The Skeleton Structure—An Improved Compression Algorithm With Perfect Reconstruction

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This article presents an improved morphological image representation that can be used for image compression, obtaining very high compression rates. The new image representation described in this work is called skeleton structure and is a natural extension of the morphologic structure. This article will present its theoretical background, introduce the new representation, and show some application examples.

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WE WILL PRESENT a new method for binary and grayscale image compression, based on mathematical morphology, which is a relatively new, and rapidly growing, nonlinear theory for image processing. For binary images mathematical morphology provides a well-founded theory for analysis and processing. Mathematical morphology involves the study of the different ways in which a structuring element interacts with a given set, modifies its shape, and extracts the resultant set. The new morphologic image representation presented here is the skeleton structure. The skeleton structure reduces the number of redundant points that exist in the structure representation. The original shape can be perfectly reconstructed.

MORPHOLOGIC OPERATORS—AN OVERVIEW

The skeleton and the structure are the main morphologic operators used for image compression. They can be calculated entirely using the fundamental morphologic operators.

The Morphologic Skeleton and the Morphologic Structure

In 1978, Lantuejoul¹ proved that the skeleton $S(X)$ of a topologically open shape X in Z^2 can be calculated by means of binary morphologic operations, in the following way:

$$S(X) = \bigcup_{n>0} S_n(X) \\ = \bigcup_{n>0} \left\{ X - nB - \bigcup_{\Delta n>0} [(X - nB) \ominus \Delta nB] \right\} \quad (1)$$

where nB , ΔnB , and B are, respectively, the discrete topologically open discs with radii, n , Δn , and 1. $S_i(X)$ represents the reunion of the centers

of the structuring elements with radius i . “ $-$ ” denotes the erosion operator and “ $\ominus$ ” denotes the opening operator. Figure 1 shows a binary image and its morphologic skeleton. The skeleton is obtained using a rhomb as structuring element. The compression rate for this example is 25. This means that the size of the resulting image is 25 times smaller than the size of the original image.

However, the reconstruction process needs additional information about the size of the structuring element for each point of the skeleton. This will reduce the compression rate to about 19. The biggest problem with the skeleton representation is the fact that it contains many redundant points. These points are not needed for reconstruction, but appear in the skeleton.

The morphologic structure $ST(X)$ of a topologically open shape X in Z^2 can be calculated by means of binary morphologic operations, in a recursive way:

- Initial conditions:

$$S(X) = X - nB - (X - nB) \ominus B \\ X^{(N)} = X \quad (2)$$

- The equations of recurrence:

$$X^{(n-1)} = X^{(n)} - ST_n \oplus nB \\ ST_{n-1} = X^{(n-1)} - nB - (X^{(n-1)} - nB) \ominus B \quad (3)$$

Reconstruction of the Image From Its Skeleton Representation

The original shape X can be perfectly reconstructed from its skeleton or structure representation in the following way:

$$X = \bigcup_{n \geq 0} S_n(X) \oplus nB \quad (4)$$

$$X = \bigcup_{n \geq 0} ST_n(X) \oplus nB \quad (5)$$

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THE SKELETON STRUCTURE

The method proposed here is based on the structure and skeleton representation. First, the structure of the binary image is computed. The structural element used for this operation must be an open disk, a square, or an elementary cross. The computation of the structure eliminates some of the redundant points from the original image. Still, the structure representation has a large number of redundant points that reduce the compression rate. Because of these redundant points, the elements of the structure are highly connected.

We can eliminate the redundant points that exist in the structure by computing its skeleton:

$$\begin{aligned}
 SST &= S(ST(X)) = \bigcup_{n>0} S_n(ST(X)) \\
 &= \bigcup_{n>0} \left\{ ST(X) - nL - \bigcup_{\Delta n>0} [(ST(X) - nL) \ominus \Delta nL] \right\}
 \end{aligned} \quad (6)$$

Since the skeleton is mainly composed of straight lines, the structuring elements used for the second skeleton computation will be horizontal, vertical, 45-degree inclined, and 35-degree inclined lines. In equation (6), L represents these structural elements used for skeleton structure

Fig 1. The original image (A) and its skeleton (B).

Fig 2. The skeleton structure.

computation. Figure 2 shows the skeleton structure of the image from Fig 1(A). It is very easy to see that the number of points in skeleton structure is a lot smaller than the number of points in the skeleton.

CONCLUSIONS

The skeleton structure is a new and improved method for removing the redundant points from the morphologic skeleton of a binary image. For the binary images used in our experiments, we have obtained compression rates up to 80 with perfect reconstruction. Because of these properties, the skeleton structure is recommended for image compression in areas where image quality is essential, where a degradation of the image due to the compression process is not accepted. The skeleton structure can be generalized grayscale images (such as x-rays).

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