Mean square stability of uncertain stochastic BAM neural networks with interval time-varying delays

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Abstract The robust asymptotic stability analysis for uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances is considered. By using the stochastic analysis approach, employing some free-weighting matrices and introducing an appropriate type of Lyapunov functional which takes into account the ranges for delays, some new stability criteria are established to guarantee the delayed BAM neural networks to be robustly asymptotically stable in the mean square. Unlike the most existing mean square stability conditions for BAM neural networks, the supplementary requirements that the time derivatives of time-varying delays must be smaller than 1 are released and the lower bounds of time varying delays are not restricted to be 0. Furthermore, in the proposed scheme, the stability conditions are delay-range-dependent and rate-dependent/independent. As a result, the new criteria are applicable to both fast and slow time-varying delays. Three numerical examples are given to illustrate the effectiveness of the proposed criteria.

Keywords Uncertain BAM neural networks · Stability · Stochastic disturbances · Interval time-varying delays · Linear matrix inequalities (LMIs)

Introduction

The bidirectional associative memory (BAM) neural networks were first introduced by Kosko and had been paid much attention in past decades due to its applications in many fields such as image and signal processing, pattern recognition, optimization and automatic control (Kosko 1987, 1988). Up to now, there are many application examples for BAM neural networks in real life. Emerging parallel/distributed architectures were explored for the digital VLSI implementation of adaptive bidirectional associative memory (BAM) neural network (Hasan and Siong 1997). Teddy and Ng (2011) proposed to use a novel local learning model of the pseudo self-evolving cerebellar model articulation controller (PSECMAC) associative memory network to produce accurate forecasts of ATM cash demands. Sudo et al. (2009) proposed a novel associative memory that operated in noisy environments and performed well in online incremental learning using self-organizing incremental neural network. Chang et al. (2008) presented a maximum-likelihood-criterion based bidirectional associative memory network which can be employed to evaluate the similarity between a template and a matching region. It is worth pointing out that such an application depends heavily on the stability of the equilibrium point of BAM neural networks. Therefore, it is important to study the stability of the equilibrium point of BAM neural networks. On the other hand, owing to the unavoidable finite switching speed of amplifiers in the electronic implementation of analog neural networks, time delays usually occur in the communication and response of neurons. However, time delay is a source of frequent instability and deteriorates system performance, and as a result, the stability analysis of time-delayed neural networks has received considerable attention over the past years (Arik 2005; Cao et al. 2007; Cao and Xiao 2007; Dong and Wang

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2011; Huang et al. 2008; Li et al. 2011; Liao and Wong 2003, 2004; Liao et al. 2002; Liu et al. 2006, 2010; Lou and Cui 2007b; Song and Cao 2007; Wan and Zhou 2007; Wang and Zhang 2011; Wang 2009; Wu et al. 2008; Zhang et al. 2010).

Besides time delays, in the applications and designs of networks, some unavoidable uncertainties which result from using an approximate system model for simplicity, parameter fluctuations, and data errors, etc., must be integrated into the system model. In recent year, the robust stability of delayed neural networks have been investigated by many researchers (Liao and Wong 2004; Lou and Cui 2006; Ozcan and Arik 2009; Park and Kwon 2008). On the other hand, in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters (Haykin 1994). Such time delays, parametric uncertainties and stochastic disturbances may significantly influence the overall properties of a dynamic system. Therefore, it is of practical importance to study the stochastic effects on the stability property of delayed neural networks. Some results related to this problem have been published, see, Blythe et al. 2001; Lou and Cui 2007a; Syed Ali and Balasubramaniam 2008; Shua et al. 2009; Wan and Zhou 2007; Wang et al. 1992; Zhang et al. 2010.

In Lou and Cui (2007a), by the approach combining the Lyapunov functional with linear matrix inequality, stochastic exponential stability for Markovian jumping BAM neural networks with time-varying delays was addressed. Syed Ali and Balasubramaniam (2008) studied the global asymptotic stability of uncertain stochastic fuzzy BAM neural networks with time-varying delays which are represented by the Takagi–Sugeno (TS) fuzzy models. Shua et al. (2009) investigated the global asymptotic stability analysis problem for a class of stochastic BAM networks with mixed time-delays and parameter uncertainties. In Wan and Zhou (2007), the stability property of stochastic hybrid BAM neural networks with discrete delays was considered. The delay-independent sufficient conditions to guarantee the exponential stability of the equilibrium solution for such networks were given by using the nonnegative semimartingale convergence theorem. In the recent literature, Bao and Cao (2012) investigated exponential stability for stochastic BAM neural networks with mixed delays. A sufficient condition is established by constructing suitable Lyapunov functional. In Yuan and Li (2010), the global robust stability for BAM neural networks with time-varying delays had been considered, and some less conservative conditions were obtained.

Recently, a special type of time delay in practical engineering systems, i.e., interval time-varying delay, was identified and investigated (He et al. 2007; Qiu et al. 2009; Rakkiyappan et al. 2008). Interval time-varying delay was a time delay that varies in an interval in which

the lower bound was not restricted to be 0. He et al. (2007) studied the stability analysis of neural networks with interval time-varying delay. By considering the relationship between the time-varying delay and its lower and upper bounds, some delay/interval-dependent stability criteria for neural networks with interval time-varying delay were proposed. In Qiu et al. (2009), the problem of robust stability of uncertain neural networks with interval time-varying delays was studied. Rakkiyappan et al. (2008) discussed stability analysis problem for uncertain stochastic neural networks with discrete interval and distributed time-varying delays. To the best of our knowledge, very few papers investigate the stability problem of uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances, which is still open.

Motivated by the above statement, in this paper, we deal with the robust stability of uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances. By choosing an appropriate Lyapunov functional, employing some free-weighting matrices and using the stochastic analysis approach, some new stability criteria are derived based on the consideration of the ranges for time-varying delays. Unlike the most existing mean square stability conditions for BAM neural networks, the supplementary requirements that the time derivatives of time-varying delays must be smaller than 1 are released and the lower bounds of time varying delays are not restricted to be 0. Furthermore, in the proposed scheme, the stability conditions are delay-range-dependent and rate-dependent/independent. As a result, the criteria are applicable to both fast and slow time-varying delays. Finally, three numerical examples are given to illustrate the effectiveness and the merit of the proposed method.

Notations The notations used throughout the paper are fairly standard. The superscript “ T ” stands for matrix transposition; R^n denotes the n -dimensional Euclidean space; the notation $A > 0$ means that A is real symmetric and positive definite; I and 0 represent identity matrix and zero matrix. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (it is right continuous and $\mathcal{F}_0$ contains all $\mathcal{P}$ -null sets); $\mathcal{L}_{\mathcal{F}_0}^p([-\tau, 0]; R^n)$ be the family of all bounded, $\mathcal{F}_0$ -measurable $\mathcal{C}([-\tau, 0]; R^n)$ -valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau \leq \theta \leq 0} E|\xi(\theta)|^p < \infty$. The mathematical expectation operator with respect to the given probability measure $\mathcal{P}$ is denoted by $E\{\cdot\}$. The shorthand $\text{diag}\{\cdot\cdot\cdot\}$ denotes the block diagonal matrix. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

Problem formulation

Consider the following uncertain BAM neural networks with interval time-varying delays:

$$\begin{cases} \frac{du_{1i}(t)}{dt} = -(a_{1i} + \Delta a_{1i})u_{1i}(t) + \sum_{j=1}^m (w_{1ji} + \Delta w_{1ji})\tilde{f}_{1j}(u_{2j}(t - \tau_{2j}(t))) + I_i, & i = 1, 2, \dots, n, \\ \frac{du_{2j}(t)}{dt} = -(a_{2j} + \Delta a_{2j})u_{2j}(t) + \sum_{i=1}^n (w_{2ij} + \Delta w_{2ij})\tilde{f}_{2i}(u_{1i}(t - \tau_{1i}(t))) + J_j, & j = 1, 2, \dots, m, \end{cases} \quad (1)$$

in which $u_{1i}(t)$ and $u_{2j}(t)$ are the activations of the i th neurons and the j th neurons, respectively; $\tilde{f}_{1j}(\cdot), \tilde{f}_{2i}(\cdot)$ denote the sig-

to simplify our proof, we shift the equilibrium point to the origin by the transformation $x_{1i}(t) = u_{1i}(t) - u_{1i}^*$, $x_{2j}(t) = u_{2j}(t) - u_{2j}^*$, $\tilde{f}_{2i}(x_{1i}(t)) = \tilde{f}_{2i}(x_{1i}(t) + u_{1i}^*) - \tilde{f}_{2i}(u_{1i}^*)$, $\tilde{f}_{1j}(u_{2j}(t)) = \tilde{f}_{1j}(u_{2j}(t) + u_{2j}^*) - \tilde{f}_{1j}(u_{2j}^*)$, which yields the following system

$$\begin{cases} \dot{x}_{1i}(t) = -(a_{1i} + \Delta a_{1i})x_{1i}(t) + \sum_{j=1}^m (w_{1ji} + \Delta w_{1ji})\tilde{f}_{1j}(x_{2j}(t - \tau_{2j}(t))), & i = 1, 2, \dots, n, \\ \dot{x}_{2j}(t) = -(a_{2j} + \Delta a_{2j})x_{2j}(t) + \sum_{i=1}^n (w_{2ij} + \Delta w_{2ij})\tilde{f}_{2i}(x_{1i}(t - \tau_{1i}(t))), & j = 1, 2, \dots, m. \end{cases} \quad (2)$$

nal functions of the j th neurons and the i th neurons, respectively; I_i, J_j denote the external inputs at time t ; a_{1i}, a_{2j} are positive numbers, and they denote the charging time constants or passive decay rate of the i th neurons and the j th neurons, respectively; w_{1ji}, w_{2ij} denote the synaptic connection weights; $\tau_{1i}(t), \tau_{2j}(t)$ are the time-varying delays in the state. Symbol Δ denotes the uncertain term.

The initial conditions associated with (1) are assumed to be

$$\begin{aligned} u_{1i}(s) &= \phi_{u_{1i}}(s), \quad t \in [-\bar{\tau}_1, 0], \quad i = 1, 2, \dots, n, \\ u_{2j}(s) &= \phi_{u_{2j}}(s), \quad t \in [-\bar{\tau}_2, 0], \quad j = 1, 2, \dots, m. \end{aligned}$$

Throughout this paper, we also assume that

Assumption 1 The neuron activation functions $\tilde{f}_{1j}(\cdot)$ and $\tilde{f}_{2i}(\cdot)$ in (1) are bounded and there exist positive numbers $l_j^{(1)} > 0$ and $l_i^{(2)} > 0$ such that

$$|\tilde{f}_{1j}(\xi_1) - \tilde{f}_{1j}(\xi_2)| \leq l_j^{(1)} |\xi_1 - \xi_2|,$$

$$|\tilde{f}_{2i}(\xi_1) - \tilde{f}_{2i}(\xi_2)| \leq l_i^{(2)} |\xi_1 - \xi_2|,$$

$$\forall \xi_1, \xi_2 \in R, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m,$$

Assume that $u_1^* = (u_{11}^*, u_{12}^*, \dots, u_{1n}^*)^T$, $u_2^* = (u_{21}^*, u_{22}^*, \dots, u_{2m}^*)^T$ are the equilibrium point of the system. In order

In compact matrix form, (2) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = -(A_1 + \Delta A_1)x_1(t) + (W_1 + \Delta W_1)f_1(x_2(t - \tau_2(t))), \\ \dot{x}_2(t) = -(A_2 + \Delta A_2)x_2(t) + (W_2 + \Delta W_2)f_2(x_1(t - \tau_1(t))), \end{cases} \quad (3)$$

where $x_1(t) = (x_{11}(t), x_{12}(t), \dots, x_{1n}(t))^T$, $x_2(t) = (x_{21}(t), x_{22}(t), \dots, x_{2m}(t))^T$, $A_1 = \text{diag}\{a_{11}, a_{12}, \dots, a_{1n}\}$, $A_2 = \text{diag}\{a_{21}, a_{22}, \dots, a_{2m}\}$, $W_1 = [(w_{1ji})_{m \times n}]^T$, $W_2 = [(w_{2ij})_{n \times m}]^T$, $f_1(x_2) = (f_{11}(x_2), f_{12}(x_2), \dots, f_{1m}(x_2))^T$, $f_2(x_1) = (f_{21}(x_1), f_{22}(x_1), \dots, f_{2n}(x_1))^T$, $\tau_1(t) = (\tau_{11}(t), \tau_{12}(t), \dots, \tau_{1n}(t))^T$, $\tau_2(t) = (\tau_{21}(t), \tau_{22}(t), \dots, \tau_{2m}(t))^T$.

Obviously, the activate functions satisfy the following properties:

$$\begin{aligned} f_1^T(x_2(t))f_1(x_2(t)) &\leq x_2^T(t)L_1^T L_1 x_2(t) \quad \text{and} \\ f_2^T(x_1(t))f_2(x_1(t)) &\leq x_1^T(t)L_2^T L_2 x_1(t), \end{aligned} \quad (4)$$

with $L_1 = \text{diag}\{l_1^{(1)}, l_2^{(1)}, \dots, l_m^{(1)}\}$ and $L_2 = \text{diag}\{l_1^{(2)}, l_2^{(2)}, \dots, l_n^{(2)}\}$.

In this section, we will consider the following uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances:

$$\begin{cases} dx_1(t) = [-(A_1 + \Delta A_1)x_1(t) + (W_1 + \Delta W_1)f_1(x_2(t - \tau_2(t)))]dt + [(C_1 + \Delta C_1)x_1(t) + (D_1 + \Delta D_1)x_2(t - \tau_2(t))]d\omega(t), \\ dx_2(t) = [-(A_2 + \Delta A_2)x_2(t) + (W_2 + \Delta W_2)f_2(x_1(t - \tau_1(t)))]dt + [(C_2 + \Delta C_2)x_2(t) + (D_2 + \Delta D_2)x_1(t - \tau_1(t))]d\omega(t), \end{cases} \quad (5)$$

where $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_l(t))^T$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$.

Assumption 2 The time-varying delays $\tau_1(t)$ and $\tau_2(t)$ satisfy

$$0 \leq \underline{\tau}_1 \leq \tau_1(t) \leq \bar{\tau}_1, \quad 0 \leq \underline{\tau}_2 \leq \tau_2(t) \leq \bar{\tau}_2, \quad (6)$$

$$\dot{\tau}_1(t) \leq \mu_1, \quad \dot{\tau}_2(t) \leq \mu_2, \quad (7)$$

where $0 \leq \underline{\tau}_1 < \bar{\tau}_1$, $0 \leq \underline{\tau}_2 < \bar{\tau}_2$, μ_1 and μ_2 are positive constants.

Assumption 3 The parameter uncertainties ΔA_i , ΔW_i , ΔC_i and ΔD_i , $i = 1, 2$, are of the form:

$$\begin{aligned} \Delta A_1 &= H_1 F_1(t) E_1, & \Delta W_1 &= H_2 F_2(t) E_2, \\ \Delta A_2 &= H_3 F_3(t) E_3, & \Delta W_2 &= H_4 F_4(t) E_4, \\ \Delta C_1 &= H_5 F_5(t) E_5, & \Delta D_1 &= H_6 F_6(t) E_6, \\ \Delta C_2 &= H_7 F_7(t) E_7, & \Delta D_2 &= H_8 F_8(t) E_8, \end{aligned} \quad (8)$$

in which H_j , E_j , $j = 1, 2, \dots, 8$ are known constant matrices with appropriate dimensions. The uncertain matrices $F_j(t)$, $j = 1, 2, \dots, 8$, satisfy

$$F_j^T(t) F_j(t) \leq I \quad \text{for } \forall t \in R. \quad (9)$$

Now, we give the following lemmas and fact that are useful in deriving our LMI-based stability criteria.

Lemma 1 (Wang et al. 1992) Let D and N be real constant matrices of appropriate dimensions, matrix $F(t)$ satisfies $F^T(t)F(t) \leq I$. Then we have (i) For any $\varepsilon > 0$, $DF(t)N + N^T F^T(t)D^T \leq \varepsilon^{-1}DD^T + \varepsilon N^T N$. (ii) For any $P > 0$, $2a^T b \leq a^T P^{-1}a + b^T P b$.

Lemma 2 (Boyd et al. 1994) [Schur complement] Given constant symmetric matrices $\sum_1$, $\sum_2$ and $\sum_3$, where $\sum_1 = \sum_1^T$ and $0 < \sum_2 = \sum_2^T$, then $\sum_1 + \sum_3^T \sum_2^{-1} \sum_3 < 0$ if and only if

$$\begin{bmatrix} \sum_1 & \sum_3^T \\ \sum_3 & -\sum_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\sum_2 & \sum_3 \\ \sum_3^T & \sum_1 \end{bmatrix} < 0.$$

Let $\mathcal{C}^{1,2,2}(R_+ \times R^n \times R^n : R_+)$ be denote the family of all non-negative functions $V(x(t), y(t), t) \in R_+ \times R^n \times R^n$ which are continuously twice differentiable in x, y and once differentiable in t .

Fact 1 (Khasminski 1980; Syed Ali and Balasubramaniam 2008) The trivial solution of a stochastic differential equation

$$d(x(t), y(t), t) = G(x(t), y(t), t)dt + H(x(t), y(t), t)d\omega(t) \\ \text{on } t \in [t_0, T]$$

with $x(t) = \phi_x(t)$ for every $t \in [-\tau, 0]$, $y(t) = \phi_y(t)$ for every $t \in [-\rho, 0]$, $G : R_+ \times R^n \times R^n \rightarrow R^n$ and $H : R_+ \times R^n \times R^n \rightarrow \mathbb{R}^{n \times m}$ is globally asymptotically stable in probability if there exist a function $V(x(t), y(t), t) \in R_+ \times R^n \times R^n$ which is positive definite in the Lyapunov sense and satisfies

$$\begin{aligned} \mathcal{L}V(x(t), y(t), t) &= \frac{\partial V}{\partial t} + \text{grad}(V)G \\ &\quad + \frac{1}{2} \text{trace}(HH^T) \text{Hess}(V) < 0. \end{aligned}$$

The matrix $\text{Hess}(V)$ is the Hessian matrix of second-order partial derivatives.

Main results

First of all, denote

$$\begin{aligned} g_1(t) &= -(A_1 + \Delta A_1)x_1(t) + (W_1 + \Delta W_1)f_1(x_2(t - \tau_2(t))), \\ g_2(t) &= -(A_2 + \Delta A_2)x_2(t) + (W_2 + \Delta W_2)f_2(x_1(t - \tau_1(t))), \\ g_3(t) &= (C_1 + \Delta C_1)x_1(t) + (D_1 + \Delta D_1)x_2(t - \tau_2(t)), \\ g_4(t) &= (C_2 + \Delta C_2)x_2(t) + (D_2 + \Delta D_2)x_1(t - \tau_1(t)), \end{aligned}$$

then, system (5) can be written

$$\begin{cases} dx_1(t) = g_1(t)dt + g_3(t)d\omega(t), \\ dx_2(t) = g_2(t)dt + g_4(t)d\omega(t). \end{cases} \quad (10)$$

In this section, the stability analysis of uncertain BAM neural networks with interval time-varying delays and stochastic disturbances described by (10) is carried out. Our first result in this paper deals with the stability of (10) with $\Delta A_i = 0$, $\Delta W_i = 0$, $\Delta C_i = 0$ and $\Delta D_i = 0$, $i = 1, 2$. For this case, the following theorem holds.

Theorem 1 For given scalars $0 \leq \underline{\tau}_1 < \bar{\tau}_1$, $0 \leq \underline{\tau}_2 < \bar{\tau}_2$, μ_1 and μ_2 , the system (10) is globally asymptotically stable in the mean square, if there exist matrices $P_i > 0$, $Q_i \geq 0$, $R_i \geq 0$, $T_i \geq 0$, $i = 1, 2$, $Z_j > 0$, $j = 1, 2, \dots, 8$, $N_j^{(i)}, M_j^{(i)}, S_j^{(i)}$, $j = 1, 2$, $i = 1, 2$, and two positive scalars α_1, α_2 , such that the following LMI (11) holds:

$$\Xi = \begin{bmatrix} \Xi_0 & \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 \\ * & -\Xi_{11} & 0 & 0 & 0 \\ * & * & -\Xi_{22} & 0 & 0 \\ * & * & * & -\Xi_{33} & 0 \\ * & * & * & * & -\Xi_{44} \end{bmatrix} < 0, \quad (11)$$

with

$$\begin{aligned} \Upsilon_2 &= [-A_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad W_1 \quad 0 \quad 0 \quad 0 \quad 0], \\ \Upsilon_3 &= [0 \quad -A_2 \quad 0 \quad 0 \quad W_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \end{aligned}$$

$$\Xi_0 = \begin{bmatrix} \Upsilon_1 & \Upsilon_2^T U_1 & \Upsilon_3^T U_2 & \Upsilon_4^T U_3 & \Upsilon_5^T U_4 \\ * & -U_1 & 0 & 0 & 0 \\ * & * & -U_2 & 0 & 0 \\ * & * & * & -U_3 & 0 \\ * & * & * & * & -U_4 \end{bmatrix}, \quad \Xi_i = \begin{bmatrix} \bar{\tau}_i N_1^{(i)} & h_i M_1^{(i)} & h_i S_1^{(i)} \\ \bar{\tau}_i N_2^{(i)} & h_i M_2^{(i)} & h_i S_2^{(i)} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad \Xi_{2+i} = \begin{bmatrix} N_1^{(i)} & M_1^{(i)} & S_1^{(i)} \\ N_2^{(i)} & M_2^{(i)} & S_2^{(i)} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} \Sigma_1 & 0 & \Sigma_5 & 0 & 0 & P_1 W_1 & M_1^{(1)} & -S_1^{(1)} & 0 & 0 \\ * & \Sigma_2 & 0 & \Sigma_6 & P_2 W_2 & 0 & 0 & 0 & M_1^{(2)} & -S_1^{(2)} \\ * & * & \Sigma_3 & 0 & 0 & 0 & M_2^{(1)} & -S_2^{(1)} & 0 & 0 \\ * & * & * & \Sigma_4 & 0 & 0 & 0 & 0 & M_2^{(2)} & -S_2^{(2)} \\ * & * & * & * & -\alpha_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\alpha_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -Q_2 & 0 \\ * & * & * & * & * & * & * & * & * & -R_2 \end{bmatrix},$$

$$\Upsilon_4 = [C_1 \ 0 \ 0 \ D_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0],$$

$$\Upsilon_5 = [0 \ C_2 \ D_2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0],$$

$$U_1 = \bar{\tau}_1 Z_1 + h_1 Z_3, \quad U_2 = \bar{\tau}_2 Z_2 + h_2 Z_4,$$

$$U_3 = P_1 + \bar{\tau}_1 Z_5 + h_1 Z_7, \quad U_4 = P_2 + \bar{\tau}_2 Z_6 + h_2 Z_8,$$

$$\Xi_{11} = \text{diag}\{\bar{\tau}_1 Z_1, h_1 Z_3, h_1(Z_1 + Z_3)\},$$

$$\Xi_{22} = \text{diag}\{\bar{\tau}_2 Z_2, h_2 Z_4, h_2(Z_2 + Z_4)\},$$

$$\Xi_{33} = \text{diag}\{Z_5, Z_7, Z_5 + Z_7\},$$

$$\Xi_{44} = \text{diag}\{Z_6, Z_8, Z_6 + Z_8\},$$

$$\Sigma_i = -P_i A_i - A_i^T P_i + Q_i + R_i + T_i + N_1^{(i)} + \left(N_1^{(i)}\right)^T,$$

$$\Sigma_{2+i} = -(1 - \mu_i)T_i + S_2^{(i)} + \left(S_2^{(i)}\right)^T - N_2^{(i)} - \left(N_2^{(i)}\right)^T$$

$$- M_2^{(i)} - \left(M_2^{(i)}\right)^T + \alpha_i L_{3-i}^T L_{3-i},$$

$$\Sigma_{4+i} = S_1^{(i)} - N_1^{(i)} - M_1^{(i)} + \left(N_2^{(i)}\right)^T,$$

$$h_i = \bar{\tau}_i - \underline{\tau}_i, \quad i = 1, 2.$$

Proof The Lyapunov functional of system (10) is defined by:

$$V(x_1(t), x_2(t)) = \sum_{i=1}^2 [V_1(x_1(t), x_2(t)) + V_2(x_1(t), x_2(t))$$

$$+ V_3(x_1(t), x_2(t))], \quad V_1(x_1(t), x_2(t)) = x_i^T(t) P_i x_i(t),$$

$$V_2(x_1(t), x_2(t)) = \int_{t-\underline{\tau}_i}^t x_i^T(s) Q_i x_i(s) ds + \int_{t-\bar{\tau}_i}^t x_i^T(s) R_i x_i(s) ds$$

$$+ \int_{t-\tau_i(t)}^t x_i^T(s) T_i x_i(s) ds,$$

$$V_3(x_1(t), x_2(t)) = \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t g_i^T(s) Z_i g_i(s) ds d\theta$$

$$+ \int_{-\bar{\tau}_i}^{\underline{\tau}_i} \int_{t+\theta}^t g_i^T(s) Z_{2+i} g_i(s) ds d\theta$$

$$+ \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t g_{2+i}^T(s) Z_{4+i} g_{2+i}(s) ds d\theta$$

$$+ \int_{-\bar{\tau}_i}^{\underline{\tau}_i} \int_{t+\theta}^t g_{2+i}^T(s) Z_{6+i} g_{2+i}(s) ds d\theta. \quad (12)$$

From the Leibniz-Newton formula, the following equalities are true for any matrices $N_j^{(i)}, M_j^{(i)}, S_j^{(i)}$, $i = 1, 2, j = 1, 2$, with appropriate dimensions:

$$0 = 2 \left[x_i^T(t) N_1^{(i)} + x_i^T(t - \tau_i(t)) N_2^{(i)} \right]$$

$$\times \left[x_i(t) - x_i(t - \tau_i(t)) - \int_{t-\tau_i(t)}^t g_i(s) ds \right. \quad (13)$$

$$\left. - \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) \right],$$

$$0 = 2 \left[x_i^T(t) M_1^{(i)} + x_i^T(t - \tau_i(t)) M_2^{(i)} \right] \\ \times \left[x_i(t - \underline{\tau}_i) - x_i(t - \tau_i(t)) - \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_i(s) ds \right. \\ \left. - \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s) \right], \quad (14)$$

$$0 = 2 \left[x_i^T(t) S_1^{(i)} + x_i^T(t - \tau_i(t)) S_2^{(i)} \right] \\ \times \left[x_i(t - \tau_i(t)) - x_i(t - \bar{\tau}_i) - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_i(s) ds \right. \\ \left. - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s) \right]. \quad (15)$$

By Lemma 1(ii), for any matrices $Z_j \geq 0$, $j = 1, 2, \dots, 8$, the following equalities hold:

$$-2\zeta^T(t) N^{(i)} \int_{t-\tau_i(t)}^t g_i(s) ds \leq \bar{\tau}_i \zeta^T(t) N^{(i)} Z_i^{-1} (N^{(i)})^T \zeta(t) \\ + \int_{t-\tau_i(t)}^t g_i^T(s) Z_i g_i(s) ds, \quad (16)$$

$$-2\zeta^T(t) M^{(i)} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_i(s) ds \leq h_i \zeta^T(t) M^{(i)} Z_{2+i}^{-1} (M^{(i)})^T \zeta(t) \\ + \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_i^T(s) Z_{2+i} g_i(s) ds, \quad (17)$$

$$-2\zeta^T(t) S^{(i)} \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_i(s) ds \leq h_i \zeta^T(t) S^{(i)} (Z_i + Z_{2+i})^{-1} \\ \times (S^{(i)})^T \zeta(t) + \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_i^T(s) (Z_i + Z_{2+i}) g_i(s) ds, \quad (18)$$

$$-2\zeta^T(t) N^{(i)} \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) \leq \zeta^T(t) N^{(i)} Z_{4+i}^{-1} (N^{(i)})^T \zeta(t) \\ + \int_{t-\tau_i(t)}^t g_{2+i}^T(s) d\omega(s) Z_{4+i} \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) \quad (19)$$

$$-2\zeta^T(t) M^{(i)} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s) \leq \zeta^T(t) M^{(i)} Z_{6+i}^{-1} (M^{(i)})^T \\ \times \zeta(t) + \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) d\omega(s) Z_{6+i} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s), \quad (20)$$

$$-2\zeta^T(t) S^{(i)} \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s) \leq \zeta^T(t) S^{(i)} (Z_{4+i} + Z_{6+i})^{-1} \\ \times (S^{(i)})^T \zeta(t) + \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}^T(s) d\omega(s) (Z_{4+i} + Z_{6+i}) \\ \times \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s), \quad (21)$$

with $N^{(i)} = \begin{bmatrix} (N_1^{(i)})^T & (N_2^{(i)})^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $M^{(i)} = \begin{bmatrix} (M_1^{(i)})^T & (M_2^{(i)})^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $S^{(i)} = \begin{bmatrix} (S_1^{(i)})^T & (S_2^{(i)})^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

It is obvious from (4) that

$$f_i^T(x_{3-i}(t - \tau_{3-i}(t))) f_i(x_{3-i}(t - \tau_{3-i}(t))) \leq x_{3-i}^T(t - \tau_{3-i}(t)) L_i^T L_i x_{3-i}(t - \tau_{3-i}(t)), \\ i = 1, 2. \quad (22)$$

Calculating the $\mathcal{LV}$ along the trajectories of the system (10), then we have:

$$\mathcal{LV}(x_1(t), x_2(t)) = \sum_{i=1}^2 \left[\mathcal{LV}_1(x_1(t), x_2(t)) \right. \\ \left. + \mathcal{LV}_2(x_1(t), x_2(t)) + \mathcal{LV}_3(x_1(t), x_2(t)) \right],$$

$$\mathcal{LV}_1(x_1(t), x_2(t)) = 2x_i^T(t) P_i [-A_i x_i(t) \\ + W f_i(x_{3-i}(t - \tau_{3-i}(t)))] + g_{2+i}^T(t) P_i g_{2+i}(t), \quad (23)$$

$$\mathcal{LV}_2(x_1(t), x_2(t)) \leq x_i^T(t) (Q_i + R_i) x_i(t) - x_i^T(t - \underline{\tau}_i) \\ \times Q_i x_i(t - \underline{\tau}_i) - x_i^T(t - \bar{\tau}_i) R_i x_i(t - \bar{\tau}_i) \\ + x_i^T(t) T_i x_i(t) - (1 - \mu_i) x_i^T(t - \tau_i(t)) \\ \times T_i x_i(t - \tau_i(t)), \quad (24)$$

$$\begin{aligned}
\mathcal{L}V_3(x_1(t), x_2(t)) &= g_i^T(t)(\bar{\tau}_i Z_i + h_i Z_{2+i})g_i(t) - \int_{t-\bar{\tau}_i}^t g_i^T(s)Z_i g_i(s)ds - \int_{t-\bar{\tau}_i}^{t-\underline{\tau}_i} g_i^T(s)Z_{2+i}g_i(s)ds \\
&\quad + g_{2+i}^T(t)(\bar{\tau}_i Z_{4+i} + h_i Z_{6+i})g_{2+i}(t) - \int_{t-\bar{\tau}_i}^t g_{2+i}^T(s)Z_{4+i}g_{2+i}(s)ds - \int_{t-\bar{\tau}_i}^{t-\underline{\tau}_i} g_{2+i}^T(s)Z_{6+i}g_{2+i}(s)ds \\
&= g_i^T(t)(\bar{\tau}_i Z_i + h_i Z_{2+i})g_i(t) + g_{2+i}^T(t)(\bar{\tau}_i Z_{4+i} + h_i Z_{6+i})g_{2+i}(t) \\
&\quad - \int_{t-\tau_i(t)}^t g_i^T(s)Z_i g_i(s)ds - \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_i^T(s)Z_{2+i}g_i(s)ds - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_i^T(s)(Z_i + Z_{2+i})g_i(s)ds \\
&\quad - \int_{t-\tau_i(t)}^t g_{2+i}^T(s)Z_{4+i}g_{2+i}(s)ds - \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s)Z_{6+i}g_{2+i}(s)ds - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}^T(s)(Z_{4+i} + Z_{6+i})g_{2+i}(s)ds.
\end{aligned} \tag{25}$$

$$\begin{aligned}
\mathcal{L}V(x_1(t), x_2(t)) &\leq \xi^T(t) \left\{ \Upsilon_1 + \Upsilon_2^T U_1 \Upsilon_2 + \Upsilon_3^T U_2 \Upsilon_3 + \Upsilon_4^T U_3 \Upsilon_4 + \Upsilon_5^T U_4 \Upsilon_5 + \sum_{i=1}^2 \left[\bar{\tau}_i N^{(i)} Z_i^{-1} \left(N^{(i)} \right)^T + h_i M^{(i)} Z_{2+i}^{-1} \left(M^{(i)} \right)^T \right. \right. \\
&\quad \left. \left. + h_i S^{(i)} (Z_i + Z_{2+i})^{-1} \left(S^{(i)} \right)^T + N^{(i)} Z_{4+i}^{-1} \left(N^{(i)} \right)^T + M^{(i)} Z_{6+i}^{-1} \left(M^{(i)} \right)^T + S^{(i)} (Z_{4+i} + Z_{6+i})^{-1} \left(S^{(i)} \right)^T \right] \right\} \xi(t) \\
&\quad + \int_{t-\tau_i(t)}^t g_{2+i}^T(s) d\omega(s) Z_{4+i} \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) + \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) d\omega(s) Z_{6+i} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s) \\
&\quad + \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}^T(s) d\omega(s) (Z_{4+i} + Z_{6+i}) \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s) - \int_{t-\tau_i(t)}^t g_{2+i}^T(s) Z_{4+i} g_{2+i}(s) ds \\
&\quad - \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) Z_{6+i} g_{2+i}(s) ds - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}^T(s) (Z_{4+i} + Z_{6+i}) g_{2+i}(s) ds,
\end{aligned} \tag{26}$$

Combining (13)–(25), we have

where $\xi(t) = [x_1^T(t)x_2^T(t)x_1^T(t-\tau_1(t))x_2^T(t-\tau_2(t))f_2^T(x_1(t-\tau_1(t)))f_1^T(x_2(t-\tau_2(t)))x_1^T(t-\underline{\tau}_1)x_1^T(t-\bar{\tau}_1)x_2^T(t-\underline{\tau}_2)x_2^T(t-\bar{\tau}_2)]^T$.

Using

$$\begin{aligned}
&\mathbb{E} \left\{ \int_{t-\tau_i(t)}^t g_{2+i}^T(s) d\omega(s) Z_{4+i} \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) \right\} \\
&= \mathbb{E} \left\{ \int_{t-\tau_i(t)}^t g_{2+i}^T(s) Z_{4+i} g_{2+i}(s) ds \right\},
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E} \left\{ \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) d\omega(s) Z_{6+i} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s) \right\} \\
&= \mathbb{E} \left(\int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) Z_{6+i} g_{2+i}(s) ds \right),
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E} \left\{ \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}^T(s) d\omega(s) (Z_{4+i} + Z_{6+i}) \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s) \right\} \\
&= \mathbb{E} \left\{ \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}^T(s) (Z_{4+i} + Z_{6+i}) g_{2+i}(s) ds \right\},
\end{aligned}$$

it follows from

$$\begin{aligned} \Xi = & \Upsilon_1 + \Upsilon_2^T U_1 \Upsilon_2 + \Upsilon_3^T U_2 \Upsilon_3 + \Upsilon_4^T U_3 \Upsilon_4 + \Upsilon_5^T U_4 \Upsilon_5 \\ & + \sum_{i=1}^2 \left[\bar{\tau}_i N^{(i)} Z_i^{-1} \left(N^{(i)} \right)^T + h_i M^{(i)} Z_{2+i}^{-1} \left(M^{(i)} \right)^T \right. \\ & + h_i S^{(i)} (Z_i + Z_{2+i})^{-1} \left(S^{(i)} \right)^T + N^{(i)} Z_{4+i}^{-1} \left(N^{(i)} \right)^T \\ & \left. + M^{(i)} Z_{6+i}^{-1} \left(M^{(i)} \right)^T + S^{(i)} (Z_{4+i} + Z_{6+i})^{-1} \left(S^{(i)} \right)^T \right] < 0, \end{aligned}$$

that

$$\mathbb{E}[dV(x_1(t), x_2(t))] = \mathbb{E}[\mathcal{L}V(x_1(t), x_2(t))dt] < 0$$

for all $x_1(t), x_2(t)$ except for $x_1(t) = x_2(t) = 0$.

By Lemma 2, it is easy to see that (11) is equivalent to $\Xi < 0$. It follows from Lyapunov stability theorem that the system (10) with both interval time-varying delays and stochastic disturbances is globally, stochastically, asymptotically stable in the mean square. This completes the proof of Theorem 1.

Remark 1 In Theorem 1, the stability criterion is delay-range-dependent and rate-dependent, i.e., Theorem 1 is dependent on the upper and lower bounds of the delays and includes the information of the derivatives of time-varying delay functions. It is noteworthy that the ranges for time-varying delays considered in (Liu et al. 2006; Lou and Cui 2006; Park and Kwon 2008; Syed Ali and Balasubramaniam 2008) are from 0 to an upper bound. However, in practice, an interval time-varying delay is often encountered, that is, a time delay varies in an interval in which the lower bound is not restricted to be 0. In this case, the criteria in the previous work (Liu et al. 2006; Lou and Cui 2006; Park and Kwon 2008; Syed Ali and Balasubramaniam 2008) may have limited use because they do not take into account the information of the lower bounds of delays.

Remark 2 The supplementary requirements of $\dot{\tau}_i(t) < 1$, $i = 1, 2$, are imposed in (Liu et al. 2006; Lou and Cui 2006; Park and Kwon 2008; Syed Ali and Balasubramaniam 2008) which means that the criteria only allows the time-delays to be slowly time-varying functions. In our Theorem 1, the constraints are relaxed by constructing a Lyapunov functional as (12) and $\dot{\tau}_i(t)$ can be any value or unknown due to Σ_{2+i} . Therefore, Theorem 1 is applicable to both cases of fast and slow time-varying delays.

Case 1 In many cases, the ranges of the time-varying delays are from 0 to an upper bound, i.e., $0 \leq \tau_1(t) \leq \bar{\tau}_1$, $0 \leq \tau_2(t) \leq \bar{\tau}_2$. Regarding this circumstance, the Lyapunov functional of system (10) is defined by:

$$\begin{aligned} V(x_1(t), x_2(t)) = & \sum_{i=1}^2 [V_1(x_1(t), x_2(t)) + V_2(x_1(t), x_2(t)) \\ & + V_3(x_1(t), x_2(t))], \quad V_1(x_1(t), x_2(t)) = x_i^T(t) P_i x_i(t), \end{aligned}$$

$$\begin{aligned} V_2(x_1(t), x_2(t)) = & \int_{t-\bar{\tau}_i}^t x_i^T(s) R_i x_i(s) ds + \int_{t-\bar{\tau}_i(t)}^t x_i^T(s) T_i x_i(s) ds, \\ V_3(x_1(t), x_2(t)) = & \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t g_i^T(s) Z_i g_i(s) ds d\theta \\ & + \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t g_{2+i}^T(s) Z_{4+i} g_{2+i}(s) ds d\theta. \end{aligned}$$

Then, similar to the proof of Theorem 1, we can yield the following corollary immediately.

Corollary 1 For given scalars $\bar{\tau}_1 > 0$, $\bar{\tau}_2 > 0$, $\underline{\tau}_1 = 0$, $\underline{\tau}_2 = 0$, μ_1 and μ_2 , the system (10) is globally asymptotically stable in the mean square, if there exist matrices $P_i > 0$, $R_i \geq 0$, $T_i \geq 0$, $i = 1, 2$, $Z_j > 0$, $j = 1, 2, 5, 6$, $N_j^{(i)}$, $S_j^{(i)}$, $j = 1, 2$, $i = 1, 2$, and two positive scalars α_1 , α_2 , such that the following LMI (27) holds:

$$\begin{bmatrix} \bar{\Xi}_0 & \bar{\Xi}_1 & \bar{\Xi}_2 & \bar{\Xi}_3 & \bar{\Xi}_4 \\ * & -\bar{\Xi}_{11} & 0 & 0 & 0 \\ * & * & -\bar{\Xi}_{22} & 0 & 0 \\ * & * & * & -\bar{\Xi}_{33} & 0 \\ * & * & * & * & -\bar{\Xi}_{44} \end{bmatrix} < 0, \quad (27)$$

with

$$\begin{aligned} \bar{\Xi}_0 = & \begin{bmatrix} \bar{\Upsilon}_1 & \bar{\Upsilon}_2^T \bar{U}_1 & \bar{\Upsilon}_3^T \bar{U}_2 & \bar{\Upsilon}_4^T \bar{U}_3 & \bar{\Upsilon}_5^T \bar{U}_4 \\ * & -\bar{U}_1 & 0 & 0 & 0 \\ * & * & -\bar{U}_2 & 0 & 0 \\ * & * & * & -\bar{U}_3 & 0 \\ * & * & * & * & -\bar{U}_4 \end{bmatrix}, \\ \bar{\Xi}_i = & \begin{bmatrix} \bar{\tau}_i N_1^{(i)} & \bar{\tau}_i S_1^{(i)} \\ \bar{\tau}_i N_2^{(i)} & \bar{\tau}_i S_2^{(i)} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \quad \bar{\Xi}_{2+i} = \begin{bmatrix} N_1^{(i)} & S_1^{(i)} \\ N_2^{(i)} & S_2^{(i)} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\bar{\Upsilon}_1 = \begin{bmatrix} \bar{\Sigma}_1 & 0 & \bar{\Sigma}_5 & 0 & 0 & P_1 W_1 & -S_1^{(1)} & 0 \\ * & \bar{\Sigma}_2 & 0 & \bar{\Sigma}_6 & P_2 W_2 & 0 & 0 & -S_1^{(2)} \\ * & * & \bar{\Sigma}_3 & 0 & 0 & 0 & -S_2^{(1)} & 0 \\ * & * & * & \bar{\Sigma}_4 & 0 & 0 & 0 & -S_2^{(2)} \\ * & * & * & * & -\alpha_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -\alpha_2 I & 0 & 0 \\ * & * & * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & * & * & -R_2 \end{bmatrix},$$

$$\bar{\Xi}_{11} = \text{diag}\{\bar{\tau}_1 Z_1, \bar{\tau}_1 Z_1\}, \quad \bar{\Xi}_{22} = \text{diag}\{\bar{\tau}_2 Z_2, \bar{\tau}_2 Z_2\},$$

$$\bar{\Xi}_{33} = \text{diag}\{Z_5, Z_5\}, \quad \bar{\Xi}_{44} = \text{diag}\{Z_6, Z_6\},$$

$$\bar{Y}_2 = [-A_1 \ 0 \ 0 \ 0 \ 0 \ W_1 \ 0 \ 0],$$

$$\bar{Y}_3 = [0 \ -A_2 \ 0 \ 0 \ W_2 \ 0 \ 0 \ 0],$$

$$\bar{Y}_4 = [C_1 \ 0 \ 0 \ D_1 \ 0 \ 0 \ 0 \ 0],$$

$$\bar{Y}_5 = [0 \ C_2 \ D_2 \ 0 \ 0 \ 0 \ 0 \ 0],$$

$$\bar{U}_1 = \bar{\tau}_1 Z_1, \quad \bar{U}_2 = \bar{\tau}_2 Z_2, \quad \bar{U}_3 = P_1 + \bar{\tau}_1 Z_5,$$

$$\bar{U}_4 = P_2 + \bar{\tau}_2 Z_6,$$

$$\bar{\Sigma}_i = -P_i A_i - A_i^T P_i + R_i + T_i + N_1^{(i)} + \left(N_1^{(i)}\right)^T,$$

$$\bar{\Sigma}_{2+i} = -(1 - \mu_i) T_i + S_2^{(i)} + \left(S_2^{(i)}\right)^T \\ - N_2^{(i)} - \left(N_2^{(i)}\right)^T + \alpha_i L_{3-i}^T L_{3-i},$$

$$\bar{\Sigma}_{4+i} = S_1^{(i)} - N_1^{(i)} + \left(N_2^{(i)}\right)^T, \quad i = 1, 2.$$

The following result provides the feasible stability criterion for uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances. The parameter uncertainties satisfy Assumption 3.

Theorem 2 For given scalars $0 \leq \underline{\tau}_1 < \bar{\tau}_1$, $0 \leq \underline{\tau}_2 < \bar{\tau}_2$, μ_1 and μ_2 , the system (10) is robustly globally asymptotically stable in the mean square, if there exist matrices $P_i > 0$, $Q_i \geq 0$, $R_i \geq 0$, $T_i \geq 0$, $i = 1, 2$, $Z_j > 0$, $j = 1, 2, \dots, 8$, $N_j^{(i)}$, $M_j^{(i)}$, $S_j^{(i)}$, $j = 1, 2$, $i = 1, 2$, two positive scalars α_1, α_2 , and eight positive scalars ε_j , $j = 1, 2, \dots, 8$, such that the following LMI (28) holds:

$$\begin{bmatrix} \Xi_0 + \Omega & \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 & \Xi_5 \\ * & -\Xi_{11} & 0 & 0 & 0 & 0 \\ * & * & -\Xi_{22} & 0 & 0 & 0 \\ * & * & * & -\Xi_{33} & 0 & 0 \\ * & * & * & * & -\Xi_{44} & 0 \\ * & * & * & * & * & -\Xi_{55} \end{bmatrix} < 0, \quad (28)$$

where

$$\Omega = \text{diag}\{(\varepsilon_1 E_1^T E_1 + \varepsilon_5 E_5^T E_5), (\varepsilon_3 E_3^T E_3 + \varepsilon_7 E_7^T E_7), \\ \varepsilon_8 E_8^T E_8, \varepsilon_6 E_6^T E_6, \varepsilon_4 E_4^T E_4, \varepsilon_2 E_2^T E_2, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\Xi_5 = \begin{bmatrix} \Omega_{A_1} & \Omega_{W_1} & \Omega_{A_2} & \Omega_{W_2} & \Omega_{C_1} & \Omega_{D_1} & \Omega_{C_2} & \Omega_{D_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{A_1} = [H_1^T P_1^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_1^T U_1 \ 0 \ 0 \ 0]^T$$

$$\Omega_{W_1} = [H_2^T P_1^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_2^T U_1 \ 0 \ 0 \ 0]^T,$$

$$\Omega_{A_2} = [0 \ H_3^T P_2^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_3^T U_2 \ 0 \ 0 \ 0]^T,$$

$$\Omega_{W_2} = [0 \ H_4^T P_2^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_4^T U_2 \ 0 \ 0 \ 0]^T,$$

$$\Omega_{C_1} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_5^T U_3 \ 0]^T,$$

$$\Omega_{D_1} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_6^T U_3 \ 0]^T,$$

$$\Omega_{C_2} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_7^T U_4]^T,$$

$$\Omega_{D_2} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ H_8^T U_4]^T,$$

$$\Xi_{55} = \text{diag}\{\varepsilon_1 I, \varepsilon_2 I, \varepsilon_3 I, \varepsilon_4 I, \varepsilon_5 I, \varepsilon_6 I, \varepsilon_7 I, \varepsilon_8 I\},$$

and the other parameters are defined in Theorem 1.

Proof By Lemma 2 and (8), the system (10) is robustly, globally, asymptotically stable in the mean square if the following inequality holds:

$$\begin{aligned} & \Xi_0 + 2\Omega_{A_1} F_1(t) \left(\Omega_{A_1}^*\right)^T + 2\Omega_{W_1} F_2(t) \left(\Omega_{W_1}^*\right)^T \\ & + 2\Omega_{A_2} F_3(t) \left(\Omega_{A_2}^*\right)^T + 2\Omega_{W_2} F_4(t) \left(\Omega_{W_2}^*\right)^T \\ & + 2\Omega_{C_1} F_5(t) \left(\Omega_{C_1}^*\right)^T + 2\Omega_{D_1} F_6(t) \left(\Omega_{D_1}^*\right)^T \\ & + 2\Omega_{C_2} F_7(t) \left(\Omega_{C_2}^*\right)^T + 2\Omega_{D_2} F_8(t) \left(\Omega_{D_2}^*\right)^T \\ & + \Xi_1 \Xi_{11}^{-1} \Xi_1^T + \Xi_2 \Xi_{22}^{-1} \Xi_2^T + \Xi_3 \Xi_{33}^{-1} \Xi_3^T + \Xi_4 \Xi_{44}^{-1} \Xi_4^T < 0. \end{aligned} \quad (29)$$

By Lemma 1(i) and (9), (29) holds if the following inequality satisfies:

$$\begin{aligned} & \Xi_0 + \varepsilon_1^{-1} \Omega_{A_1} \Omega_{A_1}^T + \varepsilon_1 \Omega_{A_1}^* \left(\Omega_{A_1}^*\right)^T + \varepsilon_2^{-1} \Omega_{W_1} \Omega_{W_1}^T \\ & + \varepsilon_2 \Omega_{W_1}^* \left(\Omega_{W_1}^*\right)^T + \varepsilon_3^{-1} \Omega_{A_2} \Omega_{A_2}^T + \varepsilon_3 \Omega_{A_2}^* \left(\Omega_{A_2}^*\right)^T \\ & + \varepsilon_4^{-1} \Omega_{W_2} \Omega_{W_2}^T + \varepsilon_4 \Omega_{W_2}^* \left(\Omega_{W_2}^*\right)^T \\ & + \varepsilon_5^{-1} \Omega_{C_1} \Omega_{C_1}^T + \varepsilon_5 \Omega_{C_1}^* \left(\Omega_{C_1}^*\right)^T \\ & + \varepsilon_6^{-1} \Omega_{D_1} \Omega_{D_1}^T + \varepsilon_6 \Omega_{D_1}^* \left(\Omega_{D_1}^*\right)^T \\ & + \varepsilon_7^{-1} \Omega_{C_2} \Omega_{C_2}^T + \varepsilon_7 \Omega_{C_2}^* \left(\Omega_{C_2}^*\right)^T \\ & + \varepsilon_8^{-1} \Omega_{D_2} \Omega_{D_2}^T + \varepsilon_8 \Omega_{D_2}^* \left(\Omega_{D_2}^*\right)^T \\ & + \Xi_1 \Xi_{11}^{-1} \Xi_1^T + \Xi_2 \Xi_{22}^{-1} \Xi_2^T + \Xi_3 \Xi_{33}^{-1} \Xi_3^T + \Xi_4 \Xi_{44}^{-1} \Xi_4^T \\ & = \Xi_0 + \Omega + \varepsilon_1^{-1} \Omega_{A_1} \Omega_{A_1}^T + \varepsilon_2^{-1} \Omega_{W_1} \Omega_{W_1}^T + \varepsilon_3^{-1} \Omega_{A_2} \Omega_{A_2}^T \\ & + \varepsilon_4^{-1} \Omega_{W_2} \Omega_{W_2}^T + \varepsilon_5^{-1} \Omega_{C_1} \Omega_{C_1}^T + \varepsilon_6^{-1} \Omega_{D_1} \Omega_{D_1}^T + \varepsilon_7^{-1} \Omega_{C_2} \Omega_{C_2}^T \\ & + \varepsilon_8^{-1} \Omega_{D_2} \Omega_{D_2}^T + \Xi_1 \Xi_{11}^{-1} \Xi_1^T + \Xi_2 \Xi_{22}^{-1} \Xi_2^T + \Xi_3 \Xi_{33}^{-1} \Xi_3^T \\ & + \Xi_4 \Xi_{44}^{-1} \Xi_4^T < 0, \end{aligned} \quad (30)$$

with

$$\Omega_{A_1}^* = [-E_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$\Omega_{W_1}^* = [0 \ 0 \ 0 \ 0 \ 0 \ E_2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$\Omega_{A_2}^* = [0 \ -E_3 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$\begin{aligned}\Omega_{W_2}^* &= [0 \ 0 \ 0 \ 0 \ E_4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \Omega_{C_1}^* &= [E_5 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \Omega_{D_1}^* &= [0 \ 0 \ 0 \ E_6 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \Omega_{C_2}^* &= [0 \ E_7 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \Omega_{D_2}^* &= [0 \ 0 \ E_8 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T,\end{aligned}$$

where $\varepsilon_j > 0$, $j = 1, 2, \dots, 8$ and $\Omega, \Omega_{A_i}, \Omega_{W_i}, \Omega_{C_i}, \Omega_{D_i}$, $i = 1, 2$ are defined in Theorem 2.

Then, by Schur complement, the inequality (30) is equivalent to the LMI (28). Thus, if the LMI given in (28) holds, the system (10) is robustly, globally, asymptotically stable in the mean square. This completes the proof.

If the system is simplified to stochastic BAM neural networks with interval time-varying delay, the model can be described as follows:

$R_i \geq 0$, $T_i \geq 0$, $i = 1, 2$, $Z_j > 0$, $j = 1, 2, \dots, 8$, $N_j^{(i)}, M_j^{(i)}, S_j^{(i)}$, $j = 1, 2$, $i = 1, 2$, two positive scalars α_1, α_2 , and six positive scalars ρ_j , $j = 1, 2, \dots, 6$, such that the following LMIs (34)–(36) hold:

$$\Psi = \begin{bmatrix} \Psi_0 & \Psi_1 & \Psi_2 & \Psi_3 & \Psi_4 \\ * & -\Psi_{11} & 0 & 0 & 0 \\ * & * & -\Psi_{22} & 0 & 0 \\ * & * & * & -\Psi_{33} & 0 \\ * & * & * & * & -\Psi_{44} \end{bmatrix} < 0, \quad (34)$$

$$P_1 \leq \rho_1 I, \quad Z_5 \leq \rho_3 I, \quad Z_7 \leq \rho_5 I, \quad (35)$$

$$P_2 \leq \rho_2 I, \quad Z_6 \leq \rho_4 I, \quad Z_8 \leq \rho_6 I, \quad (36)$$

with

$$\begin{cases} dx_1(t) = [-A_1 x_1(t) + W_1 f_1(x_2(t - \tau_2(t)))]dt + \sigma_1(t, x_1(t), x_2(t - \tau_2(t)))d\omega(t), \\ dx_2(t) = [-A_2 x_2(t) + W_2 f_2(x_1(t - \tau_1(t)))]dt + \sigma_2(t, x_2(t), x_1(t - \tau_1(t)))d\omega(t), \end{cases} \quad (31)$$

where $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_l(t))^T$ is an l -dimensional Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$. Let $\sigma(t, x, y) : R_+ \times R^n \times R^n \rightarrow R^{n \times l}$ is locally Lipschitz continuous and satisfies the linear growth condition as well. Moreover σ_1, σ_2 satisfy

$$\begin{aligned}\text{trace}(\sigma_1^T(t, x_1(t), x_2(t - \tau_2(t)))\sigma_1(t, x_1(t), x_2(t - \tau_2(t)))) \\ \leq x_1^T(t)X_1x_1(t) + x_2^T(t - \tau_2(t))Y_1x_2(t - \tau_2(t)), \\ X_1 \geq 0, \quad Y_1 \geq 0, \quad (32)\end{aligned}$$

$$\begin{aligned}\text{trace}(\sigma_2^T(t, x_2(t), x_1(t - \tau_1(t)))\sigma_2(t, x_2(t), x_1(t - \tau_1(t)))) \\ \leq x_2^T(t)X_2x_2(t) + x_1^T(t - \tau_1(t))Y_2x_1(t - \tau_1(t)), \\ X_2 \geq 0, \quad Y_2 \geq 0. \quad (33)\end{aligned}$$

Theorem 3 For given scalars $0 \leq \underline{\tau}_1 < \bar{\tau}_1$, $0 \leq \underline{\tau}_2 < \bar{\tau}_2$, μ_1 and μ_2 , the system (31) is globally asymptotically stable in the mean square, if there exist matrices $P_i > 0$, $Q_i \geq 0$,

$$\begin{aligned}\Psi_0 &= \begin{bmatrix} \Theta_1 & \Theta_2^T U_1 & \Theta_3^T U_2 \\ * & -U_1 & 0 \\ * & * & -U_2 \end{bmatrix}, \\ \Psi_i &= \begin{bmatrix} \bar{\tau}_i N_1^{(i)} & h_i M_1^{(i)} & h_i S_1^{(i)} \\ \bar{\tau}_i N_2^{(i)} & h_i M_2^{(i)} & h_i S_2^{(i)} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \\ \Psi_{2+i} &= \begin{bmatrix} N_1^{(i)} & M_1^{(i)} & S_1^{(i)} \\ N_2^{(i)} & M_2^{(i)} & S_2^{(i)} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix},\end{aligned}$$

$$\Theta_1 = \begin{bmatrix} \Pi_1 & 0 & \Pi_5 & 0 & 0 & P_1 W_1 & M_1^{(1)} & -S_1^{(1)} & 0 & 0 \\ * & \Pi_2 & 0 & \Pi_6 & P_2 W_2 & 0 & 0 & 0 & M_1^{(2)} & -S_1^{(2)} \\ * & * & \Pi_3 & 0 & 0 & 0 & M_2^{(1)} & -S_2^{(1)} & 0 & 0 \\ * & * & * & \Pi_4 & 0 & 0 & 0 & 0 & M_2^{(2)} & -S_2^{(2)} \\ * & * & * & * & -\alpha_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\alpha_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -Q_2 & 0 \\ * & * & * & * & * & * & * & * & * & -R_2 \end{bmatrix},$$

$$\begin{aligned}
\Psi_{11} &= \text{diag}\{\bar{\tau}_1 Z_1, h_1 Z_3, h_1(Z_1 + Z_3)\}, \\
\Psi_{22} &= \text{diag}\{\bar{\tau}_2 Z_2, h_2 Z_4, h_2(Z_2 + Z_4)\}, \\
\Psi_{33} &= \text{diag}\{Z_5, Z_7, Z_5 + Z_7\}, \\
\Psi_{44} &= \text{diag}\{Z_6, Z_8, Z_6 + Z_8\}, \\
\Theta_2 &= [-A_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad W_1 \quad 0 \quad 0 \quad 0 \quad 0], \\
\Theta_3 &= [0 \quad -A_2 \quad 0 \quad 0 \quad W_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\
U_1 &= \bar{\tau}_1 Z_1 + h_1 Z_3, \quad U_2 = \bar{\tau}_2 Z_2 + h_2 Z_4, \quad h_i = \bar{\tau}_i - \underline{\tau}_i,
\end{aligned}$$

$$\begin{aligned}
\Pi_i &= -P_i A_i - A_i^T P_i + Q_i + R_i + T_i + N_1^{(i)} + \left(N_1^{(i)}\right)^T \\
&\quad + (\rho_i + \bar{\tau}_i \rho_{2+i} + h_i \rho_{4+i}) X_i, \\
\Pi_{2+i} &= -(1 - \mu_i) T_i + S_2^{(i)} + \left(S_2^{(i)}\right)^T - N_2^{(i)} \\
&\quad - \left(N_2^{(i)}\right)^T - M_2^{(i)} - \left(M_2^{(i)}\right)^T + \alpha_i L_{3-i}^T L_{3-i} \\
&\quad + (\rho_{3-i} + \bar{\tau}_{3-i} \rho_{5-i} + h_{3-i} \rho_{7-i}) Y_{3-i}, \\
\Pi_{4+i} &= S_1^{(i)} - N_1^{(i)} - M_1^{(i)} + \left(N_2^{(i)}\right)^T, \quad i = 1, 2.
\end{aligned}$$

Proof The Lyapunov functional of system (31) is defined by:

$$\begin{aligned}
V(x_1(t), x_2(t)) &= \sum_{i=1}^2 \left[V_1(x_1(t), x_2(t)) + V_2(x_1(t), x_2(t)) \right. \\
&\quad \left. + V_3(x_1(t), x_2(t)) \right], \\
V_3(x_1(t), x_2(t)) &= \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t g_i^T(s) Z_i g_i(s) ds d\theta \\
&\quad + \int_{-\bar{\tau}_i}^{-\underline{\tau}_i} \int_{t+\theta}^t g_i^T(s) Z_{2+i} g_i(s) ds d\theta \\
&\quad + \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t \text{trace}(g_{2+i}^T(s) Z_{4+i} g_{2+i}(s)) ds d\theta \\
&\quad + \int_{-\bar{\tau}_i}^{-\underline{\tau}_i} \int_{t+\theta}^t \text{trace}(g_{2+i}^T(s) Z_{6+i} g_{2+i}(s)) ds d\theta,
\end{aligned}$$

with $V_1(x_1(t), x_2(t))$ and $V_2(x_1(t), x_2(t))$ are as same as the ones Theorem 1.

By Itô's formula, we can obtain the following stochastic differential:

$$\begin{aligned}
\mathcal{L}V(x_1(t), x_2(t)) &= \sum_{i=1}^2 [\mathcal{L}V_1(x_1(t), x_2(t)) + \mathcal{L}V_2(x_1(t), x_2(t)) \\
&\quad + \mathcal{L}V_3(x_1(t), x_2(t))], \\
\mathcal{L}V_1(x_1(t), x_2(t)) &= 2x_i^T(t) P_i [-A_i x_i(t) \\
&\quad + W_i f_i(x_{3-i}(t - \tau_{3-i}(t)))] \\
&\quad + \text{trace}(g_{2+i}^T(t) P_i g_{2+i}(t)),
\end{aligned} \quad (37)$$

$$\begin{aligned}
\mathcal{L}V_2(x_1(t), x_2(t)) &\leq x_i^T(t) (Q_i + R_i) x_i(t) - x_i^T(t - \underline{\tau}_i) \\
&\quad \times Q_i x_i(t - \underline{\tau}_i) - x_i^T(t - \bar{\tau}_i) R_i x_i(t - \bar{\tau}_i) + x_i^T(t) T_i x_i(t) \\
&\quad - (1 - \mu_i) x_i^T(t - \tau_i(t)) T_i x_i(t - \tau_i(t)),
\end{aligned} \quad (38)$$

$$\begin{aligned}
\mathcal{L}V_3(x_1(t), x_2(t)) &= g_i^T(t) (\bar{\tau}_i Z_i + h_i Z_{2+i}) g_i(t) \\
&\quad + \bar{\tau}_i \text{trace}(g_{2+i}^T(t) Z_{4+i} g_{2+i}(t)) + h_i \text{trace}(g_{2+i}^T(t) Z_{6+i} g_{2+i}(t)) \\
&\quad - \int_{t-\tau_i(t)}^t g_i^T(s) Z_i g_i(s) ds - \int_{t-\underline{\tau}_i}^{t-\bar{\tau}_i} g_i^T(s) Z_{2+i} g_i(s) ds \\
&\quad - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_i^T(s) (Z_i + Z_{2+i}) g_i(s) ds \\
&\quad - \int_{t-\tau_i(t)}^t \text{trace}(g_{2+i}^T(s) Z_{4+i} g_{2+i}(s)) ds \\
&\quad - \int_{t-\underline{\tau}_i}^{t-\bar{\tau}_i} \text{trace}(g_{2+i}^T(s) Z_{6+i} g_{2+i}(s)) ds \\
&\quad - \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} \text{trace}(g_{2+i}^T(s) (Z_{4+i} + Z_{6+i}) g_{2+i}(s)) ds.
\end{aligned} \quad (39)$$

Combining (13)-(22),(37)-(39) and using (32)-(33), we have

$$\begin{aligned}
\mathcal{L}V(x_1(t), x_2(t)) &\leq \xi^T(t) \left\{ \Theta_1 + \Theta_2^T U_1 \Theta_2 + \Theta_3^T U_2 \Theta_3 \right. \\
&\quad + \sum_{i=1}^2 \left[\bar{\tau}_i N^{(i)} Z_i^{-1} \left(N^{(i)}\right)^T + h_i S^{(i)} (Z_i + Z_{2+i})^{-1} \left(S^{(i)}\right)^T \right. \\
&\quad + h_i M^{(i)} Z_{2+i}^{-1} \left(M^{(i)}\right)^T + N^{(i)} Z_{4+i}^{-1} \left(N^{(i)}\right)^T \\
&\quad + S^{(i)} (Z_{4+i} + Z_{6+i})^{-1} \left(S^{(i)}\right)^T + M^{(i)} Z_{6+i}^{-1} \left(M^{(i)}\right)^T \left. \right] \xi(t) \\
&\quad + \int_{t-\tau_i(t)}^t g_{2+i}^T(s) d\omega(s) Z_{4+i} \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) \\
&\quad + \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) d\omega(s) Z_{6+i} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s) \\
&\quad + \int_{t-\bar{\tau}_i}^t g_{2+i}^T(s) d\omega(s) (Z_{4+i} + Z_{6+i}) \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s) \\
&\quad - \int_{t-\tau_i(t)}^t \text{trace}(g_{2+i}^T(s) Z_{4+i} g_{2+i}(s)) ds \\
&\quad - \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} \text{trace}(g_{2+i}^T(s) Z_{6+i} g_{2+i}(s)) ds \\
&\quad - \int_{t-\bar{\tau}_i}^t \text{trace}(g_{2+i}^T(s) (Z_{4+i} + Z_{6+i}) g_{2+i}(s)) ds.
\end{aligned} \quad (40)$$

Using

$$\begin{aligned} & \mathbb{E} \left\{ \int_{t-\tau_i(t)}^t g_{2+i}^T(s) d\omega(s) Z_{4+i} \int_{t-\tau_i(t)}^t g_{2+i}(s) d\omega(s) \right\} \\ &= \mathbb{E} \left\{ \int_{t-\tau_i(t)}^t \text{trace}(g_{2+i}^T(s) Z_{4+i} g_{2+i}(s)) ds \right\}, \\ & \mathbb{E} \left\{ \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}^T(s) d\omega(s) Z_{6+i} \int_{t-\tau_i(t)}^{t-\underline{\tau}_i} g_{2+i}(s) d\omega(s) \right\} \\ &= \mathbb{E} \left(\int_{t-\tau_i(t)}^{t-\underline{\tau}_i} \text{trace}(g_{2+i}^T(s) Z_{6+i} g_{2+i}(s)) ds \right), \end{aligned}$$

$$\begin{cases} dx_1(t) = [-A_1 x_1(t) + W_1 f_1(x_2(t - \tau_2(t)))] dt + [C_1 x_1(t) + D_1 x_2(t - \tau_2(t))] d\omega(t), \\ dx_2(t) = [-A_2 x_2(t) + W_2 f_2(x_1(t - \tau_1(t)))] dt + [C_2 x_2(t) + D_2 x_1(t - \tau_1(t))] d\omega(t), \end{cases} \quad (41)$$

$$\begin{aligned} & \mathbb{E} \left\{ \int_{t-\tau_i}^{t-\tau_i(t)} g_{2+i}^T(s) d\omega(s) (Z_{4+i} + Z_{6+i}) \int_{t-\tau_i}^{t-\tau_i(t)} g_{2+i}(s) d\omega(s) \right\} \\ &= \mathbb{E} \left\{ \int_{t-\tau_i}^{t-\tau_i(t)} \text{trace}(g_{2+i}^T(s) (Z_{4+i} + Z_{6+i}) g_{2+i}(s)) ds \right\}, \end{aligned}$$

it follows from

$$\begin{aligned} \Psi &= \Theta_1 + \Theta_2^T U_1 \Theta_2 + \Theta_3^T U_2 \Theta_3 \\ &+ \sum_{i=1}^2 \left[\bar{\tau}_i N^{(i)} Z_i^{-1} (N^{(i)})^T + h_i M^{(i)} Z_{2+i}^{-1} (M^{(i)})^T \right. \\ &+ h_i S^{(i)} (Z_i + Z_{2+i})^{-1} (S^{(i)})^T + N^{(i)} Z_{4+i}^{-1} (N^{(i)})^T \\ &\left. + M^{(i)} Z_{6+i}^{-1} (M^{(i)})^T + S^{(i)} (Z_{4+i} + Z_{6+i})^{-1} (S^{(i)})^T \right] < 0, \end{aligned}$$

that $\mathbb{E}[dV(x_1(t), x_2(t))] = \mathbb{E}[\mathcal{L}V(x_1(t), x_2(t))dt] < 0$, for all $x_1(t), x_2(t)$ except for $x_1(t) = x_2(t) = 0$.

It follows from Lyapunov stability theorem that the system (31) is globally, stochastically, asymptotically stable in the mean square. This completes the proof of Theorem 3.

Remark 3 The stability conditions in Theorem 1, Theorem 2 and Theorem 3 are both delay-rang-dependent and delay-rate-dependent. Generally speaking, the delay-rang-independent and delay-rate-independent criteria are conservative than the delay-rang-dependent and

delay-rate-dependent criteria since some information about the lower bounds and derivatives of the time-varying delay functions do not been considered.

Numerical examples

In this section, three examples are given to show the effectiveness and the merit of the proposed method.

Example 1 Consider the following BAM neural networks with both interval time-varying delays and stochastic disturbances, but without parameter uncertainties:

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \end{aligned}$$

with $f_i(x) = \frac{1}{2}(|x+1| - |x-1|)$, $\tau_1(t) = 0.9 + 0.9 \sin^2(t)$, $\tau_2(t) = 0.7 + 0.5 \sin^2(t)$. Then it is obviously that $l_i = l_j = 1$ for all i and j , i. e., $L_1 = L_2 = I$. And $\bar{\tau}_1 = 1.8$, $\underline{\tau}_1 = 0.9$, $\bar{\tau}_2 = 1.2$, $\underline{\tau}_2 = 0.7$, $\mu_1 = 0.9$, $\mu_2 = 0.5$.

Using LMI Control Toolbox, by Theorem 1, we can easily obtain feasible solution of the LMI (11). Thus the network (41) is globally stochastically asymptotically stable in the mean square. Limited to the length of the paper, we only show a part of the feasible solution here. The trajectories of variable $x_1(t)$ and $x_2(t)$ are shown in Fig. 1.

$$\begin{aligned} P_1 &= \begin{bmatrix} 1.3530 & -0.7322 \\ -0.7322 & 6.9207 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 5.9807 & -2.8486 \\ -2.8486 & 8.7211 \end{bmatrix}, \\ R_1 &= \begin{bmatrix} 0.6759 & -0.2672 \\ -0.2672 & 2.8091 \end{bmatrix}, \end{aligned}$$

Fig. 1 Trajectories of $x_1(t)$ and $x_2(t)$ of the BAM neural networks with stochastic perturbation

$$R_2 = \begin{bmatrix} 1.2778 & -0.0802 \\ -0.0802 & 2.0147 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.0529 & -0.2375 \\ -0.2375 & 1.2405 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} 0.4037 & -0.8339 \\ -0.8339 & 2.4746 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.2827 & -0.0196 \\ -0.0196 & 0.9826 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 1.2346 & -0.0011 \\ -0.0011 & 1.0711 \end{bmatrix},$$

$$\alpha_1 = 0.5372, \quad \alpha_2 = 1.7460.$$

Example 2 In this example, we consider the BAM neural networks with the following parameters:

$$P_1 = \begin{bmatrix} 6.1719 & 0.4420 \\ 0.4420 & 11.3368 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 34.4178 & -0.9030 \\ -0.9030 & 30.4409 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 6.3118 & 0.3002 \\ 0.3002 & 7.8403 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 9.0260 & -0.3447 \\ -0.3447 & 9.4511 \end{bmatrix},$$

$$Z_5 = \begin{bmatrix} 9.8040 & 0.9388 \\ 0.9388 & 9.2729 \end{bmatrix}, \quad Z_6 = \begin{bmatrix} 1.9073 & 0.1193 \\ 0.1193 & 1.8696 \end{bmatrix},$$

$$Z_7 = \begin{bmatrix} 1.0735 & 0.0507 \\ 0.0507 & 1.0172 \end{bmatrix}, \quad Z_8 = \begin{bmatrix} 17.9262 & 2.7249 \\ 2.7249 & 16.0859 \end{bmatrix},$$

$$\alpha_1 = 5.0657, \quad \alpha_2 = 14.2841, \quad \rho_1 = 11.7124,$$

$$\rho_2 = 11.8777, \quad \rho_3 = 1.3633, \quad \rho_4 = 35.2401,$$

$$\rho_5 = 2.5531, \quad \rho_6 = 21.0552.$$

$$\begin{cases} dx_1(t) = [-A_1 x_1(t) + W_1 f_1(x_2(t - \tau_2(t)))]dt + \sigma_1(t, x_1(t), x_2(t - \tau_2(t)))d\omega(t), \\ dx_2(t) = [-A_2 x_2(t) + W_2 f_2(x_1(t - \tau_1(t)))]dt + \sigma_2(t, x_2(t), x_1(t - \tau_1(t)))d\omega(t), \end{cases} \quad (42)$$

where

$$A_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1.8 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \quad X_1 = Y_1 = 0.1I,$$

$$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \quad X_2 = Y_2 = 0.02I.$$

with $f_i(x) = \frac{1}{2}(|x+1| - |x-1|)$, $\tau_1(t) = 1.3 + 1.1 \sin^2(t)$, $\tau_2(t) = 1 + 0.9 \sin^2(t)$. Then $L_1 = L_2 = I$ and $\bar{\tau}_1 = 2.4$, $\underline{\tau}_1 = 1.3$, $\bar{\tau}_2 = 1.9$, $\underline{\tau}_2 = 1$, $\mu_1 = 1.1$, $\mu_2 = 0.9$.

Therefore, we can find that the system (42) is globally, asymptotically stable in the mean square. The dynamical behavior is shown in Fig. 2.

Example 3 Consider the following uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances:

$$\begin{cases} dx_1(t) = [-(A_1 + \Delta A_1)x_1(t) + (W_1 + \Delta W_1)f_1(x_2(t - \tau_2(t)))]dt + [(C_1 + \Delta C_1)x_1(t) + (D_1 + \Delta D_1)x_2(t - \tau_2(t))]d\omega(t), \\ dx_2(t) = [-(A_2 + \Delta A_2)x_2(t) + (W_2 + \Delta W_2)f_2(x_1(t - \tau_1(t)))]dt + [(C_2 + \Delta C_2)x_2(t) + (D_2 + \Delta D_2)x_1(t - \tau_1(t))]d\omega(t), \end{cases} \quad (43)$$

Using Theorem 3 and solving LMIs (34)–(36), we get a feasible solution. Limited to the length of the paper, we only show a part of the feasible solution here.

where

$$A_1 = \begin{bmatrix} 2.6 & 0 \\ 0 & 2.1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1.1 & 1 \\ -0.2 & 0.1 \end{bmatrix},$$

Fig. 2 Trajectories of $x_1(t)$ and $x_2(t)$ of the BAM neural networks with stochastic perturbation

Fig. 3 Trajectories of $x_1(t)$ and $x_2(t)$ of the BAM neural networks with stochastic perturbation and parameter uncertainties

$$C_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.9 & 0.1 \\ -0.1 & 0.1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$H_1 = H_2 = H_3 = H_4 = 0.1I, \quad H_5 = H_6 = H_7 = H_8 = 0.2I,$$

$$E_1 = E_2 = E_3 = E_4 = 0.2I, \quad E_5 = E_6 = E_7 = E_8 = 0.4I$$

$$F_1 = F_5 = \text{diag}\{\sin(t) \quad \cos(2t)\}, \quad F_2 = F_6 = 0.1I,$$

$$F_3 = F_7 = \text{diag}\{-0.01 \cos(t) \quad -0.01 \cos(t)\}, \quad F_4 = F_8 = 0.1I.$$

with $f_i(x) = \frac{1}{2}(|x+1| - |x-1|)$, $\tau_1(t) = 1 + 0.5 \sin^2(t)$, $\tau_2(t) = 0.8 + 1.2 \sin^2(t)$. Then $L_1 = L_2 = I$ and $\bar{\tau}_1 = 1.5$, $\bar{\tau}_2 = 2$, $\bar{\tau}_1 = 0.8$, $\mu_1 = 0.5$, $\mu_2 = 1.2$.

By Theorem 2, the feasible solution of the LMI (28) is obtained. As mentioned above, we only show a part of the feasible solution here. The simulation result of the trajectories of the variables $x_1(t)$ and $x_2(t)$ is shown in Fig. 3.

$$P_1 = \begin{bmatrix} 0.2393 & 0.0083 \\ 0.0083 & 0.4475 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.4146 & 0.0031 \\ 0.0031 & 0.5130 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.1506 & 0.0046 \\ 0.0046 & 0.2167 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.2336 & 0.0079 \\ 0.0079 & 0.2442 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.1117 & 0.0151 \\ 0.0151 & 0.4475 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.0497 & 0.0068 \\ 0.0068 & 0.0801 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.0242 & 0.0005 \\ 0.0005 & 0.0467 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0.0255 & -0.0003 \\ -0.0003 & 0.0391 \end{bmatrix},$$

$$\alpha_1 = 0.1449, \quad \alpha_2 = 0.2260, \quad \varepsilon_1 = 0.2179,$$

$$\varepsilon_2 = 0.1980, \quad \varepsilon_3 = 0.2266,$$

$$\varepsilon_4 = 0.2058, \quad \varepsilon_5 = 0.2326, \quad \varepsilon_6 = 0.1850,$$

$$\varepsilon_7 = 0.3078, \quad \varepsilon_8 = 0.2395.$$

Therefore, we can find that the system (43) described by Example 3 is robustly, globally, asymptotically stable in the mean square.

Remark 3 It should be noted out that, when $W_0 = V_0 = 0$ of system in Bao and Cao (2012) and $W = V = 0$ of system in Yuan and Li (2010), i.e. without external stochastic perturbations and parameter uncertainty, systems in Bao and Cao (2012) and Yuan and Li (2010) can be transformed into the following form:

$$\begin{cases} \dot{x}_1(t) = -A_1 x_1(t) + W_1 f_1(x_2(t - \tau_2(t))), \\ \dot{x}_2(t) = -A_2 x_2(t) + W_2 f_2(x_1(t - \tau_1(t))). \end{cases} \quad (44)$$

Since $\bar{\tau}_1 \neq 0$ and $\bar{\tau}_2 \neq 0$, the results in Bao and Cao (2012) and Yuan and Li (2010) are not applicable to our Example 2 and Example 3. Additional, the criteria in the previous work (Liu et al. 2006; Lou and Cui 2006; Park and Kwon 2008; Syed Ali and Balasubramaniam 2008) fail in our examples.

Remark 4 In (44), since $\mu_1 > 1$ in Example 2 and $\mu_2 > 1$ in Example 2, the results in Bao and Cao (2012) and Yuan

and Li (2010) are not applicable to our Example 2 and Example 3. On the other hand, the criteria in the previous work (Liu et al. 2006; Lou and Cui 2006; Park and Kwon 2008; Syed Ali and Balasubramaniam 2008) also fail in our examples.

Conclusions

This paper presents some new results of robust asymptotic stability analysis for uncertain BAM neural networks with both interval time-varying delays and stochastic disturbances. Some new stability criteria are proposed to guarantee the delayed BAM neural networks to be robustly asymptotically stable in the mean square by using the stochastic analysis approach, employing some free-weighting matrices and constructing an appropriate type of Lyapunov functional which takes into account the ranges of delays. Unlike the most existing mean square stability conditions for BAM neural networks, due to the Lyapunov functional, the supplementary requirements that the time derivatives of time-varying delays must be smaller than 1 are relaxed. Furthermore, in the proposed scheme, the lower bounds of time varying delays are not restricted to be 0 and the stability conditions are delay-range-dependent and rate-dependent/independent. As a result, the new stability criteria in term of LMIs are applicable to both fast and slow time-varying delays. Finally, three numerical examples are provided to verify the effectiveness of the proposed results.

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References

- Arik S (2005) Global asymptotic stability analysis of bidirectional associative memory neural networks with time delays. *IEEE Trans Neural Netw* 16:580–586
- Bao H, Cao J (2012) Exponential stability for stochastic BAM networks with discrete and distributed delays. *Appl Math Comput* 218:6188–6199
- Blythe S, Mao X, Liao X (2001) Stability of stochastic delay neural networks. *J Franklin Inst* 338:481–495
- Boyd S, El Ghaoui L, Feron E et al (1994) Linear matrix inequalities in system and control theory. SIAM, Philadelphia
- Cao J, Xiao M (2007) Stability and Hopf bifurcation in a simplified BAM neural network with two time delays. *IEEE Trans Neural Netw* 18:416–430
- Cao J, Ho DWC, Huang X (2007) LMI-based criteria for global robust stability of bidirectional associative memory networks with time delay. *Nonlinear Anal Theory Methods Appl* 66:1558–1572
- Chang H, Feng Z, Wei X (2008) Object recognition and tracking with maximum likelihood bidirectional associative memory networks. *Neurocomputing* 72:278–292
- Dong W, Wang R (2011) Exploring human rhythmic gait movement in the role of cerebral cortex signal. *Appl Math Mech* 32(2): 223–230
- Hasan S, Siong N (1997) A parallel processing VLSI BAM engine. *IEEE Trans Neural Netw* 8(2):424–436
- Haykin S (1994) Neural networks. Prentice Hall, New York
- He Y, Liu G, Rees D et al (2007) Stability analysis for neural networks with time-varying interval delay. *IEEE Trans Neural Netw* 18:1850–1854
- Huang T, Huang Y, Li C (2008) Stability of periodic solution in fuzzy BAM neural networks with finite distributed delays. *Neurocomputing* 71:3064–3069
- Khasminski R (1980) Stochastic stability of differential equations. Sijthoff and Noordhoff, Netherlands
- Kosko B (1987) Adaptive bi-directional associative memories. *Appl Opt* 26:4947–4960
- Kosko B (1988) Bidirectional associative memories. *IEEE Trans Syst Man Cybern* 18:49–60
- Li C, Li C, Liao X (2011) Impulsive effects on stability of high-order BAM neural networks with time delays. *Neurocomputing* 74(10): 1541–1550
- Liao X, Wong KW (2003) Global exponential stability of hybrid bidirectional associative memory neural networks with discrete delays. *Phys Rev E* 67:60–65
- Liao X, Wong KW (2004) Robust stability of interval bidirectional associative memory neural network with time delays. *IEEE Trans Syst Man Cybern* 34:1142–1154
- Liao X, Yu J, Chen G (2002) Novel stability criteria for bidirectional associative memory neural networks with time delays. *Int J Circuit Theory Appl* 30:519–546
- Liu Y, Wang Z, Liu X (2006) Global asymptotic stability of generalized bi-directional associative memory networks with discrete and distributed delays. *Chaos, Solitons Fractals* 28: 793–803
- Liu Y, Wang R, Zhang Z, Jiao X (2010) Analysis on stability of neural network in the presence of inhibitory neurons. *Cogn Neurodyn* 4:61–68
- Lou X, Cui B (2006) On the global robust asymptotic stability of BAM neural networks with time-varying delays. *Neurocomputing* 70:273–279
- Lou X, Cui B (2007a) Stochastic exponential stability for Markovian jumping BAM neural networks with time-varying delays. *IEEE Trans Syst Man Cybern* 37:713–719
- Lou X, Cui B (2007b) Absolute exponential stability analysis of delayed bi-directional associative memory neural networks. *Chaos, Solitons Fractals* 31:695–701
- Ozcan N, Arik S (2009) A new sufficient condition for global robust stability of bidirectional associative memory neural networks with multiple time delays. *Nonlinear Anal Real World Appl* 10:3312–3320
- Park J, Kwon O (2008) On improved delay-dependent criterion for global stability of bidirectional associative memory neural networks with time-varying delays. *Appl Math Comp* 199:435–446
- Qiu J, Yang H, Zhang J, Gao Z (2009) New robust stability criteria for uncertain neural networks with interval time-varying delays. *Chaos, Solitons Fractals* 39:579–585
- Rakkiyappan R, Balasubramaniam P, Lakshmanan S (2008) Robust stability results for uncertain stochastic neural networks with discrete interval and distributed time-varying delays. *Phys Lett A* 372:5290–5298
- Shua H, Wang Z, Lü Z (2009) Global asymptotic stability of uncertain stochastic bi-directional associative memory networks with discrete and distributed delays. *Math Comp Simul* 80:490–505
- Song Q, Cao J (2007) Dynamics of bidirectional associative memory networks with distributed delays and reaction-diffusion terms. *Nonlinear Anal Real World Appl* 8:345–361

- Sudo A, Sato A, Hasegawa O (2009) Associative memory for online learning in noisy environments using self-organizing incremental neural network. *IEEE Trans Neural Netw* 20:964–972
- Syed Ali M, Balasubramaniam P (2008) Robust stability for uncertain stochastic fuzzy BAM neural networks with time-varying delays. *Phys Lett A* 372:5159–5166
- Teddy SD, Ng SK (2011) Forecasting ATM cash demands using a local learning model of cerebellar associative memory network. *Int J Forecast* 27:760–776
- Wan L, Zhou Q (2007) Convergence analysis of stochastic hybrid bidirectional associative memory neural networks with delays. *Phys Lett A* 370:423–432
- Wang Y (2009) Global exponential stability analysis of bidirectional associative memory neural networks with time-varying delays. *Nonlinear Anal Real World Appl* 10:1527–1539
- Wang R, Zhang Z (2011) Phase synchronization motion and neural coding in dynamic transmission of neural information. *IEEE Trans Neural Netw* 22:1097–1106
- Wang Y, Xie L, Souza CE (1992) Robust control of a class of uncertain nonlinear systems. *Syst Control Lett* 19:139–149
- Wu H, Feng W, Liang X (2008) New stability criteria for uncertain neural networks with interval time-varying delays. *Cogn Neurodyn* 2:363–370
- Yuan Y, Li X (2010) New results for global robust asymptotic stability of BAM neural networks with time-varying delays. *Neurocomputing* 74:337–342
- Zhang X, Wang R, Zhang Z (2010) Dynamic phase synchronization characteristics of variable high-order coupled neuronal oscillator population. *Neurocomputing* 73:665–2670