The Sinus Node as a Nonlinear Dynamic System

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Abstract. The Sinus Node (SN) of the heart is the natural pacemaker driving electrical impulses into the atria, an ordinary excitable medium. In order to avoid wave interference, the SN operation must be of such a nature that no reflections from the atria should occur. It is shown that such a behavior is achieved when the SN operates in a nonlinear dynamical regime situated away from relaxation.

Key words: nonlinear dynamics, relaxation regime, Sinus Node

1. Introduction

The sinoatrial node or Sinus Node (SN) of the heart is its natural pacemaker. It drives the electric impulse through the atria and into the ventricles to induce contraction. A large number of theoretical and experimental papers have appeared which describe the construction and function of the SN (see e.g. the review by Boyett et al. [1], and references therein). A major problem already posed by Joyner and van Capelle in 1986 [2] is ‘how does a small SN drive a large atrium?’. The prevalent answers to this question have been that this ability was due to: (a) the gradual increase of intracellular couplings from the center of the SN towards its periphery; or (b) the restricted contact area between the SN and the atria, or poor electrical contact there; or (c) an increase of i_f or i_{Na} at the periphery of the SN. We offer here a simple but different possible answer to this question. It is partly related to point (c) above, and is based on the noticeable difference between the typical shape of the action potential profile in the SN and the AV node, and its shape in the rest of the heart.

2. The Basic Action Potential Shapes

Figure 1 taken from Ref. [3] shows the typical shapes of the action potential in several parts of the heart. It is immediately obvious that the pulse shapes of the SN and the AV node (which is also a potential pacemaker) are different from the others, namely that the rise time for these nodes is much longer, i.e. the defibrillation velocity is much smaller. From the nonlinear dynamics point of view, the rest of the heart operates in a ‘relaxation regime’ [4], while the nodes do not. We shall

Figure 1. Typical shapes of action-potential pulses in different tissues of the heart [3]. SAN – sinoatrial node (SN); AM – atrium; AVN – AV-node; PF – Purkinje fiber; VM – ventricle.

show that this feature alone is sufficient for the ability of the SN or AV node to perform as pacemakers controlling the rest of the heart.

3. The Model

In order to concentrate on the basic behavior we treat a simple one dimensional model, namely the Fitzhugh-Nagumo (or Bonhoeffer-Van der Pol). It consists of two nonlinear differential equations:

$$\begin{aligned}\dot{v} &= Dv_{xx} + v(v-a)(1-v) - w + v_0(x)\delta(t) \\ \dot{w} &= \varepsilon(v-dw)\end{aligned}\tag{1}$$

where $v(x,t)$ stands for the *action potential*, $w(x,t)$ is an *inhibitor*, or *refractoriness* function, D is the diffusion coefficient describing the connection between cells, $a(x)$ is the *excitability* parameter, and d and $\varepsilon(x)$ are parameters, the latter specifying the ratio between the time constants of v and w . An initiating pulse

Figure 2. Waves in a divided medium: for the interior, 1, ($200 < x < 1000$), $a = -0.16$; Exterior region, 2, ($0 < x < 200$, $1000 < x < 1200$), $a = 0.1$. $\varepsilon = 0.005$ throughout (time changes vertically and x horizontally.)

$v_0(x)$ is launched at $t = 0$, its spatial shape being a narrow Gaussian centered at $x_0 = 0$. Let us recall (see e.g. [5]) that a may be either positive or negative; if a is negative below a certain threshold value, the system is in a limit-cycle regime, namely a pacemaker; if a is positive the system describes an excitable medium. The parameter $\varepsilon(x)$ controls the ‘relaxation’ behavior: very small values ($\varepsilon \ll 0.005$) correspond to a ‘relaxation’ regime (RR), while larger values (but still $\ll 1$) describe the non-relaxation (NRR) regime. The RR shows a very fast rise at the beginning of the action potential pulse. With the increase of ε , and passage to the NRR, the pulse rise becomes much slower.

4. Reflection Results

We consider a medium divided into three spatial regions, Figure 2. In the central region ($200 < x < 1000$) labeled 1, $a = -0.16$ (limit cycle regime, simulating the SN), while in both external excitable regions ($0 < x < 200$ and $1000 < x < 1200$) labeled 2, $a = 0.1$ depicting atria. The value $\varepsilon = 0.005$ (RR), is constant throughout the whole medium. An initial pulse is launched in the first region. Since this region is in a limit cycle regime, pulses are periodically generated there. The main phenomenon occurring in this situation is a reflection of waves from regions 2 into region 1. The frequency of pulses in region 1 is modified by these reflections. Figure 3 shows the periods of reflected waves in region 1 as a function of both Δa (the change in a between regions 1 and 2) and ε . Although for small Δa ,

Figure 3. Period of reflected waves as functions of Δa ($= a_2 - a_1$) and ϵ . $a_1 = -0.16$; $d = 3$; $\Delta t = 1$; The symbols designate $\circ$: $\epsilon = 0.005$; $\square$: $\epsilon = 0.01$; $\triangle$: $\epsilon = 0.02$; $\times$: $\epsilon = 0.03$.

the reflected waves have little influence on the period, this constitutes an outside interference with the SN operation. For larger Δa the change in frequency is more significant. Note that for large ϵ values (>0.02) no reflection occurs for small Δa values, and for $\epsilon > 0.05$ no reflection occurs at all, even for very large Δa .

Figure 4a depicts the 'natural' situation in the SN – atria interface, as we conceive it. The excitability profile $a(x)$ is the same as in Figure 2, while ϵ , instead of

Figure 4. Waves in a divided medium: $a(x)$ – the same as for Figure 2. 4a: $\varepsilon(x) = 0.008$ for $550 < x < 650$; $\varepsilon(x) = 0.005$ otherwise. 4b: $\varepsilon(x) = 0.002$ for $550 < x < 650$; $\varepsilon(x) = 0.005$ otherwise.

being constant throughout the whole domain as in Figure 2, presents a step profile shown at the bottom: $\varepsilon = 0.005$ for $(0 < x < 550 \text{ and } 650 < x < 1200)$, and $\varepsilon = 0.008$ for $(550 < x < 650)$ which places this small region in a NRR regime. This is sufficient to ensure the following behavior in the entire region 1 (where a is negative): (a) There is no reflection of waves from region 2 back into 1; (b) After a certain transient time, a phase synchronization builds up in the entire SN region 1; (c) The wave frequency in the whole medium is governed by that of the NRR. Similar effects arise when the NRR extends throughout the entire region 1.

Figure 4b shows a situation where, instead of a NRR central region, this segment is attributed an RR character even stronger than the rest of the medium, namely $\varepsilon = 0.002$. It is seen that additional reflections appear in this central region while the former reflections from region 2 into 1 remain intact. Note that the time interval between two successive wave profiles in Figures 2 and 4, is about 400 computational time steps (slow stroboscopy). Therefore, not all dynamical details are reproduced.

5. Discussion

It was demonstrated that a change of behavior of the SN from RR to NRR leads to the disappearance of reflections from the atria towards the SN. Moreover, the frequency of the outgoing pulse train is dominated by the SN, rendering it into a coherent pacemaker. Since a NRR behavior is observed in the SN (Figure 1), this in fact corresponds to a normal, uninterfered SN operation. A word of caution however, is due. The FitzHugh-Nagumo model used here being rather crude, cannot be ‘pushed’ beyond a certain limit. Thus, if one increases ε to values higher than ~ 0.007 , a number of arrhythmia effects become apparent, such as Wenckebach 2:1 and n:1 blocks, and even chaos. The objective of this paper was to show qualitatively that changing the character of the SN away from the relaxation regime, ensures its functioning as pacemaker.

Quantitative estimations would be possible only by using more complex, and sophisticated models. In general, the reflection and refraction of waves in excitable media present a wide interest not only in the investigation of the cardiac muscle, but also in all kinds of nonlinear waves in nature. Such phenomena have been investigated for specific systems, both experimentally and theoretically [7–10]. It should be pointed out however, that the phenomenon discussed in this work is of an entirely different type: wave reflections at a boundary between an oscillatory (limit cycle) medium, and an excitable one.

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