The possibilities of modeling neural networks in the framework of the thermodynamics of genetically disordered systems (glasses)

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Abstract. Non-spin glasses possess a number of specific features which, in structural and dynamic aspects, are close to conditions necessary for neural networks to function. In a disordered network there exists a plurality of structural parameters and a number of two-level states defined by double-well potentials. Their characteristics are specified by the conditions of glass formation, i.e. by genesis. The thermodynamic description of glass as a self-organizing system (that does not require introducing an interacting potential model) leads to an unambiguous conclusion that its frequency spectrum is predetermined by the structure, which is characterized by zero-point entropy. Glass is a natural system of oscillators which form a disordered network. In this sense, glass conforms to a known model of a disordered neural network formed by interconnected oscillators. If one assumes that in living organisms the structure of a neural network (the brain) is inherited according to a genetic mechanism, the quickness of learning and recognition of patterns, the stability of associative memory and other capabilities have to be inherited genetically. The more ordered a neural network formed by distinguishable neurons, the better its capabilities.

Key words: Neural networks, Associative memory, Brain functions, Disordered systems, Genetics, Synergetics, Self-organization, Vitreous state.

1. Introduction

The present work intends to encompass the modern theory of disordered systems (glasses), the internal dynamics self-organization and the theory of neural network functioning. As will be seen, there is a remarkable similarity between those groups of theories, although at first sight they describe classes of phenomena which are rather distant from each other and are related to subjects of different disciplines. Aware of the certain conditionality, or even impossibility of full identification of the subjects of the two models, I nevertheless believe it is useful to formulate consequences that may be obtained from such comparison at the level of the *analogy of statistical states*. For that purpose a minimum of acquaintance with the principles of neural networks and glass thermodynamics is necessary, as both of these two disciplines normally is not familiar to researchers engaged in one of them, while the readers of this work may be from both.

2. Some Principles of the Thermodynamic Description of Neural Network Functioning

Modeling of processes of image learning and recognition proceeds from the well-known structure of constituent units (parts) of brain and the data of biophysical investigations. The principal unit of brain structure is a neuron, which consists of the neuron body, dendrites and the axon. Axons make contacts (synapses) between dendrite endings, axons, or bodies of other neurons (Ref. e.g. [1, 2]). A neuron can exist in a passive (not excited) or in an active (excited) state and transmit its excitation to other neurons. Synaptic contacts regulate the transmission of a pulse, which has an electric nature, from one neuron to another by an electro-chemical mechanism. The process of image learning consists of a complex (co-operative) excitation of a system of neurons through corresponding synapses. This process is initiated by a certain stimulus (i.e. excitation) coming from information receptors – those of vision, hearing, etc. The stimulus represents a complicated system of excitations which encodes information in matrix form, so that each pattern assumes an individual reflection in the neural network. Learning leads to a stable system of network states which has an energy minimum with respect to the adjacent states. Recognition of an image is an action of a stimulus on the neural system in which some totality of Gestalts (or attractors) is recorded. The energy minimum of the system ‘stimulus – neural system’ is reached when the stimulus matches one of the images, on condition of their coherence. The process of reaching this state can be modeled as stochastic.

Application of glass thermodynamics to the problem of learning and recognition of images so far has been limited to the models of spin glasses (see e.g. [3]). The possibility of such an application is defined by accepted thermodynamic principles of neural network activity. Initially those principles were regarded just speculatively. From the early twenties on, the notion of stability of learning has been associated with concepts of energy minimum (see e.g. [4]). In 1982 Hopfield [5] suggested a thermodynamic description of processes in neural networks using the relation

$$L = -E = 1/2 \sum_{i \neq j} \sum T_{ij} V_i V_j. \quad (1)$$

L is the Lagrangian of the system. The energy of a system of neurons E is defined by the sum of the vectors V_i and V_j of the i th and j th neurons’ states (the total number of neurons is known) and of synaptic coefficients T_{ij} which correspond to the transmission of excitations. Naturally, direct transmission of excitation from a neuron back to itself is excluded ($i \neq j$), though transmission of an already transformed pulse along a returning loop in the neural network back to the initial neuron is possible. For simplicity, the value of V corresponding to the active state is encoded as a positive quantity (+1) and that corresponding to the passive state is encoded as zero.

The process of learning or recognition, i.e. the determination of an energy E minimum as dependent on V_i , V_j and T_{ij} , can be done either manually or by using a computer by the method of Metropolis et al. [6].¹ The whole system is regarded as being placed in a thermostat at a temperature T which corresponds to the level of noise in the neural network. Here we omit the questions of kinetics of these processes, which define the variation of V_i , V_j and T_{ij} with time, and the questions of the connections between T_{ij} values and the states of neurons. Those questions are quite comprehensively elucidated in the literature (see, for instance, Ref. [1]). Just note, that the relaxation of the states of a neuron and synapses is non-linear vs. time and the characteristic relaxation time of synaptic coefficients is larger than that of the states of neurons [7]. Even first attempts to determine the properties of the matrix of synaptic coefficients T_{ij} [5] have already shown that these coefficients are asymmetrical ($T_{ij} \neq T_{ji}$), or $T_{ji} = 0$ if $T_{ij} \neq 0$.

Successful application of the thermodynamics of spin glasses is owed to the undoubted identification of active and passive states of a neuron with the orientation of spins with respect to each other, to the existing bifurcation tree, to the evident association of degenerated states with spins' orientation in the vicinity of the one specified, and with other circumstances. The model of spin glasses is clarified by the procedure of step-by-step annealing, which leads to the minimization of the energy of initially non-equilibrium spin system at each temperature step while lowering. As it turns out, to the most stable state (i.e. the state of learning) corresponds a rarefied matrix of synaptic coefficients T_{ij} a substantial portion of which becomes nullified, and this matrix is ordered (see e.g. [8, 9]).

As we see, model presentations of glass physics applied to neural systems are used at the level of statistical analogues of states, but not at all in the literal sense. One cannot make a neural system from glass but one can understand (and describe) its functioning based on the vitreous state theory. In connection with that it is useful to cite Hopfield's words: 'any physical system whose dynamics in phase space is dominated by a substantial number of locally stable states to which it is attracted can therefore be regarded as a general content-addressable memory. The physical system will be a partially useful memory if, in addition, any prescribed set of states can readily be made the stable states of the system' [5].

The goal of this work is to demonstrate the appropriateness of the thermodynamics of normal (non-spin) glasses being applied to the modeling of neural networks.

3. A Neural Network as a System of Connected Oscillators

We can find the physical foundations for the application of solid state dynamics to neural network modeling in a number of works. As early as 1985 Stanley Rowlands [10], when citing the papers by Fröhlich (e.g. [11]) and by other authors on the theory of self-organization of low-frequency electrically active coherent excitations in biomolecules and noting experimental results, emphasized that 'the cell appears

less and less like a bag containing molecules dissolved in water, . . . and more and more like a solid structure Such a structure will impose a spatial order of sorts but gives no clue of how motion is ordered in relation to the structure'. In 1983 Yamada and Fujisaka [12] found conditions for the occurrence of stable (ordered) state dynamics in a model of a neural network formed from a system of connected oscillators. In 1991 Haken [13, p. 197] mathematically rigorously proved that, under the assumption that frequency-locking occurs, a network of oscillators showed the same behavior as that of a neural network which possessed associative memory.² It was also mentioned in these papers that the model of a neural network formed by oscillators not only could be a clue to constructing principally new computers (those having attention and the capability to perceive complex objects) but to creating as well the necessary theoretical basis for the foundation of such a principle of brain functioning as Ukhtomsky's dominant [14, 15].³

In a series of papers by Inoue et al. [17–20], simulating a neural network with associative memory, a model of self-organization of a disordered (chaotic) network of connected oscillators was used. The active state of a neuron is reflected by a certain degree of vibration phase synchrony. The presence of coefficients of oscillator interaction and phase distribution of vibrations allows us to introduce an energy threshold of excitation which discriminates active and passive states of the neurons. This threshold was also necessary in the preceding theories for the discrimination of information contrast at different signal-noise ratios and, consequently, for categorization.⁴ When the thermostat temperature T goes down, these two-level states are established with varying occupancy and the heat capacity of the system defined as $C = \partial \langle E \rangle / \partial T$ ($\langle E \rangle = (-\partial \ln Z / \partial \beta)$, where Z is the partition function and $\beta = 1/T$) has the form of a Schottky function:

$$C/k = \varepsilon / (kT)^2 [\exp(\varepsilon/kT)] / [1 + \exp(\varepsilon/kT)]^2,$$

where k is the Boltzmann constant and ε is the energy threshold that separates the states. In Inoue's theory, for the selection of states, as well as in the preceding theories, the Boltzmann distribution (the method of a Boltzmann machine) is applied, with the final state of the most stable learning corresponding to the solution in Hopfield's model. No annealing procedure is used in this method. To the final state of the network also corresponds the entropy minimum (the least possible entropy), which is established after the disordered states of the network has become ordered and the system has absorbed the information introduced into it. This state corresponds to the end of learning.

In [19, 20] models were considered which contained a small number of neurons. The Schottky function obtained describes the process of learning: the initial state of the system corresponds to the 'time' $T = \infty$ and the final one to $T = 0$. The real learning process starts not at $T = \infty$, but at a finite T value. At the very first moment of learning (temperature T) the state of the system acquires characteristics relating to that temperature. It was noted that, depending on the values of its coupling coefficients and synaptic coefficients, the model in general

can represent an analogue of the gaseous phase, the liquid, or the glass. In the latter case especially important is the theory of clusterization of attractors, i.e. states of minimal energy [21].

In connection with the solid-state analogy of neural systems, I should also mention here the possibility of quantum tunneling through synaptic contacts [22].

Using the concepts of statistical physics (e.g. the Boltzmann distribution), notions of energy (Hamiltonian) and entropy of a system undergoing learning, temperature, and informational entropy (in Shannon's sense) leave quite a lot of problems unsolved. For instance, Inoue et al. [19, 20] deny that Thermodynamic Laws (i.e. the Principles of Thermodynamics) directly hold as relations that are completely adequate for the theory of neural networks. There are grounds for that because so far the axiomatics of thermodynamics apparently cannot be equivalent to the axiomatic of the theory of neural systems. At the same time, the utilization of relations borrowed from thermodynamics in the theory of neural systems can be legitimate because the relations themselves may be of some wider sense as mathematical relations than their thermodynamic interpretation. The latter may appear to be special. In all probability, this is exactly what is taken into consideration in practical applications of thermodynamics to neural networks. We follow this practice further on.

4. General Grounds for the Comparison of Glasses and Neural Networks at the Level of Models

Non-spin glasses, i.e. those made from liquid, from a physical point of view represent natural disordered systems of connected oscillators (atoms or molecules). In this aspect it is rather tantalizing to analyze specific features of these natural systems, since model systems of the sort, as we have seen, are useful for studying neural systems.

At present the layout of low-energy excitations in glasses based on the use of physical research methods is known quite well (Ref. e.g. [23]). Thermodynamics gives a generalized description of energy and structural (i.e. related to entropy) changes which result in the vitreous state, while its genesis is, from a liquid [24]. Besides, it predicts not only the inevitability of anomalies in the internal dynamics of glasses compared to ordered systems (crystals) but also their quantitative character, which is defined by the genetically formed structure [25–28]. Models of neural systems are limited to small numbers of neurons whereas glass is a naturally disordered system of oscillators, the number of which is enormously large ($\sim 10^{22}$). In this view glass is closer to natural neural systems (the number of neurons in human brains is $\sim 10^{10}$). It should be noted also that in the theory of neural systems, for the condition of their functioning a sophisticated network topology and non-equivalence of the initial states of neurons are assumed [29–31]. This condition is also fulfilled in glasses in analogy.

5. The Thermodynamics of the Vitreous State and Parameters of Ordering

The vitreous state occurs by cooling a liquid as a result of freezing its structure at the glass transition temperature, T_g [24].⁵ This process is described by linear non-equilibrium thermodynamics as freezing of the relaxation of internal structural parameters ζ_i , the number of which may be large. The analysis of Prigogine-Defay Ratio values obtained from the increments of thermodynamic coefficients at T_g shows that in glasses the number of independent parameters is more than one. The parameters are not commuting, i.e., they affect the variations of the system's entropy and volume in different ways. In the general case parameters of order are tensors, but in model computations they may be coefficients defining non-linearity of the interaction forces (anharmonicity) [24].

The glass transition is not a phase transition of the 1st, nor of the 2nd order [24]. In this aspect the theory of the glass transition differs from the theory of spin glasses. However, in both cases, when the temperature goes down, disorder is fixed. In spin glasses the parameter of ordering is defined by the root-mean-square deviation of spin. In normal glasses, the parameter of ordering is defined by the non-equilibrium character (with respect to the internal equilibrium structure) of mutual orientation of molecules or bonds between atoms, i.e. by the spatial topological structure. It is the distinction of the latter structure from that reached at thermodynamic (metastable) equilibrium which defines the extent of deviation from equilibrium of the system by its energy and entropy.

Most simple thermodynamics of glass can be created if the state of crystal at 0 K is taken for the reference state.

A consequence of the non-equilibrium state of glass (g) as related to crystal (cr) is that the differences of the zero-point entropy, $\Delta S_0(g - cr)$, and the zero-point free energy (actually, enthalpy), $\Delta G_0(g - cr) = \Delta H_0(g - cr)$, of these states differ from zero. Thus, the adoption of the crystal state as a reference is legitimate. By force of the 3rd Law of Thermodynamics, both crystal and metastable liquid (msl) at $T = 0$ have to be indistinguishable by their entropy though may differ by their energy [24]. Both these values (energy and entropy) are interconnected by a universal theoretical relation which describes the genesis of the vitreous state at 0 K proceeding from the liquid state at the melting temperature T_m (Ref. Nemilov, 1981 [24]):

$$\frac{\Delta G_{0-T_m}(g, msl - cr)}{\Delta G_0(g - cr)} = (1 - \xi) \frac{\Delta H_{0-T_m}(g, msl - cr)}{\Delta H_0(g - cr)} - (1 - \xi) \frac{\Delta S_{0-T_m}(g, msl - cr)}{\Delta S_0}, \quad (2)$$

where ξ is the degree of structural equilibrium of the vitreous state,

$$\xi = 1 - (\Delta S_0 / \Delta S_m),$$

and ΔS_m is the melting entropy of the crystal (the maximum possible difference in the entropies of the liquid and the crystal states). The integrals in the numerators

of Equation (2), those like $\Delta G_{0-T_m}(g, msl - cr)$, are found as the total integrals of the free energy, the enthalpy and the entropy in the range from 0 to T_g (marked $g - cr$) and from T_g to T_m (marked $msl - cr$) and are calculated solely from the differences in the heat capacities of the glass and the crystal, $\Delta C_p(g - cr)$, in the range $0 - T_g$, and in those of the metastable liquid and the crystal, $\Delta C_p(msl - cr)$, in the range $T_g - T_m$.

Experimental data show that, with T_g lowering (slowed cooling or annealing), ΔS_0 and ΔG_0 decrease for every substance. This means that the structure of glass grows more and more ordered and approaches that of crystal though does not become the same [24].

If we denote the left-hand side of Equation (2) as $-\beta$ and the first member of the right-hand side as b , we have:

$$\beta + b = \xi. \quad (3)$$

It is rather noteworthy that the relation (3) holds for all glass types: inorganic glasses (oxide, chalcogenide, halide, etc.), organic molecular glasses (hydrocarbons, alcohols, etc.) and polymers of various nature at the same b value 0.27 ± 0.06 [24]. This indicates that the structures of Gibbs' phase space and of the Hamiltonian of glassforming liquids are the same. In the Hamiltonian, only the ratio of the numbers of quickly and slowly relaxing parameters ζ_i depends on the nature of the substance [27]. Their relaxational functions ϕ appear to be non-Debye ($\phi = \exp[-(t/\tau)^\beta]$, where t is the time and τ is the characteristic time; $0 < \beta \leq 1$) and different from each other. For instance, the entropy relaxes more slowly than the volume [24]. It is noteworthy that the existing periodicity of the network does not forbid the relations (2) and (3) holding. There is a class of material (glassy crystals; these are mainly molecular substances) that display periodicity of lattice points, e.g. there exists cubic lattice, but the molecules located in these points are oriented chaotically. Structures of the sort have a glass transition temperature. The glass transition is governed by the same laws as the glass transition of normal liquids. For these substances $b = 0.18 \pm 0.03$ [24].

It is clear that the thermodynamics of the vitreous state, which is expressed by the relation (2), generally speaking is common for all systems disregarding their molecular structure. It is just essential that the species that form the system differ in some parameters. This conclusion agrees with the theory of the 3rd Law of Thermodynamics, according to which the cause of the appearing non-zero zero-point entropy of the system in general is that the eigenfunctions of particles constituting the system differ from each other [32, p. 22, 25, 26, and others]. This conclusion is important for the purposes of the present work because, as viewed from the distribution of bonds between neurons, the brain is a rather ordered system.

It is important to note that the heat capacity C of any glass in the range from 0 to T_g differs from the heat capacity of the related crystal. There is a so-called 'excess heat capacity of glass' near the absolute temperature zero, where the Debye limit law, which is correct for crystals ($C \sim T^3$), does not hold (Figure 1).

Figure 1. Temperature variation of the difference between the heat capacities of glass and crystal $\Delta C_p(g-cr)$ (a) and the frequency distribution function $g(\omega)$ (b) for an annealed (1) and a quenched (2) glasses, $T_{g1} < T_{g2}$; ω_{max} is the maximal Debye frequency (schematic presentation).

6. Glass Structure and Specific Features of Internal Excitations; Comparison with Neural Networks

X-ray and neutron diffraction, spectroscopic, and acoustic studies of glasses produce the following picture [24]. At the glass transition a spatial network of chemical bonds is fixed, which is disordered as a whole. Depending on the degree of annealing of each material (on T_g position), the topology of the network (i.e. the correlation of prevailing structures – rings, chains, ways of return to the initial atom or molecule along the path of continuous bonding, etc.) alters – see Figure 2. Annealing results in increasing the degree of ordering. The latter apparently takes place not uniformly over the whole system but in a localized manner, though without a separate phase formation. Even in a one-component glass (i.e. in a glass formed by a single substance) there are domains different by their degree of order, their dimensions being of nanometer scale.

Only in glasses and glassforming liquids do quasilocalized collective low-energy excitations exist (the correlation radius is 1 to 2 nm and frequencies are $1/3 - 1/5$ of the Debye frequency). The density of their states depends on annealing (Fig-

Figure 2. Two-dimensional scheme of the structure of vitreous silica SiO_2 . 1 – silicon atoms, 2 – oxygen atoms. The fourth bond of the silicon atom is omitted. The number of links (1–2–1) in the rings varies. Annealing leads to clusterization of rings that differ in the number of links and to an increase in the number of rings comprising 6 links, which correspond to the crystalline structure.

Figure 3. A diagram of a double-well potential (for description see the text).

ure 1). They are reflected by a so-called boson peak in light scattering [24, Ref. also 33]. Model explanations of that phenomenon are reduced to the model of the double-well potential (Figure 3). At $T \rightarrow 0$ the excited states take place by tunneling; with rising temperature the transitions proceed above the barriers and the difference in the heat capacities of the glass and the crystal ($\Delta C_p(g - cr)$) is expressed by a Schottky-type function, which is typical for two-level systems. Because of the statistical character of structural parameters localized around the mean

values, apparently, there also exists a distribution of the parameters of double-well potentials. Acoustic experiments confirm the relaxational character of the changes in the occupation of states [34]. The potential barriers at the transition from an unexcited state to that excited and back ($\Delta g_{\rightarrow}^{\neq}$ and $\Delta g_{\leftarrow}^{\neq}$, see Figure 3), in general, may be numerous. Therefore, the life time of the excited states can vary in a wide range.

In the region of high temperatures, due to anharmonicity, the potential becomes ‘soft’ and the two minima merge into one wide minimum [35].

The above stated applies to collective excitations. At temperatures below T_g there are always non-cooperative relaxational transitions in glasses (β -relaxation) which are also described by double-well potentials. The number of such transitions and their characteristics depend essentially on the chemical type of the substance (see e.g. [36]). All relaxational transitions are united in a single common dynamic system [37]. It is important that the formation of all two-level states does not distort the topology of the glass network and the structure of interatomic bonds remains intact.

Localized low-energy excitations in glasses, as well as their topologically inhomogeneous structure itself can be successfully simulated by computer modeling (e.g. [38]).

Localized states, when the energy threshold ε as the energy difference of a two-level system is introduced (Figure 3), can be related in model terms to ‘active’ and ‘passive’ states of the neuron. In this case lowering T_g (i.e. ordering of structure) leads to an increase of the number of highly active states varying in time if ε is large, because the distribution function $g(\omega)$ corresponds to a greater probability of such states [24, 33], see Figure 1. For a neural system that would correspond to an increase of the capability to distinguish the useful signal against noise background – the capability of categorization. The presence of an energy distribution of potentials and, what is most probable, a tensor type of excitations in an analogous neural system may represent the basis of an image perception in a wide frequency range transmitted by receptors (e.g. leading to matching of an image perceived by shape and color).

It is known that a neuron most probably is capable of existing not in a single but in several states of excitation and this circumstance essentially increases informational capacity of a system [39]. Apparently, the nature of low-energy excitations in glasses corresponds as well to the mentioned feature of neural networks.

7. The Genetic Predetermination and Self-Organization of Dynamics in Glasses

Of most interest to us in the thermodynamics of glasses is the rigorous proof of the inevitability of self-organization of the internal dynamics (the distribution function $g(\omega)$) in accordance with the genetically present degree of structure disorder, i.e. according to the value of ΔS_0 . With that proof no model assumptions concerning

the structure and form of the potential are used [25–28]. The kernel is as follows. Glass and metastable liquid at the point of T_g are indistinguishable by their volume, entropy S , free energy G , and enthalpy H [24]. Therefore, the values of all thermodynamic functions at T_g calculated as differences with respect to the crystal are known numerically. At T_m , $\Delta G(msl - cr) = 0$, $\Delta H(msl - cr) = \Delta H_m$, $\Delta S(msl - cr) = \Delta S_m$, and the course of the heat capacities' variations of both liquid and crystal in the range $T_g - T_m$ is defined uniquely. Freezing of the structure at T_g makes integration constants $\Delta S_0(g - cr)$, $\Delta G_0(g - cr)$ appear in the calculated thermodynamic functionals of glasses. These correspond to the differences between the entropy and the energy of the states of glass and crystal at absolute zero (Equation (2)). Generally, their values can be found from independent (not thermodynamic) experiments, e.g. from structural and spectroscopic studies [24]. Obviously, the difference between the free energies of the glass and of the crystal at $T \leq T_g$ is defined by the relation

$$\Delta G_T(g - cr) = \Delta G_0(g - cr) + \Delta G_{0-T}(g - cr) - T \Delta S_0(g - cr), \quad (4)$$

where $\Delta G_{0-T}(g - cr)$ is the difference in the increments of the free energy of the glass and the crystal in the range from 0 to T . It is calculated only from the difference in the heat capacities of those states, $\Delta C_p(g - cr)$. The meaning of the relation (4) is that any state of either glass or crystal at the temperature T is counted from the original state at zero K . The non-equilibrium nature of the vitreous state (and the equilibrium nature of the crystal) is not violated. Thus, the integral $\Delta G_{0-T}(g - cr)$ defines the component ΔG_T , i.e. the difference between the free energies of the glass and the crystal in the specified temperature range. It is connected only with the difference in the dynamics of the internal (atomic or molecular) motions of the system. The very difference in the free energies of the states compared at T_g (ΔG_{T_g}) numerically is defined quite uniquely by the above circumstances.

According to the thermodynamics, at isothermal processes the free energy of any non-equilibrium system tends to a minimum. This corresponds to the principle of positive entropy production. In the thermodynamics of Gibbs that is achieved by the appearance of phases and redistribution of the components among the phases of the system. If the structure of the system is fixed, nothing in principle forbids the system to achieve its free energy minimum by another way, e.g. through a better organization of the internal motions spectrum (i.e. that the systems chooses the optimal $g(\omega)$ spectrum) considering the disorder existing in the system. On the other hand, the correlation between the shape of $g(\omega)$ spectrum and that of the heat capacity curve has been known since the fifties (Lifshitz, [40]).

Considering the above, it is clear that the functional $\Delta G_T(g - cr)$ in Equation (4) has to take the minimal possible value at every $T < T_g$. The boundary conditions of the problem are known; ΔS_0 in Equation (2) is known numerically, it characterizes the degree of the structure complexity; $\Delta S_{T_g-T_m}(msl - cr)$ is known

Figure 4. Illustrating the optimization of the functional $\Delta G_{0-T}(g-cr)$ (a) for the same value of $\Delta S_{0-T_g}(g-cr)$, $tg\alpha = -\Delta S_{0-T_g}(g-cr)$. Curves 1-3 correspond to curves 1-3 for $\Delta C_p(g-cr)$ (b). Only one of these curves (3) corresponds to the solution of isoperimetrical variation problem.

numerically from the genetic conditions of the state. Therefore, $\Delta S_{0-T_g}(g-cr)$ is also known numerically:

$$\Delta S_{0-T_g}(g-cr) = \Delta S_m - \Delta S_0 - \Delta S_{T_g-T_m}(msl-cr).$$

The problem of finding the optimum of $g(\omega)$ function is reduced to finding such a function $\Delta C_p(g-cr) = f(T)$ which would provide the minimal $\Delta G_{0-T}(g-cr)$ values at known boundary conditions ($\Delta C_p(g-cr) \rightarrow 0$ at $T \rightarrow 0$, which is the condition of the 3rd Law of Thermodynamics; $\Delta S_{T_g-T_m}$, ΔS_0 are known quantities) (Figure 4).

The statement of the problem and the way of its solution were determined in the papers [25–27] and a particular mathematical solution was given in the paper [28]. That was the solution to the isoperimetric problem posed for a quadratic functional that is a sum of the quadratic norms of the free energy and the entropy in the range

Figure 5. Excess heat capacity values for vitreous *o*-terphenyl (a, $T_g = 244 \text{ K}$) and selenium (b, $T_g = 304 \text{ K}$) as a function of the reduced temperature $\tau = T/T_g$. 1 – calculated as a solution to the variation problem; 2 – experimental.

0 – T_g using Hilbert's space metrics. With real substances taken as an example, the coincidence of $\Delta C_p(g - cr) = f(T)$ experimental and theoretical curves was illustrated (Figure 5) and also of $\Delta G_0(g - cr)$ values, which appear to be the minimal possible. Thus, it was proved that a disordered system self-organized the spectrum of the internal motions according to the structural disorder existing in it; that spectrum as compared to the Debye spectrum proved to be enriched with low-energy states.

8. Correlation Between the Theory of Systems Internal Dynamics Self-Organization and the Theory of Synergetics

The theory of self-organization of the frequency spectrum of glass is not a theory of synergetics. Synergetics regards self-organization of structure and of functioning of open systems, which exchange matter, energy and information with the surroundings [41]. At every moment of such self-organization the system possesses a certain internal structure which, provided the metabolism conditions are constant at the moment, assumes certain internal states. It is the dependence of the internal dynamics of the system on its internal structure formed at the moment that the elaborated theory expresses. Apparently, it should be considered as a subordinate theory in the general synergetic description of the system's evolution. There is no doubt that the suggested conclusion of the self-organization of the frequency spectrum is a particular one because it assumes only one artificially selected manner of the system order formation (glass transition). However, as in reference [25], I insist on the belief that the principle of self-organization of the dynamic spectrum of disordered systems can be applied to any sort of system including proteins, and it is only a lack of our knowledge that prevents us from generalizing this deduction.

We should remember a known work by Carnot of the early 19th century on the theory of heat machines as a result of which the 2nd Law of Thermodynamics was formulated. The latter holds for a much wider range of phenomena than the suggested primary, but far too particular model.

9. The Principle of Genetic Predetermination of Neural Network Functioning

As regards neural networks, the principle that I propose brings about an essentially new circumstance. If we assume a model of neural networks as a self-organizing chaotic net of interconnected oscillators, the functioning of the network must be defined by the fine structure (ordering) of the neural system. The brain represents an ordered system of neurons and synaptic contacts. However, for disorder to exist in such a system (as well as in the case of the mentioned glassy crystals), a mere non-equivalence of neural cells, axons and synaptic contacts is sufficient. This non-equivalence may lie in the difference of the size and the mutual orientation of the cells, of the composition of the cell liquid, and of the orientation of protein molecules, in the conformational distinctions of the latter, in their elastic stresses (Ref. e.g. [42]), and in other very fine features pertaining to structure.

The algorithm for the solution of the problem of the dynamic properties self-organization in glasses (i.e. the problem of finding $g(\omega)$ for a system at a specified structure) proved similar to that for learning patterns by a neural system with associative memory (the model of self-organization of disordered networks of connected oscillators [28]). Both models (for glass and for neural networks) are characterized by two-level states, their heat capacity is therefore defined by a Schottky-type function; at $T \rightarrow 0$ in the both models the minimal possible value of energy is attained. Glass has a natural structure which is governed by thermodynamics; the structure is able to vary just as the fine structure of a natural neural network can change depending on genetic prehistory.

In the theory of neural networks [19, 20] a selected temperature T corresponds to the beginning of the learning of the system and $T = 0$ to its finishing. In the model of glass, the system itself arises only at T_g ; at that it already has a quite well defined structure and states occupancy. Obviously, to the temperature T_g corresponds the onset of the learning of the system, which has already acquired some genetic memory in the course of its genesis.

For annealed glasses the glass transition T_g is lower than that for quenched, therefore, for such a model the interval $0 - T_g$ is narrower and the learning time (i.e. the time of information assimilating) for a genetically more ordered system appears to be less than for a less ordered system.

As we have seen, the values of threshold ε for annealed glass are higher than for quenched one, therefore, a genetically more ordered structure must distinguish the signal from the noise background more effectively and must be more capable of pattern categorization.

Finally, for a genetically more ordered neural system, the stability of image learning must be better than for a genetically less ordered: the ΔG_0 minimum grows deeper as the ordering of the disordered system increases. In Hopfield's model [5] that is reflected by a rarefied and more ordered matrix of synaptic coefficients T_{ij} . A higher ordered network is characterized by a more complete information perception as compared with a less ordered network (the former system is characterized by a lower value of entropy at $T = 0$).

If we call the whole set of the features of an associative memory, namely the quickness of learning, the stability of memory and the categorization, the capability of the system, then, for neural systems (with all the taken assumptions considered), a principle has to hold which is analogous to that already formulated by the author for information receptor systems [25–27]:

Differing functioning of one-type neural networks is only possible as long as they are disordered: such an ordered system as crystal cannot have a variable genesis. The functioning, other conditions equal, is strictly defined by the genetically formed structure of the neural network, by the degree of its order. The response of the neural system to the outer information, i.e. its capability is the better, the more ordered the neural network has been by the moment of the learning onset.

Basically, in living organisms the ordering is regulated obviously by the genetic mechanism. If this is considered correct, as well as the suggested model analogies, it should be recognized that the level of the listed capabilities of neural systems (brains) of living organisms is basically inherited genetically, i.e. is heritable under the governing of genetic laws.

The latter conclusion is especially interesting, as it conforms with practical biological and social observations. However, it should be taken into account that the limitation 'other conditions equal' is extremely vital. In reality, this corresponds to the rigid identity of not only the metabolism of the organism but of the social and other conditions too.

10. Concluding Remarks

The suggested principle of genetic predetermination of neural network functioning is just a consequence of the unification of results obtained in two fields of knowledge. On the one hand, we have the theory of neural networks functioning as disordered systems of connected oscillators. On the other hand, we have the thermodynamic, model-free theory of self-organization of the dynamic spectrum of disordered systems consisting of connected oscillators – the thermodynamics of glass. No additional assumptions of other kinds were made in this work. Nonetheless, I would rather regard the obtained result (the principle formulated) just as a hypothesis. Indeed, the possibility of adequate identification of the neural network with the system of connected oscillators looks obvious and legitimate only when the neural network is artificially constructed by man (as a worksheet model or as a technically constructed one). In this case the principle looks like an inevitability.

Regarding natural neural systems, still there are many hypotheses, based on analogies, from the physical point of view. The main circumstances have been already mentioned. Those are: the identification of the active states of the neuron and coherent excitations in a system of oscillators and the formal character of the application of thermodynamic relations to objects that perceive and treat information. In this situation nothing else is left but to accept the usefulness of such hypotheses, as the result looks very attractive and natural.

On the other hand, I consider it rather essential that a more detailed concept of ordering is introduced into the theory of neural networks than has been so far. A natural neural network has a certain, quite defined topology of a system of neurons and synaptic contacts. According to the elaborated theory, the distinctions in the functioning of already formed genetically and 'biologically correct' networks are bound to be conjugated with fine distinctions of such systems at the molecular and cell levels. Probably, these distinctions define the quickness of learning, the stability of associative memory and the standard of categorization. Probably, these properties (capabilities) can be genetically inherited. As they are defined by the structure, they cannot be cardinally altered in the very course of learning. The process of learning can only reveal and enhance the already existing available features but not change their combination, nor induce the generation of these properties. Besides that, the activity of real human brain is provided with a rather complex superposition of other mechanisms including neurochemical ones [43]. Neural cells in their activity display substantial plasticity, that is variability of parameters performed during time intervals which are longer than the duration of elementary reactions [44]. All these circumstances have not yet been taken into account in physical modeling.

A real brain is an extremely sophisticated hierarchically arranged system. The models under consideration can only claim an approximation to certain operational principles of some separate and rather limited parts of such a system. We are still very far (perhaps, fortunately) from modeling a real brain which grants us a sensation of boundless wealth and fascination of the Universe.

I should necessarily stress that the proposed theory, independent of whether it will be developed or remain just a particular hypothesis, should not be used as grounds for any humiliation of human dignity by racial, national, social, or other distinctions.

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Notes

- ¹ The method consists of a modified Monte-Carlo integration over configuration space; since the sixties it has been widely used for determination of the equations of state for substances at a specified potential of molecular interaction.
- ² Associative memory means recognition of images by incomplete information of them, which is arriving from the receptors.
- ³ The principle of dominant consists of the process of perception being a more or less stable domain of elevated excitability exists in the brain. Excitations that are newly arrived at the centers serve to enhance the excitement exactly in that domain whereas in the rest of the central neural system inhibition phenomena are widely spread [16, p. 73]. The possibility under discussion, to my mind, is correct on condition of hormone regulation being excluded.
- ⁴ Zero value of the threshold corresponds to the condition of epilepsy; infinitely large threshold corresponds to the condition of coma [15].
- ⁵ In that book many of the results stated here are generalized; the reader can find there all the references to the original papers.

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