Using Rigid Motion Constraints for the Registration of Free-Form Surfaces

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We present a new method for registration of free-form surfaces based on the iterative closest point (ICP) method and on geometric properties of reflected correspondence vectors. The method is based on computing relative gaps between reflected correspondences and between the projections of reflected correspondences along the rotation axis and using these to eliminate false matches. Experimental results on synthetic data and on real range images demonstrate that the method is robust and accurate for image registration with small motions.

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IMAGE REGISTRATION is a fundamental problem in medical image processing. Through image registration, the difference between images taken from different viewpoints at different instants and/or by different modalities can be detected and used to help diagnosis and surgery planning and execution, such as pin alignment and fixation in computer assisted orthopedic surgery.

Registration involves the three-dimensional (3D) alignment of unknown image correspondences. One of the most efficient methods to establish possible image correspondences is the iterative closest point (ICP) estimation algorithm.^{1,2} However, the algorithm also inevitably introduces a large number of false matches, since it uses only a distance constraint to define the position of the correspondences. Approaches to deal with the problem of false matches do exist, but they are mostly based on the use of invariants described in a single coordinate frame.³ In this report, we propose the use of rigid motion constraints described in different coordinate systems to deal with this problem, leading to a more accurate and robust image registration.

Our current research on the representation of

rigid body transformations based on distance, angle, and projection measurements⁴ has revealed a number of useful rigid motion constraints bridging the points described in two different coordinate frames before and after a rigid motion. All image correspondences must satisfy these motion constraints; if they do not, then they do not represent a rigid body transformation and, thus, are not image correspondences. In the method proposed here, we first use the traditional ICP criterion to establish a set of possible correspondences $(\mathbf{p}_i, \mathbf{p}_i'')$ ($i = 1, 2, \dots, n_1$) between images to be registered. Then the following constraints

$$\|p - e\| = \|p'' - e\|, \quad h^T(p + p'') = 2h^T e \quad (1)$$

are used to estimate the motion parameters essential point e and rotation axis h as:

$$\begin{pmatrix} (p_1 + p_1'')^T \\ (p_2 + p_2'')^T \\ (p_3 + p_3'')^T \end{pmatrix} e = \begin{pmatrix} \frac{p_1^T p_1 - p_1''^T p_1''}{2} \\ \frac{p_2^T p_2 - p_2''^T p_2''}{2} \\ \frac{p_3^T p_3 - p_3''^T p_3''}{2} \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} (p_1 + p_1'')^T \\ (p_2 + p_2'')^T \\ (p_3 + p_3'')^T \end{pmatrix} h = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (3)$$

where $(\mathbf{p}_i, \mathbf{p}_i'')$ ($i = 1, 2, 3$) are three arbitrary reflected correspondences. These are linear equations that can be robustly solved using the total least squares method.⁵

Normally, the number of points representing a surface in 3D space before and after the motion is very large, so for improved computational efficiency without loss of accuracy the Monte Carlo resampling technique is used. Sets of data points are sampled and the essential point e and rotation axis h are estimated for each set and the final estimated $\hat{e}$ and $\hat{h}$ are obtained by median filtering these solutions. The next step is the elimination of false matches which is described by the following procedure:

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Table 1. Experimental Results Based on Synthetic Data

Noise	Measure	$\hat{h}$ (%)	$\hat{\theta}$ (%)	$\hat{t}$ (%)	k
σ_1	μ	0.06	0.01	0.05	7.36
	σ	0.05	0.01	0.00	1.39
σ_2	μ	0.13	0.04	0.10	7.36
	σ	0.09	0.03	0.00	1.39

NOTE. The mean μ and standard deviation σ of relative calibration errors in percentage of rotation axis $\hat{h}$, rotation angle $\hat{\theta}$, and translation vector $\hat{t}$, and the number k of iterations using synthetic points data corrupted by zero mean Gaussian random noise with standard deviations $\sigma_1 = 0.03$ and $\sigma_2 = 0.06$.

1. Use Equation 1 to compute the relative gap $g1$ and the projection of any reflected correspondence along the rotation axis $\hat{h}$:

$$g1_i = \frac{\|p_i - \hat{e}\| - \|p_i'' - \hat{e}\|}{\max(\|p_i - \hat{e}\|, \|p_i'' - \hat{e}\|)}$$

$$proj_i = \hat{h}^T(p_i + p_i'')$$

Then, the gap between each projection and the one $proj_n/2$ obtained by a median filter can be computed as: $g2_i = |proj_i - proj_n/2|/|proj_n/2|$.

2. Compute the mean μ and standard deviation σ of the gaps $g1$ and $g2$:

$$\mu_{g1} = \frac{1}{n_1} \sum_{i=1}^{n_1} g1_i, \sigma_{g1} = \sqrt{\frac{1}{n_1} \sum_{i=1}^{n_1} (g1_i - \mu_{g1})^2}$$

$$\mu_{g2} = \frac{1}{n_1} \sum_{i=1}^{n_1} g2_i, \sigma_{g2} = \sqrt{\frac{1}{n_1} \sum_{i=1}^{n_1} (g2_i - \mu_{g2})^2}$$

3. Eliminate false matches: If $|g1_i - \mu_{g1}| > \kappa \sigma_{g1}$ or $|g2_i - \mu_{g2}| > \kappa \sigma_{g2}$, then the possible correspondence (p_i, p_i') is regarded as a false match.

As a result of this procedure, a set of refined correspondences is obtained. Finally, the quaternion method¹ is used for motion calibration based

Fig 1. The two range images used.

Fig 2. The evolution of registration for 200 randomly selected points with uniform distribution from the first image. Top: registration at iteration 1; middle: registration at iteration 8; bottom: after registration.

on the refined correspondences. The quaternion is initialized as $q = (\sqrt{99}/10, 0.1, 0, 0)^T$ and $t = \bar{p}' - \bar{p}$ where $\bar{p}'$ and $\bar{p}$ are the centroids of two range image points. Iteration is performed and the criterion as described in Zhang² is used as terminating condition.

In order to validate the proposed algorithm, a number of experiments based on both synthetic data and real range images have been conducted. In the experiments, the parameter κ to eliminate false matches is used as $\kappa = 1.7$. Experimental results based on synthetic data are presented in Table 1. The real range images (Fig 1) used here were downloaded from a publicly available range image database currently hosted by the Signal Analysis and Machine Perception Laboratory at Ohio State University. The size of the images are 200×200 and are made out of 6,870 and 6,874 points. The

Table 2. Experimental Results for Real Images

Points	$\bar{e}$	e_{min}	e_{max}	σ	n
Whole	0.52	0.02	5.25	0.57	5853
Selected	0.49	0.08	3.16	0.47	172

NOTE. The average $\bar{e}$, minimum e_{min} , and maximum e_{max} registration errors and the standard deviation σ , and the number n of finally established correspondences using real range images.

images were directly used in the experiments without any feature extraction and pre-processing and any knowledge about the distribution of points, occlusion, appearance and disappearance of points, and motion information. Experimental results for the real images are presented in Fig 2 and Table 2.

Experimental results based on both synthetic data and real images show that the proposed algorithm is accurate and robust. Comparing the pro-

posed algorithm with the existing ICP-based registration algorithms, it has the following advantages: (1) it can deal with occlusion, appearance, and disappearance of points; (2) it does not require the user to provide a threshold for the maximum tolerance for distance; and (3) it does not need to perform a statistical test in order to reject false matches. Thus, the proposed novel method represents a practical proposition for real applications in medical engineering.

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