Correlation Relation for the Membrane Transport Parameters L_p , σ , and ω

ARMIN KARGOL¹, MARIAN KARGOL² and STANISŁAW PRZESTALSKI³

¹ *Physics Department, Eastern Mediterranean University, Famagusta, Mersin 10, Turkey*

² *Institute of Physics, Pedagogical University, Kielce, Poland*

³ *Department of Physics and Biophysics, Agricultural University, Wrocław, Poland*

Accepted in final form 24 October 1997

Abstract. We derive a formula for the correlation of the three practical transport parameters L_p , σ , and ω appearing in Kedem-Katchalsky equations. It has a form $\omega = KL_p/\bar{v}_s(1 - \sigma)$, where $K = 0.0306$ is a universal constant independent of the choice of a membrane and a solute. It can be used to calculate the value of any of these parameters, provided the other two and the molar volume $\bar{v}_s$ of the solute are known. The formula could be very useful, in particular when measurements of the parameters are difficult or even impossible.

Key words: Membrane, Transport, Transport parameters, Correlation, Correlation constant

1. Introduction

The so-called practical Kedem-Katchalsky equations [4] are among the basic tools used in analysis of passive transport in membrane systems. They can be written as:

$$J_v = L_p \sigma \Delta \Pi + L_p \Delta P, \quad (1)$$

$$j_s = -\omega \Delta \Pi + (1 - \sigma) \bar{c} J_v, \quad (2)$$

where: J_v – volume flux, j_s – solute flux, $\Delta \Pi$ – osmotic pressure difference, ΔP – mechanical pressure difference. These equations apply to thin membranes and low-concentration solutions, and in such conditions the quantity $\bar{c}$ is given as:

$$\bar{c} = \frac{c_2 - c_1}{\log \frac{c_2}{c_1}} \approx \frac{c_2 + c_1}{2}, \quad (3)$$

where: c_1 , c_2 are the concentrations on both sides of the membrane. The equations [1, 4] describe linearized force-flow relations, and the coefficients (called the membrane transport parameters) are defined as follows:

$$L_p = \left(\frac{J_v}{\Delta P} \right)_{\Delta \Pi=0}, \quad (4)$$

$$\sigma = \left(\frac{\Delta P}{\Delta \Pi} \right)_{J_v=0}, \quad (5)$$

Figure 1. Membrane system (M – membrane, c_1, c_2 – concentrations, P_1, P_2 – mechanical pressures, J_v, j_s – fluxes, m, m – mixing devices).

$$\omega = \left(\frac{j_s}{\Delta\Pi} \right)_{J_v=0}. \quad (6)$$

The formulas above give the physical meaning of the parameters, and at the same time impose methods of their measurement [1].

The three transport parameters are treated as independent quantities, however it is not difficult to notice that there is some correlation among them. For instance, if for a given solute a membrane is semi-permeable ($\sigma = 1$), then one can see from (2) (and from experimental results) that $\omega = 0$. On the other hand if $\sigma = 0$, then $\omega_0 > 0$. If $0 < \sigma < 1$ then $\omega_0 > \omega > 0$, and membranes with larger reflection coefficients σ have smaller values of permeation coefficient ω .

Different attempts have been made to find the correlation of these parameters [4, 6–10, 12]. One can mention here a formula derived in [4, 5]:

$$\sigma = 1 - \frac{\omega \bar{v}_s}{L_p} - \frac{K_s f_{sw}}{\varphi_w (f_{sm} + f_{sw})} \quad (7)$$

where: $\bar{v}_s$ = molar volume, f_{sm}, f_{sw} = frictional coefficients, K_s = separation coefficient, φ_w = volume content of water in a membrane. The assumption was, however, that the water and the solute permeate through separate pores in a membrane.

In the present paper our goal is to find the correlation relation using standard curve-fitting techniques. We use experimental data collected from various literature sources and some of our own results.

2. Analysis

We consider a simple membrane system shown in Figure 1. The solutions on both sides of the membrane have low concentrations and are well mixed. For simplicity we also assume that $\Delta P = 0$. Then the practical Kedem-Katchalsky equations have the form:

Figure 2. Relation $\frac{J_{vw}}{L_p \Delta \Pi} = f(\sigma)$, (notation in text).

$$J_v = L_p \sigma \Delta \Pi \quad (8)$$

$$j_s = -\omega \Delta \Pi + (1 - \sigma) \bar{c} L_p \sigma \Delta \Pi. \quad (9)$$

The volume flux is the sum of water and solute volume fluxes (J_{vw} , J_{vs} , respectively):

$$J_v = j_w \bar{v}_w + j_s \bar{v}_s = J_{vw} + J_{vs}. \quad (10)$$

Here j_w , j_s are the fluxes of water and solute, respectively, and $\bar{v}_w$, $\bar{v}_s$ are their molar volumes. Combining (8), (9), and (10) we get:

$$J_{vw} = L_p \sigma \Delta \Pi + \omega \bar{v}_s \Delta \Pi - (1 - \sigma) \bar{c} \bar{v}_s L_p \sigma \Delta \Pi, \quad (11)$$

or

$$\frac{J_{vw}}{L_p \Delta \Pi} = \sigma + \frac{\omega \bar{v}_s}{L_p} - (1 - \sigma) \bar{c} \bar{v}_s \sigma. \quad (12)$$

It can be shown [3] that for low-concentration solutions the second term in (9) is much smaller than the first and can be neglected. Equation (12) can therefore be simplified to:

$$\frac{J_{vw}}{L_p \Delta \Pi} = \sigma + \frac{\omega \bar{v}_s}{L_p}. \quad (13)$$

Next we find the quantity $\frac{J_{vw}}{L_p \Delta \Pi}$ (using Equation 13) for a set of 36 membranes with coefficients L_p , σ , and ω found experimentally (see Table 1) and plot it versus σ . This is shown in Figure 2. The graph clearly suggests a linear relation of $\frac{J_{vw}}{L_p \Delta \Pi}$ and σ . We recall that if $\sigma = 1$ then $\omega = 0$. Therefore from (13) we see $\left. \frac{J_{vw}}{L_p \Delta \Pi} \right|_{\sigma=1} = 1$. We use the least squares fit with the above constraint (point B). As a result we obtain the following equation for the relation shown in Figure 2:

Table I. Values of parameters L_p , σ , and ω

No.	Membrane	Solute	$L_p \cdot 10^{12}$ [m ³ /N-s]	σ	$\omega \cdot 10^{10}$ [mol/N-s]	$v_s \cdot 10^3$ [m ³ /mol]	Reference
1	Toad skin	Acetamide	0.4	0.89	0.0041	0.052	[4]
2	Nitella translucens	Methanol	1.1	0.5	11	0.04	[4]
3	Nitella translucens	Ethanol	1.1	0.44	11	0.058	[4]
4	Nitella translucens	Isopropanol	1.1	0.4	7	0.076	[4]
5	Human red blood cell	Urea	0.92	0.62	17	0.042	[4]
6	Human red blood cell	Ethylene glycol	0.92	0.63	8	0.056	[4]
7	Visking dialysis tubing	Urea	3.2	0.013	20.8	0.115	[4]
8	Visking dialysis tubing	Glucose	3.2	0.123	7.2	0.2	[4]
9	Visking dialysis tubing	Sucrose	3.2	0.163	3.9	0.042	[4]
10	DuPond 'wet gel'	Urea	9.7	0.0016	31.6	0.042	[4]
11	DuPond 'wet gel'	Glucose	9.7	0.024	12.2	0.115	[4]
12	DuPond 'wet gel'	Sucrose	9.7	0.036	7.7	0.2	[4]
13	Dialysis membrane	Glucose	1.09	0.13	2.95	0.115	own result
14	Nephrophan	Ethanol	5	0.025	20	0.058	own result
15	Nephrophan	Glucose	5	0.065	8	0.115	own result
16	Cuprophan	Sucrose	2.88	0.28	6.7	0.2	[8]
17	GSRI I	NaCl	0.6	0.62	25.25	0.027	[13]
18	Cuprophan	Sucrose	5.5	0.157	6.89	0.2	[13]
19	Cuprophan	Urea	3.96	0.015	46.1	0.042	[7]
20	Cuprophan	Glucose	3.96	0.08	15	0.115	[7]
21	Cuprophan	Sucrose	3.96	0.157	7.05	0.2	[7]
22	Maize root	Ethanol	0.055	0.27	0.188	0.058	[12]
23	Maize root	Sucrose	0.055	0.54	0.048	0.2	[12]
24	Maize root	NaCl	0.055	0.64	0.228	0.027	[12]
25	Phaseolus coccineus root (1)	Ethanol	0.023	0.28	0.017	0.058	[11]
26	Phaseolus coccineus root (1)	KCl	0.023	0.43	0.0028	0.037	[11]
27	Phaseolus coccineus root (1)	NaCl	0.023	0.59	0.00084	0.027	[11]
28	Phaseolus coccineus root (1)	NaNO ₃	0.023	0.54	0.0014	0.037	[11]
29	Phaseolus coccineus root (2)	Ethanol	0.076	0.34	0.026	0.058	[11]
30	Phaseolus coccineus root (2)	KCl	0.076	0.54	0.0036	0.037	[11]
31	Phaseolus coccineus root (3)	Ethanol	0.032	0.47	0.028	0.058	[11]
32	Phaseolus coccineus root (4)	Methanol	0.025	0.16	0.01	0.04	[11]
33	Phaseolus coccineus root (5)	Ethanol	0.054	0.15	0.029	0.058	[11]
34	Phaseolus coccineus root (5)	Urea	0.054	0.41	0.002	0.042	[11]
35	Phaseolus coccineus root (6)	Methanol	0.058	0.34	0.024	0.04	[11]
36	Phaseolus coccineus root (7)	Urea	0.058	0.51	0.0044	0.042	[11]

$$\frac{J_{vw}}{L_p \Delta \Pi} = 0.9694\sigma + 0.0306. \quad (14)$$

Comparing (14) and (13) we get:

Table II. Comparison of experimental (ω) and theoretical (ω^t) values of the permeation coefficient

Membrane No.	1	2	3	4	5	6
$\omega \cdot 10^{10}$ [mol/N-s]	0.0041	11	11	7	17	8
$\omega^t \cdot 10^{10}$ [mol/N-s]	0.26	4.2	3.25	2.66	2.55	1.9
Membrane No.	7	8	9	10	11	12
$\omega \cdot 10^{10}$ [mol/N-s]	20.8	7.2	3.9	31.6	12.2	7.7
$\omega^t \cdot 10^{10}$ [mol/N-s]	23.0	7.5	4.1	70.6	25.2	14.3
Membrane No.	13	14	15	16	17	18
$\omega \cdot 10^{10}$ [mol/N-s]	2.95	20	8	6.7	25.25	6.89
$\omega^t \cdot 10^{10}$ [mol/N-s]	2.52	25.7	12.4	3.2	2.6	7.09
Membrane No.	19	20	21	22	23	24
$\omega \cdot 10^{10}$ [mol/N-s]	46.1	15	7.05	0.188	0.048	0.228
$\omega^t \cdot 10^{10}$ [mol/N-s]	28.4	9.7	5.10	0.211	0.039	0.2244

$$\omega = K \frac{L_p}{\bar{v}_s} (1 - \sigma), \quad (15)$$

with $K = 0.0306$. Formula (15) shows how the three parameters (L_p , σ , and ω) are correlated and K is a universal constant.

3. Conclusions

Formula (15) allows computation of one of the transport parameters (e.g., ω) provided the values of the other two are known. It is independent of any specific membrane or solute. It seems to be very useful, particularly since the measurements of L_p and σ are in general relatively easy, while the experimental determination of ω often meets with considerable difficulties, in particular for biological membranes [1].

The formula can be easily verified by computing values of ω for different membranes and solutes and comparing with experimental results. This is shown in Table 2 for selected membranes. Agreement between experimentally measured values of the permeation coefficient (ω) and values (ω^t) computed from Equation (15) is generally satisfactory. We noticed, however, significant discrepancies (up to 3 orders of magnitude) for parameter sets numbered 25–36. They refer to biological membranes for which active transport may be a factor influencing the results of measurement, as we comment below. Moreover, values of the permeation coefficient for these membranes are very small, and therefore are more sensitive to systematic experimental errors.

The above comment indicates that the main problem with the verification of Formula (15) is the quality of the available experimental data. Most of the authors of

papers from which values of L_p , σ , and ω were taken, do not give any measurement error estimates. The exception is the work by Steudle et al. [11]. The errors they give are very large, reaching 80%. We believe, therefore, that the deviations seen in Figure 2, as well as the discrepancies between calculated and measured values of ω can be attributed to experimental errors. For biological membranes they may be also caused by the passive and active transport taking place simultaneously, which obviously constitutes a serious systematic error.

Our formula is of course approximate. First, we used a simplified version of Equation (9), and second, we had only a limited number of parameter sets available. This paper is intended as a preliminary announcement. Work is in progress and upon collection of a larger set of better data, the value of a constant K may need a small adjustment. We believe, however, that even in the present stage, the formula could be applied, especially to studies of transport processes in biological membrane systems, where it is particularly difficult to measure all three parameters accurately. It also opens new research problems in situations where both passive and active transport occur.

References

1. Grygorczyk, Cz.: Measurement of practical phenomenological coefficients of membranes, *Zagadnienia Biofizyki Współczesnej* **3** (1978), 5–19 (in Polish).
2. Kargol, M.: A more general form of Kedem and Katchalsky's practical equations, *J. Biol. Phys.* **22** (1996), 15–26.
3. Kargol, M. and Kargol, A.: On solute convection in membrane stomata, *Kieleckie Studia Fizyczne* (WSP Kielce) **2** (1989), 51–59 (in Polish).
4. Katchalsky, A. and Curran, P.F.: *Nonequilibrium thermodynamics in biophysics*. Cambridge, MA: Harvard University Press, 1965, 113–132.
5. Kedem, O. and Katchalsky, A.: Thermodynamic analysis of the permeability of biological membranes to non-electrolytes, *Biochim. et Biophys. Acta* **27** (1958), 229–245.
6. Levitt, D.G.: A new theory of transport for cell membrane pores, *Biochim. et Biophys. Acta* **373** (1974), 115–131.
7. Mason, E.A., Wendt, R.P. and Bresler, E.H.: Similarity relations (dimensional analysis) for membrane transport, *J. Membr. Sci.* **6** (1980), 283–298.
8. Meyer, R.A., Hills, E.C. and Friedman, M.H.: Tracer permeabilities underestimate transmembrane solute flux under a concentration gradient, *J. Membr. Sci.* **8** (1981), 247–253.
9. Nobel, P.S.: *Introduction to biophysical plant physiology*, San Francisco: H.W. Foreman and Co., 1974, 133–157.
10. Sha'afi, R.I. and Gary-Bobo, C.M.: Water and nonelectrolytes permeability in mammalian red cell membranes, *Progress in Biophys. and Molecular Biol.* **26** (1973), 107–146.
11. Steudle, E., Oren, R. and Schultze, E.D.: Water transport in maize roots, *Plant. Physiol.* **84** (1987), 1220–1232.
12. Wendt, R.P., Mason, E.A. and Bresler, E.H.: Effect of heteroporosity on membrane rejection coefficient, *J. Membr. Sci.* **8** (1981), 69–90.

Address for correspondence: Dr Armin Kargol, Eastern Mediterranean University, Physics Department, Famagusta, N. Cyprus, Mersin 10, Turkey
Fax: +90 392 366-1604; E-mail: kargol@mozart.emu.edu.tr