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Robust Estimation of Time-of-Flight Shear Wave Speed Using a Radon Sum Transformation

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Abstract

Time-of-flight methods allow quantitative measurement of shear wave speed (SWS) from ultrasonically tracked displacements following impulsive excitation in tissue. However, application of these methods to *in vivo* data is challenging due to the presence of gross outlier data resulting from sources such as physiological motion or spatial inhomogeneities. This paper describes a new method for estimating SWS by considering a solution space of trajectories and evaluating each trajectory using a metric that characterizes wave motion along the entire trajectory. The metric used here is found by summing displacement data along the trajectory as in the calculation of projection data in the Radon transformation. The algorithm is evaluated using data acquired in calibrated phantoms and *in vivo* human liver. Results are compared to SWS estimates using a random sample consensus (RANSAC) algorithm described by Wang, *et al.* Good agreement is found between the Radon sum and RANSAC SWS estimates with a correlation coefficient of greater than 0.99 for phantom data and 0.91 for *in vivo* liver data. The Radon sum transformation is suitable for use in situations requiring realtime feedback and is comparably robust to the RANSAC algorithm with respect to outlier data.

I. Introduction

Tissue stiffness is often related to underlying pathology. For example, palpation alone is effective in detecting a variety of illnesses, including lesions, aneurysms, and inflammation. In some ailments, such as liver fibrosis, disease progression may be marked by a gradual change in tissue stiffness, leading to a spectrum of changes in material properties. The ability to non-invasively quantify tissue stiffness *in vivo* may be useful in the staging and management of such diseases.

Recently, a number of imaging methods have been developed for quantifying tissue stiffness *in vivo* using shear waves. These systems generate shear waves in tissue using either external mechanical excitation coupled to the body wall [3], [1], [2] or acoustic radiation force to remotely palpate tissue at the focal region of the acoustic beam [7], [4], [5], [6]. The propagation of these shear waves is monitored in space and time by a real-time imaging modality such as magnetic resonance imaging or ultrasound. The speed of shear wave propagation can then be related to the underlying tissue stiffness.

A fundamental challenge in quantifying tissue stiffness using shear waves is the estimation of shear wave speed (SWS) from dynamic displacement data. One method of SWS estimation is the so-called time-of-flight (TOF) approach. In this approach, a feature such as the maximum displacement or leading edge of the shear wave is identified in displacement vs. time curves at fixed lateral positions, and the SWS is calculated by assuming a linear relation between the feature arrival time and the position. This method has been successfully

used on ultrasound tracked shear wave displacement data by multiple groups [2], [8], [10], [9].

While TOF SWS estimation has been validated on simulation and phantom data [9], its use in *in vivo* patient data presents additional challenges. Physiological motion, low displacement signal-to-noise ratio (SNR) and spatial inhomogeneities in tissue can corrupt estimates of shear wave arrival times. These problems lead to gross outlier data which can skew the SWS estimate and can render “good” data inadmissible by lowering goodness of fit metrics. Approaches currently used by TOF algorithms to deal with noisy data include smoothing by averaging SWS reconstructions over multiple locations, employing goodness-of-fit criteria to remove unreliable linear regression results, and iteratively removing data with the largest residual after least squares fitting.

Recently, Wang, *et al.* [11] have applied the random sample consensus (RANSAC) algorithm [12] to the problem of TOF SWS estimation from ultrasound tracked shear wave displacements induced by acoustic radiation force. In this approach, a trial solution is found by randomly selecting a minimal set of data points required to calculate the SWS, and then enlarging this set by including all other data consistent with the initial estimate. The optimal SWS estimate is found using many trial solutions and selecting the solution with the greatest number of consistent data. Thus, in the RANSAC fitting paradigm, gross outlier data are likely to be excluded from the largest consistent solution and do not influence the SWS estimation.

While RANSAC has proven to be very robust in the presence of gross outlier data, two issues limit its usefulness for the problem of SWS estimation. First, RANSAC requires a large number of trial solutions to give a high probability of finding the optimal solution. Wang, *et al.* [11] used 5000 trial solutions and estimated the probability of selecting at least one trial solution that was free of outlier points to be greater than 0.99. This processing required 186 sec of CPU time for each patient liver acquisition. Thus, in its current implementation, RANSAC is not suitable for applications where real time feedback is required.

Second, the RANSAC algorithm depends critically on a threshold parameter used to distinguish between inlier and outlier data, and this parameter must be determined before processing the entire data set. Wang, *et al.* [11] addressed this issue by processing their *in vivo* liver data twice, once to estimate an appropriate threshold, and then a second time to determine the speeds. Thus, the need to determine the threshold adds an extra step of complexity and extra processing time to the SWS estimation.

In this paper, we describe a new method for TOF SWS estimation that is also applicable for cases with gross outlier data. The unique approach used in this method is to consider a solution space of possible shear wave trajectories and to evaluate each trajectory by computing a metric which characterizes shear wave motion along the entire trajectory. The metric used here is found by summing displacement data along each trajectory in space and time. These sums are equivalent to the calculation of projection data in the Radon transformation of an image, and we refer to our approach as a Radon sum transformation.

The Radon transformation has been used previously in a wide variety of motion related applications including blood flow [13], seismic activity [14], oceanic Rossby waves [15], ionization waves [16], and motion detection [17].

In this paper, the Radon sum transformation is described and used to estimate SWS for the case of calibrated phantom data, and *in vivo* liver data. These results are evaluated by comparison to gold standard, which is assumed here to be the RANSAC SWS estimation.

II. Radon Sum Transformation for SWS Estimation

For shear wave motion following impulsive excitation, displacement data $d(x, t)$ are tabulated as a function of lateral position x and time t . Figure 1(a) shows an example of displacement vs. time curves at fixed lateral positions for data acquired in a homogeneous elastography phantom (CIRS, Norfolk, VA, USA). Figure 1(b) shows the same data displayed as a two dimensional image. Both figures show obvious wave motion with the peak displacement moving toward greater lateral position at later time.

The Radon sum transformation described here takes advantage of the correlation between position and time for the displacement data such as shown in Fig. 1(b). To identify the trajectory which best characterizes the wave motion, we consider a solution space of linear trajectories extending from a starting position $(x_{\text{start}}, t_{\text{start}})$ to an ending position $(x_{\text{end}}, t_{\text{end}})$. For these trajectories, the position and time are related as

$$x = (t - t_{\text{start}})c + x_{\text{start}} \quad (1)$$

where the speed c associated with each trajectory is given by

$$c = \frac{x_{\text{end}} - x_{\text{start}}}{t_{\text{end}} - t_{\text{start}}}. \quad (2)$$

Only trajectories with $x_{\text{end}} > x_{\text{start}}$ and $t_{\text{end}} > t_{\text{start}}$ are considered corresponding to shear wave propagation away from the excitation region.

As suggested by Fig. 1(b), wave motion is characterized by the trajectory of the peak displacement. Thus, we can compute a metric that characterizes the wave motion along each trajectory by summing the displacement data along the trajectory in the same way that projection data are calculated in the Radon transformation of an image. Using a fixed lateral range $[x_{\text{start}}, x_{\text{end}}]$, the Radon sums $S(t_{\text{start}}, t_{\text{end}})$ for each trajectory in the solution space are calculated as

$$S(t_{\text{start}}, t_{\text{end}}) = \sum_{x_{\text{start}}}^{x_{\text{end}}} d(x, t) \quad (3)$$

where, for each position x in the sum, the time t is calculated using Eq. (1). Figure 2 shows the Radon sum image $S(t_{\text{start}}, t_{\text{end}})$ for the displacement data shown in Fig. 1(b). The optimal trajectory is identified by the peak Radon sum which is indicated by the white dot in Fig. 2. The trajectory corresponding to this peak sum is indicated by the white line in Fig. 1(b).

Like the RANSAC algorithm, SWS estimation using the Radon sum transformation is not sensitive to gross outlier data since only the Radon sums are used to compare trajectories. An outlier displacement estimate at one position and time will contribute only to the Radon sums through that point, and will not affect sums for trajectories near the optimal trajectory. Furthermore, outlier displacement data can give local maxima in displacement vs. time curves at individual lateral positions. However, these local maxima are not correlated across lateral positions and do not contribute to a peak in the Radon sum image that rivals the peak identifying the optimal trajectory.

III. Methods

A. Data Acquisition

Experimental shear wave data were acquired using a modified SONOLINE Antares ultrasound system (Siemens Healthcare, Ultrasound Business Unit, Mountain View, CA, USA) using a CH4-1 curvilinear transducer. A detailed description of the acquisition sequence is given in Wang, *et al.* [11]. Briefly, the sequence consisted of three reference A-lines, followed by a high intensity pushing line focused at a depth of 49 mm to mechanically excite the tissue, and then by 80 repeated tracking A-lines. 4:1 parallel receive [18] was implemented so that 4 tracking locations could be monitored for each push. This sequence was repeated 9 times using the same push location, but displaced track locations so that tissue motion was tracked at 36 lateral positions ranging from 0 to 21.4 mm from the push location. The pulse repetition frequency (PRF) of the reference and tracking A-lines was 4.8 kHz, providing 16.4 ms of post-push data at each tracking location.

Data were acquired in two sources. First, data were acquired on a set of six homogeneous elastic phantoms (CIRS, Norfolk, VA, USA). The Young's moduli E for these phantoms were measured by the manufacturer using an indenter system and are listed in Table I. With the assumption of a linear, isotropic, incompressible material with density ρ , the shear wave speed is given by s

$$c = \sqrt{\frac{E}{3\rho}}. \quad (4)$$

The speeds calculated using Eq. (4) for the six phantoms are also listed in Table I where the density has been assumed to equal the density of water.

Second, *in vivo* shear wave data were acquired in the livers of 123 patients before undergoing liver biopsy at Duke University Medical Center in an IRB approved study (#9328-06-12). All subjects were informed of the nature and aims of this study and signed a consent form. Imaging was performed during breathholds with the probe in either an intercostal or subcostal location. The number of independent shear wave acquisitions in each patient ranged from 6–12.

B. Preliminary Data Processing

All data processing, including the RANSAC and Radon transform SWS calculations were performed in the MATLAB (The MathWorks, Natick, MA) environment on a Linux cluster with an average CPU speed of 2.6 GHz. To insure a valid comparison of RANSAC and Radon sum speeds, the same preliminary data processing steps described in this subsection were used for both calculations.

Tissue displacements due to acoustic radiation force along each A-line were calculated by comparing pre-push reference and post-push tracking data using Loupas' method on IQ data [19]. A threshold value of 0.95 for the magnitude of the complex correlation coefficient was used to remove poor displacement estimates. A quadratic motion filter [20] was used to reduce the effect of physiological motion. This filter fits a quadratic function to the displacement at time-steps when the tissue is not perturbed by the shear wave in order to estimate underlying tissue motion. The time-steps used for fitting were the three pre-push references, and post-push tracks at times greater than 9 ms after the peak displacement when motion due to the shear wave is assumed to be negligible. Displacement data determined using the radial tracking lines were scan-converted onto a rectilinear grid with a spacing of

0.1 mm (lateral) $\times$ 0.02 mm (axial). A low-pass filter with a cutoff frequency of 1 kHz was used to remove high frequency jitter from the temporal displacement profiles. Finally, these profiles were upsampled to an effective PRF of 100 kHz to increase the temporal resolution.

Displacement data within the lateral excitation beamwidth were not used to avoid diffraction effects within this region. The excitation beamwidth is approximated by $(F/\#) \lambda = 1.4$ MM, where $F/\# = 2$ is the excitation beam f-number, and $\lambda = 0.7$ mm the excitation wavelength. Shear wave analysis was also restricted to an axial depth of field (DOF) defined by $8(F/\#)^2 \lambda = 22$ mm centered around the focal depth of the excitation beam [9], where the direction of shear wave propagation is assumed to be parallel to the lateral dimension. The 2-dimensional region-of-interest (ROI) used for SWS estimation is shown on the B-mode image in Fig. 3.

C. RANSAC SWS Estimation

SWS calculations using the RANSAC algorithm are described in detail by Wang, *et al.* [11]. Briefly, these calculations used the time-to-peak (TTP) approach described by Palmeri, *et al.* [9] to identify the peak displacement times in displacement vs. time curves at fixed lateral positions. TTP values were tabulated for all depths within the DOF and at all lateral positions outside the excitation beamwidth up to a maximum position which was determined individually for each data set as the position where the peak displacement fell below a threshold of 1 μ m. Planes were fit to the 3-dimensional (depth $\times$ lateral position $\times$ time) data sets to determine the SWS. By fitting a plane, the RANSAC fitting algorithm determined the SWS and, in addition, the depth dependence of the SWS. Wang, *et al.* [11] did not find this depth dependence to be useful in their analysis, and neglected it when reporting the SWS.

D. SWS Estimation Using the Radon Sum Transformation

After preliminary data processing, 3-dimensional data sets were averaged over the DOF to give a 2-dimensional displacement image like that shown in Fig. 1(b). Using Eq. (3), Radon sums were calculated for all $(t_{\text{start}}, t_{\text{end}})$ combinations with $t_{\text{end}} > t_{\text{start}}$. The value of x_{start} was set equal to the excitation beamwidth of 1.4 mm. The value of x_{end} was chosen as a compromise between two factors. A large value of x_{end} allows the most complete use of the measured data. However, the Radon sum approach requires the shear wave to propagate through the full lateral range included in the summation, which effectively results in a minimum detectable shear wave speed. Therefore, because the tracking time was limited to 16.4 ms, $x_{\text{end}} = 15$ mm was chosen to use a large fraction of the available range, while resulting in a minimum detectable speed of roughly 15 mm/16.4 ms $\approx$ 0.9 m/s. This range is indicated by the right edge of the ROI in Fig. 3.

When calculating the sum in Eq. (3), steps in the lateral position X were chosen to coincide with the positions of the displacement data which, after scan conversion, were tabulated with a lateral spacing of 0.1 mm. Thus, for the 1.4mm – 15mm lateral range, 137 terms were included in the sum for each trajectory, and no normalization was required to account for trajectories with different speeds. For each position x in the sum, the required time t was found using Eq. (1), and the displacement $d(x, t)$ was found by linear interpolation within the displacement vs. time curve at that position.

The optimal trajectory was obtained from the coordinates of the global maximum in the Radon sum image, and the SWS was calculated from these coordinates using Eq. (2).

IV. Results

A. Phantom Data

Twelve acquisitions were made in each of the CIRS phantoms. The transducer was repositioned between acquisitions to insure independent measurements. Figure 1 shows displacement data from one acquisition in the $E = 5.2$ kPa CIRS phantom, and the corresponding Radon sum image is shown in Fig. 2.

Figure 4 compares the speeds measured in the six CIRS phantoms using the Radon sum transformation and the RANSAC algorithm. The dashed line in Fig. 4 represents equality between the two speed estimations. Each point has horizontal and vertical error bars indicating the standard deviation of the 12 measurements. However, the error bars are smaller than the size of the symbols for all cases except the two phantoms with the greatest speed. There is good agreement between the Radon sum and RANSAC speed estimates with a correlation coefficient between the two measurements greater than 0.99. The scale factor $\alpha = c_{\text{Radon sum}}/c_{\text{RANSAC}}$ averaged over all of the phantom measurements is $\alpha = 0.975 \pm 0.028$.

Table I compares the measured SWS with the theoretical speeds calculated using Eq. (4) and the values of Young's moduli supplied by the phantom manufacturer. While the Radon sum and RANSAC SWS estimates are in good agreement, these values are systematically greater than the theoretical values by an average of 18%.

B. Patient Data

As described by Wang, *et al.* [11], a total of 1222 shear wave acquisitions were performed in 123 patients. Of these, 787 acquisitions gave valid RANSAC SWS estimates corresponding to physically realistic shear wave speeds ($0 < c < 5.8$ m/s). This number was further reduced to 605 acquisitions by manually rejecting acquisitions where visual inspection did not indicate shear wave propagation in the ROI. For these 605 RANSAC SWS estimations, the Radon sum transformation gave 561 cases with a SWS within the physically realistic limit.

Figure 5(a) shows an example of tissue displacement vs. time profiles at five lateral positions for data acquired in *in vivo* human liver. Figure 5(b) shows the same displacement data displayed as a two dimensional image. Both figures show obvious wave motion with the peak displacement moving toward greater lateral position at later time. Figure 6 displays the Radon sum transformation for the same acquisition. The peak Radon sum is indicated by the white dot in Fig. 6, and the trajectory corresponding to this peak sum is indicated by the white line in Fig. 5(b). This line extends over the lateral range 1.4 mm $\times$ 15 mm used in the calculation of the Radon sums.

Figure 7 shows a scatter plot comparison of the 561 SWS estimates determined by the Radon sum transformation and the RANSAC algorithm. In the figure, the diagonal dashed line indicates equality, and it is seen that there is generally good agreement, with a correlation coefficient of 0.91, between the two estimates. The mean difference $\langle c_{\text{Radon sum}} - c_{\text{RANSAC}} \rangle$ between the two estimates is -0.05 m/s, and the RMS difference is 0.22 m/s. The scale factor $\alpha = c_{\text{Radon sum}}/c_{\text{RANSAC}}$ averaged over the 561 SWS estimates is $\alpha = 0.967 \pm 0.115$.

V. Discussion

A motivating factor for the development of the Radon sum SWS algorithm described in this paper is the desire for real time feedback during data acquisition. As described by Wang, *et al.* [11], the currently implemented RANSAC algorithm requires 186 s for each SWS estimation and is not suitable for real time use. As currently implemented using the same

pre-processing steps used for the RANSAC estimation, the Radon sum SWS estimation requires 19 s. However, this execution time is much greater than needed because, in the RANSAC algorithm, displacement data are upsampled to a frequency of 100 kHz to increase the temporal resolution of the peak displacement times. For the Radon sum estimation, this large upsampling is not needed since larger timesteps at the endpoints of a trajectory give relatively small differences in the SWS. In fact, rerunning the SWS estimations for the CIRS phantoms using a range of upsample frequencies gives virtually no change in SWS. For example, an upsample frequency of 10 kHz gives less than a 0.8% difference in the speeds compared to the results presented in Section IV. Furthermore, decreasing the upsample frequency greatly decreases the execution time since the number of Radon sums evaluated depends quadratically on the number of t_{start} and t_{end} timepoints. Thus, with the decrease to 10 kHz upsample frequency, the Radon sum SWS estimation is obtained with less than 1 s execution time and is suitable for real time feedback during data acquisition.

Comparison of SWS estimates shown in Fig. 4 indicates that the Radon sum SWS estimates are somewhat less than the RANSAC SWS estimate for the stiffest phantoms. Also, Fig. 7 indicates that the Radon sum SWS estimates are, on average, less than the RANSAC SWS estimates for the liver data at lower speeds. While these differences are small, it would be helpful to understand the discrepancy. One possibility is suggested by the shape of the displacement vs. time plots in Fig. 1(a). These curves indicate that the effect of the motion filter is to force the displacements toward zero at later times causing the position of the maximum peak be shifted toward earlier times, and giving increased SWS estimates. This effect is particularly significant for low speeds and large lateral ranges where the peak displacements occur near the maximum time. The magnitude of this effect will be different for Radon sum and RANSAC SWS estimates because the Radon sum uses a fixed lateral range (15 mm), whereas the lateral range used for the RANSAC calculation varies depending on the magnitude of the displacement and the lateral position at which it falls below the $1\ \mu\text{m}$ threshold (see Section III(C)). Thus, depending on the specific displacement amplitudes and reconstructed speeds for each acquisition, artifacts introduced by the motion filter would influence the Radon sum and RANSAC SWS estimates differently, and could explain the small discrepancies seen in Figs. 4 and 7.

An unanticipated result is seen in the comparison of measured and theoretical shear wave speeds for the calibrated CIRS phantoms. While the Radon sum and RANSAC SWS estimates presented here are in good agreement, these results are systematically greater than the theoretical speeds by an average of 18%. However, results obtained using a different imaging protocol with a VF7-3 transducer give Radon sum SWS estimates that agree with RANSAC estimates to within an average of 4% and also agree with the theoretical speeds to within an average of 2%. There are two key differences between the two protocols. First, a lateral range of 6 mm was used with the VF7-3 protocol, and as discussed in the preceding paragraph, this smaller range reduces the possibility the motion filter would introduce shifts in the peak times giving increased speeds. Second, the VF7-3 protocol used a focal depth of 20 mm compared to the 49 mm focal depth for the CH4-1 protocol used here, and thus, the two protocols probe different physical regions within the phantom. The phantom manufacturer did not provide a measure of uncertainty for the values or uniformity of Young's moduli throughout the phantoms, and the disagreement of the calculated and measured speeds in the phantoms could be due to lack of material homogeneity (i.e., depth dependent increase in SWS) within the phantom.

It is possible to estimate the uncertainty in SWS using the Radon sum approach. Rather than averaging over the DOF to obtain one displacement image as described in Section III(D), the DOF can be divided into subsets, and SWS estimations performed on each subset. For the

phantom data, we have found that dividing the DOF into 10 subsets with individual depths randomly assigned to each subset gives 10 SWS estimates with mean and standard deviation similar to those listed in Table I. Of course, this procedure lengthens the calculation time by the number of subsets.

A strength of the Radon sum algorithm presented here is that it does not rely on the determination of adjustable parameters such as the inclusion threshold in the RANSAC method [11] or acceptance criteria used in the lateral TTP method [9]. The only adjustable parameter used in the Radon sum method is the maximum lateral range in Eq. (3), and this parameter can be determined *a priori* based on the range of acquired data, tracking time in the imaging protocol, and the minimum value of SWS to be measured.

A limitation of the Radon sum method, which is common to all TOF based methods, is the assumption that tissue behaves as a linear elastic material. However, the liver is known to exhibit viscoelastic (VE) behavior [1], [21], [23], [22], and the SWS is a function of frequency. Therefore, the broadband shear wave excited in the current studies will experience shifts in the relative phases of its constituent frequency components, and the morphology of the shear wave will change as it propagates. The measured speed of the peak displacement used here effectively represents a ‘bulk’ speed which lies between the speeds of the fastest and slowest frequency components of the shear wave. In order to capture the full VE behavior, higher order characteristics of the shear wave, in addition to the trajectory of the peak displacement, would have to be analyzed.

Another limitation of the Radon sum SWS method presented here is its spatial resolution. In the current implementation, the propagation is assumed to be homogeneous, and one SWS is calculated for the entire ROI. If the medium is not homogeneous, it would be possible to determine speeds in sub-regions of the ROI simply by averaging over smaller axial ranges and calculating the Radon sum over different lateral ranges in Eq. (3). Of course, it would be expected that reducing the axial or lateral ranges would increase the jitter in SWS estimations. This tradeoff between spatial resolution and accuracy of SWS estimations is currently under investigation.

VI. Conclusion

This paper describes a Radon sum transformation suitable for robust SWS estimation from ultrasonically tracked shear wave displacements. Like the RANSAC algorithm described by Wang, *et al.* [11], the Radon sum transformation can estimate SWS in cases where displacement data are contaminated by gross outlier data. Good agreement is found for Radon sum and RANSAC SWS estimates for data acquired in calibrated phantoms and *in vivo* human liver. To insure a fair comparison, the same preliminary data processing was used in this study for the Radon sum and RANSAC calculations. As implemented herein for comparison purposes, the upsampling was larger than necessary which increased the processing time. With minor adjustments to the currently implemented algorithm, the Radon sum transformation is suitable for use in situations requiring realtime feedback, and is comparably robust to the RANSAC algorithm with respect to outlier data.

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References

1. Huwart L, Peeters F, Sinkus R, Annet L, Salameh N, ter Beek LC, Horsmans Y, Van Beers BE. Liver fibrosis: non-invasive assessment with MR elastography. *NMR Biomed*. 2006; 19:173–179. [PubMed: 16521091]
2. Sandrin L, Fourquet B, Hasquenoph JM, Yon S, Fournier C, Mal F, Christidis C, Ziou M, Poulet B, Kazemi F, Beaupré M, Palau R. Transient elastography: a new noninvasive method for assessment of hepatic fibrosis. *Ultrasound Med Biol*. 2003; 29(12):1705–1713. [PubMed: 14698338]
3. Yin M, Talwalkar JA, Glaser KJ, Manduca A, Grimm RC, Rossman PJ, Fidler JL, Ehman RL. Assessment of hepatic fibrosis with magnetic resonance elastography. *Clin Gastroenterol Hepatol*. 2007; 5(10):1207–1213. [PubMed: 17916548]
4. Bercoff J, Tanter M, Fink M. Supersonic shear imaging: a new technique for soft tissue elasticity mapping. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2004; 51(4):396–409. [PubMed: 15139541]
5. Chen SS, Fatemi MM, Greenleaf JF. Quantifying elasticity and viscosity from measurement of shear wave speed dispersion. *J Acoust Soc Am*. 2004; 115(6):2781–2785. [PubMed: 15237800]
6. Nightingale K, McAleavey S, Trahey G. Shear-wave generation using acoustic radiation force: in vivo and ex vivo results. *Ultrasound Med Biol*. 2003; 29(12):1715–1723. [PubMed: 14698339]
7. Sarvazyan A, Rudenko OV, Swanson SD, Fowlkes JB, Emelianov SY. Shear wave elasticity imaging: a new ultrasonic technology of medical diagnostics. *Ultrasound Med Biol*. 1998; 24(9):1419–1435. [PubMed: 10385964]
8. McLaughlin J, Renzi D. Shear wave speed recovery in transient elastography and supersonic imaging using propagating fronts. *Inverse Problems*. 2006; 22(2):681–706.
9. Palmeri ML, Wang MH, Dahl JJ, Frinkley KD, Nightingale KR. Quantifying hepatic shear modulus in vivo using acoustic radiation force. *Ultrasound Med Biol*. 2008; 34(4):546–558. [PubMed: 18222031]
10. Tanter M, Bercoff J, Athanasiou A, Deffieux T, Gennisson JL, Montaldo G, Muller M, Tardivon A, Fink M. Quantitative assessment of breast lesion viscoelasticity: initial clinical results using supersonic shear imaging. *Ultrasound Med Biol*. 2008; 34(9):1373–1386. [PubMed: 18395961]
11. Wang MH, Palmeri ML, Rotemberg VM, Rouze NC, Nightingale KR. Improving the Robustness of Time-of-Flight Based Shear Wave Speed Reconstruction Methods Using RANSAC in Human Liver *in vivo*. *Ultrasound Med Biol*. 2010; 36(5):802–813. [PubMed: 20381950]
12. Fischler MA, Bolles RC. Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography. *Commun ACM*. 1981; 24(6):381–395.
13. Drew PJ, Blinder P, Cauwenberghs G, Shih AY, Kleinfeld D. Rapid determination of particle velocity from space-time images using the Radon transform. *J Comput Neurosci*. 10.1007/s10827-009-0159-1
14. Thorson JR, Claerbout JR. Velocity-stack and slant-stack stochastic inversion. *Geophysics*. 1985; 50(12):2727–2741.
15. Challenor PG, Cipollini PP, Cromwell D. Use of the 3D Radon Transform to Examine the Properties of Oceanic Rossby Waves. *J Atmos Oceanic Technol*. 2001; 18(9):1558–1566.
16. Aliverdiev AA, Ashurbekov NA. Application of the direct Radon transform for processing of a spatiotemporal dependence of spontaneous emission from a nanosecond discharge in long tubes. *Russian Physics Journal*. 2004; 47(3):331–332.
17. Marzetta TL, Shepp LA. A Surprising Radon Transform Result and Its Application to Motion Detection. *IEEE Trans Image Processing*. 1999; 8(8):1039–1049.
18. Dahl JJ, Palmeri ML, Agrawal V, Nightingale KN, Trahey GE. A parallel tracking method for acoustic radiation force impulse imaging. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2007; 54(2):301–312. [PubMed: 17328327]
19. Pinton GF, Dahl JJ, Trahey GE. Rapid tracking of small displacements with ultrasound. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2006; 53(6):1103–1117. [PubMed: 16846143]

20. Hsu SJ, Bouchard RB, Dumont DM, Wolf PD, Trahey GE. *In Vivo* Assessment of Myocardial Stiffness with Acoustic Radiation Force Impulse Imaging. *Ultrasound Med Biol.* 2007; 33(11): 1706–1719. [PubMed: 17698282]
21. Chen S, Urban M, Pislaru C, Kinnick R, Zheng Y, Yao A, Greenleaf JF. Shearwave dispersion ultrasound vibrometry (SDUV) for measuring tissue elasticity and viscosity. *IEEE Trans Ultrason Ferroelectr Freq Control.* 2009; 56(1):55–62. [PubMed: 19213632]
22. Deffieux T, Montaldo G, Tanter M, Fink M. Shear wave spectroscopy for *in vivo* quantification of human soft tissues visco-elasticity. *IEEE Trans Med Imaging.* 2009; 28(3):313–322. [PubMed: 19244004]
23. Muller M, Gennisson JL, Deffieux T, Tanter M, Fink M. Quantitative viscoelasticity mapping of human liver using supersonic shear imaging: preliminary *in vivo* feasibility study. *Ultrasound Med Biol.* 2009; 35(2):219–229. [PubMed: 19081665]

Fig. 1.

(a) Measured axial displacement vs. time profiles for data acquired in the 5.2 kPa CIRS phantom at five lateral positions indicated in the legend. These data have been averaged over the depth of field after preliminary data processing which included the motion filter, low pass filter, and upsampling. (b) Displacement data from the same acquisition displayed as a two-dimensional image in position and time. The white line indicates the optimal shear wave trajectory with $c = 1.50$ m/s identified by the peak Radon sum in Fig. 2. The lateral extent of this trajectory corresponds to the range 1.4 mm $\times$ 15 mm used in the calculation of the Radon sums.

Fig. 2.

Radon sum transformation obtained using Eq. (3) for the displacement data displayed in Fig. 1. The white dot indicates the position of the peak Radon sum and corresponds to the optimal trajectory displayed by the white line in Fig. 1(b).

Fig. 3.

In vivo B-mode image of a patient liver with an ROI indicating the range of displacement data used for shear wave analysis. The excitation occurs at a lateral position of 0 mm and is focused at a depth of 49 mm. The DOF is 22 mm centered at the focal depth. The left edge of the ROI indicates the beamwidth of 1.4 mm. The right edge of the ROI indicates the lateral range of 15 mm used in the calculation of the Radon sums in Eq. (3). For the RANSAC SWS estimations, the maximum lateral range was determined individually for each data set as the position where the peak displacement fell below a threshold of $1\ \mu\text{m}$.

Fig. 4.

Comparison of Radon sum and RANSAC SWS estimations for data acquired in the six CIRS phantoms. The dotted line indicates equality. Error bars are displayed for each data point and are smaller than the symbol size for all but two phantoms with the greatest SWS. The scale factor $\alpha = c_{\text{Radon sum}}/c_{\text{RANSAC}}$ averaged over the phantom measurements is $\alpha = 0.975 \pm 0.028$.

Fig. 5.

(a) Measured axial displacement vs. time profiles for data acquired in *in vivo* human liver at five lateral positions indicated in the legend. These data have been averaged over the DOF after preliminary data processing which included the motion filter, low pass filter, and upsampling. (b) Displacement data from the same acquisition displayed as a two-dimensional image in position and time. The white line indicates the optimal shear wave trajectory with $c = 3.27$ m/s identified by the peak Radon sum in Fig. 6. The lateral extent of this trajectory corresponds to the range 1.4 mm $\times$ 15 mm used in the calculation of the Radon sums.

Fig. 6.

Radon sum transformation for the displacement data displayed in Fig. 5. The white dot indicates the position of the peak Radon sum and corresponds to the optimal trajectory displayed by the white line in Fig. 5(b).

Fig. 7.

Scatter plot comparing the Radon sum and RANSAC SWS estimates for 561 acquisitions in *in vivo* human liver. The dashed line indicates equality. The correlation coefficient between the two estimates is 0.91. The mean difference $\langle c_{\text{Radon sum}} - c_{\text{RANSAC}} \rangle$ between the two estimates is -0.05 m/s, and the RMS difference is 0.22 m/s. The scale factor $\alpha = c_{\text{Radon sum}} / \alpha c_{\text{RANSAC}}$ average over the 561 acquisitions is $\alpha = 0.967 \pm 0.115$.

TABLE I

Measured Young's moduli and theoretical speeds calculated using Eq. (4) for the six CIRS phantoms. Also listed are the measured speeds (mean $\pm$ standard deviation) from 12 independent measurements as determined using the Radon sum and RANSAC algorithms.

Young's modulus (kPa)	calculated SWS (m/s)	Radon sum SWS estimate (m/s)	RANSAC SWS estimate (m/s)
5.2	1.32	1.53 ± 0.04	1.56 ± 0.04
9.8	1.81	2.19 ± 0.02	2.21 ± 0.03
23.9	2.82	3.38 ± 0.02	3.42 ± 0.04
44.2	3.84	4.47 ± 0.03	4.60 ± 0.04
67.3	4.74	5.48 ± 0.12	5.71 ± 0.08
107.1	5.78	6.68 ± 0.18	7.17 ± 0.22