Shape Change of the Vitreous Chamber Influences Retinal Detachment and Reattachment Processes: Is Mechanical Stress during Eye Rotations a Factor?

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PURPOSE. We aim to understand how mechanical causation influences retinal detachment and reattachment processes. In particular, myopes suffer retinal detachment more frequently than emmetropes, and following a retinal detachment, scleral buckling promotes retinal reattachment. We test the hypothesis that stresses arising from saccadic eye rotations are involved in the processes, and that the alteration in the stress due to the change in the vitreous chamber geometry is sufficient to explain the phenomena.

METHODS. The vitreous chamber of the eye has an approximately spherical shape and it is filled with vitreous humor. We developed a mathematical model, treating the vitreous chamber in emmetropic and myopic eyes as a spheroid and in eyes subjected to scleral buckling as a sphere with a circumferential indentation. We assume that the eye performs prescribed small-amplitude, periodic, torsional rotations and we solve semi-analytically for the fluid pressure, velocity, and stress distributions.

RESULTS. The shape of the vitreous chamber has a large effect on the retinal stress. The vitreous and the retina of a highly myopic eye continuously experience shear stresses significantly higher than those of an emmetropic eye. An eye fitted with a scleral buckle experiences large stress levels localized around the buckle.

CONCLUSIONS. Our results provide a mechanical explanation for the more frequent occurrence of posterior vitreous detachment and retinal detachment in myopic eyes. To understand how the stress distribution in a buckled eye facilitates reattachment, an additional model of the details of the reattachment process should be coupled to this model. (*Invest Ophthalmol Vis Sci.* 2012;53:6271-6281) DOI:10.1167/iov.11-9390

Retinal detachment (RD) is defined as a separation of the sensory part of the retina from the underlying retinal pigment epithelium, and often leads to sight loss. The most common type is rhegmatogenous retinal detachment, in

which, following liquefaction of the vitreous humor and the development of a tear in the retina, the subretinal space is filled with liquefied vitreous humor. In healthy young eyes, vitreous humor is a gel with complex mechanical properties, which have been studied experimentally by several authors.¹⁻⁶ The gel state is maintained by a network of collagen fibrils, which typically degrades with age, resulting in vitreous liquefaction. The mechanisms underlying this process are not fully understood.

The incidence of myopia is correlated with an increased risk of RD, reported to be four times higher if the refractive error is between -1 and -3 diopters (D), and up to 10 times higher if the error is more than -3 D.⁷ It is also known that both posterior vitreous detachment (PVD) and lattice degeneration, which are risk factors for RD, are more common in myopes.⁸⁻¹⁰ To our knowledge, the reasons for these findings are not fully understood. Myopia is typically associated with a shape change in the vitreous chamber; in particular, myopic eyes are on average larger than emmetropic eyes and more elongated in the anteroposterior direction.^{11,12} We postulate that this might promote the disintegration of the vitreous structure due to the generation of larger stresses in the vitreous humor and on the retina during eye movements, in turn increasing the risk of PVD and RD. The potential mechanical influence of such a shape change has been put forward by various authors in the past,^{13,14} but quantitative evaluation of the variation of the stress in the vitreous and on the retina in myopic eyes with respect to emmetropic ones has not been carried out. Evidence for mechanical causation of RD in myopes is provided by the fact that mechanical insult can induce RD in some circumstances. Furthermore, Pickett-Seltner et al.¹⁵ measured the chemical composition of the vitreous humor of chicken myopes, finding it to be similar to those with normal eyes. Finally, Meyer-Schwickerath and Gerke¹⁶ found that eyes with RD, whether myopic or emmetropic, are of different dimensions from normal eyes, another indication that mechanics plays a role.

A common treatment for RD is scleral buckling: a silicone band is placed tightly around the eyeball so as to induce an indentation in the sclera at the location of the retinal tear. This is thought to change the subretinal pressure and drive flow of the liquefied vitreous humor out from under the retina, promoting reattachment. Bains¹⁷ reviewed the types of implants that are in use in this surgery, both in terms of the materials used and in terms of the shape of the buckle. A mechanical model of the effect of scleral buckling was developed by Foster et al.,¹⁸ but a full understanding of the reattachment process is so far elusive.

Various theoretical studies exist that address the effect of eye rotations on the stress exerted by the vitreous on the retina. David et al.¹⁹ developed a mathematical model of an eye undergoing rotations, representing the vitreous chamber as a

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torsionally oscillating sphere. With this model, they calculated the fluid motion and the resulting stress on the retina. The stress grows with the radius of the eye; thus, for a given eye movement the retinas of myopes are subject to a greater shear stress, and the authors suggest this finding as a possible explanation for the increased incidence of RD in myopia.

Meskauskas et al.²⁰ investigated and compared different rheological models of the vitreous humor,²⁻⁴ paying particular attention to the possible occurrence of resonance. They computed the natural frequencies of oscillation of the vitreous using the parameter values reported by various authors and showed that, if the fluid-filled sphere were forced by torsional oscillations of the appropriate frequency and of constant amplitude, the fluid motion would be significantly stronger than in the case of nonresonant forcing.

Repetto et al.²¹ studied the effect of the shape of the vitreous chamber on the flow of a purely viscous fluid during torsional oscillations. They found that the flow in the chamber indented by the lens, and consequently also the wall shear stress, is significantly different from that in a sphere. These changes are caused by additional vortices in the flow that appear near to the indentation.

In this article, we developed a new mathematical model of the motion of the vitreous humor induced by rotations of the eyeball by combining and extending the models of Meskauskas et al.²⁰ and Repetto et al.²¹ As the eye rotates in the course of everyday activities, its motion induces flow in the vitreous humor, leading to a time- and space-dependent stress distribution within it. We model the vitreous humor as a viscoelastic fluid and the vitreous chamber as a deformed sphere, and use this to predict the stress on the retina during saccades. We use this to find how the mechanical forces on the retina due to eye motion differ when the shape of the sclera is changed. We investigate whether mechanics can be responsible for the increased incidence of RD among myopes and whether it is involved in the reattachment process during scleral buckling.

MATHEMATICAL MODEL

Formulation of the Mathematical Problem

We have developed a mathematical model consisting of a near-spherical region filled with an incompressible viscoelastic fluid of density ρ . We used the model to calculate the flow and pressure fields during idealized eye rotations, and we found, in particular, the stress on the retina. The details of the method are provided in the Appendix, and here we give a summary.

Viscoelastic fluids have a “memory,” meaning that the stress within them depends not only on the rate of strain at that time, but also on both the strain and the rates of strain at previous times, as specified by the relaxation modulus $G(t)$, which describes the strength of the memory of the state at a time t ago. It can be shown that, for small time-sinusoidal displacements and velocities, this fluid behavior can be modeled by introducing a “complex viscosity,” in place of the usual “real” viscosity used for viscous fluids.²² The real part of this complex viscosity provides a measure of the viscosity of the fluid, whereas its imaginary part quantifies the elastic component.

We describe the shape of the vitreous chamber in terms of a spatially varying radius $R(\vartheta, \varphi)$, where $(r^*, \vartheta, \varphi)$ are spherical coordinates (radial, zenithal, and azimuthal, respectively), and we assume that R is equal to a constant value, R_0 , plus a relatively small perturbation, whose maximum size is δR_0 ; thus δ (assumed small) characterizes the depth of the indentation or the height of the maximum projecting region.

Although eye rotations are not exactly periodic, in this article, for simplicity, we assume that the eye undergoes sinusoidal, torsional oscillations of angular amplitude ϵ radians (assumed small), and frequency ω . In the discussion section, we remove this assumption and consider the effect of real saccadic rotations on the results.

The pressure and flow fields of the fluid in a perfect sphere of radius R_0 can be calculated analytically as a closed form solution. This was done by David et al.¹⁹ for the case of a viscoelastic fluid described by a model with four mechanical elements and by Meskauskas et al.²⁰ for the general case. One advantage of analytical, as opposed to numerical, solutions is that the solution is found for all parameter values, leading to a better understanding of the behavior. This can enable the trends as parameters change to be found, and also any qualitative changes in behavior to be determined.

In this article, we also use an analytical method to find the leading-order deviation in the pressure and flow fields due to the change in shape for small δ , using the method developed by Repetto et al.,²¹ which involves decomposing the solution into vector spherical harmonics.²³ The flow field can be used to find the stress at the wall, which is our estimate of the force per unit area exerted by the fluid on the retina. The normal component of the stress on the boundary is due to the fluid pressure, whereas the tangential component arises due to the fluid velocity.

Model of Myopic Eyes

Atchison et al.^{11,12} studied the shape of myopic eyes using magnetic resonance imaging. They provided regression lines for the vitreous chamber width, height, and length as a function of the refractive error (best sphere correction) (Fig. 4A of their article¹²), and showed that the vitreous chamber shape can be approximated by an ellipsoid. They showed that in emmetropic eyes, the anteroposterior axis is typically shorter than the other two. As the degree of myopia increases, all axes grow in length, but the anteroposterior axis grows the most. Therefore, myopic eyes are both larger and closer to spherical in shape than emmetropic ones, whereas in severely myopic eyes the vitreous chamber may be prolate (i.e., the anteroposterior axis is the longest). Horizontal and vertical cross-sections of various eyes are illustrated in Figure 1, based on the regressions proposed by Atchison et al.¹² As can be seen, the predicted shapes are close to spherical.

In this article, we model the vitreous chamber of myopic eyes as an ellipsoid. For a given refractive error of the eye, we use the lengths of the principal radii given by Atchison et al.¹²

Model of Eyes Subjected to Scleral Buckling

Keeling et al.²⁴ studied the effect of scleral buckles that encircle the eyeball, or cerclages, on the scleral shape. They modeled the eye as a uniform spherical shell filled with a fluid and the buckle as an elastic band stretched around the equator that exerts an inward radial force on the shell. The authors calculated both the intraoperative shape and the postoperative shape, and compared their results to the shape change in an enucleated eyeball. Wang et al.²⁵ also investigated the change in shape of the eyeball due to a scleral buckle, using a finite element numerical model to calculate the predicted shape change and the change in refractive index.

In this article, we calculate the stress that the vitreous exerts on the retina during eye rotations after the implantation of a buckle, rather than the stress in the scleral wall due to changes of its curvature. Therefore, we adopt a simpler approach to describe the change in shape of the vitreous

FIGURE 1. Horizontal (a) and vertical (b) cross section of the ellipsoids fitting the vitreous chamber shape of eyes with various degrees of refractive error (0 D, -5 D, -10 D, -15 D, -20 D) based on data by Atchison et al.¹² The emmetropic eye corresponds to the thick curve.

FIGURE 2. Sagittal cross section of the domain. The three-dimensional model is constructed by rotating this cross section about the z -axis. The radius of the indentation is R_i and the axis about which the eye rotates is the y -axis.

chamber after scleral buckling, and consider an idealized, prescribed geometry.

For a cerclage, this consists of a sphere with an axisymmetric indentation. Figure 2 shows a cross section through the domain, which is a circle with radius $\hat{R}$ with two circular indentations of radius R_i on opposite sides of the circle (the lines around the intersections between the circles are smoothed). The depth of the indentation with respect to the circle of radius $\hat{R}$ is denoted by d . The three-dimensional shape is obtained by rotating this cross section about the z -axis. R_0 is chosen as the radius of the sphere with the same volume as the actual domain.

We modify the geometry of the domain by changing the value of the ratios $R_i / \hat{R}$ and $d / \hat{R}$, and investigate the role of these parameters on the stress distribution and intensity within the vitreous chamber.

We also briefly consider the effect of a segmental buckle, which extends only over the area of the retinal break.

RESULTS

Myopic Eyes

At leading order, δ^0 , we obtain the solution for the motion of a fluid in a sphere, originally found by David et al.¹⁹ and recently reconsidered by Meskauskas et al.²⁰ We focus in this article on the effect of the shape of the vitreous chamber.

In Figure 3, we show the spatial distribution on the retina of the maximum (dimensionless) tangential stress and normal stress (pressure), attained within a period of oscillation, for an emmetropic (Figs. 3a, 3b) and a myopic eye (Figs. 3c, 3d) (-20 D). Moreover, in Figures 4a and 4b we plot the distribution of the dimensionless tangential (Fig. 4a) and normal (Fig. 4b) stresses on the retina along the horizontal plane (x, z) orthogonal to the axis of rotation, y . In the figure, $\varphi = 0$ corresponds to the anterior part of the vitreous chamber and $\varphi = \pi$ to the back pole. The normal and tangential stresses are scaled by $\epsilon \rho \omega^2 R_{0,e}^2$, where $R_{0,e}$ denotes the radius of the sphere with the same volume as the emmetropic eye in all plots. Figures 3 and 4 are obtained using the rheological properties of the vitreous humor proposed by Swindle et al.⁴ Note that the maximum stress on the retina also represents the maximum value of the stress attained in the vitreous body. The tangential stress is significantly larger than the pressure, and it appears at leading order, whereas the pressure first appears at the next order (δ) (see equations [16] and [21] in the Appendix). Therefore, the normal stress is close to zero in a domain with an (almost) spherical shape.

The distribution of both normal and tangential stresses on the retina is highly variable. In particular, the peak of the tangential stress is located at the front and back poles of the vitreous chamber in the case of the emmetropic eye (Figs. 3a, 4a), which is oblate in the anteroposterior direction, whereas it peaks at the sides of the chamber in the case of highly myopic prolate eyes (Figs. 3c, 4a). Note, however, that the shear stress in the case of myopic eyes is always significantly higher than that in the case of the emmetropic eye.

In Figure 5 we plot the maximum (over time and space) values of the tangential stress (Fig. 5a) and pressure (Fig. 5b) as a function of the refractive error. In this case, the stresses are normalized with the corresponding values relative to the emmetropic eye (0 D). In the figures, we report various curves that are obtained using different values for the rheological properties of the vitreous humor, proposed in the literature.³⁻⁵ In spite of the large differences of the rheological properties of the vitreous humor provided by the authors, the curves are relatively close to one another. Figure 5a shows that the

FIGURE 3. Spatial distribution of the maximum dimensionless tangential stress (**a**, **c**) and normal stress (**b**, **d**). (**a**) and (**b**): emmetropic eye; (**c**) and (**d**): myopic eye with refractive error -20 D. The eyeball rotates about the y -axis. The stress is scaled by $\epsilon \rho \omega^2 R_{0,e}^2$, with $R_{0,e}$ being the radius of the sphere approximating the emmetropic eye (0 D). We use rheological viscoelastic parameters from Swindle et al.,⁴ and $\omega = 12.5$ rad/s. Note that, owing to the linearization of the equations, the dimensionless stress is independent of the amplitude ϵ .

maximum shear stress on the retina increases monotonically with increasing degree of myopia. On the other hand, the normal stress on the retina has a minimum for eyes with a refractive error of approximately -11 D, which corresponds to the point at which the shape of the vitreous chamber is closest to spherical.

To quantify the effect of the departure from the spherical shape on the magnitude of the stress on the boundary, in Figure 6 we plot the maximum wall shear stress in eyes of different degrees of myopia (thick curve) and that in a sphere with the same volume (thin curve). We also show by the dashed curve the maximum shear stress obtained in abnormally long eyes, in which the anteroposterior axis grows with the degree of myopia at a rate twice as large as that predicted by Atchison et al.¹² The shape has a significant effect on the shear stress intensity on the retina, and departure from sphericity invariably induces an increase of stress magnitude.

All plots presented above were obtained assuming a fixed angular frequency of eye rotations. Qualitatively similar results are obtained for different values of ω and the dependence of the maximum stress on the retina upon the frequency is shown in Figure 7. The figure shows that the wall shear stress in highly myopic eyes is significantly higher than that in emmetropic eyes for the whole range of frequencies spanned.

Eye Subjected to Scleral Buckling

In Figure 8 we show the maximum (Fig. 8a) tangential and (Fig. 8b) normal stresses at the wall, scaled by $\epsilon \rho \omega^2 R_0^2$, in a geometry approximating the vitreous chamber of an eye subjected to scleral buckling by a cerclage. The wall shear stress has intense peaks at the locations of maximum indentation of the wall. The value of the maximum shear stress is approximately twice as large as the maximum shear stress generated in a sphere, indicating that the buckle has a significant effect.

The strength of the “pumping” of fluid out of the subretinal space induced by the buckle is arguably primarily dependent on the pressure. This is because the motion of a flap of partially detached retina is primarily due to the pressure difference between the two bodies of fluid across the flap, in this case in the vitreous humor and in the subretinal space. Figure 8b shows that additional intense maxima in the pressure occur at the sides of the buckle, along the line of sutures of the silicone band onto the sclera.

To investigate the effect of the indentation depth, d , and buckle width on the stress magnitude, we consider a range of various indentations with different dimensionless depths $d / \hat{R}$ and different shapes (in each case the cross section of the indentation is an arc of a circle, but we change the value of $R_i / \hat{R}$). In the plots we report the stress scaled with $\epsilon \rho \omega^2 R_0^2$, and

FIGURE 4. Spatial distribution of the maximum dimensionless tangential (a) and normal (b) stress on the retina on the horizontal midplane orthogonal to the axis of rotation, for different degrees of myopia; $\phi = 0$ corresponds to the anterior part of the vitreous chamber and $\phi = \pi$ to the back pole. The stress is scaled by $\epsilon \rho \omega^2 R_{0,e}^2$, with $R_{0,e}$ being the radius of the sphere approximating the emmetropic eye (0 D). Rheological viscoelastic parameters are from Swindle et al.,⁴ and $\omega = 12.5$ rad/s. *Thick solid curve:* 0 D; *thin solid curve:* -10 D, *dashed curve:* -20 D.

we keep the volume of the vitreous chamber constant (i.e., R_0 has been fixed). Following the discussion reported in Keeling et al.,²⁴ this case is representative of short times after surgery; however, we also ran the model for the case in which the surface area of the vitreous chamber was kept constant. This case corresponds approximately to the state of the eye in the long term after surgery. We found very little quantitative (and no qualitative) difference on replotting Figure 9 for a fixed scleral surface area.

Figure 9 shows that there is a significant dependence of the maximum (in space and time) normal and tangential stresses on the indentation depth $d / \hat{R}$ (thick curves), and there is a slight decrease as $R_i / \hat{R}$ increases. We note that our model is not valid for very small values of $R_i / \hat{R}$, due to the asymptotic method used; see also Repetto et al.²¹ In Figure 9a we also show with thin lines the maximum (in time) of the shear stress at the back pole of the eye. This is only slightly affected by the presence of the buckle, and it is invariably significantly smaller than the stress at the indentation. Finally, we note that the pressure at the back pole of the vitreous chamber is always equal to zero.

FIGURE 5. Maximum (over time and space) of the tangential (a) and normal (b) stress on the retina as a function of the refractive error in diopters. Values are normalized with the corresponding stress in the emmetropic eye (0 D). The different curves correspond to different values of the rheological properties of the vitreous humor taken from the literature: *thick solid curve,*⁵ *thin solid curve,*⁴ *dashed curve.*³ We use $\omega = 10$ rad/s.

For segmental buckles, we consider just two examples, which are shown in Figure 10. Figure 10a shows a circumferentially orientated buckle, and Figure 10b shows a radially orientated buckle. We center both buckles on the equatorial plane, as this is the position that generates the largest deviation in the stress. The figure shows the distribution of the maximum (in time) of the pressure on the retina, which is found to be qualitatively the same as for a cerclage, and indeed for a circumferentially orientated buckle (Fig. 10a), also quantitatively similar.

DISCUSSION

Myopic Eyes

As described in the previously, it has been hypothesized that the increased frequency of occurrence of vitreous gel degradation, PVD and RD in myopic eyes could be attributed to an increased stress in the vitreous and on the retina during eye rotations. David et al.¹⁹ showed that the maximum amplitude of the wall shear stress grows with the eyeball

FIGURE 6. Maximum over time and space of the dimensionless wall shear stress plotted against the volume V (relative to the volume V_0 of an emmetropic eye). *Solid thick curve*: myopic eye shape from Atchison et al.¹²; *solid thin curve*: sphere; *dashed curve*: abnormally long eye (the anteroposterior axis grows with the refractive error twice as fast as found by Atchison et al.¹²). The stress is normalized with the shear stress of the emmetropic eye (0 D). Rheological viscoelastic parameters are taken from Swindle et al.,⁴ $\omega = 12.5$ rad/s.

diameter; however, in that article the shape of the eye was not considered, and, most crucially from the clinical point of view, the relationship between the degree of myopia and the stress on the retina was not quantified.

The maximum wall shear stress increases significantly with the refractive error, the increase being almost linear. In severely myopic eyes, the maximum wall shear stress can be more than 1.5 times larger than that in emmetropic eyes.

It is worth noting that, even though the values of the mechanical properties of the vitreous humor measured by different authors are quite far apart, the curves shown in Figures 5a and 5b are very close to each other. This suggests that, in spite of the uncertainties concerning the rheology of the vitreous humor, the model is robust in predicting the effect of the shape of the vitreous chamber on the stress on the retina.

For all shapes considered, the maximum shear stress on the retina is larger than in a comparable sphere, which shows that myopic eyes experience even higher shear stresses due to their shape, extending the result of David et al.,¹⁹ in which only spheres were considered.

We hypothesize that, because the retina and vitreous humor of a myopic eye are subject to significantly higher shear stresses during physiologically realistic rotations than in an emmetropic eye, the structure of the vitreous humor could be more easily disrupted, leading to disintegration of the collagen network, an increased risk of vitreous liquefaction and PVD, and ultimately of RD. It is difficult to prove the above speculation on the basis of clinical data; however, as discussed in the previously, this assumption has already been put forward, and the present work demonstrates that it is mechanically very plausible.

We note that other mechanisms, still related to mechanics, are also likely to promote the more frequent occurrence of RD in myopic eyes. The retina in pathological myopes is fundamentally different from that in emmetropic eyes. In particular it is thinner,²⁶ and thus more prone to damage. Also, the pumping efficiency of the retina in myopic eyes is likely to be different, and this possibly influences the strength of

FIGURE 7. Temporal and spatial maximum of the dimensionless tangential (a) and normal (b) stress versus the angular frequency of eye rotations ω , for different values of the refractive error. The stress is scaled with $\epsilon \rho \omega^2 R_{0,e}^2$, where $R_{0,e}$ is the radius of the sphere with the same volume as the emmetropic eye (0 D). Rheological viscoelastic parameters from Sharif-Kashani et al.,⁵ who measured the dependency of the vitreous rheological properties (G) on the frequency ω (Fig. 1 of their article).

adhesion between the neural retina and retinal pigment epithelium.

Eye Subjected to Scleral Buckling

Scleral buckling is known to facilitate retinal reattachment in the case of a retinal tear, which is thought to be as a consequence of mechanical processes; however, the mechanisms underlying this process are not fully understood, and only a few attempts have been made to study them.^{18,27} To this end, it is essential to understand the stress distribution on the retina in the presence of a scleral buckle. We believe that the pressure distribution is particularly relevant, as the dynamics of the detached retina are likely to be strongly influenced by the pressure difference between the two sides of the flap. Our model shows that the buckle has a significant influence on the stress exerted on the retina. In particular, we showed that the maximum values of the pressure are located at the sides of the indentation produced by the buckle; moreover, we found that, for a given indentation depth, the stress on the retina does not depend significantly on the

FIGURE 8. Maximum tangential (a) and normal (b) components of the stress on the retina of a buckled eye. The eyeball rotates about the y -axis; rheological parameters from Swindle et al.⁴ are used; geometrical parameters used are $R_i / \hat{R} = 1$, $d / \hat{R} = 0.15$, $R_0 = 0.012\text{m}$, and $\omega = 12.5\text{rad/s}$. The stress is scaled with $\epsilon\rho\omega^2 R_0^2$.

radius of the indentation produced by the buckle, which, for simplicity and generality, has been assumed to have a cross section consisting of an arc of a circle.

In this article, we have considered only one implication of the scleral buckling procedure, which is that the change in shape of the vitreous chamber has an effect on the distribution of stress exerted by the vitreous on the retina. Complete modeling of the reattachment process is beyond the scope of the present work, but results obtained with our model represent an essential component.

There is an ongoing debate on whether scleral buckling or vitrectomy is the better procedure for repairing a retinal detachment. Each of the procedures involves different mechanical effects, and we believe that mathematical modeling can contribute significantly to this debate by improving our understanding of the effects of the mechanical processes involved in the two procedures, and that our model represents a step in this direction.

Limitations of the Model and Possible Future Developments

In this article, we studied the effect of the geometry of the vitreous chamber on the stress distribution on the retina. Our work is based on several assumptions, in particular that the

FIGURE 9. Maximum (in time and space) tangential (a) and normal (b) components of the stress as a function of the dimensionless radius $R_i / \hat{R}$ of the cross section of the buckle. (a) *Thin lines* refer to the maximum (in time) tangential stress at the back pole. The stress is scaled by $\epsilon\rho\omega^2 R_0^2$. Rheological values of the vitreous humor taken from Swindle et al.,⁴ $R_0 = 0.012\text{ m}$, $\omega = 12.5\text{ rad/s}$.

eye rotations are periodic and of a small amplitude, that the shape of the vitreous chamber is approximately spherical, that the rheological properties of the vitreous humor are accurately represented by our mechanical model, and that the vitreous humor is homogeneous. We discuss the first two of these here.

We described saccadic eye rotations in a simplified way, by assuming that the eye performs small-amplitude, sinusoidal, torsional, periodic oscillations, allowing us to obtain an analytical solution of the problem. We note that our model is quite versatile, however, and, owing to the linearity of the equations, we can find the solution corresponding to more physiologically realistic (small amplitude) eye movements by superposing solutions obtained in the sinusoidal case.

As an example, we consider the case of a realistic saccadic eye rotation. Following Repetto et al.,²⁸ we model the angular velocity of the eyeball during a saccadic rotation with a fourth-order polynomial, the coefficients of which are based on the empirical relationships proposed by Becker²⁹ (see equation [1] in Repetto et al.²⁸). In Figure 11a we show the eyeball angular velocity versus time during saccades of different amplitudes.

FIGURE 10. Maximum dimensionless pressure on the retina after the implantation of a segmental buckle (scaled with $\epsilon\rho\omega^2R_0^2$). The eyeball rotates about the y -axis, and the center of the buckle is located on the equatorial plane ($y=0$). (a) Circumferential and (b) radial orientation of the buckle. Rheological values of the vitreous humor taken from Swindle et al.,⁴ $R_0 = 0.012\text{m}$, $d/R = 0.15$, $\omega = 12.5\text{ rad/s}$.

We now consider a periodic motion of the eyeball, consisting of a sequence of saccades in opposite directions, each of which has duration T , separated by a resting time of duration $19T$. We expand the angular velocity corresponding to this motion in Fourier harmonics and superimpose the solution (in terms of velocity and wall shear stress) induced by each of these harmonics. Owing to the dissipative nature of the system, the velocity in the vitreous humor decreases to very small values after each saccadic rotation; therefore, the solution can be thought of as the response to a single eye rotation (the more so the higher the separation time between two successive rotations). We note that, to study vitreous dynamics induced by this motion of the eye, we had to make an assumption concerning the dependence of the complex modulus G on the frequency ω . Following Meskauskas et al.,²⁰ we modeled the vitreous according to a two-parameter Kelvin model.

In Figure 11b, we plot the maximum (dimensional) value of the shear stress on the retina versus the amplitude of the saccadic rotation. Each curve corresponds to a different degree of myopia: emmetropic eye and myopic eyes with -10 D and -20 D refractive error. It appears that the maximum shear stress grows with the saccade amplitude and, as in the case of sinusoidal eye rotations, also grows significantly with the

FIGURE 11. (a) Angular velocity of the eyeball versus time for saccadic rotations of different amplitudes (5° , 10° , 15° , 20° , 25° , and 30°). (b) Maximum shear stress on the retina versus the saccade amplitude for different values of the degree of myopia. Rheological values of the vitreous humor taken from Swindle et al.⁴ The dependence of the complex modulus G on ω has been computed adopting a two-parameter Kelvin model for the vitreous humor behavior, as suggested in Meskauskas et al.²⁰

degree of myopia. Thus, the results are qualitatively similar to those obtained for sinusoidal eye rotations.

We also focused on idealized approximations of two different geometries of clinical significance: the myopic eye and the eye subjected to scleral buckling. In the work presented here, we neglected the indentation due to the lens. However, Repetto et al.²¹ have shown (in the case of a purely viscous fluid) that the presence of the lens predominantly influences the fluid motion in the anterior segment of the vitreous chamber, while the flow in the posterior pole is only slightly affected, meaning the predictions of our model in the posterior part of the vitreous chamber are expected to hold.

CONCLUSIONS

As mentioned previously, there is evidence that the causes underlying the increased frequency of RD in myopes are mechanical, and David et al.¹⁹ added strength to this

argument by showing that the stress at the retina increases with the size of the eye. In this article, we have shown that the nonspherical shape of the vitreous chamber is also significant, and were able to relate the degree of myopia to the excess of stress on the retina. We found that the vitreous humor and the retina in highly myopic eyes are continuously subjected to significantly larger shear stresses than in emmetropic eyes and postulate that this might provide a mechanical explanation for the more frequent occurrence of disruption of the structure of the vitreous humor, PVD, and RD in myopia.

We also showed how the change in geometry of the vitreous chamber produced by the implant of a scleral buckle modifies the stress distribution on the retina, which is needed to understand the mechanics of the reattachment process.

References

- Zimmerman RL. In vivo measurements of the viscoelasticity of the human vitreous humor. *Biophys J*. 1980;29:539-544.
- Lee B, Litt M, Buchsbaum G. Rheology of the vitreous body. Part I: viscoelasticity of human vitreous. *Biorheology*. 1992;29:521-533.
- Nickerson CS, Park J, Kornfield JA, Karageozian H. Rheological properties of the vitreous and the role of hyaluronic acid. *J Biomech*. 2008;41:1840-1846.
- Swindle K, Hamilton P, Ravi N. In situ formation of hydrogels as vitreous substitutes: Viscoelastic comparison to porcine vitreous. *J Biomed Mater Res A*. 2008;87A:656-665.
- Sharif-Kashani P, Hubschman JP, Sassoon D, Kavehpour HP. Rheology of the vitreous gel: effects of macromolecule organization on the viscoelastic properties. *J Biomech*. 2011;44:419-423.
- Picciorelli M, Bergamin O, Landau K, Boesiger P, Luechinger R. Vitreous deformation during eye movement. *NMR Biomed*. 2011;25:59-66.
- Risk factors for idiopathic rhegmatogenous retinal detachment. The Eye Disease Case-Control Study Group. *Am J Epidemiol*. 1993;137:749-757.
- Akiba J. Prevalence of posterior vitreous detachment in high myopia. *Ophthalmology*. 1993;100:1384-1388.
- Morita H, Funata M, Tokoro T. A clinical study of the development of posterior vitreous detachment in high myopia. *Retina*. 1995;15:117-124.
- Yura T. The relationship between the types of axial elongation and the prevalence of lattice degeneration of the retina. *Acta Ophthalmol Scand*. 1998;76:90-95.
- Atchison DA, Jones CE, Schmid KL, et al. Eye shape in emmetropia and myopia. *Invest Ophthalmol Vis Sci*. 2004;45:3380-3386.
- Atchison DA, Pritchard N, Schmid KL, Scott DH, Jones CE, Pope JM. Shape of the retinal surface in emmetropia and myopia. *Invest Ophthalmol Vis Sci*. 2005;46:2698-2707.
- Sebag J. *The Vitreous: Structure, Function and Pathobiology*. New York, NY: Springer and Verlag; 1989.
- Bishop PN, Holmes DF, Kadler KE, McLeod D, Bos KJ. Age-related changes on the surface of vitreous collagen fibrils. *Invest Ophthalmol Vis Sci*. 2004;45:1041-1046.
- Pickett-Seltner RL, Doughty MJ, Pasternak JJ, Sivak JG. Proteins of the vitreous humor during experimentally induced myopia. *Invest Ophthalmol Vis Sci*. 1992;33:3424-3429.
- Meyer-Schwickerath G, Gerke E. Biometric studies of the eyeball and retinal detachment. *Br J Ophthalmol*. 1984;68:29-31.
- Baino F. Scleral buckling biomaterials and implants for retinal detachment surgery. *Med Eng Phys*. 2010;32:945-956.
- Foster WJ, Dowla N, Joshi SY, Nikolaou M. The fluid mechanics of scleral buckling surgery for the repair of retinal detachment. *Graefes Arch Clin Exp Ophthalmol*. 2009;248:31-36.
- David T, Smye S, Dabbs T, James T. A model for the fluid motion of vitreous humour of the human eye during saccadic movement. *Phys Med Biol*. 1998;43:1385-1399.
- Meskauskas J, Repetto R, Siggers JH. Oscillatory motion of a viscoelastic fluid within a spherical cavity. *J Fluid Mech*. 2011;685:1-22.
- Repetto R, Siggers JH, Stocchino A. Mathematical model of flow in the vitreous humor induced by saccadic eye rotations: effect of geometry. *Biomech Model Mechanobiol*. 2010;9:65-76.
- Tanner RI. *Engineering Rheology*. New York, NY: Oxford University Press; 2000.
- Quartapelle L, Verri M. On the spectral solution of the three-dimensional Navier-Stokes equations in spherical and cylindrical regions. *Comput Phys Commun*. 1995;90:1-43.
- Keeling S, Propst G, Stadler G, Wackernagel W. A mathematical model for the deformation of the eyeball by an elastic band. *Math Med Biol*. 2009;26:165-185.
- Wang F, Lee HP, Lu C. Biomechanical effect of segmental scleral buckling surgery. *Curr Eye Res*. 2007;32:133-142.
- Hayashi K, Ohno-Matsui K, Shimada N, et al. Long-term pattern of progression of myopic maculopathy: a natural history study. *Ophthalmology*. 2010;117:1595-1611.
- Clemens S, Kroll P, Stein E, Wagner W, Wriggers P. Experimental studies on the disappearance of subretinal fluid after episcleral buckling procedures without drainage. *Graefes Arch Clin Exp Ophthalmol*. 1987;225:16-18.
- Repetto R, Stocchino A, Cafferata C. Experimental investigation of vitreous humour motion within a human eye model. *Phys Med Biol*. 2005;50:4729-4743.
- Becker W. Metrics. In: Wurtz RH, Goldberg ME, eds. *The Neurobiology of Saccadic Eye Movements*. Amsterdam, The Netherlands: Elsevier Science Publisher BV (Biomedical Division); 1989:13-67.

APPENDIX

Mathematical Details

Governing Equations. We consider the flow of an incompressible viscoelastic fluid of density ρ , occupying a near-spherical region of approximate radius R_0 . The fluid motion is governed by

$$\rho \left(\frac{\partial \mathbf{u}^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla^* \mathbf{u}^* \right) = \nabla^* \cdot \boldsymbol{\sigma}^*, \quad \nabla^* \cdot \mathbf{u}^* = 0. \quad (1a, b)$$

Here superscript asterisks are used to denote dimensional variables that will later be made dimensionless, and $\mathbf{u}^*$ is the velocity, t^* is time and $\boldsymbol{\sigma}^*$ is the Cauchy stress tensor. In a linear viscoelastic fluid the stress tensor is given by²²

$$\boldsymbol{\sigma}^* = -p^* \mathbf{I} + 2 \int_{-\infty}^{t^*} G(t^* - s) \mathbf{D}(s) ds, \quad (2)$$

where p^* is the fluid pressure, G is the relaxation modulus,

$$\mathbf{D} = \frac{1}{2} \left(\nabla^* \mathbf{u}^* + (\nabla^* \mathbf{u}^*)^T \right), \quad (3)$$

is the rate-of-strain tensor, and $\mathbf{I}$ is the identity matrix. Equations (1a,b) are to be solved subject to no-slip boundary conditions.

We assume the eye undergoes small-amplitude, sinusoidal, torsional oscillations of prescribed angular displacement $\beta(t^*)$

$= -\epsilon \cos(\omega t^*)$, where $\epsilon \ll 1$. The assumption $\epsilon \ll 1$ allows us to neglect the nonlinear inertial term in equation (1a), meaning the equations become linear and allowing us to seek solutions of the form $\mathbf{u}^*(\mathbf{x}^*, t^*) = \mathbf{U}^*(\mathbf{x}^*)e^{i\omega t^*} + c.c.$, $P^*(\mathbf{x}^*, t^*) = P^*(\mathbf{x}^*)e^{i\omega t^*} + c.c.$ and $\boldsymbol{\sigma}^*(\mathbf{x}^*, t^*) = \boldsymbol{\sigma}^*(\mathbf{x}^*)e^{i\omega t^*} + c.c.$, where *c.c.* denotes the complex conjugate. The variables are related by

$$\Sigma^* = -P^* \mathbf{I} + \frac{\tilde{G}(\omega)}{i\omega} \left(\nabla^* \mathbf{U}^* + (\nabla^* \mathbf{U}^*)^T \right), \quad (4)$$

where the complex modulus of the fluid is given by

$$\tilde{G}(\lambda) = \lambda \int_0^\infty G(s) e^{-\lambda s} ds, \quad (5)$$

and the governing equations (1a,b) become

$$\rho i \omega \mathbf{U}^* = \nabla^* \cdot \Sigma^* = -\nabla^* P^* + \frac{\tilde{G}(\omega)}{i\omega} \nabla^{*2} \mathbf{U}^*, \quad \nabla^* \cdot \mathbf{U}^* = 0. \quad (6a,b)$$

Following the approach developed by Repetto et al.,²¹ we model the vitreous chamber as a weakly deformed sphere and assume that the eyeball has radius

$$R^*(\vartheta, \varphi) = R_0 \left(1 + \delta R_1(\vartheta, \varphi) \right), \quad (7)$$

where the constant R_0 is the radius of the sphere whose volume equals that of the vitreous chamber and δ is chosen so that the maximum absolute value of R_1 is 1, and (r^*, ϑ, ϕ) are spherical coordinates fixed in space, with the line $\vartheta = 0$, π being the inferior-superior axis of the eye (the axis of rotation), $\phi = 0$ being the anterior direction, and we define $\varphi = \phi - \beta(t^*)$, so that φ is the azimuthal coordinate rotating with the domain. The origin of the shape is coincident with its center of mass.

We work in terms of the following dimensionless variables

$$\begin{aligned} r &= \frac{r^*}{R_0}, \quad t = t^* \omega, \quad \mathbf{U} = \frac{\mathbf{U}^*}{\omega R_0}, \quad P = \frac{P^*}{\rho \omega^2 R_0^2}, \\ \Sigma &= \frac{\Sigma^*}{\rho \omega R_0^2}. \end{aligned} \quad (8a-c)$$

equation (4) gives

$$\Sigma = -P \mathbf{I} + \frac{1}{\alpha_c^2} \left(\nabla \mathbf{U} + (\nabla \mathbf{U})^T \right), \quad (9)$$

where

$$\alpha_c = \sqrt{\frac{\rho \omega R_0^2}{\tilde{G}(\omega)/(i\omega)}} \quad (10)$$

is the complex Womersley number, while equations (6a,b) become

$$i \mathbf{U} = \nabla \cdot \Sigma = -\nabla P + \frac{1}{\alpha_c^2} \nabla^2 \mathbf{U}, \quad \nabla \cdot \mathbf{U} = 0. \quad (11a,b)$$

The no-slip boundary condition gives

$$\mathbf{U} = -\frac{\epsilon i}{2} (1 + \delta R_1) \sin \vartheta \mathbf{e}_\varphi \quad \text{at} \quad r = 1 + \delta R_1, \quad (12)$$

where $\mathbf{e}$ denotes the unit vector in the direction of its subscript. From a clinical point of view, it is of particular interest to evaluate the stress exerted by the vitreous humor on the retina, which is given by $\boldsymbol{\sigma}^* \mathbf{n} = \rho \omega^2 R_0^2 (\mathbf{T} e^{it^*} + c.c.)$, where $\mathbf{n}$ is the unit normal vector of the surface and

$$\mathbf{T} = \Sigma|_{r=1+\delta R_1} \mathbf{n}. \quad (13)$$

Solution for Small δ

We expand the variables as

$$\mathbf{U} = \mathbf{U}_0 + \delta \mathbf{U}_1 + O(\delta^2), \quad (14a)$$

$$P = P_0 + \delta P_1 + O(\delta^2), \quad (14b)$$

$$\Sigma = \Sigma_0 + \delta \Sigma_1 + O(\delta^2), \quad (14c)$$

$$\mathbf{T} = \mathbf{T}_0 + \delta \mathbf{T}_1 + O(\delta^2), \quad (14d)$$

and in this article neglect terms of order $O(\delta^2)$ and higher. Where necessary in the following, we denote by U , V , and W the components of $\mathbf{U}$ in the directions of increasing r , ϑ , and φ , respectively, and we use the subscripts 0 and 1 to denote their contributions to $\mathbf{U}_0$ and $\mathbf{U}_1$, respectively.

At $O(\delta^0)$, the problem is in the spherical geometry $r \leq 1$, and it was solved by David et al.¹⁹ We briefly recap the solution: The continuity equation (11b) and the r - and ϑ -components of equation (11a) decouple from the φ -component of (11a), and thus since the right-hand side of the boundary condition (12) is purely in the φ -direction, only the φ -component of the velocity is nonzero. The solution is

$$P_0 = 0, \quad \mathbf{U}_0 = -\epsilon \frac{i(\sin ar - ar \cos ar)}{2r^2(\sin a - a \cos a)} \sin \vartheta \mathbf{e}_\varphi + c.c., \quad (15a,b)$$

where $a = \alpha_c e^{-i\pi/4}$. Substituting into equations (9) and (13) gives the leading-order contribution to the stress as

$$\begin{aligned} \mathbf{T}_0 &= \Sigma_0|_{r=1} \mathbf{e}_r = \frac{1}{\alpha_c^2} \left(\frac{\partial W_0}{\partial r} - \frac{W_0}{r} \right) \Big|_{r=1} \mathbf{e}_\varphi \\ &= -\frac{\epsilon i}{2\alpha_c^2} \left(\frac{(a^2 - 3)\sin a - a \cos a}{\sin a - a \cos a} \right) \sin \vartheta \mathbf{e}_\varphi + c.c. \end{aligned} \quad (16)$$

At $O(\delta^1)$ we use the fact that any smooth scalar function can be written as a sum of scalar spherical harmonics, Y_{mn} , and a vector-valued function as a sum of vector spherical harmonics, $\mathbf{P}_{mn}$, $\mathbf{B}_{mn}$, and $\mathbf{C}_{mn}$. The reader is referred to other literature²³ for a detailed definition of the vectors $\mathbf{P}_{mn}$, $\mathbf{B}_{mn}$, and $\mathbf{C}_{mn}$, but here we remark that for a particular m and n they are pairwise orthogonal, with $\mathbf{P}_{mn}$ always in the radial direction and $\mathbf{B}_{mn}$ and $\mathbf{C}_{mn}$ spanning the zenithal and azimuthal directions. We expand the velocity and pressure as

$$\begin{aligned} \mathbf{U}_1 &= \sum_{n=0}^{\infty} \sum_{m=-n}^n \left[U_1^{mn}(r) \mathbf{P}_{mn}(\vartheta, \varphi) + V_1^{mn}(r) \mathbf{B}_{mn}(\vartheta, \varphi) \right. \\ &\quad \left. + W_1^{mn}(r) \mathbf{C}_{mn}(\vartheta, \varphi) \right], \end{aligned} \quad (17a)$$

$$P_1 = \sum_{n=0}^{\infty} \sum_{m=-n}^n P_1^{mn}(r) Y_{mn}(\vartheta, \varphi), \quad (17b)$$

where $U_1^{mn}(r)$, $V_1^{mn}(r)$, $W_1^{mn}(r)$, and $P_1^{mn}(r)$ are functions to be found.

The boundary conditions are projected onto $r = 1$, a description of which appears in the literature,²¹ and become

$$\begin{aligned}
 U_1|_{r=1} &= -R_1(\vartheta, \varphi) \left(\frac{\epsilon i \sin \vartheta}{2} + \frac{\partial W_0}{\partial r} \Big|_{r=1} \right) \mathbf{e}_\varphi \\
 &= \sum_{n=0}^{\infty} \sum_{m=-n}^n \left(\hat{V}_{mn} \mathbf{B}_{mn}(\vartheta, \varphi) + \hat{W}_{mn} \mathbf{C}_{mn}(\vartheta, \varphi) \right),
 \end{aligned} \tag{18}$$

so that $\hat{V}_{mn}$ and $\hat{W}_{mn}$ depend on the choice of shape through R_1 .

Substituting into the governing equations (11a,b) we obtain a system of ordinary differential equations that can be solved to obtain

$$P_1^{mn} = -C_1^{mn} \frac{i\alpha_c^2}{n} r^n, \tag{19a}$$

$$U_1^{mn} = C_1^{mn} r^{n-1} + C_2^{mn} \frac{J_{n+\frac{1}{2}}(ar)}{r^{\frac{3}{2}}}, \tag{19b}$$

$$V_1^{mn} = C_1^{mn} \frac{s_n}{n} r^{n-1} + \frac{C_2^{mn}}{s_n r^{\frac{3}{2}}} \left(-n J_{n+\frac{1}{2}}(ar) + ar J_{n-\frac{1}{2}}(ar) \right), \tag{19c}$$

$$W_1^{mn} = C_3^{mn} \frac{J_{n+\frac{1}{2}}(ar)}{\sqrt{r}}, \tag{19d}$$

for $n > 0$, and $P_1^{00} = U_1^{00} = 0$, where J_k denotes the Bessel function of order k , $s_n = \sqrt{n(n+1)}$, and the boundary

condition (18) implies

$$C_1^{mn} = \frac{-s_n J_{n+\frac{1}{2}}(a) \hat{V}_{mn}}{a J_{n-\frac{1}{2}}(a) - (2n+1) J_{n+\frac{1}{2}}(a)}, \tag{20a}$$

$$C_2^{mn} = \frac{s_n \hat{V}_{mn}}{a J_{n-\frac{1}{2}}(a) - (2n+1) J_{n+\frac{1}{2}}(a)}, \tag{20b}$$

$$C_3^{mn} = \frac{\hat{W}_{mn}}{J_{n+\frac{1}{2}}(a)}. \tag{20c}$$

The stress at order δ can be calculated from the formula

$$\begin{aligned}
 \mathbf{T}_1 &= \left(-P_1 + \frac{1}{\alpha_c^2} \left(2 \frac{\partial U_1}{\partial r} - \left(\frac{\partial W_0}{\partial r} - W_0 \right) \frac{\partial R_1}{\partial \varphi} \csc \vartheta \right) \right) \Big|_{r=1} \mathbf{e}_r \\
 &+ \frac{1}{\alpha_c^2} \left(\frac{\partial V_1}{\partial r} - V_1 + \frac{\partial U_1}{\partial \vartheta} \right) \Big|_{r=1} \mathbf{e}_\vartheta \\
 &+ \frac{1}{\alpha_c^2} \left(\frac{1}{\sin \vartheta} \frac{\partial U_1}{\partial \varphi} + \frac{\partial W_1}{\partial r} - W_1 \right. \\
 &\quad \left. + R_1 \left(\frac{\partial^2 W_0}{\partial r^2} - \frac{\partial W_0}{\partial r} + W_0 \right) \right) \Big|_{r=1} \mathbf{e}_\varphi.
 \end{aligned} \tag{21}$$