LTI corrections and the intensity weighted LTI

TABLE IV. Summary of ROI reconstruction errors for different lag correction algorithms.

Error (HU)	Pelvic phantom		Head phantom	
	Avg	Max	Avg	Max
Uncorrected	35	52	16	21
LTI (best)	14	27	2	7
LTI (worst)	19	35	11	14
Intensity weighted	15	31	9	12
NLCSC	11	19	3	5

algorithm. For the head phantom, the NLCSC correction provides comparable correction to the best LTI correction, but outperforms the worst case LTI correction and the intensity weighted algorithm. For the maximum ROI errors, the NLCSC algorithm outperforms the other corrections in all cases.

It is interesting to note that the NLCSC correction outperforms the intensity weighted algorithm for the CBCT reconstructions, but not the earlier FSRF data. Clearly, the rates of trap filling are more important for properly correcting the CBCT data. This is due to the much more complex irradiation history of the FP detector during CBCT data acquisition, as opposed to the FSRF experiments that had 600 frames of constant irradiation.

Centrally located 50×50 pixel ROIs were compared in the pelvic and head phantoms for the CBCT data. For the pelvic phantom, a ROI just above the dark shading was chosen in the large uniform material area. The LTI algorithm increased the standard deviation by 1.5%, while the NLCSC algorithm increased it by 1.2% over the uncorrected reconstruction. For the head phantom, the increase in standard deviation was 2.8% for both algorithms.

IV. DISCUSSION AND CONCLUSIONS

The NLCSC is a novel, nonlinear lag correction algorithm that outperforms the corresponding LTI technique. The algorithm performance is better than that of the best-case LTI performance for the pelvic phantom and has the added benefit of removing the residual blurred ring artifact seen in Fig. 7(b). Importantly, the algorithm is not sensitive to the calibration exposure intensity or step-response technique used because a range of exposure techniques and both RSRF and FSRF data are used in the calibration. The NLCSC algorithm developed is based on a deconvolution of the projection data with an IRF, but the IRF can change for each frame. The estimate of stored charge trapped by each exponential term in the current frame, $q_{n,k}(x)$, is held constant when switching from one IRF to another and determines the initial condition to be used for the subsequent deconvolution step.

Similar to the LTI algorithm, the calibration of the base lag rates $a_{1,n}$ in the NLCSC algorithm require the choice of a single exposure level. From our experience with the NLCSC algorithm, using different exposure intensities to determine

the base rate gives very similar lag correction results. Future work will explore this relationship further.

For this work, all step-response and CBCT data were collected at 15 fps for simplicity. To apply the lag correction at a different frame rate, the step-response data needed for calibration could be collected again and the fits performed accordingly. Alternatively, the step-response data collected at 15 fps could easily be converted to data at other frame rates. Previous measurements of the RSRF and FSRF on a FP detector at different frame rates were nearly identical when plotted versus time, instead of frame number.⁶ Fits could then be performed for frame-rate specific corrections since most detectors typically have a finite number of allowable frame rates. Future work is needed to confirm that this refitting method will be effective.

Lag has the effect of producing noise correlations, or smoothing, in projection data and CBCT reconstructions. This form of noise reduction may actually be desirable for applications like low-dose fluoroscopy, but is not desirable for CBCT reconstructions. Because software lag corrections remove the effect causing the noise correlation, image noise will increase in lag-corrected images. Furthermore, lag corrections with any imperfections, especially when applied on a pixel basis, may introduce their own correction noise. Since the radar artifact is such a broad area artifact, it should be possible to minimize any noise introduced by the lag correction. For the purposes of this work, the lag reduction algorithms were applied to the projection data and then reconstructed with a standard pipeline. An alternate method would be to reconstruct the CBCT projection data with and without the lag correction applied, thus doubling the computational load to produce two reconstructions. The difference between the two reconstructions corresponds to an additive lag correction term for the CBCT image. Smoothing or adaptive filtering could be used to remove noise from the lag correction term prior to addition to the uncorrected reconstruction. The key to the filtering would be preserving the sharp boundary of the lag correction, but still removing pixel noise.

In Fig. 4, the exposure-dependent lag rates were forced to go through the origin because any offset should have been measured in the base lag rate measurements with no exposure. However, Fig. 3 does not make the same constraint for $Q_n(x)$. Physically, $Q_n(x)$ is an estimate of the number of charge traps available at an exposure level x . There are many charge traps available for electrons at zero exposure in the a-Si naturally, so the model does not need to be forced to zero.

In the pelvic phantom reconstructions in Fig. 7 there is a shading artifact that appears near the center of each image. The off-center detector reconstructions for such a large pelvic phantom are particularly sensitive to scatter artifacts.²⁰ The lag corrections affect the center shading slightly, but for the most part, the artifact is fairly consistent across all reconstructions using different lag corrections.

Wieczorek developed a lag correction algorithm that also captures the nonlinear effects of the a-Si photodiode.¹⁶ However, there are some important differences between that algorithm and the one presented here. First, the equations that Wieczorek developed implicitly assume that the input is a

scaled step-response and not a time-varying signal. This is shown in more detail in Appendix A. The presented solution does not hold for a time-varying input because the time-varying case results in differential equation with variable coefficients. A possible extension of Wieczorek's work is to apply it for very short time periods, similar to our method, and then to calculate consistent initial conditions for the next time step. A second important difference between the two works is that Wieczorek defines his model in terms of the semiconductor parameters, while our algorithm derives everything from measured RSRF and FSRF data. For many end users of the a-Si FPs, the parameters necessary for the Wieczorek model may be difficult to determine; our calibration procedure requires no intimate knowledge of the semiconductor properties. Finally, the algorithm presented here is computationally efficient, while the Wieczorek algorithm is a more complex calculation since the trap states are considered to exist in a continuum.

In summary, the NLCSC lag correction algorithm presented removes the radar artifact similar to or better than LTI techniques, removes the residual blurred ring present in LTI lag corrections, and has less ambiguity in the calibration of the algorithm parameters. Future work will focus on evaluating its noise properties and mitigating any potential noise increase.

ACKNOWLEDGMENTS

This work was supported in part by "Varian Medical Systems, the Stanford-NIH Biotechnology Traineeship," National Institutes of Health (NIH) Grant Nos. EB003524 and 1 R01HL087917, and the Lucas Foundation.

APPENDIX A: WIECZOREK DIFFERENTIAL EQUATION SOLUTION

Wieczorek¹⁶ finds that the time rate of change at a specific energy level, E_{tr} , for the trap occupation function $f(E_{tr}, t)$ is the following:

$$\frac{df(E_{tr}, t)}{dt} = a f_{\text{Fermi}}(E_{tr}) - a f(E_{tr}, t), \quad (\text{A1})$$

$$\text{where } a = \nu_0 [e^{-(E_c - E_{tr})/kT} + e^{-(E_c - E_{Fn})/kT}], \quad (\text{A2})$$

$f_{\text{Fermi}}(E_{tr})$ is the Fermi function, E_{Fn} is the quasi-Fermi level, ν_0 is the attempt-to-escape frequency, and E_c is the energy level of the conduction band edge.

The solution given by Wieczorek to Eq. (A1) is

$$f(E_{tr}, t) = f(E_{tr}, 0)e^{-at} + f_{\text{Fermi}}(E_{tr})(1 - e^{-at}). \quad (\text{A3})$$

An unstated assumption is that the equation was solved for a step-response input. Or, more precisely, that a is not a function of t and that $f_{\text{Fermi}}(E_{tr}) = f_{\text{Fermi}}(E_{tr})u(t)$, where $u(t)$ is the unit step function in time, giving

$$\frac{df(E_{tr})}{dt} = a f_{\text{Fermi}}(E_{tr})u(t) - a f(E_{tr}). \quad (\text{A4})$$

Then, solving this equation by taking the Laplace transform gives

$$s F(E_{tr}, s) = a \frac{F_{\text{Fermi}}(E_{tr})}{s} - a F(E_{tr}, s) + f(E_{tr}, 0), \quad (\text{A5})$$

$$F(E_{tr}, s)[s + a] = a \frac{F_{\text{Fermi}}(E_{tr})}{s} + f(E_{tr}, 0), \quad (\text{A6})$$

$$F(E_{tr}, s) = a \frac{F_{\text{Fermi}}(E_{tr})}{s(s + a)} + \frac{f(E_{tr}, 0)}{s + a}. \quad (\text{A7})$$

When the inverse Laplace transform is taken, Wieczorek's exact solution is obtained. If a and $f_{\text{Fermi}}(E_{tr})$ are allowed to be generally time varying, then Eq. (A1) becomes

$$\frac{df(E_{tr})}{dt} = a(t) f_{\text{Fermi}}(E_{tr}, t) - a(t) f(E_{tr}, t). \quad (\text{A8})$$

Equation (A8) is not a linear constant coefficient differential equation, so it is not easily solved with Laplace transforms or other methods. Rewriting Eq. (A3) with time-varying parameters gives

$$f(E_{tr}, t) = f(E_{tr}, 0)e^{-a(t)t} + f_{\text{Fermi}}(E_{tr}, t)(1 - e^{-a(t)t}). \quad (\text{A9})$$

It is easily verifiable that Eq. (A9) is not a solution to Eq. (A8).

APPENDIX B: ANALYTICAL MODEL OF a-Si FP WITH TRAPS

For nonequilibrium carrier concentrations, the quasi-Fermi level determines what fraction of the trap states inside the bandgap are filled and will change as a function of panel irradiation (carrier concentration).²¹ The overall shape of the distribution of carriers is a Fermi function, centered at the quasi-Fermi level. For a device with a constant photocurrent, the electron occupation function in the bandgap is given by the Fermi distribution function

$$f_{\text{Fermi}}(E_{tr}) = \frac{1}{1 + e^{(E_{tr} - E_{Fn})/kT}}. \quad (\text{B1})$$

In Eq. (B1), k is the Boltzmann constant, T is temperature, E_{Fn} is the quasi-Fermi level, and E_{tr} is the specific energy level of a trap state. Because the Fermi function is nonlinear, as is the trap state distribution in the bandgap, the trap behavior may not be linear. In other words, irradiating an FP with twice the amount of x-rays may not result in twice the amount of trapped charge in the a-Si. Wieczorek has developed an analytical model of a-Si trapping from basic semiconductor principles.¹⁶ He states that the rate of electron emission, R_α , from a trap at some energy E_{tr} is proportional to several factors: the trap density, $N_t(E_{tr})$, the occupation function, $f(E_{tr})$, and the energy difference between E_{tr} and the conduction band edge, E_c . The electron trapping rate, $R_\beta(E_{tr})$, is proportional to the number of free electrons, n , and the number of available traps states. Overall, the derivative of the occupation function can be written as normalized difference between the

trapping and emission rates:

$$\frac{df(E_{tr}, t)}{dt} = \frac{1}{N_t} [R_\beta(E_{tr}) - R_\alpha(E_{tr})] \quad (\text{B2})$$

$$= af_{\text{Fermi}}(E_{tr}) - af(E_{tr}, t), \quad (\text{B3})$$

where a is defined in Eq. (A2) and v_0 is the attempt to escape frequency.

The solution to Eq. (B3) is given as Eq. (A3) by Wiczeorek.

1. Detector output in the presence of traps

We extend Wiczeorek's analytical description of trapping in a-Si to consider the output of the a-Si FP. Let $x(t)$ be the ideal, continuous (nonframe integrated) charge generation function for an element of the a-Si FP (i.e., signal generated with no traps). Let $y(t)$ be the continuous output of the detector with the effects of charge trapping. The amount of charge per unit time that interacts with the traps is the change in the trap occupation function, $f'(E_{tr})$, multiplied by the trap density and integrated over all trap energies. For example, if $f' = 0$, then there is no net change in stored charge over that time period and the output would be equal to the generated charge function, $x(t)$. The output of the detector is

$$y(t) = \left[x(t) - \int_{E_{tr}} f'(E_{tr}, t) N_t(E_{tr}) dE_{tr} \right]. \quad (\text{B4})$$

By first considering a system where traps only exist at a single energy level, E_{tr} , and substituting Eq. (B3) into Eq. (B4), $y(t)$ can be written as

$$y(t) = x(t) - a(f_{\text{Fermi}}(E_{tr}) - f(E_{tr}, t))N_t(E_{tr}). \quad (\text{B5})$$

Substituting for $f(E_{tr}, t)$ from Eq. (A3) gives

$$y(t) = x(t) - ae^{-at}(f_{\text{Fermi}}(E_{tr}) - f(E_{tr}, 0))N_t(E_{tr}). \quad (\text{B6})$$

Equation (B6) has the general form

$$y(t) = x(t) + \beta_1 e^{-at}, \quad (\text{B7})$$

where $\beta_1 = (f(E_{tr}, 0) - f_{\text{Fermi}}(E_{tr}))N_t(E_{tr})a$. Furthermore, the “ a ” term in Eq. (A2) can be broken into two terms and Eq. (B3) written as

$$a = a_{\text{thermal}} + a_{\text{exposure}}. \quad (\text{B8})$$

The first term of the right-hand side in Eq. (A2) becomes a_{thermal} and represents the part of the rate dependent only on the trap energy level, E_{tr} . The second term becomes a_{exposure} and depends on the number of electrons in the conduction band, which is directly proportional to the input signal, $x(t)$. For the case with no irradiation on the detector (i.e., $x(t) = 0$), Eq. (B7) can be simplified to

$$y(t) = \beta_2 e^{-a_{\text{thermal}}t}, \quad (\text{B9})$$

where $\beta_2 = (f(E_{tr}, 0) - f_{\text{Fermi}}(E_{tr}))N_t(E_{tr})a_{\text{thermal}}$.

While the x-ray off case and the general case both have an exponential form for the detector output, $y(t)$, the dynamics of the systems occur at different rates, a_{thermal} versus a . In fact,

whenever $x(t)$ changes, a will change, and so will the system dynamics that govern the rates of trap filling and emptying.

Equations (B5)–(B9) are for traps at a single energy level. The arguments can easily be extended for traps existing at multiple energy levels, since each level can be considered its own term with corresponding values of a and f_{Fermi} . Using only a few exponential terms has the effect of discretizing the energy spectrum.

^{a)}Electronic mail: jared.starman@gmail.com

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