Magnetostatic focal spot correction for x-ray tubes operating in strong magnetic fields using iterative optimization

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Purpose: Combining x-ray fluoroscopy and MR imaging systems for guidance of interventional procedures has become more commonplace. By designing an x-ray tube that is immune to the magnetic fields outside of the MR bore, the two systems can be placed in close proximity to each other. A major obstacle to robust x-ray tube design is correcting for the effects of the magnetic fields on the x-ray tube focal spot. A potential solution is to design active shielding that locally cancels the magnetic fields near the focal spot.

Methods: An iterative optimization algorithm is implemented to design resistive active shielding coils that will be placed outside the x-ray tube insert. The optimization procedure attempts to minimize the power consumption of the shielding coils while satisfying magnetic field homogeneity constraints. The algorithm is composed of a linear programming step and a nonlinear programming step that are interleaved with each other. The coil results are verified using a finite element space charge simulation of the electron beam inside the x-ray tube. To alleviate heating concerns an optimized coil solution is derived that includes a neodymium permanent magnet. Any demagnetization of the permanent magnet is calculated prior to solving for the optimized coils. The temperature dynamics of the coil solutions are calculated using a lumped parameter model, which is used to estimate operation times of the coils before temperature failure.

Results: For a magnetic field strength of 88 mT, the algorithm solves for coils that consume 588 A/cm². This specific coil geometry can operate for 15 min continuously before reaching temperature failure. By including a neodymium magnet in the design the current density drops to 337 A/cm², which increases the operation time to 59 min. Space charge simulations verify that the coil designs are effective, but for oblique x-ray tube geometries there is still distortion of the focal spot shape along with deflections of approximately 3 mm in the radial and circumferential directions on the anode.

Conclusions: Active shielding is an attractive solution for correcting the effects of magnetic fields on the x-ray focal spot. If extremely long fluoroscopic exposure times are required, longer operation times can be achieved by including a permanent magnet with the active shielding design.

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Key words: XMR, active shielding, x-ray production, magnetic fields, optimization

I. INTRODUCTION

I.A. Background

Using hybrid x-ray/MR systems has become more prevalent for image guidance during interventional procedures, with various vendors offering dual-modality suites.¹⁻⁶ These systems combine the three-dimensional imaging capabilities and excellent soft tissue contrast offered by MR with the high spatial/temporal resolution and device tracking ability of projection x-ray. The suites typically have a long

travel distances between MR and x-ray systems, on the order of 5 to 10 m. The long travel distances eliminate concerns about system compatibility and hardware modifications. However, switching between the modalities requires shuttling the patient from one system to the other. Typically, the patient has critically placed intravenous lines for drug delivery and anesthesia along with various monitoring systems attached. Thus, the process becomes cumbersome and is an impediment for repeated switching between the modalities.

With digital flat panel x-ray detectors replacing x-ray image intensifiers, close proximity hybrid system geometries are now feasible because flat panel detectors offer more resistance to magnetic fields. The first clinical implementation of a fully colocalized solution (i.e., where the MR and x-ray imaging field of views are in the same location) was implemented at Stanford University.⁷ A static-anode x-ray tube and flat panel detector were integrated into the gap of a split bore 0.5 T interventional MR system (Signa SP, GE Milwaukee, WI). The x-ray tube and detector were oriented such that anteroposterior projections could be acquired of patients in the supine position. The biggest advantage of the colocalized solution was that switching between modalities requires no patient motion. The system was used successfully for various clinical applications such as placement of transjugular intrahepatic portosystemic shunts (TIPS), chemoembolization of hepatic tumors, brain biopsy, hysterosalpingograms, and loopograms.^{8–11} However, the system is limited by the static anode x-ray tube output and the split bore 0.5 T magnet design. Having a static anode design limits the instantaneous heat loading capabilities of the anode target, which limits the maximum x-ray flux. This is problematic for several high flux angiographic procedures and abdominal interventions in larger patients. Furthermore, the Signa SP split bore magnet design is not prevalent worldwide, and the main field strength (0.5 T) limits the maximum achievable signal-to-noise ratio (SNR).

To overcome the limitations presented by the colocalized hybrid solution another system design was proposed¹² called closed bore XMR (CBXMR). The x-ray imaging hardware for the CBXMR system consists of a rotating anode x-ray tube and digital flat panel x-ray detector are mounted on opposite ends of a c-arm. The MR system consists of a 1.5 T closed bore magnet with standard gradient coil design. In this approach the x-ray and MR imaging field of views (FOVs) are separated by a distance of 2 m or less, which implies that the x-ray c-arm has to be placed no more than 1 m from the face of the MR shroud. The primary differences in hardware between the CBXMR system and the colocalized solution are that the x-ray tube has a rotating anode and the main field strength of the MR system is 1.5 T. These differences allow the CBXMR to be applicable to a broader range of clinical applications than the colocalized solution. Furthermore, because the systems are still in close proximity to each other rapid switching between the modalities is feasible.

To minimize the movement of critically placed catheters, guide wires, and other interventional tools we propose a CBXMR system geometry where the isocenter of the x-ray system is as close as 30 cm from the face of the MR shroud (Fig. 1). This geometry presents several compatibility issues for the x-ray source that stem from the MR fringe field, which is the magnetic field created by the MR system that is not confined to the interior of the bore. As shown by Brzozowski *et al.* the fringe field can degrade the performance of the x-ray tube induction motor, as well as alter the position and shape of the focal spot on the anode. Studies have been performed to solve the problems with the x-ray tube motor using an innovative nonmagnetic motor design that is immune to static

FIG. 1. The close proximity CBXMR geometry with x-ray isocenter 30 cm from the face of the MR shroud. The distance from the center of the MR imaging FOV to the face of the shroud is 96 cm.

external fields and can utilize the MR fringe field to generate torque.¹³ Work has also been performed on developing correction mechanisms for the changes in the focal spot. By using active shielding local cancellation of the MR fringe field in the region between the anode and cathode can be achieved, preserving the shape and position of the focal spot on the anode target. An active shielding design was implemented by placing resistive coils outside the x-ray tube housing, and experimentally evaluated for fringe field values on the order of 7 mT.¹⁴ However, for the system geometry proposed in Fig. 1 the fringe field magnitude can be as high as 100 mT. To locally cancel these higher field magnitudes we propose placing shielding coils inside the tube housing adjacent to the x-ray tube insert. Furthermore, the current density in the shielding coils has to be increased drastically. The increase in current limits the operation times of the shield because the conductor windings reach a temperature that can melt the conductor insulation. The limits on coil operation times will directly limit the fluoroscopic exposure times as well. The aims of this study are to design shielding geometries that are practical for fringe field magnitudes on the order of 100 mT, and to estimate the feasible operation times of the active shield geometries before temperature failure.

I.B. System geometry parameters

The exact geometry of the active shield design will depend on the specific value of the fringe field at the position of the x-ray tube, which depends on the system geometry. Figure 2(a) provides a qualitative sense of the behavior of the magnetic field lines due to the MR bore (1.5 T GE conquest short bore magnet) at the location of the x-ray tube (represented by the orange cylinder). The fringe field data are provided by the magnet manufacturer as a representative field map for a short bore 1.5 T magnet design. The data are axisymmetric and is plotted over a 40 cm by 40 cm region that contains the potential x-ray tube locations in Fig. 3 below.

We define the x-ray tube axis as the direction electrons travel from the cathode to the anode. From the orientation of the field lines it is evident that there is a large component of the field perpendicular to the tube axis. The relevant dimensions for the rendering in Fig. 2(a) are shown in the cross sectional view in Fig. 2(b). This specific geometry is suitable for neurological interventional procedures where most device

FIG. 2. (a) Magnetic field lines from a 1.5 T GE conquest short bore scanner with x-ray tube placed adjacent to the MR shroud. (b) Cross section of the system geometry for neuro procedures with no rotation of the c-arm. (c) Cross section of the system geometry for cardiac procedures which displays three possible system orientations, two of which require rotation of the c-arm.

guidance can be accomplished using anteroposterior or lateral views. However, cardiac applications require angulation of the x-ray system toward and away from the shroud of the MR scanner to obtain oblique views of the cardiac vessels. The geometric parameters for these procedures are shown in Fig. 2(c). The x-ray tube experiences different field conditions due to the spatial variability of the MR fringe field. In the standard neuro and cardiac geometries (i.e., no rotation of the x-ray system) the radial component of the fringe field (B_R) is perpendicular to the x-ray tube axis, while the longitudinal component (B_Z) is parallel to it. Both cardiac oblique geometries rotate the tube axis with respect to the fringe field lines (Fig. 4). Hence, the component of the magnetic field that is orthogonal to the electron trajectories (R_{ROT} direction) is composed of contributions from both B_R and B_Z . The same is true for the component parallel to the trajectories (Z_{ROT} direction). A summary of typical fringe field values for the different system geometries is shown in Table I.

R represents radial distance from the MR imaging FOV, while Z represents the longitudinal distance. All distances and all field values tabulated above are in centimeters and mT, re-

spectively. The first oblique geometry refers to when the x-ray tube is tilted CCW from the MR shroud, while the second is when the tube is tilted CW toward the shroud. The standard neuro and the second cardiac oblique geometries have the highest magnitude of field values, which are highlighted in bold. In this study these are the two geometries that will provide primary design constraints, but remarks will be made on the other geometries in the discussion.

II. METHODS

II.A. Charged particle motion in electric and magnetic fields

The motion for a charged particle in a combined electric and magnetic field environment is given by the Lorentz force equation,

$$\frac{d\vec{p}}{dt} = q \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right). \quad (1)$$

FIG. 3. Plots of the longitudinal and radial components for fringe field data for a 1.5 T short bore magnet over a 40 by 40 cm region outside of the magnet bore.

TABLE I. CBXMR system geometry and field parameters.

Geometry	Parameters	Distance from FOV (cm)	B_R (mT)	B_Z (mT)	B_R ROT (mT)	B_Z ROT (mT)	$\ B\ $ (mT)
Neuro	$R = 66$ cm	$Z = 126$	88.1	-7.9	88.1	-7.9	88.5
Cardiac	$R = 66$ cm	$Z = 150$	40.1	8.2	40.1	8.2	40.9
Cardiac oblique (1)	$R = 57$ cm	$Z = 182$	15.7	12.3	19.7	2.8	19.9
Cardiac oblique (2)	$R = 57$ cm	$Z = 128$	102.2	24.7	76.2	72.5	105.1

In the above expression $\vec{p} = m\vec{v}/\sqrt{1 - \|\vec{v}\|^2/c^2}$ is the relativistic momentum, $\vec{E}$ is the electric field vector, $\vec{B}$ is the magnetic field vector, q is the electron charge, $\vec{v}$ is the velocity, and c is the speed of light. The x-ray tube will be placed adjacent to the MR bore as shown in Fig. 1. This creates a crossed field environment (Fig. 5), where $\vec{E}$ represents the electric field created by the potential difference between the anode and the cathode, and $\vec{B}$ represents the radial component of the fringe field from the MR bore. Given magnetic field strengths given in Table I and typical voltages used in fluoroscopic procedures (from 70 to 120 kVp) we find that $|\vec{B}| > |\vec{E}|/c$. If we assume $\vec{E}$ and $\vec{B}$ are both static in time and uniform in space we know from the solution to Eq. (1) that electrons moving in this crossed field environment will experience a “drift” velocity in the $\vec{E} \times \vec{B}$ direction,¹⁵ which is the direction perpendicular to the RZ plane shown in Fig. 5. The drift velocity direction will be along the circumference of the anode leading to deflection of the focal spot in this direction. If the radial component of $\vec{B}$ (the component perpendicular to $\vec{E}$) is large enough, the electrons may be deflected outside the target area on the anode. The purpose of this study is to design and evaluate magnetostatic schemes to actively shield for the radial component of the MR fringe field, thus correcting for the deflection of the focal spot on the anode. In the above description, the effect of the longitudinal component of the fringe field is omitted as its magnitude is usually smaller than the radial component. However, once the radial component of the fringe field is cor-

rected for there will only be a longitudinal component remaining that is parallel to the electric field. It is known that this field environment will produce spiraling of the electrons around the field lines along with acceleration along the field lines. The analytic solution for the electron trajectories for this case are given in Wen *et al.*¹⁶ In this study we do not attempt to correct for the effects of the longitudinal magnetic field but we do include its presence in the electron trajectory simulations which are described in Sec. II.D.

II.B. Design problem description and optimization formulation

The given problem description is to design resistive active shielding coils that will create a magnetic field in the direction opposite to the radial fringe field generated by the superconducting MR bore at the x-ray tube. The design problem consists of determining the following criteria:

- Number of coils,
- Current values for each coil,
- Position of coils relative to x-ray tube,
- Practical dimensions for each coil.

Furthermore, we want to solve for these criteria in an “optimal” way. Qualitatively speaking, the number and power consumption of the coils needs to be as small as possible, while satisfying constraints related to the positions and dimensions of the coils.

The algorithm proposed by Xu *et al.*¹⁷ is an attractive starting point for this optimization problem because it proposes a minimum power design and flexibility in its geometric

FIG. 4. (a) X-ray tube geometry when the tube is rotated toward (CW) the MR shroud. The helical structure represents the filament. (b) Geometry with the radial direction orthogonal to the electron trajectories. (c) Geometry when the tube is rotated away (CCW) from the MR shroud.

FIG. 5. Diagram of the electric and magnetic field environment inside the X-ray tube. The direction of electron drift is into the page for this case.

constraints. It formulates the design problem using a linear matrix approach, i.e.,

$$\vec{b} = A\vec{i}, \quad (2)$$

where $\vec{b}$ is the field at the M target points, $\vec{i}$ is the current values at the N possible coil locations, and A_{mn} is the magnetic field generated at the m th target point due to a unit of current at the n th possible coil location. To compute the system matrix A we must determine which magnetic field components will be optimized. The algorithm only considers axis-symmetric coils, which implies that the field from each coil can be completely described by its radial ($B_{r \text{ coil}}$) and longitudinal ($B_{z \text{ coil}}$) components. We propose to use the *longitudinal* component from the coils to shield for the *radial* component from the MR bore. $B_{r \text{ coil}}$ is ignored in the following formulation because it will have close to zero magnitude at the target points. This allows us to proceed with the computation of A_{mn} from the analytical solution to the Biot-Savart law as follows:

$$A_{mn} = \left(\frac{\mu_0}{2\pi((r_n + \rho_m)^2 + (\zeta_m - z_n)^2)^{1/2}} \right) \times \left(K(\kappa_{mn}^2) + \frac{r_n^2 - \rho_m^2 - (\zeta_m - z_n)^2}{(r_n - \rho_m)^2 + (\zeta_m - z_n)^2} E(\kappa_{mn}^2) \right), \quad (3)$$

where K and E are the complete elliptic integrals of the first and second kind, respectively, and the argument κ_{mn}^2 , commonly referred to as the modulus, is defined as

$$\kappa_{mn}^2 = \left(\frac{4r_n\rho_m}{(r_n + \rho_m)^2 + (\zeta_m - z_n)^2} \right). \quad (4)$$

In Eqs. (3) and (4) r_n and z_n refer to the radius and longitudinal location of the n th coil, the coordinate pair (ρ_m, ζ_m) refers to the location of the m th target point, and μ_0 is the magnetic permeability of vacuum ($4\pi \cdot 10^{-7}$ N/A²).

For practical design problems the number of target points, M , will be much smaller than the number of possible coil locations, N , which implies that the system is underdetermined and has no unique solution. To obtain the solution that minimizes power consumption while imposing field homogeneity inequality constraints Xu's L1-norm optimization formulation can be used and is given below

$$\begin{aligned} \min : & \sum_{n=1}^N r_n |i_n| \\ \text{s.t.} : & \vec{b}_0(1 - \varepsilon) \leq A\vec{i} \leq \vec{b}_0(1 + \varepsilon). \end{aligned} \quad (5)$$

Here $\vec{b}_0$ and ε refer to the magnetic field value we are trying to correct for (i.e., the radial component of the MR bore fringe field at the x-ray tube) and the tolerance in the correction that we can allow for, respectively. The magnetic field being corrected for is treated as a constant because there is a ± 1 mT variation over the homogeneity region. By performing an auxiliary variable substitution it is possible to recast the L1-norm optimization problem stated in Eq. (5) into standard linear programming form [Eq. (A1)], for which many efficient numerical solution packages are available.¹⁸ The optimization

FIG. 6. Diagram of the optimization problem. The grids above and below the x-ray tube show candidate coil positions. The blue dotted region is where the field homogeneity constraints are applied.

problem is summarized in Fig. 6, which demonstrates the discretization of the candidate coil space surrounding the x-ray tube. One of the advantages to Xu's approach, which derives from the L1-norm minimization, is that the number of nonzero elements in the solution for $\vec{i}$ (i.e., the number of unique coils needed) is sparse. This property of the L1-norm minimization makes implementation of the coils practical, and henceforth we will refer to solutions to Eq. (5) as L1-norm minimization or L1-norm solutions.

The complete design algorithm requires iteration of the optimization formulation stated above. In the first iteration it is assumed that the system matrix A is computed for ideal current loops that exist at (r_n, z_n) , and that each current loop can have infinite current if necessary (i.e., there is no limit on i_n). The output from the first iteration gives initial positions and current values for a set of coils that satisfy the field homogeneity inequalities while consuming minimum power. Subsequent iterations differ from the initial iteration in two important ways. First, the possible coil locations (r_n, z_n) are restricted further to local areas centered around the positions that were solved for in the previous iteration, and the search grid is refined so that the L1-norm solution gives more accurate positions for the coil locations. Second, the assumption that each possible coil location is an ideal current loop is no longer used, which makes the computation of the system matrix A slightly more involved. Instead of ideal current loops, the coils have a rectangular cross sectional area with a fixed aspect ratio (W_i/L_i) and current density (i_i/W_iL_i). These parameters are held constant over all coils, and the distribution of the current is uniform across the cross sectional area of the coil. Using this approach each coil is modeled as a two-dimensional array of ideal current filaments, where the total current in the coil is spread equally over all of the ideal current filaments. As a result, each entry in the system matrix A is a sum over the individual ideal current filaments. The

FIG. 7. Diagram of the geometry constraints for finite sized coils. The inner radius of coil 1 must be large enough to not truncate the imaging FOV (blue cone). Both coils must also have 1 cm of separation from the glass envelope.

exact expression for the revised A_{mn} is shown in the Appendix [Eq. (A2)].

II.C. Adaptation to optimization formulation

A disadvantage in the above formulation is that the current density and aspect ratio of the candidate coils are free parameters in the optimization and have to be preset before computing the L1-norm solution. The current density and the aspect ratio are necessary precursors for determining the exact dimensions of the solution coils. Once the dimensions are computed, the solution is checked to determine if it satisfies feasibility geometry constraints (Fig. 7). There must be adequate space between the coil and the glass insert (i.e., the length of the coils in the Z direction is not too long). There are also constraints on the inner radii that depend on the x-ray FOV being imaged (represented by the blue cone in Fig. 7), and the outer radii that depend on the size of the tube housing. If a particular L1-norm solution cannot be physically realized because it violates one of the above constraints the optimization procedure is rerun with different values for the current density or aspect ratio. As a last resort stronger restrictions are placed on the coil location space (r_n, z_n) . This process is carried out iteratively until a feasible L1-norm solution is reached, which is guaranteed to be optimal in the minimum power sense for the given values of current density and aspect ratio. However, *there is no guarantee* that the values chosen for the current density and aspect ratio are optimal. To achieve this we must first define what is meant by “optimal” for these parameters.

Qualitatively speaking lower current densities are preferable because they produce lower joule heating leading to lower average conductor temperature and smaller resistance changes. The combination of these effects allows for longer continuous operation times of the coil, which broadens the range of procedures the x-ray tube can be used for. Since the overall current in the coil is fixed as an output of the L1-norm optimization, the only method available to lower the current density is to increase the cross sectional area of the coil. The area can be enlarged by either increasing the width (W) or the length (L), but not necessarily by the same percentage

FIG. 8. Cross section and associated dimensions of a possible coil, along with possible enlarged cross section to decrease current density.

(Fig. 8). The best choice for W_n and L_n for a particular coil (i.e., the aspect ratio) will allow for the maximum cross sectional area without violating any geometric constraints, while still satisfying all field homogeneity constraints specified in the L1-norm minimization.

To put the above description into a mathematical formulation let us define a vector $\vec{x}$ as follows:

$$\vec{x} = \begin{bmatrix} \vec{W} \\ \vec{L} \end{bmatrix}, \quad (6)$$

where the elements of $\vec{W}$ and $\vec{L}$ consist of the widths and lengths of the solution coils, respectively. Then the sum of the cross sectional area of the solution coils is defined as

$$\text{area}(\vec{x}) = \sum_{i=1}^s W_i L_i, \quad (7)$$

where s is the total number of coils for a particular solution. Using these definitions it is possible to formulate the nonlinear optimization problem shown below

$$\begin{aligned} \max : & \text{area}(\vec{x}) \\ \text{s.t.} : & G\vec{x} \leq \vec{d}, \\ & c(\vec{x}) = \vec{0}, \\ & \vec{x}_{\min} \leq \vec{x} \leq \vec{x}_{\max}. \end{aligned} \quad (8)$$

The formulation shown above is a nonlinear optimization problem with linear inequality and nonlinear equality constraints on $\vec{x}$. The matrix G represents all of the geometry constraints that can be written as linear inequalities, and the vector $\vec{d}$ contains the bounds for these inequalities. The function c is a vector-valued function that is nonlinear in $\vec{x}$ and only comes into play when the LP solution contains two or more coils. The purpose of c is to constrain the current densities in each of the solution coils such that they are equal, which is a necessary condition for any set of series connected coils that have a common current supply. The definitions of G , $\vec{d}$, and c are shown in the Appendix [Eqs. (A4)–(A6)].

Finally, $\vec{x}_{\max}$ and $\vec{x}_{\min}$ represent the upper and lower bounds that we are willing to allow for coil dimensions. Various methods exist to solve constrained nonlinear optimization problems, and the choice of method depends on the number of unknowns in the vector $\vec{x}$ and the number of constraints. The sparse nature of the L1-norm solution works to our benefit because it solves for the minimum number of necessary coils. Hence, the sizes of $\vec{x}$, G , and c are relatively small, and a solution method that employs sequential quadratic programming (SQP) can be used. These types of algorithms try to mimic Newton's method for nonlinear constrained optimization problems.¹⁹ Henceforth, we will refer to solutions to Eq. (8) as nonlinear programming or NLP solutions.

Up to this point two optimization schemes have been described. The L1-norm optimization gives current values and coordinate positions of coils that minimize power consumption, while the NLP optimization determines the coil geometry that maximizes the sum of the cross sectional area, which effectively minimizes the current density. The criteria for the design problem that were listed at the beginning of the optimization formulation can now be refined as follows:

- (i) Minimize power consumption and current density,
- (ii) Determine current, position, and aspect ratio,
- (iii) Satisfy field homogeneity and geometry constraints,
- (iv) Satisfy equal current density across coils.

To solve this problem we propose interleaving the optimization schemes of Eqs. (5) and (8) as shown in Fig. 9. We start with the first iteration of the L1-norm optimization because, due to the ideal current loop assumption, no information about the exact geometry of the coils is required. With the ideal current loop solution, the first iteration of the NLP

optimization can be run using the L1-norm information to compute G , $\vec{d}$, and c . The NLP solution will give the first geometry estimate for the coils, and all subsequent LP iterations no longer use the ideal current loop assumption, instead using discrete approximations for the coils. The exact discrete approximation used will change depending on the output from the NLP iteration. Updating the geometry estimate for the coils can cause fluctuations in the output from the L1-norm solution. Hence, we must iterate between the two optimization procedures. Interleaving the iterations is carried out until the value for the current density stabilizes. This approach was used to compute coil solutions for the two geometries with the largest magnetic field magnitudes, as listed in Table I.

II.D. Finite element validation of magnetostatic correction

To validate the coil design produced by the combined optimization algorithm (Fig. 9) a finite element model of the focusing cup and anode was used to perform a 3D space charge simulation (OPERA 3D v 11.0, Cobham). The outputs of this simulation have been validated experimentally at lower magnetic field values, as described in Ref. 20. The simulation solves the electrostatic version of Poisson's equation in the finite element geometry shown in Fig. 10. The geometry of the focusing cup and anode in the simulation were based on measurements from a standard fluoroscopy rotating anode x-ray tube. Specific details of the simulation can be found in the reference given above.

Electron trajectories were calculated using the model for the following three cases: no magnetic fields, only the magnetic field from the MR bore, and the combined magnetic field from the MR bore and the correction coils. The fringe field from the MR system was treated as a constant in the

FIG. 9. Iterative optimization scheme where the L1-norm and NLP optimizations are interleaved to generate feasible solutions for the design problem.

FIG. 10. Finite element model to simulate electron trajectories in the x-ray tube. Outputs are used to validate solutions from the optimization.

model because the value at the edges of the mesh varied ± 1 mT compared to the value at the center of the mesh. The electric potential difference between anode and cathode was held constant at 70 kV for all simulations. For the purposes of evaluating different shielding coil designs the value for the potential difference between anode and cathode is not critical. Although the proposed system will be operated over a range of diagnostic voltages (from 70 to 120 kVp) the shielding design will not change based on the operating voltage of the tube and the focal spot characteristics will be similar over these voltages. The correction coils were implemented in the model as finite size conductors with constant current density, with values for the geometry and current density of the coils derived from the optimization for both the neuro and second cardiac oblique system geometries. Ideally, we do not want to implement new coil geometries for each x-ray tube position. It is far simpler to implement one set of coils that is optimized for the most challenging fringe field environment, and decrease the current density for less challenging cases. To demonstrate feasibility of this strategy more simulations are presented where the coil solution that is optimized for the neuro geometry is used for all three cardiac system geometries. In these cardiac simulations the current density of the neuro coils is scaled in proportion to the field values.

II.E. Combination of correction coils with a permanent magnet

Correcting for magnetic fields on the order of 100 mT using resistive coil pairs will always raise concerns about the amount of heat being generated. A potential method to achieve lower current density in the coils, and hence less heat generation, is to design a solution that combines a permanent magnet with high remnant magnetization and high coercivity along with the shielding coils. The permanent magnet would be placed outside the x-ray tube insert on the side opposite from the output port with 1 cm of clearance between the insert and the face of the permanent magnet, and would be on-axis with any shielding coils that are present. The clearance allows oil that is circulating between the x-ray tube housing and a heat exchanger to come in contact with the surface of the insert, which is necessary for cooling purposes. The permanent magnet we choose for this study is a N40 grade neodymium (NdFeB) cylindrical disk (4 in. diameter, 2 in. height). The specific geometry is chosen because it is one of the largest Nd-FeB magnets that is commercially available (Amazing Magnets, Irvine, CA), therefore providing higher field values at long distances relative to a smaller geometry, and it does not have to be custom manufactured. The reasons for this choice are that the N40 grade has high nominal remanent magnetization (1.265 T) and high nominal coercivity (919 kA/m). The curie temperatures (T_C) of NdFeB magnet grades are above 300 °C, and at higher temperatures the permanent magnet material begins to lose its magnetization. The permanent magnet will not be exposed to temperatures in this range when inside the x-ray tube housing so there are no concerns about temperature demagnetization.

II.E.1. Permanent magnet demagnetization model

The shielding geometry described above will position the permanent magnet in the MR fringe field, and typically the radial component of the fringe field will be antiparallel to the polarization direction of the permanent magnet. These conditions can cause demagnetization of the neodymium magnet and it is necessary to numerically calculate these effects before computing an optimized shielding solution. The demagnetization effects are calculated using a magnetostatic finite element model (COMSOL ver. 4.2), where the following system of equations is solved

$$\begin{aligned}\vec{\nabla} \times \vec{H} &= \vec{J}, \\ \vec{B} &= \vec{\nabla} \times \vec{A}, \\ \vec{B} &= \vec{B}_R + \mu_0 \cdot \mu_r(\vec{H}) \cdot \vec{H}.\end{aligned}\quad (9)$$

In the above formulation the first equation is the time invariant version of Ampère's law, the second is the definition of the magnetic vector potential $\vec{A}$, and the third is the constitutive relation between $\vec{H}$ and $\vec{B}$ for general nonlinear materials. The unknown quantities in the system of equations are the magnetic field $\vec{H}$, the flux density $\vec{B}$, and the vector potential $\vec{A}$. These unknown vector fields are the superposition of contributions from the MR bore and from the permanent magnet. The known quantities are the current density $\vec{J}$, the remanent magnetization $\vec{B}_R$, the relative permeability tensor μ_r , and the magnetic permeability of vacuum μ_0 . It is assumed that all quantities are time invariant so that a static solution can be found. However, it should be noted that $\vec{B}_R$ and μ_r are material dependent. For air $\vec{B}_R$ is equal to $\vec{0}$ and μ_r is the identity matrix, but in the permanent magnet domain of the model $\vec{B}_R$ is nonzero and μ_r is derived from the BH curve for the specific magnet material. The data used for the permanent magnet properties were taken from manufacturer data specific for the N40 magnet grade.²¹ The system of equations shown in Eq. (9) is used to derive a single equation where the only unknown quantity is $\vec{A}$ [Eq. (A7)], and with appropriate boundary conditions a solution for $\vec{A}$ is obtained from which solutions for $\vec{B}$ and $\vec{H}$ can be derived.

II.E.2. Modeling the MR fringe field using ideal current loops

The $\vec{B}$ and $\vec{H}$ field variables in the demagnetization model are a superposition of the fields created by the permanent magnet ($\vec{B}_{PM}$, $\vec{H}_{PM}$) and the fringe fields created by the MR bore ($\vec{B}_{MR}$, $\vec{H}_{MR}$). The exact values of the permanent magnet fields will be dependent on the amount of demagnetization that occurs. When specifying the physics in the model, software restrictions do not allow the MR fringe field to be specified as an external variable. To work around this inconvenience, we use ideal current loops in the modeling software to create a magnetic field that mimics the fringe field (Fig. 11). The field created by the current loops is matched to the MR fringe field within certain tolerances (± 2.5 mT) throughout the mesh volume that includes and immediately surrounds the permanent magnet, which is denoted by the

FIG. 11. Diagram of the set of coils used to model the MR fringe field. Each pair of dots centered around the coil axis of symmetry represents one coil. The dotted line on the bottom left displays the boundary for the permanent magnet region in the model. The arrows represent the magnetic field vectors generated from the ideal current loops.

blue dotted region in Fig. 11. The radii sizes, positions, and currents in the loops are determined using the ideal current loop L1-norm formulation, with slight alterations to the algorithm:

- (i) The fringe field is not spatially uniform so the components of b_0 in Eq. (5) are no longer constant.
- (ii) The system matrix A_{mn} is changed to reflect the fact that we are modeling the *radial* component of the fringe field. We ignore the longitudinal component because it is much smaller and is not antiparallel to the polarization direction of the permanent magnet. The revised expression for A_{mn} is

$$A_{mn} = \left(\frac{\mu_0(\zeta_m - z_n)}{2\pi\rho_m((r_n + \rho_m)^2 + (\zeta_m - z_n)^2)^{1/2}} \right) \times \left(-K(\kappa_{mn}^2) + \frac{r_n^2 + \rho_m^2 - (\zeta_m - z_n)^2}{(r_n - \rho_m)^2 + (\zeta_m - z_n)^2} E(\kappa_{mn}^2) \right), \quad (10)$$

where it should be noted that as $\rho_m \rightarrow 0$, $A_{mn} \rightarrow 0$, and the definition for κ_{mn}^2 is the same as in Eq. (4). The number of coils needed to model the fringe field varies depending on the size of the region being modeled and on the tolerances allowed in the constraints for Eq. (5).

II.E.3. Computing optimized coils with the permanent magnet

After running the modified L1-norm algorithm described above for a single iteration the geometric parameters and current values of the ideal current loops are inserted into the FEM as edge currents. The software solvers are then used to compute the solutions for two independent cases, once with the permanent magnet and once without the permanent magnet. The former case solves for the complete $\vec{B}$ and $\vec{H}$ fields, while the latter solves for just the MR fringe fields, $\vec{B}_{MR}$ and $\vec{H}_{MR}$. By subtracting the MR fringe fields from the complete fields,

FIG. 12. Flow diagram for the active shielding optimization procedure when trying to derive coil solutions with the presence of a partially demagnetized permanent magnet field. Initially, the MR fringe field is modeled using ideal current loops. These current loops are inserted into the FEM as edge currents, and the FEM is then used to compute the demagnetized permanent magnet field. The requested shielding coil field can be derived from the permanent magnet field, and is fed into the iterative L1-norm/NLP solver to compute the optimized coil solution.

it is possible to isolate for the fields created just by the permanent magnet, $\vec{B}_{PM}$ and $\vec{H}_{PM}$. It should be noted that the permanent magnet fields calculated from this method include the demagnetization effects from the MR fringe fields. At this point the requested shielding coil field can be computed from the difference between the MR fringe field and the permanent magnet field, and can be fed in as a magnetic field constraint into the iterative L1-norm/NLP optimization scheme shown in Fig. 9. The requested shielding coil field will not be spatially uniform because it will be designed to complement the permanent magnet field so that their sum is uniform. As a result the optimization scheme is slightly modified to accommodate this change. The individual steps for the entire coil optimization procedure with the presence of a permanent magnet are illustrated in Fig. 12 for clarity. To summarize, the superposition of the optimized correction coil field with the permanent magnet field will work together to shield the x-ray tube from the radial component of the MR fringe field. The procedure was performed only for the neuro geometry (first row in Table I) as it has the largest magnitude of field in the radial direction and hence would receive the most benefit from the additional field provided by the permanent magnet.

II.F. Coil heat modeling

A lumped capacitance method²² is used to model the temperature dynamics of the shielding coils. The differential equation governing the heat transfer is shown in Eq. (11), with definitions and values of all parameters as listed in Table II.

$$I^2 R_0 (1 + \alpha(T - T_0)) - (hA(T - T_\infty) + \varepsilon\sigma(T^4 - T_\infty^4)) = \frac{d}{dt}(\rho V C_p T) \quad (11)$$

TABLE II. Lumped capacitance heat model parameters.

Parameter	Definition	Value
T	Temperature	Varies with time
T_∞	Ambient air temperature	297.3 °K
T_0	Coil temperature@ $t = 0$	298.5 °K
I	Single wire current	Coil dependent
R_0	Coil resistance@ $t = 0$	Coil dependent
α	Resistance temperature coeff.	$3.9 \times 10^{-3} \text{ K}^{-1}$
h	Convection coeff.	$15 \text{ W m}^{-2} \text{ K}^{-1}$
ε	Emissivity of copper	0.05
σ	Stefan-Boltzmann constant	$5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$
ρ	Density of copper	8940 kg m^{-3}
V	Coil volume	Coil dependent
A	Coil surface area	Coil dependent
C_p	Specific heat capacity	385 J kg K^{-1}

A spatially uniform temperature distribution is assumed to exist inside the coil at any moment in time. The assumption is valid because the Biot number for our coils, which is the ratio of heat conduction to heat convection, is much less than 0.1. Equation (11) is numerically integrated to solve for the temperature (T) of the coil as a function of time (t) using a variable time step Runge-Kutta method. As mentioned previously it is far simpler to implement one set of coils that is optimized for the most challenging fringe field environment, and decrease the current density for less challenging cases. Hence, the coil solution that is optimized for the neuro geometry is used for each set of temperature calculations where two coils are present, and the current density is scaled in proportion to the field values. This enables computation of a temperature solution for each x-ray tube position in Table I even though coil geometries are not explicitly optimized for the cardiac or cardiac oblique (1) geometries. However, a single coil geometry is reoptimized for the case that includes the neodymium permanent magnet, and the temperature solution for this case is based on the new geometry.

III. RESULTS

III.A. Optimized coil solutions

Optimized shielding coil solutions are computed for the standard neuro and the second cardiac oblique geometries. As stated earlier in general we shield for the radial component of the MR fringe field with the longitudinal component of the coil field. In the cardiac oblique case we shield for the com-

ponent of the MR fringe field that is parallel to the rotated radial direction (R_{ROT}). Hence, for the neuro geometry we optimize the solution to shield for 88 mT, and for 76 mT in the cardiac oblique geometry. For both solutions we impose the homogeneity constraint that the shielding coil field is within 0.5 mT of the requested value. This constraint is applied along the ten target points that are used for the optimization. The target points are on a plane that is positioned equidistant between the target area on the anode and the focusing cup. The points are distributed linearly on this plane with a separation distance of 1 mm. The results from the optimization procedure for both field conditions are summarized in Table III. Each solution contains a pair of coils, one on either side of the x-ray tube insert. The solutions for the on-axis fields produced by the optimized coils along with the convergence plots for the current density are shown in Fig. 13. As expected the field plots are within ± 0.5 mT of the requested field values. Given the current densities required by the coil pairs, a 5 kW constant current dc power supply would be more than sufficient to drive the coils, and these supplies are readily available from multiple manufacturers.

The need for optimization of the coil solutions can be demonstrated by comparing a simple one coil solution for the neuro geometry to the two coil optimized solution. In a one coil solution the geometry is similar to that of coil 1 shown in Fig. 7, and the parameters for a feasible one coil solution are shown in Table IV below. This particular solution consumes 1.21 kW of power compared to the 2.28 kW consumed by the two coil optimized solution. However, the field homogeneity of the one coil solution is far worse than the two coil solution, with a ± 9 mT deviation over the homogeneity region compared to ± 0.5 mT.

III.B. Space charge finite element simulation

Space charge simulations are performed for each coil solution in Table III to calculate the electron trajectories in the shielded environment created by the coils. For comparison trajectories are also calculated for the following two cases: no magnetic fields present, and only the MR fringe field present. Whenever the MR fringe field is included in these simulations, both the radial and longitudinal effects are included. The solutions for the electron trajectory simulations are shown in Fig. 14. The model of the cathode used in the calculations contains two filaments, which is standard in current rotating anode x-ray tubes. For the following calculations the filament with the smaller diameter is kept active while

TABLE III. Coil solution parameters for the neuro and cardiac oblique (2) geometries.

Geometry	B field (mT)	R_C (cm)	Z_C (cm)	W (cm)	L (cm)	I (A)	Weight (kg)	Current density (A/cm ²)
Neuro	88.1 ± 0.5	7.3 7	− 12.60	8.00	6.80	31 964	22.5	587.6
		9.5 5	5.70	5.12	4.20	12 635	11.5	587.6
Cardiac oblique (2)	76.2 ± 0.5	7.4 4	− 12.60	8.00	6.80	27 289	22.7	501.6
		9.6 2	5.70	5.24	4.20	11 049	11.9	501.6

FIG. 13. (a) On-axis field generated by the optimized coil solution for the neuro geometry. The dashed line represents the requested field. (b) On-axis field generated for the cardiac oblique (2) geometry. (c) Convergence plot for the current density over several iterations of the optimization algorithm.

the larger filament is inactive. As mentioned previously the homogeneity constraints are enforced on a plane that lies in between the cathode and the anode. However, the corrective field variation over the entire focusing cup volume is limited to less than 1.5 mT which allows the optimized coil solution to be viable for both filaments in the x-ray tube.

For the trajectories shown in Figs. 14(a) and 14(d) we compute the (x, y) coordinate of the intersection of each trajectory with the anode (Fig. 15). This enables us to visualize the deflection of the shielded trajectories shown in (d) relative to the control trajectories shown in (a). We see that there is a -2.75 mm deflection in the x direction and a 3.0 mm deflection in the y direction. This procedure is not done for the trajectories in Fig. 14(b) or 14(c) because for the former the electron beam does not reach the anode, and for the latter the coordinates of intersection are nearly identical to the control case. Space charge simulations are also computed for all three of the cardiac geometries using the shielding coils optimized for the neuro system geometry and scaling the current densities according to the corresponding field values. The current densities used for the cardiac, cardiac oblique (1) and cardiac oblique (2) geometries were 267.5, 131.4, and 508.2 A/cm²,

TABLE IV. Coil solution parameters for a single coil.

Parameter	Value
B field (mT)	88.1 ± 9
R_C (cm)	7.32
Z_C (cm)	6.92
W (cm)	6.6
L (cm)	6.6
I (A)	26 100
Current density (A/cm ²)	600

FIG. 14. (a) Calculated trajectories in the presence of no magnetic fields and 70 kV potential difference between cathode and anode. Color encoding represents relative displacement of electrons in the x direction. (b) Calculated trajectories in the presence of both radial and longitudinal component of the MR fringe field with no shielding coils. Fringe field values are taken for the neuro geometry. (c) Calculated trajectories for the neuro geometry including the presence of both components of the MR fringe field and shielding coils. (d) Calculated trajectories for the second cardiac oblique geometry including the presence of both components of the MR fringe field and shielding coils.

respectively. The results from these simulations are shown in Fig. 16.

III.C. Permanent magnet demagnetization model

III.C.1. Modeling the MR fringe field using ideal current loops

As mentioned previously the radial component of the MR fringe field (B_R) is modeled using ideal coil pairs with an L1-norm minimization based upon Eq. (10) to determine where to position the coils. The coils do not model the fringe field over the whole volume, but instead only try to match the behavior of the fringe field over a local volume in the vicinity of the permanent magnet. The field map for the values of B_R that the coil pairs are trying to emulate is shown in Fig. 17(a), while the map of the radial field created by the ideal coil pairs in the finite element model is shown in Fig. 17(b). The black rectangles in the figures indicate the cross section. Both the

FIG. 15. Projection of the intersection points of electron trajectories with the anode onto the $x-y$ plane. The points on the right are calculated from the trajectories in Fig. 13(a), while the points on the left are calculated from the trajectories in Fig. 14(d). The black dotted line demonstrates the relative shift in the centroid of all trajectories.

FIG. 16. (a) Calculated trajectories for the cardiac geometry using the neuro optimized coils with the current density set to 267.5 A/cm². (b) Calculated trajectories for the cardiac oblique (1) geometry using the neuro optimized coils with the current density set to 131.4 A/cm². (c) Calculated trajectories for the cardiac oblique (2) geometry using the neuro optimized coils with the current density set to 508.2 A/cm².

MR fringe field that is being modeled and the field generated by the ideal coil pairs are axis-symmetric, which implies that if the fields are in agreement over the two-dimensional areas shown in Fig. 17 then they agree for the entire three-dimensional volume. The difference map between the MR fringe field and the ideal coil pair field is shown in Fig. 17(c) showing ± 1 mT agreement over the central portion of the permanent magnet. The tolerance falls to ± 3 mT at the edges of the domain.

FIG. 17. (a) Radial fringe field map that is being modeled from the manufacturer data. (b) Radial fringe field map that is generated from the ideal coil pairs using optimization procedure. (c) Difference between field maps showing good relative agreement between (a) and (b).

TABLE V. Coil solution parameters from permanent magnet optimization.

Parameter	V alue
B field (mT)	88.1 ± 0.7
R_C (cm)	6.91
Z_C (cm)	5.81
W (cm)	8
L (cm)	4.42
I (A)	11 923
Current density (A/cm ²)	337.2

III.C.2. Optimized coil solutions with permanent magnets

An optimized shielding coil solution with the presence of the permanent magnet field is computed for the neuro geometry. The solution requires only one coil in addition to the permanent magnet, and the computed parameters are shown in Table V. It should be noted that the field homogeneity constraints for this solution are relaxed from ± 0.5 to ± 0.7 mT. The extra tolerance is added to improve the behavior of the algorithm so that it converges to coil solutions that are more practical. The current density for this solution is 57% of the current density in the two coil solution that does not include the permanent magnet.

The solution for the on-axis field values from the shielding coil and from the permanent magnet are shown in Fig. 18(a). This plot demonstrates that the optimization algorithm converges to a coil solution that complements the permanent magnet field, i.e., the shielding coil output compensates for the decreasing permanent magnet field output. The sum of the field outputs is shown in Fig. 18(b) to show that the sum of the fields does satisfy the field homogeneity constraints. A schematic diagram of the positioning of the solution coil and permanent magnet relative to the x-ray tube is shown in Fig. 19 below.

III.D. Lumped capacitance heat model

Temperature solutions for the shielding coils are calculated for each potential x-ray tube position. The temperatures are tabulated at 10 and 20 min, along with the time it takes the coils to reach 200 °C (Table VI). It is known that copper

FIG. 18. (a) On-axis field outputs from the optimized coil and the NdFeB cylindrical disk permanent magnet. (b) Sum of the on-axis fields from (a). The value of the radial component of the MR fringe field that we are trying to shield for is shown as the black line. The summation of the coil and permanent magnet stay within ± 7 G of the fringe field value.

magnet wire that is insulated with polyimide, which is one of the best heat insulators for this application, can operate under continuous load safely up to temperatures slightly above 200 °C, and can be used intermittently at 300 °C.²³ For the first four geometries we only list the data for the larger coil in the pair because it is known that it will be the limiting factor because it will experience quicker heating. The geometry labeled “Neuro using P.M.” is the permanent magnet optimized solution that contains only one shielding coil. The coils for the second cardiac oblique geometry never reach 200° C because of their low current density.

IV. DISCUSSION

IV.A. Optimized shielding coils

The coil solutions shown in Table III for the neuro and second cardiac oblique x-ray tube positions both consist of two coils each, which demonstrates that it is not possible to satisfy the field homogeneity constraints using only one coil for either geometry. Furthermore, for both geometries the coil that is closer to the focal spot in z has a smaller cross sectional area and less overall current. Intuitively this seems reasonable as the closer coil requires fewer amp-turns to gener-

ate a given amount of magnetic field at the focal spot than the far coil. The maximum allowable dimensions for the coil cross section in the optimization are 8 cm in each direction. These upper bounds are subjective and are enforced to limit the overall mass of the coils and the size of the x-ray tube housing. The weights for the optimized coil solutions are listed in Table III and the total weight for a coil pair is slightly more than 30 kg. This weight is comparable to the weight of a typical rotating anode x-ray tube assembly,²⁴ and can be counter-balanced on the detector side of the c-arm to achieve balanced rotation. Studies will need to be performed to evaluate the impact of the added weight on performance of the c-arm.

From the convergence plot it is seen that the algorithm initially underestimates the current density. This occurs because the initial values in the system matrix A_{mn} using the ideal current loop assumption are overestimated. As the values of system matrix become more accurate and the positions of the coils stabilize, the current density also reaches a stable value over subsequent iterations of the algorithm. Fluctuations on the order of less than 1% are seen in the current density values due to submillimeter adjustments in the exact coil locations between iterations.

Compared to the two coil solution, a simple single coil solution consumes less overall power by approximately half. However, as the current density is comparable for both approaches the feasible continuous operation times of the coils will also be comparable. The disadvantage of using the single coil solution is that the homogeneity of the field is far

FIG. 19. Schematic diagram of the permanent magnet solution geometry with a single coil relative to the x-ray tube insert.

TABLE VI. Coil heat model solutions for various field conditions.

Geometry	Flux density (mT)	$c.d.$ (A/cm ²)	$T@10$ mins (°C)	$T@20$ mins (°C)	t to 200 °C (mins)
Neuro	88.1	58 7.6	137	286	15
Cardiac	40.1	26 7.5	46	65	1 06
Cardiac oblique (1)	19.7	13 1.4	28	31	N/A
Cardiac oblique (2)	76.2	50 8.2	101	194	21
Neuro using P.M.	88.1	33 7.2	58	88	59

worse than in the two coil case. A ± 9 mT variation over the homogeneity region would cause the shape of the focal spot to be distorted. This distortion has been demonstrated in previous work where a ± 5 mT variation causes “shearing” of the focal spot that can have a negative impact on imaging performance.²⁰

IV.B. Electron trajectory simulations

The trajectories in Fig. 14(a) represent the control case where there are no magnetic fields and the electric field created by the focusing cup directs the trajectories toward the center of the anode. Once the MR fringe field is added into the simulation [Fig. 14(b)] the trajectories curve significantly in the $-x$ direction demonstrating the need for some form of correction to achieve a reasonable focal spot on the anode. The trajectories for the neuro geometry in Fig. 14(c) display near perfect correction even with the presence of -8 mT in the z direction.

Up to this point we have ignored the longitudinal component of the MR fringe field (B_z) and have designed our coils to only shield for the radial component. By including the effects of the longitudinal component the trajectory calculations will produce a more realistic estimate of where the electrons will intersect the anode. These effects are evident for the trajectories in Fig. 14(d) where there is a relatively large component (72.5 mT) of the fringe field in the z direction. In this case the trajectories do not intersect with the anode at their usual position as in Figs. 14(a) and 14(c). Instead, the electrons are constricted by the presence of the longitudinal magnetic field and travel parallel to the field lines in a helical motion, where the radius of the helix is inversely proportional to the magnitude of the longitudinal component of the field. This behavior causes the focal spot shift in the $-x$ direction seen in Fig. 15. The shift in the y direction occurs because the focusing cup is designed such that in the vicinity of the smaller diameter filament it creates an electric field component in the $-x$ direction. This electric field is orthogonal to the magnetic field in the z direction and together they will produce a $\vec{E} \times \vec{B}$ drift in the y direction. However, both the x and y focal spot shifts in this case are small enough (on the order of 3 mm). Using a standard fluoroscopy x-ray tube collimator it was demonstrated that a 3 mm focal spot shift produces a 6 mm FOV shift on the detector, and that a 5 mm shift was necessary to create FOV truncation.²⁵ Using a precalibration procedure it is possible to know the true FOV for all tube positions such that the FOV shift will not impact imaging performance. Since the focal spot size is not increased the resolving power of the x-ray system will not be degraded.

The simulations in Figs. 16(a) and 16(b) are virtually identical to those shown in Fig. 14(c), while the simulation in Fig. 16(c) is virtually identical to the simulation in Fig. 14(d). The agreement between the simulations demonstrates that it is not necessary to optimize new coil parameters for the different system geometries. Instead, using the neuro optimized coils and scaling the current densities appropriately according to the radial magnetic field magnitude is a feasible approach.

IV.C. Permanent magnet solution

The difference map shown in Fig. 17(c) demonstrates that in the worst case the radial component of the ideal coil pair field is within ± 3 mT of the MR fringe field, but for the central region of the permanent magnet the tolerance is within ± 1 mT. The agreement between the field maps is distinctly worse in the left-hand region of the permanent magnet domain. This is the case because this area is closer to the position of the coils in the finite element model, and the spatial variation of the fields is much larger at these positions making the finite element model outputs less accurate. However, a ± 3 mT difference in the field values will not make a significant difference to the permanent magnet field that is solved for by the model, as it takes fields that are orders of magnitude larger to get substantial demagnetization in the model. The magnetic field from the shielding coil is ignored in the demagnetization model as it is small at the location of the permanent magnet (5 mT on average) and is parallel to the magnetization direction.

The primary benefit from including the permanent magnet is the reduced current density in the shielding coil, which lowers the heat load leading to longer operation times. The cost of the neodymium magnet is approximately \$1000 and is on same order of magnitude as the cost for having one of the shielding coils constructed. The major trade-off from including the permanent magnet is that it makes the active shield more inflexible. With resistive coil pairs it is possible to dial down the current density by adjusting the power supply settings if lower field strengths are required. However, it is not possible to adjust the field strength of the permanent magnet without subjecting it to further demagnetization. Another option would be to remove the permanent magnet for system geometries where the x-ray tube is placed at lower field strengths, but this is also infeasible since permanent magnet is positioned inside the x-ray tube housing so that it is as close to the tube insert as possible. A last resort is to reverse bias the shielding coils so that they cancel a portion of the permanent magnet field. For example in the first cardiac oblique geometry the x-ray focal spot is 182 cm from the MR imaging FOV so the effective radial component ($B_{R\text{ ROT}}$) that needs to be shielded is only 20 mT. The permanent magnet creates on average 40 mT in the 1 cm region near the focal spot. Thus, the shielding coil needs to be reverse biased to generate approximately 20 mT so that the permanent magnet would only shield for the necessary 20 mT. These calculations are just crude approximations since all of the fields being considered have spatial dependence, and the effectiveness of the shield would vary from position to position. Further work is necessary to calculate the spatial nonuniformity of the shield as well as to explore the possibility of introducing more resistive coils to improve the uniformity properties for these low field cases.

IV.D. Heat model

The coil heat solutions show that the operating times will be limited under continuous load. In the neuro geometry we get 15 min of continuous operation before reaching the

200 °C limit, while in the second cardiac oblique geometry we get 21 min of operation. The operation times for the other geometries are significantly longer because the required current densities are lower. It is difficult to predict the exact timings that the coils are required to be active for a given interventional procedure as there are patient specific variables that will determine the amount of fluoroscopy time needed. The data from the TIPS study utilizing hybrid image guidance performed by Kee *et al.* can be used to provide estimates of how long the coils would need to be active. For patients in this study an average of 22.3 min of fluoroscopy time was required over an average total procedure time of 2.5 h. These would be feasible heat loading conditions for the optimized coils even when positioned in the neuro geometry as the loading is not continuous and is spread over a longer period of time giving the coils time to cool between exposures.

The operation time estimates for the coil solutions are conservative because they are calculated using a convection coefficient that is appropriate for simple air cooling. If the coils are inserted into the x-ray tube housing they will benefit from the presence of the circulating oil and the heat exchanger which will increase the convective heat transfer from the coils yielding longer operation times. A custom housing would have to be built for this approach such that there would be appropriate space for the coils and circulating oil. Additional modeling is necessary to achieve accurate temperature estimates for the oil cooled boundary condition and to determine what efficiency heat exchanger is needed to keep the oil below its maximum working temperature of 65 °C.²⁶ Furthermore, the coil designs in this study are meant to be solutions for the first prototype of the system to demonstrate proof of concept. Future iterations of the CBXMR system could include coils with water cooling internal to each individual conducting loop volume, a common approach for high current density applications. In this approach the coil temperatures are controlled by the flow rate of the water and the efficiency of the heat exchanger.

V. CONCLUSIONS

An iterative optimization scheme was implemented to solve for minimum power active shielding coils for an x-ray tube operating in a strong magnetic field. The optimization scheme interleaves between linear and nonlinear programming stages. Linear programming is used to compute the active shielding coil currents and positions in space, while nonlinear programming is used to compute the cross sections of the coils. The performance of the coil solutions from the optimization scheme was verified using a space charge finite element model that simulates the electron trajectories inside the x-ray tube. The simulations demonstrate that the deflection of the focal spot can be controlled to approximately 3 mm in each direction for field strength magnitudes on the order of 100 mT.

To alleviate heating concerns the optimization scheme was modified to compute coil solutions in the presence of a permanent magnet field. A commercially available neodymium

magnet was chosen for these calculations because of its high energy product. The fields from the permanent magnet, along with any demagnetization effects, were calculated using a magnetostatic finite element model. The current density of the new coil geometry computed with the presence of the permanent magnet field was found to decrease dramatically. The current densities and coil geometries from the optimized solutions were fed into a lumped capacitance temperature model, which estimated how long the coils can be operated before reaching the failure point of the insulation material. If longer fluoroscopic exposure times are required then the permanent magnet solution becomes more attractive. However, there are concerns about the flexibility of the permanent magnet approach for low magnetic field cases.

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APPENDIX: DETAILS OF OPTIMIZATION FORMULATION

1. L1-norm optimization reformulation

The constrained optimization shown in Eq. (A1) is the canonical linear programming formulation of the L1-norm optimization shown in Eq. (5):

$$\begin{aligned} \min : & \sum r_n t_n \\ \text{s.t.} : & A\vec{i} \leq B_0(1 + \varepsilon), \\ & -A\vec{i} \leq -B_0(1 - \varepsilon), \\ & |i_n| \leq t_n, n = 1, \dots, N. \end{aligned} \quad (\text{A1})$$

Here we have substituted the auxiliary variables t_n for the currents i_n while adding the constraint $|i_n| \leq t_n$. This enables us to remove the L1-norm from the functional being minimized. Since there are no other constraints on t_n when the weighted sum of t_n is minimized it is guaranteed that the $|i_n| \leq t_n$ constraint will be strictly satisfied.

2. System matrix definition for finite coil sizes

Equation (A2) gives the definition of the system matrix for a finite cross section coil. Here $r_n + \Delta r_i$ and $z_n + \Delta z_j$ represent the radial and longitudinal location of the (i,j) ideal current loop within the thick approximation of the coil. S is the number of ideal current loops that the thick approximation of the coil is composed of in the radial direction, while T is the number of ideal current loops in the longitudinal direction. We normalize the contribution from each ideal current loop by the product ST because the current for the finite size coil is assumed to be spread equally among all of the ideal current loops. To compute the revised system matrix we also have to

recompute the modulus as shown in Eq. (A3).

$$A_{mn} = \frac{1}{ST} \sum_{j=1}^T \sum_{i=1}^S \left(\frac{\mu_0}{2\pi((r_n + \Delta r_i) + \rho_m)^2 + (\zeta_m - (z_n + \Delta z_j))^2)^{1/2}} \right) \times \left(K(\kappa_{mn}^{ij})^2 + \frac{(r_n + \Delta r_i)^2 - \rho_m^2 - (\zeta_m - (z_n + \Delta z_j))^2}{((r_n + \Delta r_i) - \rho_m)^2 + (\zeta_m - (z_n + \Delta z_j))^2} E(\kappa_{mn}^{ij})^2 \right), \quad (\text{A2})$$

$$\kappa_{mn}^{ij} = \left(\frac{4(r_n + \Delta r_i)\rho_m}{((r_n + \Delta r_i) + \rho_m)^2 + (\zeta_m - (z_n + \Delta z_j))^2} \right). \quad (\text{A3})$$

3. Nonlinear optimization constraint definitions

In the nonlinear programming optimization formulation we had linear inequality constraints in the form of $G\vec{x} \leq \vec{d}$, as well as nonlinear equality constraints in the form $c(\vec{x}) = 0$. As a reminder the unknown vector $\vec{x}$ was defined as $\vec{x} = [\vec{W}^T \vec{L}^T]^T$. The definitions of G , $\vec{d}$, and c will depend on the exact number of coils contained in the solution output by the L1-norm solver. For the x-ray tube geometries considered in this work that do not include permanent magnets the L1-norm solution generally consist of two coils, one on either side of the x-ray tube. For these cases G will be a 4×4 matrix and $\vec{d}$ will be a 4×1 vector, as there are four unknowns in the vector $\vec{x}$ and 4 linear inequality constraints on the unknowns. The definitions of G and $\vec{d}$ for the two coil case are given below in Eqs. (A4) and (A5), respectively.

$$G = \begin{bmatrix} 1/2 & 0 & l_D/4l_{\text{FSD}} & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}, \quad (\text{A4})$$

$$\vec{d} = \begin{bmatrix} r_1 - z_1 l_D/2l_{\text{FSD}} \\ r_2 \\ z_1 - l_{C_1} \\ -l_{C_2} - z_2 \end{bmatrix}. \quad (\text{A5})$$

The definitions for the variables used in the definitions of G and $\vec{d}$ are given in Table VII. The clearance parameters refer

TABLE VII. Definitions for parameters used in nonlinear optimization procedure.

Parameter	Definition
r_1	Radial location of coil 1
r_2	Radial location of coil 2
z_1	Longitudinal location of coil 1
z_2	Longitudinal location of coil 2
l_D	Detector length
l_{FSD}	Focal spot to detector length
l_{C_1}	Clearance of coil 1 from focal spot
l_{C_2}	Clearance of coil 2 from focal spot

to the minimum distance required from the x-ray focal spot to the resistive coils.

The nonlinear equality constraint function $c(\vec{x})$ for the two coil case is given below in Eq. (A6). This constraint computes the difference in current densities between the two coils and sets the difference equal to 0. This approach is justified because the two coils will be series connected to the same power supply unit, which implies that their current densities are equal

$$c(\vec{x}) = \frac{i_1}{W_1 L_1} - \frac{i_2}{W_2 L_2} \quad (\text{A6})$$

4. Permanent magnet finite element model formulation

Equation (A7) is the differential equation for the magnetic vector potential in vector notation that is solved in the permanent magnet finite element model. The equation is derived by taking the curl of both sides of the constitutive relation given in the system of equations shown in Eq. (9), then making the appropriate substitutions using Ampère's law and the definition of the magnetic vector potential,

$$\vec{\nabla} \times \left(\frac{1}{\mu_r(\vec{H})} \cdot \vec{\nabla} \times \vec{A} \right) = \mu_0 \left(\vec{J} + \vec{\nabla} \times \frac{\vec{B}_R}{\mu_0 \mu_r(\vec{H})} \right). \quad (\text{A7})$$

Equation (A8) is the boundary condition on the magnetic vector potential that is applied at the edge of the finite element mesh. This specific boundary condition is referred to as magnetic insulation and sets the tangential component of the vector potential at the boundary equal to 0,

$$\vec{n} \times \vec{A} = 0. \quad (\text{A8})$$

The boundary condition is applied on an ellipsoid volume that encapsulates the entire simulation (i.e., the ellipsoid serves as a bounding box). Studies were performed that altered the dimensions of the ellipsoid to determine if the boundary condition had significant impact on the results. It was found that increasing the dimensions of the ellipsoid only created differences on the order of fractions of mT.

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¹ T. J. Vogl *et al.*, "Hybrid MR interventional imaging system: combined MR and angiography suites with single interactive table. Feasibility study in vascular liver tumor procedures," *Eur. Radiol.* **12**(6), 1394–1400 (2002).

- ²M. W. Wilson *et al.*, "Experimental renal artery embolization in a combined MR imaging/angiographic unit," *J Vasc Interv. Radiol.* **14**(9 Pt 1), 1169–1175 (2003).
- ³K. S. Rhode *et al.*, "Registration and tracking to integrate x-ray and MR images in an XMR facility," *IEEE Trans. Med. Imaging* **22**(11), 1369–1378 (2003).
- ⁴J. Kucharczyk *et al.*, "Cost-efficacy of MR-guided neurointerventions," *Neuroimaging Clin. N. Am.* **11**(4), 767–772 (2001).
- ⁵R. M. Chu *et al.*, "Minimally invasive procedures. Interventional MR image-guided functional neurosurgery," *Neuroimaging. Clin. N. Am.* **11**(4), 715–725 (2001).
- ⁶K. S. Rhode *et al.*, *XMR Guided Cardiac Electrophysiology Study and Radio Frequency Ablation* (SPIE, 2004).
- ⁷R. Fahrig *et al.*, "A truly hybrid interventional MR/x-ray system: Feasibility demonstration," *J. Magn. Reson. Imaging* **13**(2), 294–300 (2001).
- ⁸S. T. Kee *et al.*, "MR-guided transjugular intrahepatic portosystemic shunt creation with use of a hybrid radiography/MR system," *J. Vasc. Interv. Radiol.* **16**(2 Pt 1), 227–234 (2005).
- ⁹R. Fahrig *et al.*, "Truly hybrid interventional MR/x-ray system: Investigation of *in vivo* applications," *Acad. Radiol.* **8**(12), 1200–1207 (2001).
- ¹⁰R. Fahrig *et al.*, "First use of a truly-hybrid X-ray/MR imaging system for guidance of brain biopsy," *Acta Neurochir. Suppl. (Wien)* **145**(11), 995–997 (2003).
- ¹¹A. Ganguly *et al.*, "Truly hybrid x-ray/MR imaging: Toward a streamlined clinical system," *Acad. Radiol.* **12**(9), 1167–1177 (2005).
- ¹²L. Brzozowski *et al.*, "Compatibility of interventional x-ray and magnetic resonance imaging: Feasibility of a closed bore XMR (CBXMR) system," *Med. Phys.* **33**(8), 3033–3045 (2006).
- ¹³P. Lillaney, R. Bennett, and R. Fahrig, "Motor design for an MR-compatible rotating anode x-ray tube," *Proc. 18th ISMRM*, 2010.
- ¹⁴J. A. Bracken *et al.*, "Closed bore XMR (CBXMR) systems for aortic valve replacement: Active magnetic shielding of x-ray tubes," *Med. Phys.* **36**(5), 1717–1726 (2009).
- ¹⁵J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).
- ¹⁶Z. Wen *et al.*, "Investigation of electron trajectories of an x-ray tube in magnetic fields of MR scanners," *Med. Phys.* **34**(6), 2048–2058 (2007).
- ¹⁷H. Xu *et al.*, "Homogeneous magnet design using linear programming," *IEEE Trans. Magn.* **36**(2), 476–483 (2000).
- ¹⁸S. Mehrotra, "On the implementation of a primal-dual interior point method," *SIAM J. Control Optim.* **2**, 575–601 (1992).
- ¹⁹M. C. Biggs, L. C. W. Dixon, and G. P. Szegö, "Constrained minimization using recursive quadratic programming," in *Towards Global Optimization* (North-Holland, Amsterdam, 1975). pp. 341–349.
- ²⁰P. Lillaney *et al.*, *Development of an MR Compatible Rotating Anode X-Ray Tube* (SPIE, 2008).
- ²¹Neodymium-Iron-Boron Magnet Catalogs. Properties for different neodymium magnet grades. Available from: http://www.arnoldmagnetics.com/Neodymium_Literature.aspx.
- ²²F. P. Incropera, *Fundamentals of Heat and Mass Transfer*, 6th ed. (Wiley, Hoboken, NJ, 2007), Vol. xxv, p. 997.
- ²³D. M. Jewett, "Electrical heating with polyimide-insulated magnet wire," *Rev. Sci. Instrum.* **58**(10), 1964–1967 (1987).
- ²⁴Dunlee DR 1400 Rotating Anode X-Ray Tube Assembly, 2007; X-Ray Tube Specifications Sheet. Available from: http://www.dunlee.com/resources/content/1/0/8/0/documents/LF_DR1400.pdf.
- ²⁵J. A. Bracken *et al.*, "Closed-bore XMR (CBXMR) systems for aortic valve replacement: X-ray tube imaging performance," *Med. Phys.* **36**(4), 1086–1097 (2009).
- ²⁶Industrial X-Ray Tube. Data related to working temperature of oil inside x-ray tube housing. Available from: <http://www.varian.com/media/xray/products/pdf/eg50.pdf>.