Statistical CT noise reduction with multiscale decomposition and penalized weighted least squares in the projection domain

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Purposes: The suppression of noise in x-ray computed tomography (CT) imaging is of clinical relevance for diagnostic image quality and the potential for radiation dose saving. Toward this purpose, statistical noise reduction methods in either the image or projection domain have been proposed, which employ a multiscale decomposition to enhance the performance of noise suppression while maintaining image sharpness. Recognizing the advantages of noise suppression in the projection domain, the authors propose a projection domain multiscale penalized weighted least squares (PWLS) method, in which the angular sampling rate is explicitly taken into consideration to account for the possible variation of interview sampling rate in advanced clinical or preclinical applications.

Methods: The projection domain multiscale PWLS method is derived by converting an isotropic diffusion partial differential equation in the image domain into the projection domain, wherein a multiscale decomposition is carried out. With adoption of the Markov random field or soft thresholding objective function, the projection domain multiscale PWLS method deals with noise at each scale. To compensate for the degradation in image sharpness caused by the projection domain multiscale PWLS method, an edge enhancement is carried out following the noise reduction. The performance of the proposed method is experimentally evaluated and verified using the projection data simulated by computer and acquired by a CT scanner.

Results: The preliminary results show that the proposed projection domain multiscale PWLS method outperforms the projection domain single-scale PWLS method and the image domain multiscale anisotropic diffusion method in noise reduction. In addition, the proposed method can preserve image sharpness very well while the occurrence of “salt-and-pepper” noise and mosaic artifacts can be avoided.

Conclusions: Since the interview sampling rate is taken into account in the projection domain multiscale decomposition, the proposed method is anticipated to be useful in advanced clinical and preclinical applications where the interview sampling rate varies. © 2012 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4745564>]

Key words: CT, noise reduction, statistical noise reduction, multiscale decomposition, penalized weighted least squares

I. INTRODUCTION

With increasing concerns about the biological risks posed by radiation exposure in diagnostic x-ray computed tomography (CT),¹ tremendous effort has been devoted to developing radiation dose reduction solutions for CT imaging.^{2–15} Among all the efforts, the strategy to suppress noise in CT imaging with algorithmic solutions in either the image or projection domain is of relevance. For example, the anisotropic diffusion-based method, which can be implemented in either the image^{2,14–18} or projection domain,^{3,4} is commonly utilized in practice. The image domain-based anisotropic diffusion solutions^{19,20} are in general computationally efficient, in which a noise map is usually employed to guide the anisotropic diffusion for noise reduction.¹⁷ However, the image domain-based noise reduction approaches may suffer from streak artifacts

that are common in CT imaging, particularly in the scenarios wherein x-ray exposure is relatively low.²¹ In addition, due to the intrinsic features of anisotropic diffusion, mosaic effects may appear in the CT images with noise reduced, which may necessitate the application of blending techniques to restore the image's visual fidelity to radiologists.¹⁵ In the projection domain-based noise reduction approaches, at least two strategies are currently in use. One is to minimize an objective function through an iterative forward- and back-projection,⁵ while the other conducts an iterative operation in the projection domain only. Since the *a priori* information in both the image and projection domains can be utilized, the former can achieve a better performance in noise reduction. However, an iteration involving both the image and projection domains is computationally demanding. Recognizing these factors, we believe that, from the viewpoints of noise reduction, artifact

avoidance, and visual fidelity, the statistical noise reduction methods with operations in the projection domain only are preferable.

Based on an experimental investigation of the noise properties in CT projection data,⁶ a number of projection domain-based iterative noise reduction algorithms have been developed,^{7–12} wherein either the method of maximum *a posteriori* (MAP) estimation or penalized weighted least squares (PWLS) is employed. The PWLS has been recognized as one of the best methods for statistical CT noise reduction in projection domain,¹³ and its combination with multiscale decomposition can further enhance its performance.¹¹ In the existing projection domain multiscale PWLS method for statistical noise reduction, the multiscale decomposition is usually carried out using wavelet-based methods.¹¹ A series of projection views are acquired at an interview sampling rate that is comparable to the intraview sampling rate,¹¹ in which the angular interval determining the interview sampling rate is usually not taken into account in the multiscale decomposition. In this paper, we propose a novel projection domain PWLS method for statistical noise reduction, in which the multiscale decomposition is carried out using isotropic diffusion. As such, the interview sampling rate can play an explicit role in the projection domain multiscale decomposition. With the projection data simulated by computer and acquired by a CT scanner, we conduct an experimental investigation to evaluate and verify the performance of the proposed projection domain multiscale PWLS method for noise reduction in CT imaging and its robustness over applications.

II. MATERIALS AND METHODS

The algorithm starts by converting the partial differential equation (PDE) of isotropic diffusion in the image domain into its counterpart in the projection domain, wherein a multiscale decomposition is implemented. At each scale, the PWLS method associated with an objective function is utilized for noise reduction. Overall, the algorithm consists of five major steps: (i) Conversion of the isotropic diffusion from image domain into projection domain; (ii) multiscale decomposition in projection domain; (iii) statistical noise reduction with the PWLS method in projection domain; (iv) multiscale composition in projection domain; and (v) image reconstruction from the noise-reduced projection data. A detailed description of the first three algorithmic steps is given below.

II.A. Multiscale decomposition in image domain

In image domain, the multiscale decomposition is implemented using an isotropic diffusion PDE.²² Letting $\mathbf{x} = (x_1, x_2)$ denote a point in the two-dimensional (2D) image domain and $f(\mathbf{x})$ denote an object function with a finite support, the corresponding isotropic diffusion PDE can be expressed as²²

$$\frac{\partial}{\partial t} f(\mathbf{x}, t) = d \Delta f(\mathbf{x}, t), \quad t \geq 0, \quad (1)$$

where t denotes the time, d is a conduction constant, and Δ is the Laplacian operator. At $t = 0$, we define

$$f(\mathbf{x}, 0) = f(\mathbf{x}). \quad (2)$$

According to Ref. 22, the solution to Eq. (1) is

$$f(\mathbf{x}, t) = G(\mathbf{x}, \sigma^2) \otimes f(\mathbf{x}, 0), \quad \sigma^2 = 2dt, \quad (3)$$

where character “ $\otimes$ ” denotes a 2D convolution with respect to $\mathbf{x}$. $G(\mathbf{x}, \sigma^2)$ is called the Gaussian convolution kernel function and is defined as^{22,23}

$$G(\mathbf{x}, \sigma^2) = \frac{1}{2\pi\sigma^2} \exp(-\|\mathbf{x}\|_2^2 / (2\sigma^2)), \quad (4)$$

where $\|\cdot\|_2$ denotes the l_2 -norm. Applying a Fourier transform on both sides of Eq. (4), we get the Fourier transform of the Gaussian convolution kernel function

$$\tilde{G}(\mathbf{v}, \sigma^2) = \exp(-\|2\pi\mathbf{v}\|_2^2 \sigma^2 / 2), \quad (5)$$

where $\mathbf{v}$ represents a vector in frequency domain.

In the field of image processing, the multiscale decomposition using the Gaussian convolution kernel function has become a common practice,²⁴ and presented in Fig. 1(a) is a flow chart to show its detailed algorithmic steps.

FIG. 1. The flow charts showing the algorithmic steps of (a) image domain multiscale noise reduction and (b) projection domain multiscale noise reduction, in which the noisy projection data are denoted by superscript n , while the decomposed projection data at each scale are represented by the corresponding scale as subscript.

II.B. Radon transform of an object function's Laplacian transform

Let $\theta = (\theta_1, \theta_2) = (\cos\theta, \sin\theta)$ denote a unit vector in 2D space. The Radon transform, i.e., the projection of an object function $f(\mathbf{x})$ in the parallel-beam geometry, can be expressed as²⁵

$$\begin{aligned} \mathbf{R}f(\theta, s) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\mathbf{x}) \delta(s - \mathbf{x} \cdot \theta) d\mathbf{x} \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x_1, x_2) \\ &\quad \times \delta(s - x_1 \cos\theta - x_2 \sin\theta) dx_1 dx_2, \end{aligned} \quad (6)$$

where θ and s denote the view-angle and distance from the central ray, respectively, and $\delta(\cdot)$ is the Dirac function. Applying the Hilbert transform on both sides of Eq. (6) and letting $p(\theta, s)$ denote $\mathbf{R}f(\theta, s)$, we get²⁵

$$\mathbf{H}p(\theta, s) = p.v. \int_{-\infty}^{+\infty} \frac{p(\theta, s')}{\pi(s - s')} ds', \quad (7)$$

where $p.v.$ denotes the Cauchy principal value integral. Furthermore, the central slice theorem can be written as²⁵

$$\mathbf{R}f^\wedge(\theta, \rho) = f^\vee(\rho\theta), \quad (8)$$

where characters “ $\wedge$ ” and “ $\vee$ ” denote the 1D and 2D Fourier transforms with respect to variables s and $\mathbf{x}$, respectively.

In the parallel-beam geometry, there exists the following relationship between the partial derivative of projection $\mathbf{R}f(\theta, s)$ and the Laplacian transform of object function $f(\mathbf{x})$:²⁶

$$\begin{aligned} &\frac{1}{2\pi} \int_0^\pi \mathbf{H} \frac{\partial}{\partial s} \frac{\partial^{2m}}{\partial s^{2m}} \mathbf{R}f(\theta, \mathbf{x} \cdot \theta) d\theta \\ &= \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{+\infty} \frac{\text{sgn}(\rho)}{i} (i2\pi\rho)^{2m+1} \mathbf{R}f^\wedge(\theta, \rho) e^{i2\pi\rho\mathbf{x} \cdot \theta} d\rho d\theta \\ &= \int_0^\pi \int_{-\infty}^{+\infty} \text{sgn}(\rho) \rho (i2\pi\rho)^{2m} f^\vee(\rho\theta) e^{i2\pi\rho\mathbf{x} \cdot \theta} d\rho d\theta \\ &= \int_0^\pi \int_{-\infty}^{+\infty} (i2\pi\rho)^{2m} f^\vee(\rho\theta) e^{i2\pi\rho\mathbf{x} \cdot \theta} |\rho| d\rho d\theta \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ((i2\pi v_1)^2 + (i2\pi v_2)^2)^m F(\mathbf{v}) e^{i2\pi\mathbf{v} \cdot \mathbf{x}} dv_1 dv_2 \\ &= \Delta^m f(\mathbf{x}), \end{aligned} \quad (9)$$

where $\text{sgn}(\cdot)$ is the sign function, and $F(\mathbf{v})$ represents the 2D Fourier transform of $f(\mathbf{x})$. Essentially, Eq. (9) indicates that the filtered back-projection (FBP) of projection $\partial^{2m} \mathbf{R}f(\theta, s) / \partial s^{2m}$ is equal to $\Delta^m f(\mathbf{x})$, so we have

$$\mathbf{R}\Delta^m f(\theta, s) = \frac{\partial^{2m}}{\partial s^{2m}} \mathbf{R}f(\theta, s). \quad (10)$$

If $m = 1$, Eq. (10) becomes

$$\mathbf{R}\Delta f(\theta, s) = \frac{\partial^2}{\partial s^2} \mathbf{R}f(\theta, s). \quad (11)$$

In current CT scanners for clinical applications, the third generation fan-beam geometry is exclusively employed. In fact, the following relationships hold between the equiangular

fan-beam and parallel-beam geometries:²⁷

$$\theta = \xi + \gamma, \quad s = R_0 \sin \gamma, \quad \gamma \in (-\pi/2, \pi/2), \quad (12)$$

$$\xi = \theta - \tan^{-1} \left(s / \sqrt{R_0^2 - s^2} \right),$$

$$\gamma = \tan^{-1} \left(s / \sqrt{R_0^2 - s^2} \right), \quad s \in (-R_0, R_0), \quad (13)$$

where ξ , γ , and R_0 denote the view-angle, fan-angle, and distance from the x-ray source to rotation axis, respectively. Subsequently, we obtain

$$\mathbf{R}f(\theta, s) = g(\xi, \gamma), \quad (14)$$

$$\frac{\partial}{\partial s} \mathbf{R}f(\theta, s) = \frac{1}{\sqrt{R_0^2 - s^2}} \left(\frac{\partial}{\partial \gamma} g(\xi, \gamma) - \frac{\partial}{\partial \xi} g(\xi, \gamma) \right), \quad (15)$$

$$\begin{aligned} \frac{\partial^2}{\partial s^2} \mathbf{R}f(\theta, s) &= \frac{1}{R_0^2 \cos^3 \gamma} \left(\cos \gamma \left(\frac{\partial^2}{\partial \gamma^2} g(\xi, \gamma) \right. \right. \\ &\quad \left. \left. - 2 \frac{\partial}{\partial \xi} \frac{\partial}{\partial \gamma} g(\xi, \gamma) + \frac{\partial^2}{\partial \xi^2} g(\xi, \gamma) \right) \right. \\ &\quad \left. + \sin \gamma \left(\frac{\partial}{\partial \gamma} g(\xi, \gamma) - \frac{\partial}{\partial \xi} g(\xi, \gamma) \right) \right), \end{aligned} \quad (16)$$

where $g(\xi, \gamma)$ represents the projection of object function $f(\mathbf{x})$ in the equiangular fan-beam geometry. For simplicity of expression, we denote the right side of Eq. (16) as $\Theta(\xi, \gamma)$. Accordingly, Eq. (11) can be rewritten as

$$\mathbf{R}\Delta f(\theta, s) = \Theta(\xi, \gamma), \quad (17)$$

which is the Radon transform of the Laplacian transform of object function $f(\mathbf{x})$ in the equiangular fan-beam geometry.

II.C. Multiscale decomposition in projection domain

II.C.1. Decomposition scheme

Applying the Radon transform on both sides of Eq. (1), the isotropic diffusion PDE, we have

$$\frac{\partial}{\partial t} \mathbf{R}f(\theta, s, t) = d \mathbf{R}\Delta f(\theta, s, t). \quad (18)$$

Substituting Eqs. (14) and (17) into Eq. (18), we obtain

$$\frac{\partial}{\partial t} g(\xi, \gamma, t) = d \Theta(\xi, \gamma, t). \quad (19)$$

It should be indicated that, from Eqs. (1) to (19), we convert the isotropic diffusion PDE in image domain into its counterpart in projection domain. However, Eq. (19) usually does not result in a closed form formula in the projection domain that is similar to Eq. (3). Hence, a numerical approach is needed, in which the 1st and 2nd order partial derivatives

are approximated by the two- and three-point difference operations, respectively. In fact, Eq. (19) can be converted into

$$g[i, j, k+1] \approx g[i, j, k] + \Delta_t d\Theta[i, j, k],$$

$$k = 0, \dots, \quad (20)$$

$$g[i, j, k] = g(i\Delta_\xi, j\Delta_\gamma, k\Delta_t), \quad i = 1, \dots, I,$$

$$j = 1, \dots, J, \quad (21)$$

$$g[i, j, 0] = g(i\Delta_\xi, j\Delta_\gamma, 0) = g(i\Delta_\xi, j\Delta_\gamma) = g[i, j], \quad (22)$$

where Δ_ξ , Δ_γ , and Δ_t are the iteration steps in variables ξ , γ , and t , respectively, and k denotes the iteration index with respect to variable t . The readers are referred to the Appendix for the detail in getting $\Theta[i, j, k]$ [Eq. (A1)]. For convenience of expression, $\Delta_t d$ is represented by λ henceforth.

An iterative running of Eq. (20) for k times in projection domain is approximately equivalent to a convolution in the image domain between the object function $f(\mathbf{x})$ and the Gaussian convolution kernel function $G(\mathbf{x}, \sigma^2)$ with the variance determined by²²

$$\sigma^2 = 2dt = 2k\Delta_t d = 2k\lambda. \quad (23)$$

Subsequently, in a way similar to the multiscale decomposition in image domain,²⁴ a multiscale decomposition in the projection domain can be conducted as

$$g_l^n[i, j] = g^n[i, j, K_l] - g^n[i, j, K_{l-1}],$$

$$l = 1, \dots, L-1, \quad (24)$$

$$g_L^n[i, j] = g^n[i, j, K_{L-1}], \quad (25)$$

$$g^n[i, j, K_0] = g^n[i, j, 0] = g^n[i, j], \quad (26)$$

where $g_l^n[i, j]$ represents the noisy projection data and K_l denotes the total number of iteration at scale l (see Sec. II.C.2 for more detail), respectively. L is the number of total scales used in the decomposition. Inclusively, the algorithmic steps of the projection domain multiscale decomposition are detailed in the flow chart presented in Fig. 1(b). It should be noted that the algorithmic structure presented in Fig. 1(b) is essentially the same as that in Fig. 1(a), except for the difference that the former is in projection domain but the latter in image domain.

II.C.2. Parameter determination

The total number of iteration at scale l in the projection domain multiscale decomposition can be determined as²⁴

$$K_l = \begin{cases} K_0 = 0, & \text{if } l = 0, \\ \rho^{2(l-1)}r, & \text{if } l = 1, \dots, L-1 \end{cases}, \quad (27)$$

which is intentionally designed to make the standard deviation of the corresponding Gaussian convolution kernel function at

scale l in the form²⁴ [also see Eq. (23)]

$$\sigma_l = \sqrt{2K_l\lambda} = \sqrt{2\rho^{2(l-1)}r\lambda} \propto \rho^l, \quad l = 1, \dots, L-1. \quad (28)$$

In this work, L is empirically determined as 3, i.e., the projection data (sinogram) is decomposed into three scales (small, medium, and large).

Using Eq. (5), the response at the medium scale that corresponds to the reference frequency $\mathbf{v}_r$ can be defined as²²

$$\begin{aligned} \tilde{h}(r\lambda) &= \tilde{G}(\mathbf{v}_r, \sigma_1^2) - \tilde{G}(\mathbf{v}_r, \sigma_2^2) \\ &= \exp(-\|2\pi\mathbf{v}_r\|_2^2 \sigma_1^2/2) - \exp(-\|2\pi\mathbf{v}_r\|_2^2 \sigma_2^2/2) \\ &= \exp(-\|2\pi\mathbf{v}_r\|_2^2 K_1\lambda) - \exp(-\|2\pi\mathbf{v}_r\|_2^2 K_2\lambda) \\ &= \exp(-\|2\pi\mathbf{v}_r\|_2^2 r\lambda) - \exp(-\|2\pi\mathbf{v}_r\|_2^2 \rho^2 r\lambda). \end{aligned} \quad (29)$$

Theoretically speaking, parameter ρ can be any value that is positive and real. In this work, ρ is chosen as 2, since a power of 2 is usually used in the multiscale decomposition.¹¹ As a result, we can reach the maximum of $\tilde{h}(r\lambda)$

$$\rho^{-2/(\rho^2-1)} - \rho^{-2\rho^2/(\rho^2-1)} = 0.4725, \quad (30)$$

at

$$r\lambda = 2 \ln \rho / ((\rho^2 - 1) \|2\pi\mathbf{v}_r\|_2^2) = 0.0117 / \|\mathbf{v}_r\|_2^2. \quad (31)$$

The reference frequency $\mathbf{v}_r$ is assumed to be approximately equal to the frequency at 80% modulation transfer function (MTF) of a CT system,²² which is typically 0.6 lp/mm. Hence, empirically, we set $\mathbf{v}_r = (v_{1r}, v_{2r}) = (0.6, 0)$ lp/mm, which results in $r\lambda \approx 0.0325$ mm². In addition, the stability of Eq. (20) is dependent on parameter λ , which requires

$$\lambda < \frac{\pi R_0^2 \cos^2 \gamma}{\Delta_\xi^2 + \Delta_\gamma^2} \Delta_\xi^2 \Delta_\gamma^2, \quad \forall \gamma \in [\gamma_{\min}, \gamma_{\max}]. \quad (32)$$

Consequently, via a trial-and-error process, λ is heuristically set as 10^{-2} , while parameter r is set as 3.

As a reference for performance evaluation, the noise reduction with an image domain multiscale decomposition [Eqs. (1)–(5) and Fig. 1(a)] is also investigated in this work, in which the parameter λ satisfies

$$\lambda < \frac{\Delta_x^2 \Delta_y^2}{2(\Delta_x^2 + \Delta_y^2)}. \quad (33)$$

In addition, to assure the stability of Eq. (1), parameters λ and r are set as 10^{-2} and 3, respectively.

II.D. Statistical noise reduction

In general, an objective function is associated with the projection domain multiscale PWLS method for statistical noise reduction in CT imaging, wherein an estimation of variance in the projection data at each scale is required. Below is a detailed description of the algorithmic steps.

II.D.1. Variance estimation

We denote the variance of the noisy projection data $g^n[i, j]$ at location $[i, j]$ as $V[i, j]$. An estimation of $V[i, j]$ can be directly obtained by gauging the standard deviation of the projection data within the eight-element neighborhood of the pixel at location $[i, j]$, represented by $N_8[i, j]$. However, such a direct estimation may be severely influenced by the object's boundaries. Based on theoretical analysis and experimental verification, a modeling method to estimate the variance $V[i, j]$ of projection data has been proposed in the literature.⁶ We adopt this method with a minor modification

$$V[i, j] \approx \exp \left(\sum_{m=0}^4 a_m (j - J/2 - 0.5)^{2m} + g^n[i, j]/b \right), \quad (34)$$

where the 4th-order polynomial on the right side is to account for the x-ray beam shaping effect (e.g., the bow-tie filtration in a CT scanner), and parameter b is to account for the influence on the variance due to the interscan x-ray tube current fluctuation. Parameters $\mathbf{a} = [a_0 \ a_1 \ a_2 \ a_3 \ a_4]$ and b can be acquired by a least square method.

Rewriting the noisy counterpart of Eq. (20) as

$$g^n[i, j, k+1] = g^n[i, j, k] + \lambda \Theta^n[i, j, k], \quad (35)$$

we can obtain an approximate relationship in variance between the k th and $(k+1)$ th diffusions

$$V[i, j, k+1] \approx V[i, j, k] W_1[i, j], \quad (36)$$

where

$$\begin{aligned} W_1[i, j] = & 1 + \frac{\lambda^2}{R_0^4 \cos^4 \gamma_j} \\ & \times \left(\frac{6}{\Delta_\xi^4} + \frac{9}{\Delta_\xi^2 \Delta_\gamma^2} + \frac{\tan \gamma_j}{\Delta_\gamma^3} + \frac{9}{\Delta_\gamma^4} \right. \\ & \left. + \left(\frac{\tan^2 \gamma_j}{2} - \frac{4}{\lambda} R_0^2 \cos^2 \gamma_j \right) \left(\frac{1}{\Delta_\xi^2} + \frac{1}{\Delta_\gamma^2} \right) \right), \end{aligned} \quad (37)$$

is defined as the variance transfer coefficient over iterations, and Eqs. (35) and (A1) are used in obtaining Eq. (37). Hence, in general, we have

$$V[i, j, k] = V[i, j, 0] W_1^k[i, j] = V[i, j] W_1^k[i, j]. \quad (38)$$

At scale l , according to Eq. (24), the variance at location $[i, j]$ can be approximately obtained as

$$V_l[i, j] = V[i, j, K_l] + V[i, j, K_{l-1}], \quad l = 1, \dots, L-1, \quad (39)$$

$$V_L[i, j] = V[i, j, K_{L-1}], \quad (40)$$

$$V[i, j, K_0] = V[i, j, 0] = V[i, j]. \quad (41)$$

II.D.2. The PWLS associated with Markov random field objective function (PWLS-MRF)

At scale l , the PWLS-MRF is carried out by minimizing the objective function,⁸

$$\begin{aligned} \Phi_l(\mathbf{g}_l) = & (\mathbf{g}_l^n - \mathbf{g}_l)' \Lambda_l^{-1} (\mathbf{g}_l^n - \mathbf{g}_l) + \beta_l \|\mathbf{R} \mathbf{g}_l\|_2^2, \\ & l = 1, \dots, L, \end{aligned} \quad (42)$$

where the prime character denotes a transpose operation, and $\mathbf{g}_l^n$ and $\mathbf{g}_l$ represent the noisy projection data at scale l and its counterpart with noise reduced, respectively. Λ_l is a diagonal variance matrix with $V_l[i, j]$, which can be estimated using the approach specified in Sec. II.D.1, as its entry. Furthermore, we define

$$\mathbf{R} \mathbf{g}_l = \sum_{[i, j]} \sum_{[i', j'] \in N_8[i, j]} w_{i'j'} (g_l[i, j] - g_l[i', j'])^2, \quad (43)$$

$$w_{i'j'} = \begin{cases} 1, & \text{if } \|[i', j'] - [i, j]\|_2 = 1, \\ 1/\sqrt{2}, & \text{if } \|[i', j'] - [i, j]\|_2 = \sqrt{2}, \end{cases} \quad (44)$$

and then an iterative version of Eqs. (42)–(44) can be written as

$$\begin{aligned} g_l^{(m)}[i, j] = & \left(g_l^n[i, j] + \beta_l V_l[i, j] \right. \\ & \times \left. \sum_{[i', j'] \in N_8[i, j]} w_{i'j'} g_l^{(m-1)}[i', j'] \right) / \\ & \left(1 + \beta_l V_l[i, j] \sum_{[i', j'] \in N_8[i, j]} w_{i'j'} \right), \\ & m = 1, \dots, M, \end{aligned} \quad (45)$$

$$g_l^{(0)}[i, j] = g_l^n[i, j]. \quad (46)$$

II.D.3. The PWLS associated with soft thresholding objective function (PWLS-ST)

At scale l , the PWLS-ST is carried out by minimizing the objective function,²⁸

$$\begin{aligned} \Phi_l(\mathbf{g}_l) = & (\mathbf{g}_l^n - \mathbf{g}_l)' \Lambda_l^{-1} (\mathbf{g}_l^n - \mathbf{g}_l) + \beta_l \|\mathbf{g}_l\|_1, \\ & l = 1, \dots, L, \end{aligned} \quad (47)$$

where $\|\cdot\|_1$ denotes the l_1 -norm. In addition, we have²⁸

$$g_l[i, j] = S_{\beta_l \sqrt{V_l[i, j]}}(g_l^n[i, j]), \quad (48)$$

where the soft thresholding function is defined as

$$S_\beta(g) = \begin{cases} g + \beta/2 & \text{if } g \leq -\beta/2, \\ 0 & \text{if } |g| \leq \beta/2, \\ g - \beta/2 & \text{if } g \geq \beta/2. \end{cases} \quad (49)$$

FIG. 2. The flow chart showing the algorithmic steps of edge enhancement with multiscale decomposition. Note that “Recon” and “DoG” denote the FBP reconstruction and the “Difference of Gaussian” operator, respectively.

II.E. Edge enhancement

It has been well-known that a noise reduction method in general cannot fully preserve the edge of an object in the images with noise reduced. Hence, once the methods introduced above are applied to attain a CT image with noise reduced (namely, a processed CT image), we carry out an edge enhancement. The algorithmic steps of edge enhancement are detailed in the flow chart presented in Fig. 2, which consists of the following major steps:

- (i) Pixelwise subtracting ($\oplus$) the noise reduced projection data at each scale from the original projection data at the corresponding scale to obtain difference projection data;
- (ii) Reconstructing (Recon) the difference images from the difference projection data at each scale;
- (iii) A boundary map is acquired by calculating the gradient’s amplitude (i.e., boundary detection) of the processed CT image. Pixelwise multiplying ($\odot$) the boundary map with the corresponding difference image processed by a Difference of Gaussian (DoG) operator²⁹ at each scale to acquire the edge images with noise removed;
- (iv) Adequately weighting the edge images at each scale with parameter η_l , and adding all of them back to the processed CT image.

A trial-and-error analysis shows that any value within the range $1 < \eta_l < 5$ is adequate for such an edge enhancement operation, and we set $\eta_l = 3.3$ in this work.

III. EXPERIMENTAL EVALUATION

The projection data simulated by computer and acquired by a CT scanner are used to evaluate and verify the performance of the proposed projection domain multiscale PWLS method for statistical noise reduction in CT imaging. In the computer simulation, a thoracic phantom is used, in which

two bony rods at diameter 30 mm are placed on the left and right sides to simulate the arms, while a small lesion at ~ 40 HU contrast is positioned at the central area between the two rods. The thoracic phantom is assumed as an elliptical cylinder made of water, with its major and minor axes equal to 200 mm and 100 mm, respectively, while the small lesion is assumed as a cylinder with its diameter equal to 5 mm. In the projection data acquired by the CT scanner, a sheep is scanned. To demonstrate the advantage of the proposed projection domain multiscale PWLS method for statistical noise reduction, the anisotropic diffusion-based noise reduction method in the image domain² is also investigated in this study. In addition, to show the necessity of multiscale decomposition, the PWLS noise reduction method without multiscale (i.e., single-scale) decomposition in the projection domain is also investigated.

III.A. Parameters in data acquisition and image reconstruction

In the data acquisition of both the computer simulation and CT scan, the distance from x-ray focal spot to the axis of rotation (R_0) is 570 mm. A total of 1160 projection views distribute along a circular source trajectory. The full fan-angle of x-ray beam in the computer simulation is 54.89° , corresponding to 888 detector cells, while that of the CT scanner is 52.10° , corresponding to 672 detector cells. In simulating the data acquisition of the thoracic phantom, the x-ray techniques are 120 kVp and effectively 150 mAs. The quantum and electronic noise are modeled to observe the Poisson and Gaussian distributions, respectively, in a way similar to that published in the literature.³⁰ The x-ray techniques used in the CT scanner (Siemens, Forchheim, Germany) to acquire the projection data of sheep are 100 kVp and 150 mAs. Image reconstruction is carried out with a FBP algorithm. The reconstructed images of the thoracic phantom are in 512×1024 matrices, corresponding to field of view (FOV) of $200 \times 400 \text{ mm}^2$, while those of the sheep are in 1024×1024 matrices, corresponding to a FOV of $291 \times 291 \text{ mm}^2$.

III.B. Parameters for the projection domain multiscale PWLS noise reduction

It is experimentally determined that, in the PWLS-MRF method, the number of total iterations $M = 10$ is adequate. In the PWLS-ST method, the algorithm is noniterative, i.e., $M = 1$.

Three parameters β_1 , β_2 , and β_3 are required by the projection domain multiscale PWLS method. As indicated above, in general, a noise reduction method inevitably compromises the image’s sharpness. The ultimate goal to use the proposed methods for noise reduction in CT images is to maximize the noise reduction while minimizing the compromise in image sharpness. Hence, the parameters β , β_1 , β_2 , and β_3 are determined by optimizing the trade-off between noise and image sharpness through a trial-and-error process, in which the noise is measured as the standard deviation (SD) of the image

pixels within a ROI, while the image sharpness is evaluated using the method described below.

A CT scanner with the FBP algorithm for image reconstruction is, in principle, a linear system and the MTF can be utilized to assess image sharpness. However, the proposed projection domain multiscale PWLS method is, in principle, nonlinear, and thus the MTF is no longer an adequate metric for image sharpness assessment. In a way similar to that practiced in the literature,^{7–13} we assess the proposed projection domain multiscale PWLS method's influence on image sharpness using an object with a sharp edge. Since a nonlinear noise reduction method may cause degradation anisotropically, the image sharpness should be assessed at least along the horizontal and vertical directions (see the example illustrated later in Fig. 7). Initially, the edge spread function (ESF) is measured, followed by a derivative operation to acquire the line spread function (LSF). Via a Gaussian fitting, the full width at half maximum (FWHM) of the LSF is measured as the metric to assess image sharpness. Such an ESF $\rightarrow$ LSF $\rightarrow$ FWHM process to measure the image sharpness is illustrated in Fig. 3.

The process to determine the parameters β_1 , β_2 , and β_3 for the projection domain multiscale PWLS-MRF method corresponding to the simulated thoracic phantom is illustrated in Fig. 4. It is readily observable that, with increasing value in parameters β_1 and β_2 , the trade-off in the projection domain

FIG. 3. The profiles showing the assessment of image sharpness in the CT images with noise reduced by the projection domain multiscale PWLS methods: (a) Edge spread function (ESF) and (b) Line spread function (LSF) [see Fig. 7(a) for the lines along which the measurement is made].

FIG. 4. The trade-off between image sharpness (FWHM) and noise (SD) corresponding to the multiscale PWLS-MRF as a function over parameters β_1 ($10^{2.0}$, $10^{2.5}$, $10^{3.0}$, $10^{3.5}$, $10^{4.0}$) and β_2 (see the legends) while β_3 is fixed at $10^{0.0}$, compared with that of the single-scale PWLS-MRF as a function over parameter β ($10^{1.0}$, $10^{1.2}$, $10^{1.4}$, $10^{1.5}$, $10^{1.6}$): (a) Horizontal and (b) Vertical.

multiscale PWLS-MRF method between noise, gauged as SD (mm^{-1}), and image sharpness, gauged as FWHM (mm), along the horizontal and vertical direction asymptotically approaches a destination. Hence, given various parameter β_3 , we can determine the optimal values of β_1 and β_2 through a trial-and-error process, with an optimization criterion that the optimal value makes the trade-off between noise and image sharpness along the horizontal and vertical directions approach the asymptotic destination simultaneously. Via such an optimization process, the parameters for the simulated thoracic phantom are determined as $\beta_1 = 10^3$, $\beta_2 = 10^3$, and $\beta_3 = 10^0$, while those for the sheep scanned by a CT are $\beta_1 = 10^2$, $\beta_2 = 10^2$, and $\beta_3 = 10^1$.

The process to determine the parameters β_1 , β_2 , and β_3 for the projection domain multiscale PWLS-ST method corresponding to the simulated thoracic phantom is illustrated in Fig. 5, in which a trial-and-error optimization is also carried out. It is readily observable that, though there is no asymptotic destination, given $\beta_2 \approx 10^{-1}$, certain values in β_1 make the noise and image sharpness reach an optimal trade-off along the horizontal and vertical directions simultaneously. This trial-and-error process is repeated over various values in β_3 . In such a way, the parameters for the simulated thoracic

FIG. 5. The trade-off between image sharpness (FWHM) and noise (SD) corresponding to the multiscale PWLS-ST as a function over parameters β_1 ($10^{-1.4}$, $10^{-1.2}$, $10^{-1.0}$, $10^{-0.8}$, $10^{-0.6}$) and β_2 (see the legends) while β_3 is fixed at $10^{-1.0}$, compared with that of the single-scale PWLS-MRF as a function over parameter β ($10^{1.0}$, $10^{1.2}$, $10^{1.4}$, $10^{1.5}$, $10^{1.6}$): (a) Horizontal and (b) vertical.

phantom are determined as $\beta_1 = 10^{-1}$, $\beta_2 = 10^{-1}$, and $\beta_3 = 10^{-1}$, while those for the sheep scanned by a CT are $\beta_1 = 6 \times 10^{-2}$, $\beta_2 = 6 \times 10^{-2}$, and $\beta_3 = 6 \times 10^{-2}$.

As a reference, the trade-offs between noise and image sharpness corresponding to the projection domain single-scale PWLS-MRF are also plotted in Figs. 4 and 5. It is observed that, without a projection domain multiscale decomposition, the trade-off between noise and image sharpness due to the statistical noise reduction using the PWLS-MRF varies contradictorily along the horizontal and vertical directions.

IV. EXPERIMENTAL RESULTS

IV.A. Projection data generation and image reconstruction

Examples of the simulated noiseless and noisy projection data of the thoracic phantom are presented in Figs. 6(a) and 6(b), respectively, while the difference between them is in Fig. 6(c). Shown in Figs. 7(a)–7(c) are the transverse images reconstructed from the simulated projection data corresponding to Figs. 6(a)–6(c). Severe noise, particularly along the horizontal direction, is observed in the image reconstructed from the noisy projection data (Fig. 6), which is employed to eval-

FIG. 6. The noiseless projection (a), noisy projection (b), and their difference (c) of the simulated thoracic phantom, at display window $[0.0, 10.81]$, $[0.0, 10.81]$, and $[0.0, 0.3]$, respectively.

FIG. 7. Transverse images of the thoracic phantom reconstructed from the noiseless projection (a), noise projection (b), and their difference (c) at display window $[0, 150]$ HU, $[0, 150]$ HU, and $[-250, 250]$ HU, respectively. Note that image sharpness is gauged along the horizontal and vertical black line segments as illustrated in (a).

FIG. 8. The variance estimated in the projection domain corresponding to a detector cell over projection angles (a) and the detector array over view angles (b).

uate and verify the performance of the proposed statistical CT noise reduction methods.

IV.B. Variance estimation in projection data

The variance of the simulated projection data estimated by both the modeling-based method specified in Eq. (34) and direct numerical estimation is presented in Fig. 8. Through an optimization process, the modeling parameters are obtained as $\mathbf{a} = [-14.6, 26.0, -56.6, 45.4, -11.2]$ and $b = 0.85$. A close inspection of Fig. 8(a) shows that, as for the variance profiles corresponding to detector cell 222 as a function over projection view angle, the modeling-based method can estimate the variance very well at the thoracic phantom's structures where an abrupt structural transition occurs, whereas the direct numerical method almost fails. Similar observation can be made in the view-wise variance profiles as illustrated in Fig. 8(b). This means that the modeling-based estimation method outperforms the direct numerical estimation method, providing a solid foundation to carry out the proposed projection domain multiscale PWLS method for statistical noise reduction in CT images. As for the projection data acquired by a CT scanner, the modeling parameters of variance estimation are obtained as $\mathbf{a} = [-9.8, 13.8, -42.3, 49.3, -19.7]$ and $b = 1.18$.

IV.C. Statistical noise reduction in CT images reconstructed from simulated projection data

The projection domain multiscale PWLS-MRF and PWLS-ST methods demonstrate comparable performance in noise reduction in CT images. Due to space limitation, only the results corresponding to the former are presented in Fig. 9. To illustrate how the projection domain multiscale PWLS-MRF method works at each scale, the details at all three scales are presented. It is clearly observed that the reduction of noise, including the streaks, mainly occurs at the small and medium scales.

To demonstrate the advantage of carrying out the statistical noise reduction in projection domain over that in image domain, we compare the images corresponding to the cases of image domain multiscale anisotropic diffusion, projection domain multiscale PWLS-MRF and PWLS-ST in Fig. 10, and that without noise reduction is also presented as a reference. It is observed that both the PWLS-MRF and PWLS-ST can suppress noise very well, regardless of whereabouts of the noise. However, though a majority of the noise is reduced substantially by the image domain multiscale anisotropic diffusion, residual noise with an appearance similar to the so-called "salt-and-pepper" noise and mosaic effect exist, especially in the area where the original noise is severe, as illustrated in the zoomed view presented in Figs. 10(a')–10(d').

IV.D. Statistical noise reduction in CT images reconstructed from data acquired by a CT scanner

The performance of the projection domain multiscale PWLS-MRF and PWLS-ST methods is evaluated with the projection data of a sheep acquired by a CT scanner. Again, due to space limitation, only the results corresponding to the former are presented in Fig. 11, in which the noise reductions at all three scales are illustrated. Similar to what has been demonstrated in Fig. 9 wherein the thoracic phantom was employed, the reduction of noise in the image reconstructed from the projection data acquired by the CT scanner mainly occurs at the small and medium scales.

To demonstrate the advantage of carrying out the statistical noise reduction with projection domain multiscale decomposition over that with the image domain multiscale decomposition, we present the results corresponding to the projection domain multiscale PWLS-MRF and PWLS-ST methods in Figs. 12(b') and 12(c'), respectively, while those of the image domain multiscale anisotropic diffusion method are shown in Fig. 12(a'). Meanwhile, their corresponding differences against the reconstructed image without noise reduction are presented in Figs. 12(a'')–12(c''). It is clearly observed that, with the projection domain multiscale decomposition, both the PWLS-MRF and PWLS-ST methods outperform the image domain multiscale anisotropic diffusion in terms of image sharpness preservation. It is also observed that, though the same projection domain multiscale decomposition is carried out in both the PWLS-MRF and PWLS-ST methods, the former outperforms the latter from the viewpoint of image sharpness preservation.

FIG. 9. Transverse images of the simulated thoracic phantom reconstructed from the noisy projection (column a–d), the projection with noise reduced by PWLS-MRF (column a'–d'), and their difference (column a''–d''), at small scale (row a–a''), medium scale (row b–b''), large scale (row c–c''), and composite (row d–d'') {display window corresponding to (c), (c'), (d), and (d'): [0, 150] HU; others: [–250, 250] HU}.

FIG. 10. Transverse images of the simulated thoracic phantom reconstructed from the noisy projection data: (a, a') without noise reduction, (b, b') with noise reduced by the image domain multiscale anisotropic diffusion, (c, c') by the projection domain multiscale PWLS-MRF, and (d, d') by the projection domain multiscale PWLS-ST. The images in (a'–d') are zoomed view of the ROIs defined in (a–d) {display window: [0, 150] HU}.

IV.E. Edge enhancement

In comparison to the transverse image reconstructed from the original projection data [Fig. 13(a)], the sharpness of the image with noise reduced by the projection domain multiscale PWLS-MRF [Fig. 13(a')] is slightly degraded. However, such a slight degradation in image sharpness can be remedied by edge enhancement, as demonstrated in the image presented in Fig. 13(b'). It is observed that the image in Fig. 13(b') looks sharper than that in Fig. 13(a'), while the noise does not change. The efficacy of the edge enhancement can be more readily appreciated by inspecting the corresponding difference images presented in Figs. 13(a'') and 13(b'').

V. DISCUSSIONS

By converting the isotropic diffusion PDE in image domain into its counterpart in projection domain, the statistical CT noise reduction methods with multiscale decomposition and PWLS in projection domain are derived in this work. By adopting the MRF or soft thresholding objective function, the projection domain multiscale PWLS method suppresses noise at each scale, followed by an edge enhancement to remedy the degradation in image sharpness that is unavoidably associated with any noise reduction solution. The performance of the proposed projection domain multiscale PWLS method is experimentally evaluated and verified using the projection data simulated by computer and acquired by a CT scanner. To summarize the results obtained in this work, a detailed discussion is provided below.

FIG. 11. Transverse images of a sheep reconstructed from the noisy projection data (column a–c), noise reduced projection data (column a'–c'), and their difference (column a''–c'') at small scale (row a–a''), medium scale (row b–b''), and large scale (row c–c'') (display window of (c) and (c'): $[-750, 250]$ HU; others: $[-250, 250]$ HU).

The primary benefit of starting the derivation of the proposed statistical noise reduction methods in the image domain is to take the interview sampling rate into account in the multiscale decomposition. In practice, advanced clinical and preclinical applications may demand that the interview sampling rate be significantly different from the intraview sampling rate. Accordingly, the interview sampling rate may have to be taken into account in the multiscale decomposition. Otherwise, the performance of the projection domain multiscale PWLS methods for statistical noise reduction may be compromised. An inspection of Eqs. (32), (36) and (37), and (A1) shows that, Δ_ξ , the variable representing the angular sampling interval, indeed plays a role in determining the variance transfer coefficient $W_1[i, j]$ over iteration in the projection domain multiscale decomposition. This feature distinguishes the proposed projection domain multiscale PWLS method from other sinogram-based 2D filtering methods, in which the interview filtering is straightforwardly conducted without taking the an-

gular sampling interval Δ_ξ into account. Therefore, it is believed that the proposed method can be applicable in more challenging scenarios, e.g., in the clinical and preclinical applications wherein the projection data can only be acquired at nonuniform or sparse angular sampling rate.

The estimation of variance in projection data is the key to enable a statistical noise reduction method, which is, in principle, a nonlinear operation, outperforms any linear filtering method. Though the direct numerical approach is an option to estimate the variance in projection data, the modeling-based method is preferable, because the interference caused by an object's boundaries can be avoided. Moreover, the estimation of variance at each scale is also of importance, because the noise at each scale is different and thus should be processed separately.

It is shown in Fig. 10 that, in comparison to the image domain multiscale anisotropic diffusion method for noise reduction in CT images, the projection domain multiscale PWLS

FIG. 12. Transverse images reconstructed from the noisy projection data (column a–c) and projection data with noise reduced by the image domain multiscale anisotropic diffusion (a'), the projection domain multiscale PWLS-MRF (b'), and the projection domain multiscale PWLS-ST (c'). Column (a''–c'') shows the corresponding difference between (a–c) and (a'–c') {display window in (a''–c''): [–250, 250] HU; others: [–750, 250] HU}.

method is of advantage in the avoidance of “salt-and-pepper” noise and mosaic artifacts. In the cases where the noise in CT images is severe, the image domain anisotropic diffusion may not be able to effectively remove all noise, especially those streaks, by anisotropic “diffusing.” Consequently, the residual noise may appear like “salt-and-pepper,” because of its abrupt variation against the diffused background that is relatively smooth, or mosaic artifacts appear because the streaks are treated as edges in the diffusion. Though the projection domain multiscale PWLS method may also leave “salt-and-pepper” noise or mosaic artifacts in projection data, they usually do not translate themselves into reconstructed images, because they are smeared out in the back-projection process.

A careful reader may have already noticed that streak artifacts still exist in the images with the proposed projection domain multi-scale PWLS-MRF method applied [Fig. 9(d')]. This is because the simulated thoracic phantom represents an

extreme case in projection data acquisition, wherein severe photon starvation occurs along the horizontal direction. We would like to point out that the proposed projection domain multiscale PWLS-MRF method is aimed at suppressing the statistical noise, other than the streak artifacts caused by photon starvation, in CT images. In fact, these streak artifacts have been well investigated in the literature^{21,31} and can be readily reduced, if not eliminated, by adequately trimming the outliers induced by the photon starvation in the projection data, prior to the application of the proposed projection domain multiscale PWLS-MRF method.

A reinspection of Figs. 4 and 5 shows that, in comparison to their single-scale (i.e., without multiscale decomposition) counterpart, the projection domain multiscale PWLS method provides a better trade-off between noise and image sharpness. In particular, without a multiscale decomposition, the image sharpness along the vertical direction is compromised

FIG. 13. Transverse image of the sheep without noise reduction (a, b), with noise reduced by the PWLS-MRF but without edge enhancement (a'), and with noise reduced by the PWLS-MRF and edge enhancement (b'). (a'') and (b'') are the difference between (a) and (a'), and (b) and (b'), respectively {display window in (a''–b''): [–250, 250] HU; others: [–750, 250] HU}.

severely. This is because, along the horizontal direction, the thickness of human body is larger and results in more noise in the projection data. Thus, based on the information obtained from variance estimation, the projection domain PWLS method works harder in these projection views and degrades image sharpness more. It should be noted that this situation is common, in practice, because the human body is generally in an oval shape. However, such a risk can be mitigated by the multiscale decomposition, because the variance estimated at each scale can guide the noise reduction to avoid unnecessary operation at the scale where little noise exists. Hence, the importance of multiscale decomposition in maintaining an isotropic spatial resolution in the CT images with statistical noise reduction can never be overstated.

In terms of noise reduction, the projection domain multiscale PWLS-ST method performs as well as its counterpart associated with the MRF objective function. The former does not need iteration and thus is substantially more efficient in computation. However, as demonstrated in Figs. 12(b'') and 12(c''), the projection domain multiscale PWLS-MRF method outperforms the projection domain multiscale PWLS-ST method from the perspective of edge preservation. Hence, the projection domain multiscale PWLS-MRF method is a preferred statistical CT noise reduction solution. Usually, a noise reduction method may compromise the image sharpness to an extent that varies over algorithms.^{7–13} Fairly speaking, the projection domain multiscale PWLS method investigated in this work maintains the image sharpness well while reducing noise in CT images substantially. However, we still propose to carry out an edge enhancement following

the statistical noise reduction, such that the image sharpness can be well preserved.

In general, a number of parameters are usually associated with a statistical noise reduction method and an optimization of the parameters can influence its performance. In addition, the parameters may vary over various applications, e.g., imaging of the head, thorax, or abdomen. A careful reader may have already noticed that the parameters used for noise reduction in the CT images of the simulated thoracic phantom are different from those in the case wherein a sheep is scanned. Fortunately, however, as demonstrated in Figs. 3 and 4, the variation in the trade-off between noise and image sharpness of the proposed projection domain multiscale PWLS method is relatively smooth, which warrants a robust performance in the trade-off between noise and image sharpness over a relatively large range of parameters.

A CT scanner is, in principle, a linear system. It is a basic understanding that a χ time increase in the x-ray dose results in a $\chi^{1/2}$ time decrease in the noise, and vice versa, if the CT scanner works within its linear dynamic range. This means that a χ time saving in the x-ray dose can occur if the noise in a CT image is reduced $\chi^{1/2}$ times by linear noise reduction methods. In general, a nonlinear noise reduction method makes the relationship between the noise and x-ray dose more complicated. Nevertheless, a nonlinear method usually outperforms a linear one in noise reduction. Hence, it still should be rational to use this relationship as a “rule of thumb” to project the potential saving of x-ray dose in the applications wherein the proposed projection domain multiscale PWLS method is utilized.

VI. CONCLUSIONS

In this work, we proposed and evaluated the projection domain multiscale PWLS methods for statistical noise reduction in CT imaging. The statistical CT noise reduction methods are derived by converting an image domain isotropic diffusion PDE into its counterpart in the projection domain. As a result, the interview sampling rate is taken into account in the projection domain multiscale decomposition. Using the projection data simulated by computer and acquired by a CT scanner, the performance of the proposed methods is experimentally evaluated and verified. The preliminary result shows that the proposed projection domain multiscale PWLS methods outperform the projection domain single-scale PWLS methods and the image domain multiscale anisotropic diffusion methods in noise reduction and sharpness maintenance. Therefore, the proposed methods are anticipated to be useful in advanced clinical and preclinical applications where the interview sampling rate varies.

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APPENDIX: RADON TRANSFORM OF AN OBJECT FUNCTION'S LAPLACIAN TRANSFORM

In the numerical implementation, the 1st and 2nd order partial derivatives are approximated by the 2- and 3-point difference operations, respectively. In this way, one has

$$\begin{aligned} \Theta[i, j, k] = & -\frac{2}{R_0^2 \cos^2 \gamma_j} \frac{g[i, j, k]}{\Delta_\gamma^2} - \frac{2}{R_0^2 \cos^2 \gamma_j} \frac{g[i, j, k]}{\Delta_\xi^2} + \frac{2}{R_0^2 \cos^2 \gamma_j} \frac{g[i, j+1, k]}{\Delta_\gamma^2} \\ & + \frac{\tan \gamma_j}{R_0^2 \cos^2 \gamma_j} \frac{g[i, j+1, k]}{2\Delta_\gamma} + \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i, j-1, k]}{\Delta_\gamma^2} - \frac{\tan \gamma_j}{R_0^2 \cos^2 \gamma_j} \frac{g[i, j-1, k]}{2\Delta_\gamma} \\ & - \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i+1, j+1, k]}{2\Delta_\xi \Delta_\gamma} + \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i-1, j+1, k]}{2\Delta_\xi \Delta_\gamma} + \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i+1, j-1, k]}{2\Delta_\xi \Delta_\gamma} \\ & - \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i-1, j-1, k]}{2\Delta_\xi \Delta_\gamma} + \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i+1, j, k]}{\Delta_\xi^2} - \frac{\tan \gamma_j}{R_0^2 \cos^2 \gamma_j} \frac{g[i+1, j, k]}{2\Delta_\xi} \\ & + \frac{1}{R_0^2 \cos^2 \gamma_j} \frac{g[i-1, j, k]}{\Delta_\xi^2} + \frac{\tan \gamma_j}{R_0^2 \cos^2 \gamma_j} \frac{g[i-1, j, k]}{2\Delta_\xi}. \end{aligned} \quad (A1)$$

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