Ideal-observer detectability in photon-counting differential phase-contrast imaging using a linear-systems approach

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(Received 28 September 2011; revised 15 June 2012; accepted for publication 11 July 2012; published 10 August 2012)

Purpose: To provide a cascaded-systems framework based on the noise-power spectrum (NPS), modulation transfer function (MTF), and noise-equivalent number of quanta (NEQ) for quantitative evaluation of differential phase-contrast imaging (Talbot interferometry) in relation to conventional absorption contrast under equal-dose, equal-geometry, and, to some extent, equal-photon-economy constraints. The focus is a geometry for photon-counting mammography.

Methods: Phase-contrast imaging is a promising technology that may emerge as an alternative or adjunct to conventional absorption contrast. In particular, phase contrast may increase the signal-difference-to-noise ratio compared to absorption contrast because the difference in phase shift between soft-tissue structures is often substantially larger than the absorption difference. We have developed a comprehensive cascaded-systems framework to investigate Talbot interferometry, which is a technique for differential phase-contrast imaging. Analytical expressions for the MTF and NPS were derived to calculate the NEQ and a task-specific ideal-observer detectability index under assumptions of linearity and shift invariance. Talbot interferometry was compared to absorption contrast at equal dose, and using either a plane wave or a spherical wave in a conceivable mammography geometry. The impact of source size and spectrum bandwidth was included in the framework, and the trade-off with photon economy was investigated in some detail. Wave-propagation simulations were used to verify the analytical expressions and to generate example images.

Results: Talbot interferometry inherently detects the differential of the phase, which led to a maximum in NEQ at high spatial frequencies, whereas the absorption-contrast NEQ decreased monotonically with frequency. Further, phase contrast detects differences in density rather than atomic number, and the optimal imaging energy was found to be a factor of 1.7 higher than for absorption contrast. Talbot interferometry with a plane wave increased detectability for 0.1-mm tumor and glandular structures by a factor of 3–4 at equal dose, whereas absorption contrast was the preferred method for structures larger than ~ 0.5 mm. Microcalcifications are small, but differ from soft tissue in atomic number more than density, which is favored by absorption contrast, and Talbot interferometry was barely beneficial at all within the resolution limit of the system. Further, Talbot interferometry favored detection of “sharp” as opposed to “smooth” structures, and discrimination tasks by about 50% compared to detection tasks. The technique was relatively insensitive to spectrum bandwidth, whereas the projected source size was more important. If equal photon economy was added as a restriction, phase-contrast efficiency was reduced so that the benefit for detection tasks almost vanished compared to absorption contrast, but discrimination tasks were still improved close to a factor of 2 at the resolution limit.

Conclusions: Cascaded-systems analysis enables comprehensive and intuitive evaluation of phase-contrast efficiency in relation to absorption contrast under requirements of equal dose, equal geometry, and equal photon economy. The benefit of Talbot interferometry was highly dependent on task,

in particular detection versus discrimination tasks, and target size, shape, and material. Requiring equal photon economy weakened the benefit of Talbot interferometry in mammography. © 2012 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4739195>]

Key words: differential phase-contrast imaging, Talbot interferometry, cascaded-systems analysis, NEQ, detectability index, dose, photon economy, mammography, photon counting

I. INTRODUCTION

Medical x-ray imaging is often limited by small contrast differences and high noise, caused by tight dose constraints. This is particularly so for mammography where low-contrast tumors constitute a major detection target, and a large number of tumors are missed or misdiagnosed due to difficulties in detection or discrimination.^{1–3}

Phase-contrast imaging is a well-known technique to image low-contrast objects in optical microscopy,⁴ but has relatively recently emerged as a promising substitute or adjunct to conventional absorption contrast in medical x-ray imaging.^{5–7} In particular, four potential benefits of phase contrast have been identified in a medical imaging context: (1) phase contrast bears promise to increase the signal-to-noise ratio because the phase shift in soft tissue is in many cases substantially larger than the absorption; (2) phase contrast has different energy dependence than absorption contrast, which changes the conventional dose-contrast trade-off and higher photon energies may be optimal with a resulting lower dose and higher output from the x-ray tube; (3) phase contrast is a new contrast mechanism that enhances other target properties than absorption contrast, which may be beneficial in some cases; (4) some phase-contrast geometries provide, in addition to phase and absorption contrast, additional information on the small-angle scattering properties of the target, referred to as dark-field imaging or extinction contrast. The present study will focus on potential benefits (1)–(3), whereas (4) is not included in our stringent definition of phase contrast.

Phase shifts cannot be measured, but have to be inferred indirectly by intensity differences. Four methods dominate research today:

- (1) Interferometer-based imaging was the first x-ray phase-contrast method to be investigated.⁸ The method requires highly coherent radiation and a complex setup, and is in practice restricted to synchrotron facilities.^{5,6}
- (2) Diffraction-enhanced imaging (DEI) (Refs. 9–12) has shown promising results on imaging of breast specimens, for instance in the detection of infiltrating lobular carcinoma.^{13,14} In addition, the technique has been implemented in small-scale setups for mammography applications.^{15,16}
- (3) Free-space propagation is relatively uncomplicated in its configuration,^{17–19} and a small-scale mammography system based on the technique was the first phase-contrast system to be commercially available.²⁰ A clinical trial of the system measured no significant difference in recall and cancer detection rates, however,²¹ possibly due to requirements on a reasonable imaging

time. Nevertheless, clinical and preclinical investigations of free-space propagation with synchrotron radiation have shown promising results.^{22–24}

- (4) Talbot interferometry, also known as grating interferometry, grating-based phase-contrast imaging, or differential phase-contrast imaging, relies on a number of gratings that are placed in the beam path.^{25–28} Introduction of gratings yields a relatively complex setup, which may be sensible to vibrations, grating imperfections, etc., and gratings that are placed after the object reduce dose efficiency. Nevertheless, the requirements on coherence are relatively low, the setup is compact, phase and absorption images can be distinctly separated, and an array of small sources may be used for illumination, which improves photon economy compared to a single small source. Specimen imaging in a small-scale Talbot interferometer has shown improved fine-structure visualization.²⁹

One additional method is based on so-called coded apertures.³⁰ Much research is focused on three-dimensional phase-contrast imaging, in particular CT,^{31–34} but also breast tomosynthesis.³⁵

The purpose of the present study is to quantitatively compare phase-contrast imaging in relation to conventional absorption-contrast imaging, which is currently the gold standard in x-ray radiology. The focus is on mammography, which has been identified as one of the most promising areas for phase-contrast imaging.^{5–7} For two reasons, we have chosen to investigate a setup based on differential phase-contrast imaging (Talbot interferometry): (1) The technique has some attractive features for small-scale imaging as outlined above; (2) the grating geometry fits the linear detector array of an existing photon-counting mammography system,^{36–38} which facilitates the comparison to absorption contrast and makes clinical feasibility probable. The terms “differential phase contrast imaging” and “Talbot interferometry” will be used interchangeably in the following, but the latter term will be preferred to avoid confusion with the differential phase signal.

Despite the growing interest in phase-contrast imaging, evaluations that take into account the full imaging chain and task are relatively few. General ideal-observer analyses do exist,³⁹ but there is a lack in the literature of quantitative comparisons to dedicated absorption contrast with focus on the restrictions that appear in medical imaging (dose, geometry, and photon economy). The present study is a task-specific comparison of Talbot interferometry and conventional absorption contrast in a clinically relevant mammography setting, under equal-dose and equal-geometry constraints, and using familiar Fourier-based metrics such as the noise-power

FIG. 1. (Left panel) Schematic of the generic absorption-contrast system that was used as gold standard in the study and with geometry loosely based on the Philips MicroDose Mammography system. It is equipped with photon-counting silicon strip detectors that are scanned across the object to acquire an image. (Right panel) Schematic of the Talbot interferometer with equal geometry as the generic absorption-contrast system. A beam splitter (phase grating) illuminated by an x-ray source induces interference fringes in the x -direction. The fringes are displaced by phase gradients in an object, which can be located either before or after the beam splitter. A fine-pitch analyzer grating can be used to demodulate the high-frequency fringes into lower frequencies so that the fringe displacement and hence the object phase gradient can be measured by the coarser detector elements (phase stepping). To cover the full field-of-view, the strip detectors are scanned in the y -direction. The figure is not to scale.

spectrum (NPS), modulation transfer function (MTF), and noise-equivalent number of quanta (NEQ). Further, the framework allows for inclusion of the coherence-flux trade-off, i.e., reduced photon economy in phase contrast caused by requirements on source size and spectrum bandwidth. Analytical results have been verified by wave-propagation simulations.

II. METHODS

II.A. Background

II.A.1. A generic absorption-contrast mammography system

The Philips MicroDose Mammography system (Philips Digital Mammography AB, Solna, Sweden) is a commercially available photon-counting digital mammography system.^{36–38} It comprises a tungsten-target x-ray tube, a precollimator, and an image receptor, all mounted on a common arm. The image receptor consists of several modules of photon-counting silicon strip detectors with corresponding slits in the precollimator. To acquire an image, the arm is rotated around the center of the source so that the detector modules and precollimator are scanned across the object. Scatter shields between the modules block detector-to-detector scatter, and scattered radiation in the object is efficiently rejected by the multislit geometry.³⁸ A low-energy threshold in the de-

tector rejects virtually all electronic noise. The pixel size is $50 \times 50 \mu\text{m}^2$.

As a basis of comparison for our study, we use a hypothetical absorption-contrast system, which is shown as a schematic in the left-hand panel of Fig. 1. This setup will in the following be referred to as the “generic absorption-contrast system.” It is similar, but not identical, to the MicroDose system; while distances are not equal in detail, the overall geometry is the same. Using the MicroDose system as a model for the generic absorption-contrast system ensures that we as far as possible obey the limitations, in particular geometrical constraints, that are put on a clinical system.

In Fig. 1 and henceforth, x refers to the detector strip direction and y to the scan direction. The term “pixel” will be used interchangeably with “detector element” in the following. Although not generally the same, the two terms represent the same size for all cases considered here.

II.A.2. The Talbot interferometer

To make a fair and intelligible comparison of systems dedicated to, respectively, phase and absorption contrast, we have investigated a Talbot interferometer with geometry identical to the generic absorption-contrast system except that gratings are introduced in the beam path. This particular implementation is depicted in the right-hand panel of Fig. 1. A phase

TABLE I. Glossary of variables and symbols.

Subscripted A ; T_Φ , $T_{\Phi'}$, T_A	Indices for generic absorption contrast; phase, differential-phase, and absorption contrast in Talbot interferometry
$\mathbf{r} = (x, y), \varrho, z$	Spatial coordinates in the grating (detector-strip), scan, radial, and beam-propagation directions
$\mathbf{f} = (f_x, f_y), f_\varrho$	Spatial frequencies in the x -, y -, and ϱ -directions
$\lambda; k = 2\pi/\lambda; E$	X-ray wave length; wave vector; photon energy
$E_d, \lambda_d; E^*$	Interferometer design energy and design wavelength; optimal energy
$\Delta E/\bar{E}, \bar{E}; \text{kVp}$	X-ray spectrum energy resolution and mean energy; x-ray tube acceleration voltage
$n = 1, 3, 5, \dots; \eta = 2$	Talbot order; beam-splitter parameter for a π -phase-shifting beam splitter
d_n, D_n	Talbot distances for spherical and plane waves
SCD; L	Source-to-precollimator distance; source-to-beam-splitter distance
Λ	Fraction of d_n from object to detector: $\Lambda = 1 \rightarrow$ object at/before beam splitter, $\Lambda = 0 \rightarrow$ object at detector
$s; s_0$	Source size; source-grating slit width ($s_0 = s$ in case of no source grating)
$p_0; p_1; p_2, P_2$	Source-grating pitch; beam-splitter pitch; analyzer-grating pitch for spherical/plane waves
$p_f, P_f; \Delta p_f$	Fringe period for spherical/plane waves; fringe displacement
Φ, Φ'	Object phase shift and differential phase shift
$N_0; N$	Counts incident on the object per pixel; counts per pixel including object absorption
Θ	Count rate
$\psi; \Psi, \Psi_{\max}, \Psi_{\min}$	Fringe function; phase-step function with maximum and minimum
$\square(\theta); \wedge(\theta)$	Periodic square function; periodic triangle function;
m, M	Step index and number of steps for a phase-stepping scan
κ	Index for object location: $\kappa = 0 \rightarrow$ object before beam splitter, $\kappa = 1 \rightarrow$ object after beam splitter
$\Gamma_0; \Gamma_1; \Gamma_2; \Gamma_{1,2} \equiv \Gamma_1^{1-\kappa} \Gamma_2$	Average transmission: source grating; beam splitter; analyzer grating; beam splitter + analyzer grating
$G_{2m}(x)$	Transmission function of the analyzer grating
$j; \Pi[(x - x_j)/p_d]; x_j; p_d$	Pixel index; pixel function; pixel location; pixel size
$V_\psi; V_\Psi$	Modulation of the fringe function; modulation of the phase-step function
$V_2; V_s$	Modulation contributions from analyzer-grating transmission; from projected source size;
V_r, V_{rE}	due to deviations from the design energy over spectrum (bandwidth) and as a function of energy (monochromatic)
$I(x, y); \Delta s; C$	Image signal; target-to-background signal difference; target-to-background contrast
$\mu; n_r = 1 - \delta + i\beta$	Linear absorption; complex refractive index
$t_b; t_r, r_t$	Object thickness; target thickness, and radius
h	Target (hypothesis) function
$\text{MTF}(f); S(f)$	Modulation-transfer function (MTF); quantum noise-power spectrum (NPS)
$\text{NEQ}(f); W(f); F(f)$	Noise-equivalent number of quanta (NEQ); task function; signal template
$d', \hat{d}'$	Detectability index and detectability benefit ratio
$u(x, z), U(f_x, z); b(x), B(f_x)$	Wave field in the spatial and Fourier domains; wave propagator in the spatial and Fourier domains

grating, denoted beam splitter, introduces an effect known as Talbot self-images, which are interference fringes that appear at periodic distances from the grating and parallel with the grating strips. For a spherical wave induced by a point source, the so-called Talbot distances are^{32,40}

$$d_n = \frac{D_n L}{L - D_n}, \text{ where } D_n = \frac{np_1^2}{2\eta^2\lambda_d}, \quad n = 1, 3, 5, \dots \quad (1)$$

Here, D_n is the Talbot distance for a plane wave, L is the source-to-grating distance, n is the Talbot order, p_1 is the pitch of the beam splitter, λ_d is the design wavelength of the interferometer, and η is a parameter that depends on the beam-splitter type. We will assume a π phase-shifting beam splitter in the following, which implies $\eta = 2$. For a $\pi/2$ -shifting beam splitter, $\eta = 1$, and for an absorption grating, $\eta = 1$ with the maxima in Eq. (1) occurring at even instead of odd Talbot orders.

The period of the interference fringes is

$$p_f = \frac{P_f L}{L - D_n}, \text{ where } P_f = \frac{p_1}{\eta}. \quad (2)$$

Again, $P_f = p_f(L \rightarrow \infty)$ is the fringe period for a plane incident wave.

If the source is covered by an absorption grating, denoted source grating, with slits that are s_0 wide and with a pitch of

$$p_0 = p_f \frac{L}{D_n}, \quad (3)$$

the Talbot images generated from the different slits coincide and generate a higher flux (Talbot-Lau geometry).²⁸ This scheme increases the available flux substantially, and is one of the major assets of differential phase-contrast imaging in a medical imaging context.

Variables and symbols that are used in the above and following are summarized in Table I.

II.A.3. Detection of phase and absorption

When a phase-shifting object is introduced in the beam, the beam is refracted an angle $\alpha = \Phi'\lambda/2\pi$, where Φ' is the differential phase shift of the object.²⁸ For small α , the refraction causes a fringe displacement

$$\Delta p_f \approx \Lambda d_n \times \alpha = \Lambda d_n \frac{\lambda}{2\pi} \Phi' \quad (4)$$

at a distance Λd_n from the object, where Λ ranges from 0 for an object placed in contact with the detector to 1 for an object at or upstream of the beam splitter.

The fringes are periodic as a function of x , and for a square-shaped beam splitter, the fringes are square with a fringe function

$$\psi(x) = \Gamma_1^{1-\kappa} \times N_0 \exp(-\mu(x)t_b(x)) \times \left[1 + V_\psi \times \Pi \left(2\pi \frac{x + \Delta p_f(x)}{p_f} \right) \right], \quad (5)$$

where $\Gamma_1 \in [0, 1]$ is the average transmission of the beam splitter and the value of κ indicates whether the object is located upstream ($\kappa = 0$) or downstream ($\kappa = 1$) of the beam splitter. N_0 is the number of counts incident on an object with thickness t_b and linear attenuation μ , and absorption is calculated with the Beer-Lambert law. V_ψ is the fringe modulation, which will be discussed in Sec. II.A.4. The fringe modulation is the Michelson (peak-to-peak) contrast of the fringe function, and is commonly referred to as “visibility” in the phase-contrast literature. We refrain to use that term here, however, because ambiguities can arise in the context of image quality. The square function is defined here as $\Pi(\theta) \equiv \text{sgn}[\sin(\theta)]$, which is a periodic function not to be confused with the rect function. For a beam splitter with a realistic degree of imperfections, a sinusoid might be a better approximation of the fringe function,²⁷ in which case $\Pi()$ in Eq. (5) can simply be substituted for $\sin()$.

According to Eq. (5), a phase gradient in the object is phase modulated on the fringe function, i.e., the phase gradient causes a shift of the fringes, which can be measured to obtain Φ' . According to Eq. (2), however, the fringe-function period (p_f) is in the same order as the beam splitter pitch (p_1), which is in the order of microns for realistic setups, and detectors with enough resolution to sample ψ may be difficult to procure. The high requirements on detector resolution can, however, be exchanged for high mechanical precision in the form of a stepwise precision scan of a fine-pitch absorption grating (analyzer grating) located in front of the detector. This procedure is known as phase stepping.⁴⁰

The intent of phase stepping is to demodulate a relatively low-frequency variation in Δp_f from the high-frequency oscillations of the fringe function. The analyzer grating is moved in a series of $M \geq 3$ steps at fractions of the fringe period. At each step, an image is acquired with signal

$$\Psi_{mj} = \int \psi(x) G_{2m}(x) \Pi \left(\frac{x - x_j}{p_d} \right) dx, \quad (6)$$

where $G_{2m}(x)$ is the grating transmission function and $\Pi[(x - x_j)/p_d]$ is the pixel function of pixel j , ideally a rect function located at x_j with pixel size p_d . Ψ will be referred to as the phase-step function in the following. The pitch of the analyzer grating needs to match the fringe period, i.e.,

$$p_2 = p_f = \frac{P_2 L}{L - D_n}, \text{ where } P_2 = \frac{p_1}{\eta}. \quad (7)$$

In other words, the intensity transmitted by the analyzer grating is ideally uniform at each scan step m for an unperturbed beam. A shift of the fringe function, caused by a phase gra-

dient in the object, results in intensity variations behind the analyzer grating, which are in the same order as the size of the object phase gradient. Hence, the detector now limits spatial resolution rather than limiting phase-detection efficiency.

For a square-shaped analyzer grating and uniform steps with a length of p_2/M ,

$$G_{2m}(x) = \Gamma_2 \left(1 + \left[\frac{1}{\Gamma_2} - 1 \right] \times \Pi \left[2\pi \left(\frac{x}{p_2} + \frac{m}{M} \right) \right] \right), \quad (8)$$

where $\Gamma_2 \geq 0.5$ is the average transmission of the grating and accounts for leakage through the grating ridges. Some leakage is reasonable to expect in the practical case, which then implies $\Gamma_2 > 0.5$ and accuracy of the fringe detection is reduced. Other conceivable scenarios may be included in the analysis by a simple modification of Eq. (8). For instance, some material may be left in the grating slits ($\Gamma_2 < 0.5$), and grating slits may deviate from $p_2/2$, either because of etching imperfections or by a desire to change the trade-off between absorption- and phase-contrast efficiency.

Using Eqs. (5) and (8), Eq. (6) evaluates to

$$\Psi_{mj} = \Gamma_{1,2} \times N_{0m} \exp(-\mu t_b|_j) \times \left(1 + V_\psi \times \wedge \left[2\pi \left(\frac{\Delta p_{fj}}{p_2} + \frac{m}{M} \right) \right] \right), \quad (9)$$

where $\Gamma_{1,2} \equiv \Gamma_1^{1-\kappa} \Gamma_2$, N_{0m} is the incident number of counts per phase step, and $V_\psi \propto V_\psi$ is the modulation. We define the periodic triangle function as $\wedge(\theta) \equiv 2/\pi \times \int_0^\theta \Pi(\phi) d\phi - 1$.

As described by Eq. (8), transmission of the grating ridges reduces the modulation of the phase-step function according to

$$V_\psi \propto V_2 \equiv \left[\frac{1}{\Gamma_2} - 1 \right], \quad (10)$$

where V_2 is the modulation contribution from the analyzer grating transmission. Phase-step modulation is further determined by the projected source size and spectrum bandwidth, which will be discussed in Sec. II.A.4. The impact of modulation on fringe-detection efficiency will be evident in Sec. II.B.

The differential phase shift (Φ') can be calculated via Eq. (4) with the fringe displacement deduced by solving the phase-step function in Eq. (9) for Δp_f (i.e., inverting the function). In order to compensate for inhomogeneities in the beam and other irregularities in the practical case, Δp_f is preferably measured relative a reference measurement of an unperturbed beam. The phase-contrast signal, which we define as the phase shift ($I_{T_\Phi} \equiv \Phi$), is in turn obtained by integration of Φ' , which in a discrete detector array is approximated by a sum. Hence, the signal in pixel j is

$$I_{T_\Phi j} = \sum_{i=0}^j \Phi'_i \times d_p \sim \int_0^{x_j} \Phi'(x) dx, \quad (11)$$

where

$$\Phi'_i = \frac{2\pi}{\Lambda d_n \lambda} p_2 \times (\Psi_i^{-1}(\Delta p_{fi}) - \Delta p_{fi}|_{\text{ref}}). \quad (12)$$

Subscript *ref* indicates reference measurement, $\Psi \equiv \sum_{m=1}^M \Psi_m$, and superscript -1 denotes the inverse of Ψ (i.e., solved for Δp_f). Hence, it is implicitly assumed that Ψ is invertible with respect to Δp_f . This assumption is valid only for $M \geq 3$ because there are two additional unknowns in Eq. (9), namely, absorption $[\exp(-\mu t_b)]$ and an offset of the periodic function caused by scattering [not explicitly included in Eq. (9)]. Equation (12) is a general formulation and does not detail the method of inverting, which could be done for instance by least-square fitting or Fourier analysis. It should be noted that there is an upper limit on the detectable fringe displacement; when Δp_f exceeds p_f , so-called phase wrapping occurs, which leads to ambiguities in reconstructing Φ' . Nevertheless, it is the derivative and not the phase itself that is restricted, and thick objects, therefore, do not in general constitute a problem.

Object absorption affects the number of detected quanta, i.e., the amplitude of the phase-step function. In order to have proportionality to μt_b , which is in analogy with the phase-contrast signal, the Talbot absorption-contrast signal (subscript T_A) is taken to be the logarithm of the detected number of quanta, i.e.,

$$\begin{aligned} I_{T_A} &= \ln[\Psi_j] = \ln \left[\sum_{m=1}^M \Psi_{mj} \right] \\ &= \ln[\Gamma_{1,2} \times N_0 \exp(-\mu t_b |_j)]. \end{aligned} \quad (13)$$

The image signal for each pixel in the generic absorption-contrast system (subscript A) is also taken to be the logarithm of the number of detected photons in order to be proportional to μt_b and comparable to the Talbot phase- and absorption-contrast signals

$$I_{A_j} = \ln[N] = \ln[N_0 \exp(-\mu t_b |_j)], \quad (14)$$

which is similar to Eq. (13), but there are no gratings and no phase stepping.

An area detector may be used to cover the full field-of-view in the Talbot interferometer. Another option, which is depicted in Fig. 1, is to scan strip detectors in the y direction, similar to the MicroDose system. In the latter case, it is possible to accommodate phase-stepping and image acquisition into a single scan movement, which is clearly advantageous from a technical point-of-view. The following discussion does, however, no assumption on the type of detector or implementation of phase stepping and is not restricted to any of these cases.

II.A.4. Effects of source size and spectrum bandwidth

The modulation of the phase-step function is defined as

$$V_\Psi = \left\langle \frac{\Psi_{\max} - \Psi_{\min}}{\Psi_{\max} + \Psi_{\min}} \right\rangle, \quad (15)$$

where $\Psi_{\max}$ and $\Psi_{\min}$ are the intensity maxima and minima, respectively.³² In addition to grating transmission, projected source size and x-ray spectrum bandwidth (essentially spatial and temporal coherence) affect the modulation of the phase-step function according to

$$V_\Psi \propto V_s \times V_t \times V_2, \quad (16)$$

where V_s and V_t are the contributions from source size and bandwidth, and $V < 1$ for a finite source size or spectrum bandwidth. The source size in this context refers either to a single small source or to the slit width of a source grating. V_2 is the contribution from grating transmission and was defined in Eq. (10). We have assumed V_s , V_t , and V_2 to be decoupled.

The phase-step function for a finite source is a convolution of Eq. (6) with the projected source function. For a square-shaped analyzer grating and a rect-shaped source of width s_0 , the maximum and minimum of the convolution ($\Psi_{\max}$ and $\Psi_{\min}$) yield

$$V_s = 1 - \frac{s_0}{p_f} \times \frac{D_n}{L} = 1 - \Gamma_0 \text{ for } \Gamma_0 < 1, \quad (17)$$

where $s_0 D_n / p_f L$ is the projected source size at the detector plane normalized to the size of the interference fringes, which happens to equal the average transmission of the source grating (Γ_0), if such is used. As opposed to the analyzer grating, which is taken to have a fill factor of exactly 0.5 [$\Gamma_2 > 0.5$ implies transmission through the grating ridges in Eq. (8)], we define Γ_0 as being the fill factor of the source grating and assume perfect absorption, i.e., $\Gamma_0 = s_0/p_0$ with p_0 according to Eq. (3). We note that a small source/small fill factor and/or long source-to-object distance increases the modulation, i.e., $V_s \rightarrow 1$ as $s_0 \rightarrow 0$, $\Gamma_0 \rightarrow 0$, or $L \rightarrow \infty$. Conversely, $V_s \rightarrow 0$ as $\Gamma_0 \rightarrow 1$. A similar expression for the modulation with a Gaussian source and a sinusoidal phase-step function has been derived by Weitkamp *et al.*³²

Spectrum bandwidth reduces fringe modulation because the Talbot distance cannot be uniquely defined according to Eq. (1) for a spectrum of energies. If the phase shift of the beam splitter is approximated to be independent of wavelength (λ), the change in modulation with λ at a fixed distance from the beam splitter is, according to Eq. (1), equivalent to the change in modulation along the optical axis for a fixed λ . For simplicity, we assume that the variation is triangular and the monochromatic modulation as a function of energy becomes

$$V_{tE} = 1 + \bigwedge \left(2\pi n \frac{E_d}{E} \right), \quad (18)$$

where E is the photon energy (inversely proportional to λ) and E_d is the design energy of the interferometer. The effective modulation is the mean of the monochromatic modulation over the spectrum bandwidth (ΔE), and the phase-step modulation is proportional to the fringe modulation ($V_\Psi \propto V_\Psi$). Hence, the modulation for a rect-distributed x-ray spectrum

with mean energy $\bar{E} = E_d$ becomes

$$V_t = \frac{1}{\Delta E} \int_{\bar{E}-\Delta E/2}^{\bar{E}+\Delta E/2} V_{tE} dE$$

$$= 1 + n \frac{\bar{E}}{\Delta E} \times \ln \left[1 - \frac{1}{4} \left(\frac{\Delta E}{\bar{E}} \right)^2 \right] \text{ for } E \geq n \frac{\bar{E}}{2} \quad (19a)$$

$$\approx 1 - n \frac{1}{4} \frac{\Delta E}{\bar{E}}, \quad (19b)$$

where $\Delta E/\bar{E}$ is the spectrum energy resolution. The approximation in Eq. (19b) is valid for $\Delta E/\bar{E} \ll 2$. We note that $V_t \rightarrow 1$ as $\Delta E/\bar{E} \rightarrow 0$.

II.B. Cascaded-systems analysis

II.B.1. Ideal-observer model

The NEQ is an efficient and widespread metric to assess the performance of medical imaging systems^{41–45}

$$\text{NEQ}(f) = \frac{\langle I \rangle^2 \text{MTF}^2(f)}{S(f)}, \quad (20)$$

where $f = (f_x, f_y)$ is the spatial frequency vector, $\langle I \rangle$ is the expected image signal, MTF is the modulation transfer function, and S is the NPS. The present analysis will treat quantum noise only and does not include other noise sources, such as electronic noise or anatomical noise, where the former in any case is low for the photon-counting detector. In general, the shift-invariance requirement of linear-systems theory is violated in digital systems at the level of the pixel size, i.e., the response will vary with subpixel shifts in the impulse location.^{46,47} This issue is resolved, however, by use of the presampled or expectation MTF in Eq. (20), and by approximating any transfer function with the average over a large number of different input positions relative to the pixel matrix.

For task-specific system performance, we can define an ideal-observer detectability index according to^{41–45}

$$d'^2 = \int_{\text{Ny}} \text{NEQ}(f) \times W^2(f) df. \quad (21)$$

The integral is taken over the Nyquist region, which for sampled quantities is equivalent to integration over the entire frequency space. W is a binary task function, i.e., the Fourier difference between two hypotheses: $W = |\mathcal{F}\{h_1\} - \mathcal{F}\{h_2\}|$, where $\mathcal{F}\{\}$ denotes the Fourier transform. Detection tasks represent the hypothesis of signal present versus signal absent and the task function reduces to $W_{\text{detection}} = C \times F$, where $C = \Delta s / \langle I \rangle$ is the target-to-background contrast for the peak signal difference $\Delta s = \max\{|I_{\text{background}}(\mathbf{r}) - I_{\text{target}}(\mathbf{r})|\}$, where $\mathbf{r} = (x, y)$ is the coordinate vector. F is the signal template, which represents the frequency content of the target and integrates to the area for a flat target (unity contrast).

In Secs. II.B.2–II.B.4, we will derive expressions of the quantities in Eq. (20) for generic absorption contrast and Talbot interferometry. The NEQ and d' will be used as figures-of-merit to evaluate and compare the two techniques.

FIG. 2. Thickness-normalized phase- and absorption-contrast signals ($k \times \delta$ and μ) as a function of energy and for average breast tissue. The absorption-contrast signal is additionally divided into photoelectric and Compton-scattering components, i.e., $\mu = \mu_{PE} + \mu_C$.

II.B.2. Signal

The absorption-contrast signal difference between target material t and surrounding tissue b in the generic absorption-contrast system is calculated via Eq. (14),

$$\Delta s_A = |\langle I_{At} \rangle - \langle I_{Ab} \rangle| = |\mu_t - \mu_b| t_t. \quad (22)$$

Similarly, Eq. (13) yields the Talbot absorption-contrast signal difference,

$$\Delta s_{TA} = |\langle I_{TA_t} \rangle - \langle I_{TA_b} \rangle| = |\mu_t - \mu_b| t_t. \quad (23)$$

The phase-contrast signal difference is calculated with Eq. (11) according to

$$\Delta s_{T\Phi} = |\langle I_{T\Phi_t} \rangle - \langle I_{T\Phi_b} \rangle| = |\langle \Phi_t \rangle - \langle \Phi_b \rangle| = k |\delta_t - \delta_b| t_t, \quad (24)$$

where $k = 2\pi/\lambda$ is the wave number, and δ_t and δ_b are the decrements from unity of the real part of the complex refractive index for the respective materials. Note that beam hardening was ignored in these schematic formulas, i.e., attenuation and refractive index are both taken to be effective or else the beam is assumed to be monochromatic.

Figure 2 plots thickness-normalized phase- and absorption-contrast signals for average breast tissue and as a function of energy. Away from absorption edges, the signals follow approximately⁴⁸

$$k \times \delta \propto E^{-1} \rho \quad \text{and} \quad \mu \propto \begin{cases} E^{-3} Z^{3.2} \rho & \text{at low } E \\ \rho & \text{at high } E \end{cases}, \quad (25)$$

where ρ is the mass density and Z represents atomic number. Linear absorption is divided into two regions dominated by photoabsorption and Compton scattering at, respectively, low and high energies. The crossing between the two interaction processes depends on material properties.

II.B.3. Spatial resolution

Spatial resolution in both the generic absorption-contrast system and the Talbot interferometer is affected by, e.g., source size, detector aperture, and the readout step in the y -direction. We do not treat the details of these common blurring effects here, but simply state that all are captured in

the presampling MTF of the generic absorption-contrast system, which we denote $\text{MTF}_A(f)$.

The Talbot interferometer differs from the generic absorption-contrast system only by the gratings. In systems with extremely high resolution, gratings and phase stepping may add substantial blurring,⁴⁰ and it is also reasonable to believe that shift invariance would be violated by phase stepping. Nevertheless, for most applications in medical imaging and for all cases that we will consider, the source, detector aperture, and readout step are 1–2 orders of magnitude larger than any grating pitch, and hence dominate resolution. We will therefore disregard blurring by gratings on the MTF and on any transfer functions (the gratings appear as uniform absorption filters), and the Talbot absorption-contrast (T_A) MTF is approximately

$$\text{MTF}_{T_A}(f) = \text{MTF}_A(f). \quad (26)$$

Accordingly, spatial resolution is not determined by the slit width (s_0) in a source grating, if any, but by the actual x-ray source size (s), i.e., effectively the distance between the outermost grating slits.

The impulse response in differential phase-contrast (T_Φ), is the derivative of an impulse $\delta(\mathbf{r})$ (not to be confused with δ for the real part of the refractive index). The MTF transfer function is the Fourier transform ($\mathcal{F}\{\}$) of the impulse response, and the differential phase-contrast MTF becomes

$$\text{MTF}_{T_\Phi}(f) \propto \left| \mathcal{F} \left\{ \frac{\partial}{\partial x} \delta(\mathbf{r}) \right\} \right| \times \text{MTF}_A(f) \propto f_x \times \text{MTF}_A(f), \quad (27)$$

where it is assumed that $\delta(\mathbf{r})$ is presampled and hence continuous. The frequency term in Eq. (27) causes edge enhancement. We will, however, consider phase-contrast (T_Φ) in the following, in which the derivative is canceled by integration according to Eq. (11) so that the MTF is

$$\text{MTF}_{T_\Phi}(f) \propto \left| \mathcal{F} \left\{ \int \frac{\partial}{\partial x} \delta(\mathbf{r}) dx \right\} \right| \times \text{MTF}_A(f) \propto \text{MTF}_A(f). \quad (28)$$

Because of normalization to unity at zero spatial frequency, the proportionality in Eq. (28) implies that the MTFs for generic absorption contrast and Talbot phase contrast are approximately equal within this framework ($\text{MTF}_{T_\Phi} = \text{MTF}_A$).

II.B.4. Quantum noise

The digital NPS measured directly on the detector signal and truncated to the region X by Y is⁴⁵

$$S_N(f) = \frac{p_d^2}{XY} \times \langle |\mathcal{F}\{N(\mathbf{r}) - \langle N \rangle\}|^2 \rangle, \quad (29)$$

where $N(\mathbf{r}) \equiv N_0 \exp(-\mu(\mathbf{r})t_b(\mathbf{r}))$ is the detected number of quanta. $\mathbf{r}$ is in this case discrete (representing pixel locations), $\mathcal{F}\{\}$ denotes a discrete Fourier transform, and $\mathbf{f}$ is valid at sampled frequencies. Under assumptions of linearity, S_N is transferred to the image through the expected squared derivative of the image function [see, e.g., Ref. 49, in particular Eq. (A7)].

The image signal in the generic absorption-contrast system (I_A) is a function of N according to Eq. (14), and the image-signal NPS (S_A) becomes

$$S_A(f) = \left\langle \left(\frac{\partial I_A}{\partial N} \right)^2 \right\rangle \times S_N(f) = \frac{1}{N^2} \times S_N(f), \quad (30)$$

where we have assumed small signal differences so that the logarithm in Eq. (14) is approximately linear. For uncorrelated detector elements, $S_N = N$ and $S_A = 1/N$, but no such assumption is applied in the following unless explicitly stated.

In Talbot interferometry, the NPS is transferred through phase stepping as the sum of individual contributions over the series. Hence,

$$\begin{aligned} S_\Psi(f) &= \sum_{m=1}^M S_{\Psi_m}(f) = \frac{p_d^2}{XY} \times \sum_{m=1}^M \langle |\mathcal{F}\{\Psi_m(\mathbf{r}) - \langle \Psi_m \rangle\}|^2 \rangle \\ &= \sum_{m=1}^M \frac{\Psi_m}{N_m} \times S_{N_m} = \frac{\Psi}{N} \times S_N = \Gamma_{1,2} \times S_N, \end{aligned} \quad (31)$$

where N_m and S_m are the per-phase-step number of detected photons and NPS, respectively, and $\Psi \equiv \sum_{m=1}^M \Psi_m$.

In Talbot absorption contrast, the detected number of quanta after phase stepping is transferred to image signal through Eq. (13). Analogous to Eq. (30), the transfer function to image-signal NPS becomes⁴⁹

$$\left\langle \left(\frac{\partial I_{T_A}}{\partial \Psi} \right)^2 \right\rangle = \frac{1}{(\Gamma_{1,2}N)^2}. \quad (32)$$

Therefore,

$$\begin{aligned} S_{T_A}(f) &= \left\langle \left(\frac{\partial I_{T_A}}{\partial \Psi} \right)^2 \right\rangle \times S_\Psi(f) \\ &= \frac{1}{\Gamma_{1,2}} \times S_A(f), \end{aligned} \quad (33)$$

where we have chosen to express the NPS in terms of S_A to facilitate comparison to the generic absorption-contrast system. Note that the difference boils down to absorption in the gratings.

The detected number of quanta after phase stepping is transferred to differential phase contrast through Eq. (12), which is built up of Eq. (4) (transfer from Δp_f to differential phase) and Eq. (9) (detection of Δp_f from the phase-step function). In analogy with Eq. (30), the corresponding NPS transfer functions become

$$\left\langle \left(\frac{\partial \Phi'}{\partial \Delta p_f} \right)^2 \right\rangle = \left(\frac{2\pi}{\lambda \Delta d_n} \right)^2 \quad (34)$$

and

$$\left\langle \left(\frac{\partial \Psi}{\partial \Delta p_f} \right)^2 \right\rangle = \left(\Gamma_{1,2} N V_\Psi \times \frac{2}{\pi} \times \square \times \frac{2\pi}{p_2} \right)^2, \quad (35)$$

where we have used $\partial/\partial\theta \wedge (\theta) = 2/\pi \times \square(\theta)$.

Using Eqs. (34) and (35), the differential phase-contrast (T_Φ) NPS evaluates to

$$\begin{aligned} S_{T_\Phi}(f) &= \left\langle \left(\frac{\partial \Phi'}{\partial \Psi} \right)^2 \right\rangle \times S_\Psi \\ &= \left\langle \left(\frac{\partial \Phi'}{\partial \Delta p_f} \right)^2 \right\rangle \times \left\langle \left(\frac{\partial \Delta p_f}{\partial \Psi} \right)^2 \right\rangle \times S_\Psi \\ &= \pi^2 \times \left(\frac{\lambda_d}{\lambda} \right)^2 \times \frac{1}{\Lambda^2 n^2 p_2^2} \times \frac{1}{\Gamma_{1,2} V_\Psi^2} \times S_A(f), \end{aligned} \quad (36)$$

where we have used $\Gamma^2 = 1$. Further, we used $\partial \Delta p_f / \partial \Psi = 1/(\partial \Psi / \partial \Delta p_f)$, which is valid for invertible functions by the inverse function theorem. Ψ is invertible under restrictions discussed in Sec. II.A.3. Sinusoidal gratings yield a different value on $\partial \Delta p_f / \partial \Psi$, which would increase the NPS slightly.

The differential phase-contrast signal is integrated to phase contrast in Eq. (11). The corresponding NPS transfer function adds a frequency dependence to the NPS according to

$$\begin{aligned} S_{T_\Phi}(f) &= \frac{p_d^2}{XY} \times \left\langle \left| \mathcal{F} \left\{ \int_0^x \Phi'(r) - \langle \Phi' \rangle dx \right\} \right|^2 \right\rangle \\ &= \frac{1}{(2\pi f_x)^2} \times S_{T_\Phi} \\ &= \frac{1}{4} \times \left(\frac{\lambda_d}{\lambda} \right)^2 \times \frac{1}{\Lambda^2 n^2 p_2^2} \times \frac{1}{\Gamma_{1,2} V_\Psi^2} \times \frac{1}{f_x^2} \times S_A(f). \end{aligned} \quad (37)$$

II.C. Comparison of phase and absorption contrast

II.C.1. Optimal energy

δ and μ both decrease monotonically with energy [Eq. (25)], and so does the deposited dose to the breast, which is related to absorption. Therefore, we can expect a maximum in detectability per unit dose at a certain incident energy (the optimal energy, denoted E^*), which provides an optimal compromise between high contrast at low energies and low noise (high transmission) at high energies. This is a well-known effect for absorption contrast,⁵⁰ and it is reasonable to have the same expectations on phase contrast, as will be illustrated with the following back-of-the-envelope calculation.

The energy dependence of absorption- and phase-contrast signals can be approximated by inserting Eq. (25) into Eqs. (22) and (24),

$$\Delta s_A(E) \propto E^{-3} \quad \text{and} \quad \Delta s_{T_\Phi}(E) \propto E^{-1}, \quad (38)$$

where we have assumed dominance by the photo-electric effect in absorption contrast. With uncorrelated noise, monochromatic radiation, and a setup optimized at each energy ($\lambda_d = \lambda$), Eqs. (30) and (37) yield the NPS energy dependence

$$S_A(E) \propto S_{T_\Phi}(E) \propto \frac{1}{N_0(E) \times \exp(-C_1 \times E^{-3})}, \quad (39)$$

where C_1 is a constant. Hence, phase- and absorption-contrast NPS have identical energy dependencies for this

case. If we approximate the dose deposition as a function of energy [AGD(E)] in the considered energy range to be inversely proportional to the photon energy, we arrive at $N_0(E) \propto 1/\text{AGD}(E) \propto E$ for equal dose at each energy. The detectability index [Eq. (21)] thus follows:

$$\begin{aligned} d_A^2(E) &\propto \exp(-C_1 \times E^{-3}) \times E^{-5} \quad \text{and} \\ d_{T_\Phi}^2(E) &\propto \exp(-C_1 \times E^{-3}) \times E^{-1}. \end{aligned} \quad (40)$$

Note that detectability in Talbot absorption contrast follows $d_{T_A}^2(E) \propto d_A^2(E)$.

A maximum of d^2 with respect to energy (i.e., E^*) can be found with differentiation, i.e., by setting $\partial d^2 / \partial E = 0$, which evaluates to

$$E_A^* \propto (3/5 \times C_1)^{1/3}, \quad E_{T_A}^* = E_A^*, \quad \text{and} \quad E_{T_\Phi}^* \propto (3 \times C_1)^{1/3}. \quad (41)$$

The back-of-the-envelope calculation in Eq. (41) serves to illustrate the trends, but a more elaborate numerical model that is used to draw final conclusions will be presented in Sec. II.C.5.

II.C.2. Comparison under equal dose

We define the detectability benefit ratio as

$$\hat{d}' = \max\{d'_{T_\Phi}, d'_{T_A}\} / d'_A, \quad (42)$$

hence the maximum detectability in Talbot interferometry (from either the phase or the absorption image) normalized to the detectability in generic absorption contrast. $\hat{d}'$ was used as a figure of merit in this study and was in all cases evaluated at equal dose for Talbot interferometry and generic absorption contrast, which is indicated by subscript *iso-dose*. The NPS calculations in Sec. II.B.4 are all presented in terms of S_N , and a direct comparison of the equations for S and calculation of $\hat{d}'$ is hence at iso-dose for a given incident spectrum $[N_0(E)]$. For a fair evaluation of $\hat{d}'_{\text{iso-dose}}$, it is, however, necessary to consider the respective optimal incident energy for phase and absorption contrast according to Sec. II.C.1. This requires different input spectra, and therefore numerical models for dose, attenuation and refraction, which will be introduced in Sec. II.C.5.

II.C.3. Comparison under equal photon economy

Complementary to dose, it is important also to consider photon economy, which takes the imaging process *before the object* into account, as opposed to the detectability analysis in Sec. II.B that evaluated the imaging process *in and after the object*, given a certain incident spectrum. Liouville's theorem implies that any gain in coherence comes at the cost of reduced photon economy, which causes a trade-off for techniques that rely on coherence, such as Talbot interferometry, and phase-contrast imaging in general. Poor photon economy leads to prolonged imaging time and/or increased x-ray tube loading. Requiring equal photon economy for phase and absorption contrast implies a "relative photon economy" of unity, i.e., $\Theta_{T_\Phi} / \Theta_A = 1$, where Θ is the count rate incident

on the object. This constraint will be indicated by subscript *iso-flux* on the detectability benefit ratio ($\hat{d}'_{\text{iso-dose, -flux}}$).

In order to illustrate the coherence-flux trade-off for Talbot interferometry, the following back-of-the-envelope calculation briefly examines the factors that impact photon economy. First, the incident count rate in Talbot interferometry is reduced by grating absorption compared to the generic absorption-contrast system. Second, if we assume that filtered mammographic energy spectra over a reasonable range of acceleration voltages are equal in shape and free of characteristic radiation, the rate of produced photons (the integral over the spectrum) should be approximately proportional to the width (ΔE) of the x-ray spectrum. For a fixed minimum energy ($E_{\min}$), ΔE is proportional to the acceleration voltage (kVp) of the x-ray tube because the maximum energy ($E_{\max}$) equals the maximum kinetic energy of the accelerated electrons. Third, if we require the empirical relationship to hold that the rate of produced photons from an x-ray tube is proportional to kVp squared, we arrive at the conclusion that the rate of produced photons per energy increment (the spectrum “height”) is approximately proportional to kVp. The kVp is in turn proportional to $E_{\max}$ and accordingly to the mean energy of the spectrum ($\bar{E}$) for fixed $E_{\min}$. In summary,

$$\begin{aligned}\Theta_{T_\Phi} &= \Theta_{T_A} \propto \Theta_A \times \Gamma_0 \Gamma_1^\kappa, \quad \text{where} \\ \Theta &\propto \text{kVp}^2 \propto \frac{\Delta E}{\bar{E}} \times \bar{E}^2.\end{aligned}\quad (43)$$

Equation (43) is in terms of $\bar{E}$ and $\Delta E/\bar{E}$ to facilitate comparison to Eqs. (17) and (19). Note that Eq. (43) is for illustration only; a more elaborate numerical model will be presented in Sec. II.C.5 and used to draw final conclusions.

II.C.4. Initial comparison of phase and absorption contrast

Numerical results from the cascaded-systems analysis will be presented in Sec. III, but we make the following initial observations:

- Equation (25) shows that Δs_{T_Φ} probes differences in density. Δs_A and Δs_{T_A} are sensitive to differences in atomic number when the photoelectric effect dominates, but have equal material dependence as Δs_{T_Φ} when Compton scattering is more likely.
- Integration of differential phase-contrast ($T_{\Phi'}$) to phase-contrast (T_Φ) moves the inherent frequency dependence from the MTF to the NPS, but does not change the NEQ. Hence, edge enhancement is converted to power-law noise (brown noise), i.e., a strong noise increase toward lower spatial frequencies. This noise behavior is in accordance with several other studies.^{31,33,34,52,53}
- Brown noise favors detection of small targets. For uncorrelated quantum noise, absorption-contrast detectability for a disc follows $d_A'^2 \propto A \propto r_t^2$, where A is the target area and r_t is the radius. In Talbot phase contrast, the f_x^{-2} noise adds an r_t^{-2} dependence, which cancels the size dependence in one direction and $d_{T_\Phi}^2 = d_{T_\Phi,x}' \times d_{T_\Phi,y}' \propto r_t^0 \times r_t = r_t$. Hence, the detectability benefit ratio for disc targets can be expected to follow $\hat{d}' = d_{T_\Phi}'/d_A' \propto \sqrt{r_t/r_t^2} = r_t^{-0.5}$. Pillbox targets (upright cylinder) will exhibit the same dependence because phase and absorption contrast have equal thickness dependence.
- The NEQ and ideal-observer analysis could be performed directly on the detected raw data with equal results if conversion to phase contrast and log normalization are assumed linear.³⁹ Nevertheless, we prefer to work on reconstructed phase and absorption image data to better understand the relative influence of signal and noise, and to be able to measure NPS and MTF in meaningful images.
- The phase- and absorption-contrast NPS have equal dose dependence, which is in accordance with other studies.⁵⁴ There is hence no reason to consider several dose configurations in the evaluation; the iso-dose case is sufficient to describe the regions of benefit for phase and absorption contrast.
- Increased transmission of the analyzer-grating ridges ($\Gamma_2 > 0.5$) reduces the modulation according to Eq. (10). Transmission also increases the number of detected quanta, however, and the reduced phase-detection efficiency [Eq. (37)] is to some extent compensated for by reduced noise in simultaneous absorption-contrast detection [Eq. (33)].
- It may be advantageous to locate the object after the beam splitter (which implies $\kappa = 1$ and $\Lambda < 1$) in order to keep the setup compact, and because the beam-splitter absorption (Γ_1) may not be negligible. The cost for doing so is reduced phase-contrast efficiency, however, because a short propagation distance yields a small fringe displacement in Eq. (4). Accordingly, the NPS increases indefinitely as $\Lambda \rightarrow 0$, i.e., for objects close to the detector.
- The difference in signal between phase and absorption contrast boils down to the difference between $k \times \delta$ and μ (Fig. 2). At diagnostic energies, Δs_{T_Φ} is approximately two orders of magnitude larger than Δs_{T_A} and Δs_A . Since p_2 is small, however, S_{T_Φ} [Eq. (37)] is substantially larger than S_A and S_{T_A} [Eqs. (30) and (33)], which takes away much of the expected gain of phase-contrast signal-to-noise compared to absorption contrast.
- Δs is dimensionless and S is in units of length squared (l^2 , e.g., [mm²]), which yields an NEQ in l^{-2} . F is in units of l^2 , which makes d' dimensionless.
- Equation (41) illustrates that the optimal incident energy in phase contrast is higher than for absorption contrast (a factor of $5^{1/3} \sim 1.7$) because the energy dependence of δ is weaker than that of μ . We further note that E^* is independent of target thickness (t_t) and material (μ or δ), but only depends on breast properties [$C_1(t_b, \mu_b)$]. This is a well-known effect for absorption contrast⁵¹ and has been indicated for phase contrast.³⁴
- Equation (41) assumes that the photoelectric effect dominates absorption contrast. When Compton scattering is more likely, the energy dependence of δ is steeper than

that of μ and we can expect the optimal energy to be higher in absorption contrast. In average breast tissue, the crossing between photo absorption and Compton scattering occurs at approximately 25 keV (cf. Fig. 2).

- The optimal energy for Talbot interferometry in Eq. (41) assumes that the setup is designed for a monochromatic incident spectrum, i.e., $E_d = E$. If the incident energy deviates from E_d (e.g., in a polychromatic spectrum), phase-contrast detection is influenced in several ways, including: (1) The modulation is reduced ($V_\psi < 1$) because the Talbot interferometer can only be optimized for a single energy [cf. Eq. (19)]. (2) λ_d/λ in Eq. (37) does not cancel, which reduces noise toward lower energies, whereas noise at higher energies is amplified. (3) The beam-splitter phase shift will deviate from π .
- Equation (43) illustrates that photon economy in Talbot interferometry is improved by increased source-grating fill factor (Γ_0) and a broader energy spectrum ($\Delta E/\bar{E}$), but this is in trade-off with the modulation according to Eqs. (17) and (19). A higher mean energy ($\bar{E}$) improves photon economy in both phase and absorption contrast, which favors Talbot interferometry because of the higher optimal energy according to Eq. (41).
- The dominant source of distraction in medical imaging, particularly in mammography, is often not quantum noise, but the variability of the anatomical background.^{55–57} This so-called anatomical noise is commonly assumed to follow a power law, i.e., similar to S_{T_b} . Inclusion of anatomical noise in the cascaded-systems framework is not within the scope of the present paper, but is part of ongoing research. It can be expected to propagate similarly through phase and absorption contrast, but to affect absorption-contrast detectability to a larger extent since the differential phase-contrast NEQ already drops at low spatial frequencies. A simulation method and initial observer-model evaluation of anatomical noise in a mammography system for free-space propagation has recently been presented.⁵⁸

II.C.5. Numerical investigation

Phase- and absorption-contrast imaging were compared numerically using two different beam geometries: (1) a plane monochromatic wave was used to illustrate ideal conditions and intrinsic limitations; (2) a spherical polychromatic wave was used to investigate the impact of finite source size and spectrum bandwidth on detectability and photon economy. The system parameters used for the plane- and spherical-wave geometries are listed in Table II. In both cases, the detector and gratings were assumed ideal, i.e., quantum efficiency, grating imperfections, and absorption in the beam splitter were not taken into account, and gratings were assumed thin so that the angular acceptance was not limited.

A 5-cm 50%-glandularity breast, including a total of 1 cm of skin, imaged with an average glandular dose (AGD) of 1 mGy was considered throughout the study. Tumors, microcalcifications, glandular structures, and air cavities were

TABLE II. Parameters used for the plane- and spherical-wave geometries.

	Plane wave	Spherical wave
Setup:		
Talbot order	$n = 1$	$n = 1$
Design energy	$E_d = 16\text{--}60$ keV	$E_d = 38$ keV
Talbot distance	$D_n = 26\text{--}97$ mm	$d_n = 68$ mm
Source-to-beam-splitter	$L \rightarrow \infty$	$L = 600$ mm
Source-to-precollimator	SCD $\rightarrow \infty$	SCD = 550 mm
Object-to-detector	$\Lambda = 1$	$\Lambda = 1$
Source:		
Shape	–	Point – rect
Size	–	$s = 500$ μm
Grating slit width	–	$s_0 = 0\text{--}29$ μm
Grating pitch	–	$p_0 = 29$ μm
Grating fill factor	–	$\Gamma_0 = 0\text{--}1$
Spectrum	Monochromatic	Monochromatic, rect, or tungsten
Energy resolution	$\Delta E/\bar{E} = 0$	$\Delta E/\bar{E} = 0\text{--}1$
Beam-splitter:		
Grating pitch	$p_1 = 4.0$ μm	$p_1 = 4.0$ μm
Grating transmission	$\Gamma_1 = 1$	$\Gamma_1 = 1$
Analyzer-grating:		
Grating pitch	$P_2 = 2.0$ μm	$p_2 = 2.2$ μm
Grating transmission	$\Gamma_2 = 0.5$	$\Gamma_2 = 0.5$
Detector:		
Pixel size	$d_p = 50$ μm	$d_p = 50$ μm

embedded in the breast. Task functions for Eq. (21) were generated similar to the procedure in Ref. 43. Detection tasks (sphere and pillbox), size discrimination tasks (sphere vs sphere of different size), and shape discrimination tasks (sphere vs pillbox) were considered. The shape discrimination task has been used in the past to model discrimination between “smooth” and “spiculated” targets.⁵⁹

All numerical calculations were implemented using the MATLAB software package (The MathWorks Inc., Natick, MA). Published linear absorption coefficients⁶⁰ and semiempirical data on atomic scattering factors⁶¹ were used to find the complex refractive index. δ was extrapolated to out-of range values by assuming E^{-2} dependence. X-ray spectra were generated with published models,⁶² and the AGD was calculated with Monte Carlo-simulated dose coefficients.⁶³

II.D. Wave-propagation model

II.D.1. Background

A simulation model based on wave propagation was developed as a complement to the geometrical-optics approach presented so far. The model is similar to what we have previously used for x-ray optics; it is summarized below, but for more details we refer to Ref. 64.

The parabolic wave equation for a field u that is traveling in the z -direction,

$$-2ikn_{r0}\frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} + k^2(n_r^2 - n_{r0}^2)u = 0, \quad (44)$$

can be derived from Maxwell's equations under the paraxial approximation. In Eq. (44), n_r is the complex refractive index and n_{r0} is a reference refractive index (1 for vacuum). Assuming uniform refractive index over the propagation distance ($n_r = n_{r0}$), a Fourier transform of Eq. (44) with respect to x yields $-2ikn_{r0}dU/dz - f_x^2 U = 0$, with U being the Fourier transform of u . The solution to this differential equation is

$$U = U_0 \exp\left[\frac{i}{2kn_{r0}} f_x^2 z\right] \equiv U_0 B. \quad (45)$$

The inverse Fourier transform is a convolution, $u = (u_0 * b)(x)$, known as the paraxial Kirchhoff diffraction integral, which yields the field u at any point x , given an initial field u_0 . The Kirchhoff diffraction integral can be solved directly in the Fourier domain, according to Eq. (45), to speed up the calculation.

The intensity at any point in the setup was found as the square of the absolute value of the field, i.e., $N(x, z) = |u(x, z)|^2$. Compton scattering was treated as absorption and added after propagation. Hence, perfect scatter rejection was assumed, which is a reasonable assumption for the scanning geometry.³⁸ Poisson-distributed quantum noise with expectation value and standard deviation determined from the intensity at the detector was optionally added after propagation.

A polychromatic finite x-ray source was invoked in the simulation by propagating the fields from a large number of monochromatic point sources. The point sources were treated as incoherent relative to each other, i.e., the intensity as a function of energy at any point in the setup [$N_E(x, z)$] was found by summing intensities from point sources at different positions (x_0): $N_E(x, z) = \sum |u_{E,x_0}(x, z)|^2$.

Fringe displacement was measured by least-square fitting to the fringes in a flat-field image, i.e., an image of the unperturbed beam. This method proved more robust in our case than Fourier methods.

II.D.2. Numerical investigation

The wave-propagation model was used to validate the analytical formulas of MTF and NPS, and to generate synthetic images for illustration. The plane-wave geometry was assumed in all cases.

The presampled MTF was measured without quantum noise in the simulation (i.e., representing a “high-dose” case). An edge method similar to the one in Ref. 37 was used, but the procedure differed in some respects and it is therefore briefly summarized here. A semitransparent 20 μm thick silicon edge test device was used rather than a completely opaque object to avoid reconstruction errors. Further, instead of slanting the edge device in a two-dimensional image, an oversampled edge-spread function was generated by simulating a number of one-dimensional images with the device moved to subpixel locations. The axial line-spread function along the x axis was taken as the derivative of the edge-spread function, and the MTF was calculated with a one-dimensional fast Fourier transform of the line-spread function. Reducing the

FIG. 3. Target shapes: (Top) A sphere. (Bottom) The target that was used for wave-propagation simulations [Eq. (46)] with a constant derivative in the phase-contrast direction.

measurement to one dimension requires uncorrelated detector pixels in the y direction, which is a valid assumption for instance for a scanning system.^{37,49}

Similar to the MTF, the NPS measured in one-dimensional simulated images was taken as the axial NPS along the x axis. The general definition of the NPS in Eq. (29) was used for the calculation with the expectation value taken as the average over a large number of noise-only simulated images. Windowing is required to avoid spectral leakage when measuring a power-law NPS and a Hann data taper was, therefore, applied to each image prior to the calculation.^{55,56}

To generate synthetic images, we used a target according to

$$h(\mathbf{r}) = \sqrt{r_t^2 - x^2 - |y|}, \text{ for } x^2 + y^2 = r_t^2, \quad (46)$$

which is similar to a sphere, but with a constant derivative (slope) in the phase-contrast direction. Figure 3 shows the target in Eq. (46) compared to a sphere. The motivation for this target shape was that a sphere is singular in its derivative at the edge, which may give rise to phase wrapping and artifacts in the simulation, as opposed to the constant slope of the target in Eq. (46), which results in a binary differential phase-contrast signal. In real life, perfect spheres are rare, and this problem is unlikely to occur.

III. RESULTS AND DISCUSSION

III.A. Plane-wave geometry

III.A.1. NPS and MTF

The quantum NPS in the 5-cm breast, calculated with Eqs. (30), (33), and (37), is illustrated with a 2D plot in Fig. 4, left. Figure 4, center, shows the x -directional axial NPS from the left plot together with the measured NPS from 100 noise-only images simulated with wave propagation. Good agreement was found between the simulations and the analytical expressions. Wave propagation was also used to verify the influence of $V_\psi < 1$ and $\Lambda < 1$ on the NPS, but this is not shown in the figure.

FIG. 4. (Left) 2D plot of the noise-power spectrum (NPS) in the Talbot interferometer and in the generic absorption-contrast system. The phase-contrast NPS has a $1/f_x^2$ dependence, but is flat in the y direction. The absorption-contrast NPS is flat in both systems and both directions, but slightly higher in Talbot interferometry because of grating absorption. (Center) Axial plot of the NPS in the x direction. Wave-propagation simulations (markers) are compared to analytical results (lines). (Right) The axial modulation-transfer function (MTF), which is expected to be equal in all cases.

In Talbot interferometry, the absorption-contrast NPS was flat according to Eq. (33), but the phase-contrast NPS went as $1/f_x^2$ (brown noise) as expected from Eq. (37). The NPS of the generic absorption-contrast system was flat in both directions and a factor of 2 lower than for Talbot absorption contrast (because of no analyzer grating). Note that the large difference in NPS magnitude is accompanied by a large difference in signal between phase and absorption contrast.

Wave-propagation simulations of the MTF is shown in the right-hand plot of Fig. 4 as the axial MTF in the x direction together with analytical results from Eqs. (26) and (28). There was, again, good agreement between the simulations and analytical expressions, which supports the finding that the MTF is largely unaffected by phase contrast according to Eqs. (26) and (28). The slight differences are caused in part by edge diffraction that appeared in the simulation but was not taken into account in the analytical expressions. Note that for the plane-wave geometry, which is considered in Fig. 4, the MTF is simply the detector aperture (sinc) function.

III.A.2. Optimal incident energy

The plots in Fig. 5 illustrate the energy dependence of phase and absorption contrast. The detectability index was calculated as a function of photon energy for four different 200- μm -diameter spherical targets: (1) a tumor structure, (2) a glandular structure, (3) a microcalcification, and (4) an air cavity. The optimal energies (E^*) for phase and absorption contrast are indicated with circles and squares, respectively.

In agreement with other studies,⁵¹ and expected from the back-of-the-envelope calculation in Sec. II.C.1, there is little variation in optimal energy between common mammo-

graphic targets in absorption contrast; the tumor target, glandular structure, and microcalcification all exhibit maxima at 22 keV. For the air cavity, however, there is a large difference in density relative surrounding tissue and Compton scattering therefore becomes the dominant contrast mechanism in systems with efficient scatter rejection [cf. Eq. (25) and Fig. 2]. Compton scattering has substantially weaker energy dependence than photoabsorption, which results in a higher optimal energy.

The optimal energy in phase-contrast imaging is located at 38 keV for all targets, including the air cavity, because δ is made up of only a single energy-dependent function [cf. Eq. (25)]. As predicted by Eq. (41) and in agreement with the discussion in Sec. II.C.4, the optimal energy in phase contrast is a factor of $38/22 = 1.7$ higher than in absorption contrast, which may compensate in part for the reduced photon economy in the phase-contrast detection process according to Eq. (43). Note, however, that quantum efficiency of the detector would punish high energies if it were taken into account.

III.A.3. Task dependence

Figure 6 illustrates the dependence on target size and material for phase and absorption contrast. The detectability benefit ratio ($\hat{d}'_{\text{iso-dose}}$) is plotted as a function of target size at equal dose and optimal energy for Talbot interferometry over generic absorption contrast. The benefit ratio can never drop below d'_{T_A}/d'_A , i.e., the efficiency of the absorption-contrast image that is recorded along with the phase-contrast image. The incident photon energy of the Talbot interferometer was, however, optimized for phase contrast, which is suboptimal

FIG. 5. Detectability index as a function of photon energy for four different 200- μm -diameter spherical targets: (1) a tumor structure, (2) a glandular structure, (3) a microcalcification (MC), and (4) an air cavity. The optimal energies for phase and absorption contrast are indicated with circles and squares, respectively.

FIG. 6. The impact of target size and material on phase and absorption contrast for (1) a tumor structure, (2) a glandular structure, (3) a microcalcification (MC), and (4) an air cavity. The detectability benefit ratio at optimal energy and equal dose for phase over absorption contrast ($\hat{d}_{\text{iso-dose}}$) is plotted as a function of target diameter.

for Talbot absorption contrast and reduced $\hat{d}_{\text{iso-dose}}$ in addition to the $\sqrt{\Gamma_{1,2}} = \sqrt{0.5} = 0.7$ caused by grating absorption.

For all targets, the frequency dependence of S_{T_Φ} caused the benefit of phase contrast to increase for smaller target radii (r_t) according to $\hat{d}_{\text{iso-dose}} \propto r_t^{-0.8}$, which shows up as a linear trend in the log-log plot in Fig. 6. Phase contrast improved detectability for tumor and glandular structures with a diameter less than ~ 0.5 mm in the order of a factor of 3–4 at $100 \mu\text{m}$, which corresponded approximately to the resolution limit (two times the pixel size) in the considered system. This result is in qualitative agreement with previous clinical findings that spiculated lobular carcinomas are better detected by phase contrast.^{13,14} The benefit for microcalcification detection was lower and did not reach above unity at the resolution limit. This smaller benefit comes from a relatively large difference in atomic number rather than density, which is favored by absorption contrast. For a gas, on the other hand, there is a large density difference compared to solid materials, and phase contrast was clearly advantageous for the air cavity, even at relatively large target diameters.

Figure 7 illustrates the integration to detectability index in Eq. (21). The signal-difference-to-noise-ratio squared, defined here as $\text{SDNR}^2(f_\theta) \equiv \text{NEQ}(f_\theta) \times C^2$, is plotted for phase contrast (SDNR_{T_Φ}) and generic absorption contrast (SDNR_A) with the signal template for detecting a 0.4-mm spherical tumor structure. Figure 7 is plotted in radial coordinates with radial frequency f_θ , and the signal template multiplied with $2\pi f_\theta$ to convert from Cartesian coordinates. Radial symmetry is used here for illustration, but was not assumed in any of the calculations. The detectability index is, more or less, obtained by multiplying the plots in Fig. 7 and integrating over the Nyquist region. The crossing between the curves for phase and absorption contrast appears around $f_\theta = 1.2 \text{ mm}^{-1}$, and phase contrast is advantageous at frequencies above this point. The tumor signal template does have much frequency content below 1.2 mm^{-1} , but the magnitude of SDNR_{T_Φ} is substantially larger than that of SDNR_A . In agreement with Fig. 6, integration therefore yields approximately equal results for phase and absorption contrast in this case.

FIG. 7. The squared signal template (F^2) for a 0.4-mm spherical tumor structure, and the squared signal-difference-to-noise ratio (SDNR^2) for phase and absorption contrast. The plot is in radial coordinates and the signal template was multiplied with $2\pi f_\theta$ so that an integral over the product of these plots illustrates calculation of the detectability index in Eq. (21).

Figure 8 illustrates the dependence on target “sharpness” (sphere or pillbox) for detecting a tumor target and shows the difference between detection and discrimination tasks. The detectability benefit ratio is plotted as a function of target size for four different cases: (1) Detection of a sphere (“smooth” structure), (2) detection of a pillbox (“sharp” or “spiculated” structure), (3) size discrimination between a sphere and another sphere with half the radius, and (4) shape discrimination between a pillbox and a sphere with equal signal power.

The larger benefit of phase contrast for “sharper” targets is caused by a larger amount of high-frequency components and manifests itself as different size dependencies for the different targets in Fig. 8: $\hat{d}_{\text{iso-dose}} \propto r_t^{-0.8}$ for spherical targets (as in Fig. 6) and $\hat{d}_{\text{iso-dose}} \propto r_t^{-0.5}$ for pillbox targets (as expected from the back-of-the-envelope calculation in Sec. II.C.4). The size-discrimination task had approximately the same size dependence as detection of the spherical target, but with a 50% positive offset. The shape-discrimination

FIG. 8. The detectability benefit ratio at optimal energy and equal dose for phase over absorption contrast ($\hat{d}_{\text{iso-dose}}$) as a function of target diameter. The impact of “sharpness” on detection tasks is illustrated by two kinds of target shapes: (1) A sphere represents smooth targets and (2) a pillbox represents sharp or spiculated targets. The difference between detection and discrimination is illustrated by two different tasks: (1) Discrimination between two spheres with a factor of 2 difference in size, and (2) discrimination between a pillbox and a sphere.

FIG. 9. The squared signal templates (F^2) for the detection and discrimination tasks in Fig. 8. As in Fig. 7, the signal templates were multiplied with $2\pi f_0$ to convert from Cartesian to radial coordinates. The plots are normalized for convenient comparison.

task had approximately the same size dependence as detection of the pillbox target, but also with a positive offset. For target diameters approaching the pixel size, all features are lost and the detectability benefit ratio levels off. Note that using the presampled or expectation MTF in Eq. (20) implies that the results, in particular for targets approaching the pixel size, should be interpreted as the average over a large number of positions relative to the pixel matrix.

Figure 9 shows the signal templates (F^2) for the detection and discrimination tasks used in Fig. 8. Sharper targets (pillboxes) fall off slower toward higher frequencies than do smoother targets (spheres). Discrimination tasks weigh middle frequencies higher than detection tasks. The human eye has a sensitivity peak at middle frequencies, and the discrimination tasks may in this case be a better representation of real observer performance than the detection tasks.

III.A.4. Synthetic images

Figure 10 shows simulated images of targets in two different size ranges: the left panel shows tumor structures/spicula with diameters 100–500 μm , and the right panel contains 1–5-mm-diameter solid tumors. Since the target signal in Eq. (46) scales linearly with thickness for both phase and absorption contrast, the target thickness could be chosen freely to reach above the detection threshold in print without losing generality. The field-of-view in Fig. 10 was adapted for each case so that the targets are zoomed to equal size. Images were in all

FIG. 10. Simulated images that illustrate the impact of target size on phase and absorption contrast. (Left panel) Tumor structures/spicula with diameters 100–500 μm . (Right panel) Solid tumors with diameters 1–5 mm. Images with Talbot phase contrast and generic absorption contrast are in the top and bottom rows of the respective panels. The targets are zoomed to equal display size within each panel.

FIG. 11. Simulated images that illustrate the impact of target material on phase and absorption contrast. (Left column) A 300- μm -diameter air cavity; (Right column) A 300- μm -diameter microcalcification. Phase and absorption-contrast images are in the top and bottom rows of the respective columns.

cases acquired at an AGD of 1 mGy and incident energy $E = 26$ keV (i.e., between the optima for phase and absorption contrast).

The difference between phase and absorption contrast in Fig. 10 boils down to the different noise patterns. Absorption-contrast images exhibit conventional quantum noise, whereas phase-contrast images have noise correlation in the vertical (x) direction that shows up as streaks. This is the brown noise ($1/f_x^2$ dependence) in Fig. 4, which is caused by integration of quantum noise. For the small targets in Fig. 10, the correlated noise is less disturbing than uncorrelated quantum noise because of the longer “range.” For the large targets, the streaks are more evident and disturbs detection so that absorption contrast is more efficient.

Figure 11 shows 300- μm -diameter symmetric air and microcalcification targets. Phase-contrast imaging is clearly advantageous for detecting air cavities (cf. Fig. 6), and the absorption-contrast image does not reach over the detection threshold. For microcalcifications, on the other hand, absorption contrast is beneficial, although the target is detected by both methods.

It should be noted that phase images, as in Figs. 10 and 11, may not be optimal to display phase contrast. Another option would be to display the differential-phase images, which are edge-enhanced (cf. Fig. 3) and exhibit uncorrelated quantum noise. The NEQ and the ideal-observer detectability index are, however, independent of the choice of display.

FIG. 12. (Left) Simulated phase-step modulation (V_ψ) as a function of the spectrum energy resolution ($\Delta E/\bar{E}$) and normalized projected source size (equivalent to source-grating fill factor, Γ_0). (Center) Modulation along the axes of the surface plot (markers) compared to analytical results (lines) for perfect energy resolution, a point source, and a $4\text{-}\mu\text{m}$ source. (Right) Relative photon economy in phase and absorption contrast (Θ_{T_ϕ}/Θ_A) as a function of modulation with dotted and dashed lines for two different spectrum widths (“wide” and “narrow”).

III.B. Spherical-wave geometry

III.B.1. Source size and spectrum bandwidth

The surface plot in Fig. 12, left, shows the phase-step modulation (V_ψ), simulated as a function of the energy resolution ($\Delta E/\bar{E}$) and of the source size projected at the detector plane and normalized to the size of the interference fringes ($s_0 D_n/p_f L$). Energy spectrum and source were both rect distributed. Since the spherical wave geometry assumed a source grating, s_0 refers to the slit width and $s_0 D_n/p_f L$ is equivalent to the source-grating fill factor (Γ_0). The plots in Fig. 12, left and center, would, however, be identical for a single small source of size s_0 .

Figure 12, center, shows the simulated modulation along the axes of the surface plot, i.e., for perfect energy resolution but finite source size, and for a point source but with finite energy resolution. Also shown are analytical calculations according to Eqs. (17) and (19), and the x axis contains additional information on the bandwidth (ΔE) and source size (s_0) in this particular geometry (cf. Table II). The simulation of a finite source size deviated slightly from Eq. (17) at small sources and did not reach unity, but the asymptotic behavior at sizes $\geq 4\text{ }\mu\text{m}$ was in agreement. This is likely an artifact of the simulation, due to, e.g., the paraxial approximation. The bandwidth dependence is therefore plotted both for a point source, with the trend following Eq. (19) but with a constant offset, and for a $4\text{-}\mu\text{m}$ source, which is in good agreement with Eq. (19).

In summary, the analytical expressions in Eqs. (17) and (19) may well be used as a first approximation of expected modulation. The sensitivity to spectrum bandwidth was found to be low, and even standard mammography spectra may yield acceptable modulation. The sensitivity to source size was high, on the other hand, and the modulation dropped quickly with increased source size.

III.B.2. Photon economy

Figure 12, right, illustrates the tradeoff between modulation and photon economy; relative photon economy (Θ_{T_ϕ}/Θ_A) is plotted as a function of modulation. The curves in Fig. 12, right, were calculated numerically, but the trend is

expected from the combination of Eqs. (17), (19), and (43). Two different realistic x-ray spectra were used for the Talbot interferometer: (1) A 60-kV-tungsten spectrum with 1.3-mm-aluminum filtration and 3-mm-polymethyl-methacrylate (PMMA) compression-plate filtration; (2) A 52-kV-tungsten spectrum filtered with 5 mm of aluminum and 3 mm of PMMA. Both spectra had a mean energy of 38 keV (optimum for phase contrast), but represent, respectively, broad and narrow energy spectra with different photon energy resolution. A source grating was assumed, and modulation was altered along the curves by different degrees of grating fill factor. For the generic absorption-contrast system, we used a 31-kV-tungsten spectrum with 0.5-mm-aluminum filtration and 3 mm of PMMA.

Judging from the two curves in Fig. 12, right, the advantage of the narrow spectrum was minute also at very small source sizes, and it proved impossible to obtain a relative photon economy above unity by adjusting the fill factor. Hence, the additional filtering and reduced acceleration voltage used to narrow the spectrum was less efficient in the tradeoff between modulation and photon economy than reducing the source size.

Restricting the spherical-wave geometry to equal photon economy implies that source size and spectrum bandwidth are finite, which affects d'_{T_ϕ} in two ways: (1) A realistic MTF_A reduces the benefit of phase-contrast imaging because primarily higher frequencies are affected. (2) Reduced modulation according to Fig. 12, right, increases noise and the plots in Figs. 6 and 8 for $\hat{d}'_{\text{iso-dose}}$ are shifted in parallel if plotted for $\hat{d}'_{\text{iso-dose, -flux}}$. This is illustrated in Fig. 13, which

FIG. 13. The detectability benefit ratio at equal dose and equal photon economy ($\hat{d}'_{\text{iso-dose, -flux}}$) as a function of target size. The targets and tasks were picked from Fig. 8.

shows $\hat{d}_{\text{iso-dose, -flux}}$ for detection and discrimination of tumor targets, similar to Fig. 8. Detection tasks were barely improved by phase contrast compared to absorption contrast, but discrimination tasks were still improved by about 70%.

IV. CONCLUSIONS

We have presented a theoretical framework for cascaded-systems analysis of differential phase-contrast imaging and evaluated the potential benefit relative absorption contrast. Specifically, the study compared a photon-counting phase-contrast mammography system based on Talbot interferometry to a generic absorption-contrast system, while taking dose, setup geometry, and to some degree, photon economy into account. Results from analytical calculations were verified by wave-propagation simulations.

Differential phase-contrast imaging did not exhibit a general signal-difference-to-noise improvement relative to absorption contrast at equal dose, but the performance was found to be highly task dependent. In particular, three effects were seen: (1) The optimal imaging energy for phase contrast was higher than for absorption contrast and independent of target. (2) The intrinsic detection of the phase differential caused the NPS to decrease rapidly with spatial frequency so that phase contrast was beneficial for small and sharp targets, e.g., tumor spicula rather than solid tumors, and for discrimination tasks rather than detection tasks. (3) Phase contrast favored detection of materials that differ in density compared to the background tissue, rather than materials with differences in atomic number that are probed by absorption contrast. For instance, the improvement offered by phase contrast in microcalcification detection was less than the improvement for tumor and glandular structures of the same size.

Talbot interferometry was largely independent of spectrum bandwidth, but the projected source size needs to be considered to a larger extent. The higher optimal energy in phase-contrast imaging compensated for some of the loss of flux in the Talbot interferometer, but photon economy remains to be an issue.

One effect that was not included in the presented framework is dark-field imaging, which comes as a bonus from the phase detection in Talbot interferometry and diffraction-enhanced imaging.^{7,12,65–67} This contrast mechanism probes scattering effects on microstructures and is expected to yield a major detection improvement, in particular for tumors.

ACKNOWLEDGMENTS

The authors would like to thank Thomas Koehler and Ewald Roessl at Philips Research Laboratories, Hamburg, Germany for rewarding discussions on phase-contrast imaging. This research was mainly funded by the Swedish agency for innovation systems (VINNOVA). Financial support was

also received from the U.S. National Institutes of Health under Grant No. 2R01-CA-127444.

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