A Frictional Interpretation of the Phenomenological Coefficients of Membrane Permeability for Multicomponent Non-ionic Solutions

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Abstract. This paper presents the physical interpretation of phenomenological coefficients of transmembrane transport for single and neutral membrane and multicomponent non-ionic solutions. In the framework of this model the expressions linking the permeability parameters of the membrane (L_p , σ , ω_{ik}) with friction coefficients (f_{ik}) for n -component, binary and ternary solutions and the method of calculation of friction coefficients for binary and ternary non-electrolytes are also presented.

Key words: Membrane transport, Friction model equations, Friction coefficients

1. Introduction

One of the fundamental treatments of transmembrane transport processes is Spiegler's frictional model equations based on the principle of a balance between gross thermodynamic forces acting on the system and frictional interactions between the components of the system [1]. The main advantage of this model is that it can characterize the interactions between all pairs of components, including a membrane, by resistance (r_{ij}) and friction (f_{ik}) coefficients. Spiegler's frictional model equations were used by Kedem and Katchalsky [2] to give a physical interpretation of the phenomenological coefficients of membrane (L_p , σ , ω) and by Meares and co-worker [3] to give a mechanistical interpretation of the molecular interaction by means of the macroscopic friction coefficient (R_{ik}). The relations between the resistance and friction coefficients and the hydration numbers of permeating species and the membrane are presented by Koter [4]. The frictional model is used to describe the transmembrane transport of electrolyte [3–5] and non-electrolyte [6] solutions. The method of generalization of the Spiegler-Kedem-Katchalsky frictional model equations (SKK model) for multicomponent non-ionic solutions was proposed by Ślęzak and co-worker [7]. The diffusive permeability and reflection coefficients of transport of electrolyte and non-electrolyte was derived by Vaidhyanathan [8, 9] on the basis of the statistical mechanical expression for the mean force acting on a molecule in a multicomponent system.

In the present paper, the aim of this theoretical study is to generalize the Spiegler-Kedem-Katchalsky frictional model equations for membrane systems containing

multicomponent, non-ionic and homogeneous solutions. The expressions relating the phenomenological coefficients of the membrane with the friction coefficients for n -component, ternary and binary non-ionic solutions are presented. The method of calculation of the friction coefficients for binary and ternary solutions is also proposed.

2. Basic assumptions

The typical starting point for consideration is that in steady state flow the thermodynamic forces ($\mathbf{X}_k$) are counterbalanced by the sum of mechanical frictional forces ($\mathbf{F}_k$) acting on one mole of the k -th type of component of the n -component solution

$$\mathbf{X}_k = -\mathbf{F}_k \quad (1)$$

where $\mathbf{X}_k$ is the thermodynamic force, $\mathbf{F}_k = \sum_{j \neq k} \mathbf{F}_{kj}$, $j \neq k = w, m, 1, 2, \dots, n$ is the frictional force acting on the k -th component of solution and is equal to the sum of interaction forces between the k -th and j -th species, with index w for water and m for membrane.

The frictional character of intermolecular interactions is illustrated by the following expression

$$\mathbf{F}_{kj} = -f_{kj}(\mathbf{v}_k - \mathbf{v}_j) \quad (2)$$

where f_{kj} is the friction coefficient between one mole of the k -th and j -th components and $\mathbf{v}_k$ and $\mathbf{v}_j$ are the velocities of the k -th and j -th components, respectively; the minus sign means, that the frictional force is opposite to the velocity of the moving components. In order to simplify the treatment, we consider the one-dimensional case only while deleting the index vector.

The flux $J_k(x)$ of the k -th substance passing normal to a unit area of the surface of the membrane at point x is expressed by

$$J_k(x) = \frac{1}{S} \frac{dn_k}{dt}(x) = \tilde{C}(x)v_k(x) \quad (3)$$

where n_k is the number of moles of the k -th component, S is the effective surface of the membrane, v_k is the velocity of k -th component and $\tilde{C}(x)$ is the average concentration of k -th substance at point x .

3. The Spiegler-Kedem-Katchalsky frictional model equations for n -component non-ionic solutions

In the steady state $J_k(x)$ is dependent on x and equal to the macroscopic flow. In an equation joining the thermodynamic forces (X_w, X_s) and fluxes (J_w, J_s) by means of the friction tensor R we have

$$\begin{bmatrix} X_w \\ X_s \end{bmatrix} = R \begin{bmatrix} \frac{J_w}{\bar{C}_w} \\ \frac{J_s}{\bar{C}_s} \end{bmatrix} \quad s = 1, 2, \dots, n \quad (4)$$

where the friction tensor R is defined by the following expression

$$R \equiv \begin{bmatrix} f_{wm} + \sum_{s=1}^n f_{ws} & -f_{w1} & \cdots & -f_{wn} \\ -f_{1w} & f_{1m} + \sum_{s \neq 1}^n f_{1s} & \cdots & -f_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ -f_{nw} & -f_{n1} & \cdots & f_{nm} + \sum_{s \neq n}^n f_{ns} \end{bmatrix} \quad (5)$$

J_w is the water flux and J_s is the molar flux of the s -th substance. Rewriting the friction tensor R in a form which allows us to introduce the C_k concentration and using Onsager's reciprocal relations we obtain

$$f_{kj} = \frac{\bar{C}_j}{\bar{C}_k} f_{jk}. \quad (6)$$

A consequence of the above relation is a relationship between minors of the friction coefficient matrix

$$\det R_{kj} = \frac{\bar{C}_k}{\bar{C}_j} \det R_{jk} \quad (7)$$

where $k, j = w, 1, 2, \dots, n$. Equation (4) has a solution in the form of the Cramer formulas

$$\frac{J_s}{\bar{C}_s} = (\det R)^{-1} \left[X_w \det R_{ws} (-1)^{s+2} + \sum_{l=1}^n X_l \det R_{ls} (-1)^{l+s} \right], \quad (8)$$

$$\frac{J_w}{\bar{C}_w} = (\det R)^{-1} \left[X_w \det R_{ww} + \sum X_l \det R_{lw} (-1)^{l+2} \right], \quad (9)$$

where $X_w \equiv -\frac{d\mu_w}{dx} = -\bar{V}_w \left[\frac{dP}{dx} - \sum_{k=1}^n \left(\frac{dn_k}{dx} \right) \right]$, $X_s \equiv -\frac{d\mu_s}{dx} = -\left[\bar{V}_s \frac{dP}{dx} + \frac{1}{\bar{C}_s} \frac{d\pi_s}{dx} \right]$ and μ_w and μ_s are the chemical potentials of water and s -th substance respectively, $P(x)$ and $\pi(x)$ are defined by quantities related to the solutions existing outside the membrane. In order to find relations between the concentration outside of the membrane ($\bar{C}_s(x)$) and the concentration in the membrane ($\tilde{C}_s(x)$) corresponding to x , we have to introduce the distribution coefficient with the expression $K_s = \tilde{C}_s(x)/\bar{C}_s(x)$ [2]; $s, l, k = 1, 2, \dots, n$.

Taking into consideration the definition of volume flux ($J_v = J_w \bar{V}_w + \sum_{s=1}^n J_s \bar{V}_s$), where J_w is the water flux, $\bar{V}_w$ is the partial molar volume of water, J_s is the flux of the s -th substance, $\bar{V}_s$ is the partial molar volume of the s -th substance), and Equations (8) and (9), we obtain

$$J_v = \frac{1}{\det R} \left\{ -\ell \bar{V}_w \frac{dP}{dx} + \sum_{l=1}^n \left[\frac{d\pi_l}{dx} \left(\ell \bar{V}_w - \frac{1}{\tilde{C}_l} \mathfrak{S}_l \right) \right] \right\}, \quad (10)$$

where $\ell = \varphi_w \det R_{ww} + \sum_{s=1}^n [(-1)^{s+2} \varphi_s \det R_{ws}]$,
 $\mathfrak{S} = \varphi_w (-1)^{l+2} \det R_{lw} + \sum_{s=1}^n [\varphi_s (-1)^{l+s} \det R_{ls}]$, $\varphi_s = \bar{V}_s \tilde{C}_s$, and $\varphi_w = \bar{V}_w \tilde{C}_w$.

We write the generalized Kedem-Katchalsky equation for volume flux in a local form [10]

$$J_v = L_p \left[-\frac{dP}{dx} + \sum_{l=1}^n \left(\sigma_l \frac{d\pi_l}{dx} \right) \right], \quad (11)$$

where L_p and σ_l are the local hydraulic permeability and reflection coefficients respectively, $l = 1, 2, \dots, n$.

Combining Equations (10) and (11) we obtain expressions for the local hydraulic permeability and reflection coefficients, respectively

$$L_p = \frac{\ell \tilde{V}_w}{\det R}, \quad (12)$$

$$\sigma_l = 1 - \frac{\mathfrak{S}_l}{\ell \bar{V}_w \tilde{C}_l}. \quad (13)$$

From Equation (13) we derive following relation

$$\frac{1 - \sigma_l}{1 - \sigma_s} = \frac{\mathfrak{S}_l \tilde{C}_s}{\mathfrak{S}_s \tilde{C}_l}, \quad (14)$$

where $l, s = 1, 2, \dots, n$.

Equation (8) may be rewritten as

$$\frac{J_s}{\tilde{C}_s} = \frac{1}{\det R} \left[\mathfrak{R}_s \frac{dP}{dx} + \sum_{l=1}^n \wp_l^s \frac{d\pi_l}{dx} \right], \quad (15)$$

where $\mathfrak{R}_s = \bar{V}_w (-1)^{s+1} \det R_{ws} + \sum_{l=1}^n [\bar{V}_l (-1)^{l+s+1} \det R_{ls}]$,

$\wp_l^s = \bar{V}_w (-1)^{s+2} \det R_{ws} + \frac{1}{\tilde{C}_l} (-1)^{l+s+1} \det R_{ls}$.

Using in Equation (15) and the differential pressure (dP/dx) calculated from Equation (11), we find the following expression

$$J_s = J_v \tilde{C}_s (1 - \sigma_s) + \sum_l^n \omega_{ls} \frac{d\pi_l}{dx} \quad (16)$$

where $\omega_{ls} = \frac{\Re_s + \sigma_l \varphi_l^s}{\det R} \tilde{C}_s$ is the diffusive permeability coefficient of the membrane for s -th substance passing under the influence of thermodynamic forces X_s and X_l .

After integrating Equations (11) and (16) over the thickness of the membrane we obtain

$$J_v = L_p \left[\Delta P - \sum_{l=1}^n \sigma_l \Delta \pi_l \right], \quad (17)$$

$$J_s = J_v (1 - \sigma_s) \tilde{C}_s + \sum_{k=1}^n \omega_{ks} \Delta \pi_k, \quad (18)$$

where $L_p = \frac{\bar{l} \cdot \bar{V}_w}{\Delta x \cdot \det R}$, $\sigma_l = 1 - \left(\frac{\bar{\mathfrak{S}}_l}{\bar{\ell} \bar{c}_l} \right) \frac{1}{\bar{V}_w}$, $\omega_{ls} = \frac{\overline{\tilde{C}_s \Re_l} + \overline{\tilde{C}_s \varphi_s} \sigma_l}{\Delta x \cdot \det R}$, $\Delta \pi = \pi_o - \pi_{\Delta x}$, $\Delta P = P_o - P_{\Delta x}$.

Examination of these equations leads to the following conclusions:

1. For $s = 1$ (binary solution), Equation (13) can be transformed to the form obtained by Spiegler and Kedem [10]

$$1 - \sigma_s = \frac{\mathfrak{S}_s}{\bar{V}_w \bar{c}_s} = \frac{K_w}{K_s} \cdot \frac{1 + \frac{f_{wm}}{f_{sw}} \frac{\bar{V}_s}{\bar{V}_w}}{1 + \frac{f_{sm}}{f_{sw}}}, \quad (19)$$

where $\mathfrak{S}_s = f_{ws} \varphi_w + (f_{ws} + f_{wm}) \varphi_s$.

The equation showed that the reflection coefficient is the product of kinetic term, which depends on the molar friction coefficients and the exclusion term, which is the quotient of the distribution coefficients of solute (K_s) and solvent (K_w). The expression is transformed to the following form

$$1 - \sigma_s = \frac{K_s f_{sw}}{\varphi_w (f_{sw} + f_{sm})} - \frac{\omega \bar{V}_s}{L_p}, \quad (20)$$

where $L_p = \frac{\varphi_w \bar{V}_w}{\Delta x \cdot f_{wm}}$, $\omega = \frac{K_s}{\Delta x (f_{sw} + f_{sm})}$, $K_s = \frac{\bar{c}_s}{\tilde{C}_s}$.

Equation (20) was obtained by Kedem and Katchalsky [2]. The transformation Equation (19) to (20) was presented by Nakao and Kimura [11].

2. For $s = 2$ (ternary solution), Equation (5) can be represented as

$$R = \begin{bmatrix} f_{w1} + f_{w2} + f_{wm} & -f_{w1} & -f_{w2} \\ -f_{1w} & f_{1w} + f_{12} + f_{1m} & -f_{12} \\ f_{2w} & -f_{21} & f_{2w} + f_{21} + f_{2m} \end{bmatrix}. \quad (21)$$

The reflection coefficient defined by Equation (13) is expressed as

$$1 - \sigma_1 = \frac{1}{\ell \bar{V}_w \tilde{C}_1}, \quad (22)$$

where $\ell = \varphi_w \det R_{ww} - \varphi_1 \det R_{w1} + \varphi_2 \det R_{w2}$.

If we assume that $\varphi_1, \varphi_2 \ll 1$ (the solutions are diluted and the concentration of substance in the membrane is small), e.g. for the substance denoted by index '1', we obtain

$$1 - \sigma_1 = \frac{K_1}{\varphi_w} \cdot \frac{f_{1w}(f_{2w} + f_{2m} + f_{21}) + f_{2w}f_{12}}{(f_{1w} + f_{12} + f_{1m})(f_{2w} + f_{21} + f_{2m}) - f_{12}f_{21}}. \quad (23)$$

For a hydrophilic membrane, for which the interactions between solute and membrane occur by water and by interactions between substance '1' and '2', Equation (14) can be expressed as

$$\frac{1 - \sigma_1}{1 - \sigma_2} \approx \frac{K_1}{K_2}. \quad (24)$$

The diffusive permeability coefficient of membrane, which is represented by Equation (19), we can write now, e.g. for a substance denoted by index '1', in following form

$$\omega_{11} = \frac{\Re_1 + \sigma_1 \wp_{11}}{\det R} \tilde{C}_1, \quad (25)$$

where $\Re_1 = \bar{V}_w \det R_{w1} - \bar{V}_1 \det R_{11} + \bar{V}_2 \det R_{21}$, $\wp_{11} = -\bar{V}_w \det R_{w1} - \frac{1}{\tilde{C}_1} \det R_{11}$. If we assume that $\varphi_1, \varphi_2 \ll 1$, we can write

$$\omega_{11} = \frac{K_1}{\Delta x \cdot (f_{1w} + f_{1m})} \left[1 - f_{12} \frac{f_{wm}(f_{2w}f_{wm}) + f_{2m}(f_{w1} + f_{w2})}{\det R} \right]. \quad (26)$$

In this equation the elements in square brackets are related to the diffusive permeability coefficient ω_{12} :

$$\omega_{12} = \frac{\Re_2 + \sigma_1 \wp_{12}}{\det R} \tilde{C}_2, \quad (27)$$

where $\Re_2 = -\bar{V}_w \det R_{w2} + \bar{V}_1 \det R_{12} - \bar{V}_2 \det R_{22}$, $\wp_{12} = \bar{V}_w \det R_{w2} + \frac{1}{\tilde{C}_1} \det R_{12}$.

Assuming that $\varphi_1, \varphi_2 \ll 1$ and using expression (26), we obtain

$$\omega_{12} = -\frac{K_1 f_{12}}{f_{1w} + f_{1m}} \cdot \frac{\tilde{C}_2}{\tilde{C}_1} \cdot \frac{f_{wm}(f_{1w} + f_{1m}) + f_{2m}(f_{w1} + f_{w2})}{\det R}. \quad (28)$$

Hence the diffusive permeability coefficient for straight transport is dependent on a suitable coefficient for mutual transport, because from the relationship between the diffusive permeability coefficient in mutual transport [12], we can calculate ω_{21}

$$\omega_{21} = \omega_{12} \frac{\tilde{C}_1}{\tilde{C}_2} = \frac{\Re_1 + \sigma_2 \wp_{21}}{\det R} \tilde{C}_1, \quad (29)$$

where $\wp_{21} = -\bar{V}_w \det R_{w1} + \frac{1}{\bar{C}_2} \det R_{21}$.

If the relation (6) is fulfilled and $K_1 = K_2$, the following relation for the molar friction coefficient is found

$$\frac{f_{2m}}{f_{2w}} = \frac{f_{1m}}{f_{1w}}. \quad (30)$$

4. The equations for friction coefficient in ternary solution

When $K_1 = K_2 = K$, the expressions describing the interactions between the dissolved substance and the water and the membrane are as follows

$$f_{12} = -\frac{K\gamma_2}{1 + \frac{\bar{C}_1\gamma_2}{\bar{C}_2\gamma_1}}, \quad (31)$$

$$f_{1w} = \frac{\gamma_4 K \left[\gamma_1 \gamma_2 K + f_{12} \left(\gamma_2 \frac{\bar{C}_1}{\bar{C}_2} + \gamma_1 \right) \right]}{\gamma_1 K + f_{12} \left(\frac{\gamma_1}{\gamma_2} + \frac{\bar{C}_1}{\bar{C}_2} \right)}, \quad (32)$$

$$f_{wm} = \frac{\Delta x \cdot \omega_{12} R \gamma_2 - f_{1m} f_{12} \frac{\bar{C}_1}{\bar{C}_2} (f_{1w} + f_{2w})}{f_{12} \gamma_2 K}, \quad (33)$$

$$f_{2w} = f_{1w} \frac{\gamma_1}{\gamma_2}, \quad (34)$$

$$f_{1m} = K\gamma_2 - f_{1w}, \quad (35)$$

$$f_{2m} = K\gamma_1 - f_{2w}, \quad (36)$$

where $\gamma_1 = -\frac{1}{\Delta x}(\omega_{21} - \omega_{11})(\omega_{11}\omega_{22} - \omega_{12}\omega_{21})^{-1}$, $\gamma_2 = -\frac{1}{\Delta x}(\omega_{12} - \omega_{22})(\omega_{11}\omega_{22} - \omega_{12}\omega_{21})^{-1}$, $\omega_{11}\omega_{22} \neq \omega_{12}\omega_{21}$, $\gamma_3 = \frac{1-\sigma_1}{1-\sigma_2} - 1$, $\gamma_4 = \frac{\varphi_w(1-\sigma_1)}{K}$.

It appears from above, that $\gamma_1 > 0$ and $\gamma_2 > 0$. For $\sigma_1 > \sigma_2$, $\gamma_3 < 0$ and for $\sigma_1 < \sigma_2$, $\gamma_3 > 0$. If $\sigma_1 = \sigma_2$, that $\gamma_3 = 1$. If we assume that $0 \leq \sigma_1 < 1$ we obtain $\gamma_4 > 0$. For $\sigma_1 = 1$, $\gamma_4 = 0$ and for $\sigma_1 > 1$ (anomalous osmosis), $\gamma_4 < 0$. The above conditions, and Equations (31)–(36) show that $f_{12} < 0$ and $f_{21} < 0$. If we assume that $|\gamma_1 \gamma_2 \gamma_4 K^2| \leq \left| f_{12} \left(\gamma_2 \frac{\bar{C}_1}{\bar{C}_2} + \gamma_1 \right) \right|$ and $|\gamma_1 K| \leq \left| f_{12} \left(\frac{\gamma_1}{\gamma_2} + \frac{\bar{C}_1}{\bar{C}_2} \right) \right|$, $f_{1w} > 0$. Also $f_{2w} > 0$.

If we assume that $K\gamma_2 > f_{1w}$ or $K\gamma_2 < 0$ we obtain $f_{1m} > 0$ or $f_{1m} < 0$ respectively. It is acceptable that $f_{1m} = 0$, because the orders of magnitude of $K\gamma_2$ and f_{1w} are identical. Therefore also $f_{2m} > 0$, $f_{2m} < 0$ and $f_{2m} = 0$. Because $f_{12}\gamma_2 K < 0$, and if $\Delta x \omega_{12} R \gamma_2 < f_{1m} f_{12} \frac{\bar{C}_1}{\bar{C}_2} (f_{1w} + f_{2w})$, then $f_{wm} > 0$. If $\Delta x \omega_{12} R \gamma_2 > f_{1m} f_{12} \frac{\bar{C}_1}{\bar{C}_2} (f_{1w} + f_{2w})$, then $f_{wm} < 0$.

Besides the coefficient R , which is the determinant of matrix friction coefficients, we can calculate

Table I. Values of the parameters of Nephrophane and Visking DT membranes for aqueous glucose (index 1) and/or ethanol (index 2) solutions

Coefficient	Value	
	Nephrophane	Visking DT
$L_p \cdot 10^{12} [\text{m}^3/\text{Ns}]$	5.0 ± 0.2	2.7
$\sigma_1 \cdot 10^2$	6.8 ± 0.3	8.3
$\sigma_2 \cdot 10^2$	2.5 ± 0.1	–
$\omega_{11} \cdot 10^{10} [\text{mole}/\text{Ns}]$	8.0 ± 0.4	7.14
$\omega_{22} \cdot 10^{10} [\text{mole}/\text{Ns}]$	2.0 ± 0.05	–
$\omega_{21} \cdot 10^{13} [\text{mole}/\text{Ns}]$	8.1 ± 2.5	–
$\omega_{12} \cdot 10^{12} [\text{mole}/\text{Ns}]$	2.0 ± 0.9	–
$\Delta x \cdot 10^4 [\text{m}]$	2.0 ± 0.02	0.8
φ_w	0.71 ± 0.06	0.68
K_1	0.53 ± 0.12	0.6
K_2	0.56 ± 0.13	–
$\tilde{c}_1 [\text{mole}/\text{l}]$	$0.0125 \div 0.1$	0.05
$\tilde{c}_2 [\text{mole}/\text{l}]$	$0.05 \div 0.4$	–
$\tilde{V}_w \cdot 10^6 [\text{m}^3/\text{mole}]$	18.05	18.05
$\tilde{V}_1 \cdot 10^6 [\text{m}^3/\text{mole}]$	120.2	120.2
$\tilde{V}_2 \cdot 10^6 [\text{m}^3/\text{mole}]$	58.3	–

$$R = \frac{K\varphi_w \mathbf{V}_w}{\Delta x \cdot L_p} \left[\gamma_1(\gamma_2 K + f_{12}) + \gamma_2 f_{12} \frac{\tilde{C}_1}{\tilde{C}_2} \right]. \quad (37)$$

The Equations (31)–(35) are the complete set of equations for the friction coefficients for transport in ternary solutions.

5. Results and discussion

The friction coefficients for binary and ternary solutions (aqueous glucose and/or ethanol solutions) and the Nephrophane flat dialyser membrane were calculated on basis of Equations (31)–(35). Calculations were made concerning the dependence of coefficients f_{ik} from $\tilde{C}_1$ with fixed (steady) value $\tilde{C}_2$ and from concentration of $\tilde{C}_2$ with fixed value of $\tilde{C}_1$ concentration. Suitable values of the permeability parameters of Nephrophane and Visking DT membranes are listed in Table I. The friction coefficients for Nephrophane membrane are listed in Table II and in Figures 1 and 2. For comparison, in Table II the values of friction coefficients for Visking DT membrane calculated by Ginzburg and Katchalsky [6] are included. The results of both calculations for binary solution are comparable. Results obtained show that the values of coefficients f_{1w} , f_{2w} , f_{1m} and f_{2m} are independent from both binary and ternary solutions concentrations.

Table II. Values of the frictional coefficients f_{ik} for Nephrophane and Visking DT membranes and aqueous glucose and/or ethanol (w-water, 1-glucose, 2-ethanol)

Coefficient	binary		Value	
			ternary	
	Nephrophane	Visking DT	$\tilde{C}_1 = 0.05$ mole/l	$\tilde{C}_2 = 0.1$ mole/l
$f_{1w} \cdot 10^{-13}$ [Ns/mole·m]	0.6	1.89	0.4	0.4
$f_{2w} \cdot 10^{-13}$ [Ns/mole·m]	1.7	—	1.6	1.7
$f_{1m} \cdot 10^{-13}$ [Ns/mole·m]	0.55	0.23	−0.06	−0.06
$f_{2m} \cdot 10^{-13}$ [Ns/mole·m]	0.5	—	−0.22	−0.22
$f_{wm} \cdot 10^{-13}$ [Ns/mole·m]	0.002	0.009	Figure 1	Figure 1
$f_{12} \cdot 10^{-13}$ [Ns/mole·m]	—	—	Figure 2	Figure 2
$f_{21} \cdot 10^{-13}$ [Ns/mole·m]	—	—	Figure 2	Figure 2
$R \cdot 10^{-36}$ [(Ns/mole·m) ³]	0.87	—	1.75	1.75

Figure 1. Glucose concentration ($\tilde{C}_1$) dependencies of friction coefficient $(f_{wm})_2$ in 0.1 mole/l aqueous ethanol solution (plot 1) and ethanol concentration ($\tilde{C}_2$) dependencies of $(f_{wm})_1$ in 0.05 mole/l aqueous glucose solution (plot 2).

Figure 1 shows the dependence of f_{wm} coefficient on the glucose concentration ($\tilde{C}_1$) in 0.1 mole/l aqueous ethanol solution (plot 1) and on concentrations of glucose in 0.05 mole/l aqueous glucose solution (plot 2). Plot 1 shows that the value of $(f_{wm})_2$ coefficients increases lineally together with the increase of the concentra-

Figure 2. Glucose concentration ($\tilde{C}_1$) dependencies of friction coefficients $(f_{12})_2$ and $(f_{21})_2$ in 0.1 mole/l aqueous ethanol solution (plots 1 and 2 respectively) and ethanol concentration ($\tilde{C}_2$) dependencies of $(f_{12})_1$ and $(f_{21})_1$ in 0.05 mole/l aqueous glucose solution (plots 3 and 4 respectively).

tion of glucose in 0.1 mole/l aqueous ethanol solution. This means the interaction between water and membrane increases. Consequently, value of $(f_{wm})_1$ coefficient decreases hyperbolically when the concentration of ethanol in 0.05 mole/l aqueous glucose solution. As the result of that the interaction between water and membrane decreases.

Figure 2 shows the dependence of coefficients f_{12} and f_{21} on glucose concentrations ($\tilde{C}_1$) in 0.1 mole/l aqueous ethanol solution (plots 1 and 2) and on ethanol concentrations in 0.05 mole/l aqueous glucose solution (plots 3 and 4). Plots 1 and 2 show that if the concentration of glucose in 0.1 mole/l aqueous ethanol solution increases the value of the coefficient $(f_{12})_2$ decreases and coefficient $(f_{21})_2$. Plots 3 and 4, on the other hand, show that if the concentration of ethanol in 0.05 mole/l aqueous glucose solution increases, the values of the coefficients f_{12} and f_{21} are reflected in the intensity of frictional relations between solution compounds. Moreover, looking at the dependencies presented on the figure it may be concluded that: for $\tilde{C}_1 = 0.0375$ mole/l and $\tilde{C}_2 = 0.15$ mole/l, $(f_{21})_1 \approx (f_{21})_2 = -0.11 \cdot 10^{13}$ [Ns/mole·m] and $(f_{12})_1 \approx (f_{12})_2 = -0.31 \cdot 10^{13}$ [Ns/mole·m]. For $\tilde{C}_1 = 0.1$ mole/l and $\tilde{C}_2 = 0.4$ mole/l $(f_{21})_2 \approx (f_{12})_2 = -0.27 \cdot 10^{13}$ [Ns/mole·m] and for $\tilde{C}_1 = 0.0125$ mole/l and $\tilde{C}_2 = 0.05$ mole/l, $(f_{12})_1 \approx (f_{21})_1 = -0.27 \cdot 10^{13}$

[Ns/mole·m]. Also the friction coefficients for binary and ternary solutions in Table II show that:

1. The values of the friction coefficients for interactions of glucose-water (f_{1w}) are greater in binary than ternary solutions and the values of the friction coefficients for interactions of ethanol-water (f_{2w}) are on an average equal in binary and ternary solutions.
2. The values of the friction coefficients for interactions of glucose-membrane (f_{1m}) and ethanol-membrane (f_{2m}) are greater in binary than ternary solutions. The values of these coefficient are negative for ternary solutions.
3. The values of the friction coefficient for interactions of water-membrane (f_{wm}) is constant in binary solutions and in ternary solutions the values of f_{wm} are dependent on glucose ($\tilde{C}_1$) and ethanol ($\tilde{C}_2$) concentrations.
4. The values of the friction coefficients f_{1w} , f_{2w} , f_{1m} and f_{2m} for both binary and ternary solutions are independent of glucose ($\tilde{C}_1$) and ethanol ($\tilde{C}_2$) concentrations.
5. The values of the friction coefficients f_{12} and f_{21} are negative and dependent on glucose ($\tilde{C}_1$) and ethanol ($\tilde{C}_2$) concentrations.

The observed results can be explained in the following way. Sugars are hydrophilic and are not dissolved in alcohol. Therefore that glucose particles will always attach water particles because of hydration, ejecting ethanol particles away (close to the membrane). This seems to be the reason why coefficients f_{2m} and f_{1m} achieve lower values for ternary solutions as compared with analogous coefficients for binary solutions. Moreover, ethanol shows properties of contraction. As a result it should glucose particles while being transported through the membrane since its concentration gradient is higher. This conclusion is confirmed by the analysis of friction force sign ($F_{ij} = -f_{ij}(v_i - v_j)$, $v_i = J_i/\tilde{C}_i$) which depends on the particles relative velocity.

6. Conclusion

The application of the SKK frictional model equation to physicochemical and biological systems is restricted by the difficulty of measuring values for the average or local concentrations of the permeating species in the membrane. These values are required for the calculation of the friction coefficients from the measured experimental parameters [6]. The SKK model equation [2, 13] was used for the interpretation of the parameters of the membrane (L_p , σ and ω). The field of application of this model is limited to binary solutions. Generalization of this model presented in this paper for systems containing multicomponent, non-electrolyte solutions increases the field of applications, particularly to biosystems. In the framework of this model, the frictional interpretation of coefficient L_p , σ and ω_{ik} and the expressions linking these coefficients with molar friction coefficients f_{ik} for multicomponent solutions are presented. Besides the methods of calculation of

f_{ik} the coefficients for ternary solutions on the basis of experimentally measured phenomenological coefficients are presented.

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