Published in final edited form as:

J Biomech. 2012 June 26; 45(10): 1803–1808. doi:10.1016/j.jbiomech.2012.04.018.

A Three-Compartment Muscle Fatigue Model Accurately Predicts Joint-Specific Maximum Endurance Times for Sustained Isometric Tasks

Frey-Law Laura A., PhD, PT, Looft John M., BS, and Heitsman Jesse, BS

The University of Iowa

Abstract

The development of localized muscle fatigue has classically been described by the nonlinear intensity – endurance time (ET) curve (Rohmert, 1960; El Ahrache et al., 2006). These empirical intensity-ET relationships have been well-documented and vary between joint regions. We previously proposed a three-compartment biophysical fatigue model, consisting of compartments (i.e. states) for active (M_A), fatigued (M_F), and resting (M_R) muscle, to predict the decay and recovery of muscle force (Xia and Frey Law, 2008). The purpose of this investigation was to determine optimal model parameter values, fatigue (F) and recovery (R), which define the “flow rate” between muscle states and to evaluate the model’s accuracy for estimating expected intensity – ET curves. Using a grid-search approach with modified Monte Carlo simulations, over 1 million F and R permutations were used to predict the maximum ET for sustained isometric tasks at 9 intensities ranging from 10 – 90% of maximum in 10% increments (over 9 million simulations total). Optimal F and R values ranged from 0.00589 (F_{ankle}) and 0.0182 (R_{ankle}) to 0.00058 (F_{shoulder}) and 0.00168 (R_{shoulder}), reproducing the intensity-ET curves with low mean RMS errors: shoulder (2.7s), hand/grip (5.6s), knee (6.7s), trunk (9.3s), elbow (9.9s), and ankle (11.2s). Testing the model at different task intensities (15 – 95% maximum in 10% increments) produced slightly higher errors, but largely within the 95% prediction intervals expected for the intensity-ET curves. We conclude that this three-compartment fatigue model can be used to accurately represent joint-specific intensity-ET curves, which may be useful for ergonomic analyses and/or digital human modeling applications.

Keywords

Rohmert’s curve; intensity-ET curve; Monte Carlo simulation; mathematical modeling; Ergonomics

© 2012 Elsevier Ltd. All rights reserved.

Corresponding Author: Laura Frey-Law, PhD, PT, 1-252 Medical Education Bldg., Iowa City, IA 52242, Phone: 319-335-9804, Facsimile: 319-335-9707, laura-freylaw@uiowa.edu.

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Conflict of interest Statement

None.

Introduction

Muscle fatigue is considered a risk factor for musculoskeletal injury (Ding et al., 2000), yet few predictive tools are available to model this ubiquitous phenomenon. The development of localized muscle fatigue has classically been described by the intensity – endurance time (ET) curve in the ergonomics literature (Rohmert, 1960). In the past 50+ years, several authors have proposed updated versions of this classic nonlinear curve (Monod and Scherrer, 1965; Hagberg, 1981; Huijgens, 1981; Rose et al., 2000; Garg et al., 2002). More recently, comparisons between several of these models observed that equations varied by joint region (El Ahrache et al., 2006; Ma et al., 2009; Ma et al., 2011). Likewise, a large meta-analysis of 194 publications involving experimental fatigue data confirmed joint-specific intensity-ET relationships (Frey Law and Avin, 2010). Thus, ETs vary as a function of task intensity, but also based on the joint region involved.

These intensity-ET statistical relationships have been well documented and provide one practical tool to predict maximum ET. However this statistical approach is unable to predict fatigue outcomes beyond this single measure, such as the time course of fatigue development, the relative decline in force producing capability for a given task duration, or force recovery during rest periods. Thus more comprehensive, flexible models are needed to represent the complexity of localized muscle fatigue behavior.

We recently proposed a three-compartment, predictive fatigue model, consisting of active (M_A), fatigued (M_F), and resting (M_R) muscle states, to predict the decay and recovery of muscle force (Xia and Frey Law, 2008). We adapted this approach from a similar model first proposed by (Liu et al., 2002) with the addition of a feedback controller, $C(t)$, and a variation of the ‘flow patterns’ between muscle states. These adaptations allow sub-maximal contractions and rest intervals to be modeled, yet preserve the use of only two constant parameters to define overall model behavior (fatigue, F , and recovery, R). While this model was shown to qualitatively reproduce expected curvilinear intensity-ET relationships (Xia and Frey Law, 2008), it has yet to be validated against experimental data.

There are many possible fatigue measures that could be used to test the accuracy of this model for predicting localized muscle fatigue. However, well-characterized intensity- ET models (Frey Law and Avin, 2010) provide useful metrics as a first step in evaluating fatigue during simple, sustained isometric contractions. Further, sustained isometric tasks are commonly used to study fatigue behavior in humans, e.g., (Bystrom and Sjogaard, 1991; Shahidi and Mathieu, 1995; Hunter et al., 2002), as they provide a well-defined methodology. Thus, the primary purposes of this study were to: 1) determine a single set of optimal model parameter values, F and R , to predict several joint-specific intensity – ET curves for sustained isometric contractions, and 2) assess the accuracy of the model using these parameter values.

Methods

A global optimization search strategy was used to determine one set of optimal parameter values for the three-compartment, biophysical fatigue model for each joint region, based on joint-specific empirical intensity-ET curves (Frey Law and Avin, 2010). A series of 9 relative task intensities were considered, referred to as “optimization intensities”. After determining these joint-specific optimal F and R parameter values, the biophysical model was then evaluated using a new set of task intensities (i.e., “test intensities”).

Biophysical Model

The three-compartment model (Xia and Frey Law, 2008) involves three differential equations (Appendix Eq 1 – 3) describing the rate of “flow” from one muscle state to the adjacent state (Figure 1). While the original (Liu et al., 2002) model proposed fatigued muscle would recover back into the active state, we have operationally defined the model such that fatigued muscle recovers to the resting state. In addition, we have added a feedback proportional controller to define the activation and deactivation of active muscle ($C(t)$). These changes alter the defining differential equations from the original equations proposed by (Liu et al., 2002) and require discrete analyses to solve for the three muscle states at each time point. See the Appendix for a review of the model equations first described by (Xia and Frey Law, 2008).

The previously published model was developed using Simulink (Xia and Frey Law, 2008) but was revised into Matlab (Mathworks, Natick, MA) for greater flexibility for this study. The differential equations were solved using the built in Matlab function, ‘c2d,’ to transform them into discrete space, in 1 sec time intervals (1 Hz). Thus, the precision of the model is limited to ± 1 sec using this step size.

While the model could be employed at multiple levels of fidelity, ranging from single fiber to whole muscles, it is employed here at the ‘joint-level’. Similarly, most human fatigue studies are performed *in vivo* at the ‘joint-level’. The joint-level model assumes each muscle state represents the relative proportion of the total joint torque capability (% max intensity). While this could be considered to be the proportion of total motor units available about a joint, technically different motor unit types produce different levels of torque. In addition, biomechanical influences such as muscle length and joint moment arm influence total torque production. Thus generalizing the model to represent relative torque (% max) rather than simply motor unit activation or muscle force provides a means to better represent the various biomechanical and physiological influences on joint torque, and matches experimental methodologies in the fatigue literature (i.e., tasks performed at relative intensities).

Endurance time was used as the primary model outcome variable due to the extensive data available in the literature (see Appendix) on ET as a function of relative task intensity. Predicted ET was operationally defined as the time at which the active muscle state could no longer maintain the target intensity (i.e., when the sum of the resting and active states fall below target levels, $M_R + M_A < TL$).

Parameter Optimization

To determine an optimal set of F and R parameter values for each joint region, a global optimization search strategy was employed. While gradient-based optimization techniques are often useful for minimizing differences between predicted and expected results as a function of time, this approach was not feasible for optimizing a single outcome assessed across multiple simulations (i.e. task intensities). Repeated model simulations with varying F and R values were performed, with the resulting model predictions compared to expected values.

A two-stage grid-search strategy was used. The first stage involved a course grid of potential F and R values; each possible permutation was used to predict maximum ET for 9 task intensities ranging from 10 – 90% of maximum effort in 10% increments (“optimization intensities”). This initial grid used 1000 F and 1000 R values ranging from 0.0001 to 0.1 in increments of 0.0001, for a total of 9,000,000 simulations (1,000,000 F and R combinations for each of the 9 task intensities). The second stage utilized a more refined grid of possible F and R values focusing in on the optimal regions observed for each joint in the first stage of simulations. A smaller range of F and R values were selected around the initial “best” F and

R values (joint-specific), in increments of 0.00001. This produced 8572 additional simulations for each of the 9 relative intensities, for a total of 77,148 simulations in the second stage. Thus a total of 9.08 million simulations were run assessing maximum ET for each F and R combination.

The optimal F and R parameter values from the each search stage were determined as those producing the least error across the 9 optimization intensities compared to the criterion intensity-ET relationships (see below) available for six joint regions: ankle, knee, trunk, shoulder, elbow, hand/grip, as well as a “general” intensity-ET curve (Frey Law and Avin, 2010). Error was calculated as the root mean square (RMS) between these predicted and expected ETs (i.e., criterion intensity-ET relationships) across the 9 optimization intensities. Optimal F and R values were defined as those producing the least mean RMS error. In addition, we assessed whether the model predictions at each of the 9 optimization intensity levels fell within the previously observed 95% prediction intervals, demonstrating expected variability (Frey Law and Avin, 2010). We chose *a priori* to define the minimum criterion for “adequate” model predictions as having a majority of the 9 optimized ETs fall within the 95% prediction intervals for each joint region (i.e., minimum of 5 of 9 intensities).

Criterion Intensity-ET Relationships

Numerous authors have proposed intensity-ET models, or statistical fatigue models, to represent the single fatigue outcome, maximum ET (Monod and Scherrer, 1965; Hagberg, 1981; Huijgens, 1981; Rose et al., 2000; Garg et al., 2002). However, these models were based on fatigue studies involving relatively small sample sizes, e.g., ranging from 5 to 38 participants. To ascertain a ‘gold standard,’ or expected intensity-ET relationship, for comparison to the predicted ETs from our biophysical model, we used the statistical, empirical models developed previously based on data from a meta-analysis of the available literature on sustained isometric contractions (Frey Law and Avin, 2010). The previously reported models (Appendix Eq 7) provided by (Frey Law and Avin, 2010) were used to determine the expected results and the prediction intervals for both the optimization and test intensities (see Appendix for more details).

Parameter Analyses

Finally, the ratios between the optimal F and R parameter values (fatigue to recovery rate ratios) were calculated for each joint region and a sensitivity analysis was performed to examine the influence of changes in F and R on the intensity-ET predictions. While our model differs somewhat from the original model proposed by (Liu et al., 2002) we tested whether an analogous equation, based on the equation proposed by (Liu et al., 2002), relating the F and R values to an intensity asymptote (Eq 8) matched our simulated model predictions. The simulated asymptote for each joint was determined by running the model at decreasing task intensities until an ET could no longer be observed (i.e., the active muscle state plateaued at a level just above the task intensity).

$$\text{asymptote (\% max)} = (1/(F/R+1)) * 100\% \quad (8)$$

Model Evaluation

Using the joint-specific optimized parameter values, the model was then evaluated across a new set of 9 task intensities, the “test intensities” (15 – 95 % max, in increments of 10%). This evaluated the resulting model accuracy using conditions not involved in the optimization process. The mean RMS errors (sec) and mean relative errors (%) were determined as described previously for the optimization intensities.

Sensitivity Analysis

A sensitivity analysis of F and R values was performed to assess the relative effects of changes in each parameter on model output behavior. The behavior of F and R about one optimal starting point was considered as a representative example (i.e., for the elbow joint).

Results

Optimal, joint specific, F and R parameters were found as a result of the global optimization strategy (Table 1). The resulting model predictions for ET well surpassed our minimum criterion for each joint, with a minimum of 7 out of 9 ET predictions falling within the expected 95% prediction intervals (using the ‘optimization task intensities’). Across each joint, the highest intensities (> ~80% max) were the most challenging to maintain within the 95% prediction intervals, typically under-predicting expected ETs (Figure 2).

Relatively minor differences were observed in ET errors between the different joint regions and task intensities. The resulting ET errors at the original optimization intensities (i.e., 10 – 90% max, by 10% increments) were small (Table 2). When evaluating the model accuracy at a separate set of test intensities (i.e. 15 – 95% max, by 10% increments) the errors increased compared to the optimization intensities, but remained below expected variation (Figure 2). Thus, the model performed nearly equally well across different muscle groups despite varying fatigue properties (Table 2).

The observed and predicted intensity asymptotes were in agreement with one another across the joint regions (Table 1). This finding indicates the relationship between F and R parameter values and the ET asymptote proposed for the original (Liu et al., 2002) biophysical fatigue model holds for our modified model, despite the changes made and lack of an analytical solution. Intensity asymptotes ranged from 6 to 10% max for both the observed, simulated and theoretical predictions based on the optimal F and R values.

The fatigue rate parameter, F, had the largest influence on the curvature of the intensity-ET relationship, whereas the recovery rate parameter, R, refined the ET response within that curvature (Figure 3). Changes in F caused relatively large changes in ET at the lower intensities (about a fixed R value), thus altering the inflection points of the intensity-ET curves (and ultimately the intensity asymptote). The effects of changes in R (about a fixed F value) predominately influenced the spread of ETs, particularly at the lowest contraction levels.

Discussion

The main finding of this study is that a parsimonious three-compartment biophysical fatigue model can accurately predict fatigue, i.e., maximum ETs, for sustained isometric contractions across a wide range of task intensities for several distinct joint regions. While this fatigue model is inherently able to predict multiple aspects of muscle fatigue, including the time course of fatigue development and recovery, we were able to validate the model using the outcome variable, ET, due to the extensive normative data available for comparison.

The joint-specific optimal fatigue and recovery parameter values reproduced the expected nonlinear intensity-ET relationships reasonably well. The mean RMS and relative errors (i.e., errors relative to expected ETs, Table 2) ranged from 3 to 28 sec and 2 to 20%, respectively, which is well within expected ET variation. Our prior meta-analysis demonstrated the expected variation in ET (i.e., the percent difference between the 95% PI curve and the mean expected curve for ET) ranged from 29–47% (Frey Law and Avin,

2010). Thus, all of the mean model errors are well-below the normal variation expected between groups of individuals. Further, this estimate of normal variation in fatigue development is based on the culmination of multiple study means which is likely less than the underlying variation between individuals.

The F and R values generally varied as one might predict based on the expected fatigue resistance between joint regions, suggesting good construct validity for the model. For example, the most fatigue-resistant muscle group region we considered is the ankle (Frey Law and Avin, 2010), where the soleus and tibialis anterior muscles about the ankle have a greater proportion of slow-twitch, type I muscle fibers than observed in most other muscles (Johnson et al., 1973). Analogously the optimal F value for the ankle was the lowest of the 6 joints, indicative of a slow fatigue rate for the biophysical model. The most fatigable joint region we considered is the shoulder based on experimental data (Frey Law and Avin, 2010), which likewise had the fastest fatigue rate (i.e., highest F value) of the 6 joints. Thus, the model is able to realistically represent between-joint differences in fatigue rates through the optimal parameter values.

In the original three-compartment fatigue model proposed by (Liu et al., 2002), the use of constant F and R values were assumed to be valid only for very short duration tasks. However, our findings suggest constant F and R values may be viable under a wide range of conditions, particularly low intensity tasks that can be maintained for extended periods of time. Constant model parameters are likely an oversimplification to explain the underlying complex muscle fatigue system, but balancing between model fidelity and usefulness, sometimes referred to as the Law of Parsimony is often desirable. Future research could investigate the use of varying F and R values at high and low task intensities. However this may be counter to the goal of finding the most parsimonious model solution and run the risk of “over-fitting” the model to expected results.

Overall, the fatigue to recovery rate ratios varied from 10 to 15, indicating muscles can be thought of as fatiguing 10 to 15 times faster than they are able to recover during a sustained contraction. Accordingly, the predicted intensity asymptotes ranged from 6 to 9 % of maximum. This is consistent with previous statistical intensity-ET models, which observed asymptotes ranging from 4 to 15 % of maximum (Rohmert, 1960; Monod and Scherrer, 1965; Huijgens, 1981; Sato et al., 1984; Ma et al., 2011), but different from those based on F and R values determined by Liu, et al. (2002) for maximum intensity tasks (22 – 33% max). Conversely, the intensity-ET power equations fit to the meta-analyses data (Frey Law and Avin, 2010) do not have explicit asymptotes and little experimental fatigue data exists for these very low intensity contractions. Thus, it remains unclear whether a true asymptote exists or whether task failure can eventually occur even at very low task intensities (albeit at very long durations). However, the Cinderella hypothesis posits that even very low intensity contractions can lead to musculoskeletal injury (Visser and van Dieen, 2006), suggesting even if no ET is predicted, fatigue may remain a concern.

Several different approaches have been used to model muscle fatigue. In addition to the statistical ET models already described, other analytical models have been developed which predict muscle force decay over time. Typically these are used to represent single muscles and rely on estimates of stimulus impulse inputs (Ding et al., 2003; Marion et al., 2010). These approaches, however, are better intended for clinical applications where electrical stimulation is employed on an individual basis (Marion et al., 2010). This compartmental, biophysical modeling approach has gained interest with applications including: maximal isometric contractions (Liu et al., 2002) and aerobic, endurance activities (Ng et al., 2011) in addition to the submaximal isometric contractions modeled in this study.

While these results suggest a certain level of validity for this model, this finding should not be over-generalized. These results were obtained using data for sustained isometric contractions only and may or may not translate to other task conditions (e.g. isometric tasks with intermittent rest intervals or dynamic contractions). A second caveat is that the optimized parameter values observed in this study were dependent on the expected endurance time-intensity equations reported (Frey Law and Avin, 2010). Both the mean curves and the 95% prediction intervals were influenced by the number and spread of data points that were available in the literature. Lastly, as our goal was to determine optimal ET predictions (minimizing ET error in sec) using a single pair of F and R values across task intensities, the RMS error was minimized. This choice of criterion weights the errors at the lower task intensities more than the higher intensities. This process could be repeated using relative error if desired for a particular application or different goal in mind. However, this alternate approach would produce exponentially greater absolute errors at low task intensities.

While this fatigue model could be used to represent single muscles, or individuals at the joint level, that would require experimental data collection for single muscles and/or individuals across a range of contraction intensities. We believe the greater value of this model is to predict normative fatigue behavior, so that no additional experimental data is needed. Accordingly, the model has potential to be used in isolation (when task intensity is readily estimated) or incorporated into digital human models that can estimate required joint torques (and task intensities) for the completion of a task.

In summary, the simplistic three-compartmental model produces heuristically accurate maximum endurance time predictions across a wide range of contraction intensities. Expected joint differences are well reproduced by differences in optimal F and R parameter values. Future work is needed to ascertain whether this simple modeling approach can be extended to predict fatigue when brief rest periods are allowed, during dynamic tasks, or how well it can predict recovery times following the development of fatigue. However, this study demonstrates this biophysical model is a valid means of predicting fatigue development during sustained isometric tasks across a wide range of task intensities.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

Acknowledgments

This research was supported in part by the United States Council for Automotive Research, Southfield, MI; the National Institutes of Health, K01 AR056134 and a University of Iowa Heartland Center Graduate Student Fellowship.

Appendix Materials

Model Summary

$$dM_R/dt = -C(t) + R * M_F \quad (1)$$

$$dM_A/dt = C(t) - F * M_A \quad (2)$$

$$dM_F/dt = F * M_A - R * M_F \quad (3)$$

Where:

$C(t)$ = the controller denoting the muscle activation-deactivation drive;

F = fatigue parameter defining the rate of change between the active and fatigued compartments; and

R = recovery parameter defining the rate of change between the fatigued and resting compartments.

The controller attempts to match the active muscle compartment (M_A) to the task requirements (e.g. target task intensity, TL , as a percent of maximum, % max), which for this investigation ranged from 10% to 95% max. The muscle activation or deactivation by the controller is defined for three possible conditions (Eq 4 – 6), showing the dependence of $C(t)$ on the relative “volumes” of the three compartments relative to the target task intensity level (TL), see (Xia and Frey Law, 2008) for more detail.

$$C(t) = L * (TL - M_A), \quad \text{if } M_A < TL \text{ and } M_R > (TL - M_A), \quad (4)$$

$$C(t) = L * M_R, \quad \text{if } M_A < TL \text{ and } M_R < (TL - M_A), \quad (5)$$

$$C(t) = L * (TL - M_A), \quad \text{if } M_A \geq TL, \quad (6)$$

Where: L is an arbitrary constant tracking factor to ensure good system behavior. We used a value of 10 for all simulations based on our previous sensitivity analysis (Xia and Frey Law, 2008).

Intensity-ET Curves

The results of (Frey Law and Avin, 2010) have been well-described, but will be summarized briefly here. A total of 194 publications were found that reported maximum ET for relative intensity tasks (i.e., % of maximum) about a single joint for voluntary contractions. These studies included 369 distinct intensity-ET data points as several studies reported ETs for multiple joints and/or task intensities. All studies included healthy young adults (18 – 55 yrs), with a range of reported activity levels (untrained to elite athletes). Sufficient data was available to curve-fit the intensity-ET relationship for six joint regions: ankle, knee, trunk, shoulder, elbow, and hand/grip and an overall general model (all joint regions combined). Total sample sizes, summed across studies for each joint were as follows: ankle ($n=207$, 20 studies, 40 data points), knee ($n=875$, 56 studies, 93 data points), trunk ($n=307$, 17 studies, 33 data points), shoulder ($n=176$, 13 studies, 17 data points), elbow ($n=838$, 60 studies, 126 data points), and hand/grip ($n=754$, 40 studies, 58 data points). Power curves best fit the intensity-ET relationships for each joint (Eq 7) with mean and 95% prediction intervals determined using Matlab Optimization Toolbox.

$$ET = a * (\text{task intensity})^b \quad (7)$$

Where: a and b are the empirical joint-specific model parameters that define the intensity-ET curves, reported by (Frey Law and Avin, 2010)

Muscle fatigue varies between joints as well as across individuals. The order of the most fatigue-resistant joint to the most fatigable joint was: ankle, trunk, knee, elbow, hand/grip, and shoulder, respectively (Frey Law and Avin, 2010). Inherent between subject variability

was demonstrated by the large 95% prediction interval widths for each joint and each task intensity (Frey Law and Avin, 2010).

References

- Bystrom S, Sjogaard G. Potassium homeostasis during and following exhaustive submaximal static handgrip contractions. *Acta Physiologica Scandinavica*. 1991; 142(1):59–66. [PubMed: 1877366]
- Ding J, Wexler AS, Binder-Macleod SA. A predictive model of fatigue in human skeletal muscles. *Journal of Applied Physiology*. 2000; 89:1322–1332. [PubMed: 11007565]
- Ding J, Wexler AS, Binder-Macleod SA. Mathematical models for fatigue minimization during functional electrical stimulation. *Journal of Electromyography and Kinesiology*. 2003; 13(6):575–588. [PubMed: 14573372]
- El Ahrache K, Imbeau D, Farbos B. Percentile values for determining maximum endurance times for static muscular work. *International Journal of Industrial Ergonomics*. 2006; 36(2):99–108.
- Frey Law LA, Avin KG. Endurance time is joint specific: A modelling and meta-analysis investigation. *Ergonomics*. 2010; 53(1):109–129. [PubMed: 20069487]
- Garg A, Hermann K, Schwoerer B, Kapellusch J. The effects of maximum voluntary contraction on endurance times for the shoulder girdle. *International Journal of Industrial Ergonomics*. 2002; 30(2):103–113.
- Hagberg M. Work load and fatigue in repetitive arm elevations. *Ergonomics*. 1981; 24(7):543–555. [PubMed: 7333271]
- Huijgens, JMM. A model for quantifying static load, incorporating muscle fatigue. In: Buskirk, WC., editor. *Biomechanics Symposium*. Boulder, CO: American Society of Mechanical Engineers; 1981. p. 97-99.
- Hunter SK, Ryan DL, Ortega JD, Enoka RM. Task differences with the same load torque alter the endurance time of submaximal fatiguing contractions in humans. *Journal of Neurophysiology*. 2002; 88(6):3087–3096. [PubMed: 12466432]
- Johnson MA, Polgar J, Weightman D, Appleton D. Data on the distribution of fibre types in 36 human muscles: An autopsy study. *Journal of the Neurological Sciences*. 1973; 18(1):111–129. [PubMed: 4120482]
- Liu J, Brown R, Yue G. A dynamic model of muscle activation, fatigue, and recovery. *Biophysical Journal*. 2002; 82:2344–2359. [PubMed: 11964225]
- Ma L, Chablat D, Bennis F, Zhang W. A new simple dynamic muscle fatigue model and its validation. *International Journal of Industrial Ergonomics*. 2009; 39(1):211–220.
- Ma L, Chablat D, Bennis F, Zhang W, Hu B, Guillaume F. A novel approach for determining fatigue resistance of different muscle groups in static cases. *International Journal of Industrial Ergonomics*. 2011; 41(1):10–18.
- Marion MS, Wexler AS, Hull ML. Predicting fatigue during electrically stimulated nonisometric contractions. *Muscle & Nerve*. 2010; 41(6):857–867. [PubMed: 20229581]
- Monod H, Scherrer J. The work capacity of a synergic muscular group. *Ergonomics*. 1965; 8(1):329–338.
- Ng LJ, Sih BL, Stuhmiller JH. An integrated exercise response and muscle fatigue model for performance decrement estimates of workloads in oxygen-limiting environments. *European Journal of Applied Physiology*. 2011; 112(4):1229–1249. [PubMed: 21769737]
- Rohmert W. Determination of the recovery pause for static work of man. *Internationale Zeitschrift Fur Angewandte Physiologie, Einschliesslich Arbeitsphysiologie*. 1960; 18:123–164.
- Rose L, Ericson M, Ortengren R. Endurance time, pain and resumption in passive loading of the elbow joint. *Ergonomics*. 2000; 43(3):405–420. [PubMed: 10755662]
- Sato H, Ohashi J, Iwanaga K, Yoshitake R, Shimada K. Endurance time and fatigue in static contractions. *Journal of Human Ergology (Tokyo)*. 1984; 13(2):147–154.
- Shahidi AV, Mathieu PA. Endurance time characteristics of human ankle dorsiflexors and plantarflexors. *European Journal of Applied Physiol Occupational Physiology*. 1995; 71(2–3):124–130.

- Visser B, van Dieen JH. Pathophysiology of upper extremity muscle disorders. *J Electromyogr Kinesiol.* 2006; 16(1):1–16. [PubMed: 16099676]
- Xia T, Frey Law LA. A theoretical approach for modeling peripheral muscle fatigue and recovery. *Journal of Biomechanics.* 2008; 41(14):3046–3052. [PubMed: 18789445]

Figure 1.

A schematic drawing of the three compartment model, adapted from (Xia and Frey Law, 2008).

Figure 2.

Model predictions using optimal F and R parameter values (optimization intensities = circles, test intensities = triangles) by joint: A) ankle, B) trunk, C) elbow, D) knee, E) hand/grip, and F) shoulder relative to expected intensity-endurance time (ET) curves: modeled mean $\pm$ 95% prediction intervals (PI) reported by (Frey Law and Avin, 2010). Note the change in ET scaling between panels. Insets enlarge these curves, highlighting the task intensities > 40% of maximum. The ETs which fall outside of the 95% PI consistently occur only at the higher task intensities.

Figure 3.

Example sensitivity analyses for F and R parameter values spanning the optimal ranges observed across joints: A) F varying from 0.002 to 0.016 in increments of 0.002, with R fixed to optimal value for elbow (0.00094) and B) R varying from 0.0002 to 0.0016 in increments of 0.0002, with F fixed to optimal value for elbow (0.00912). Note changes in F strongly influence the inflection point of the intensity-ET relationship (e.g. asymptote of the curve), whereas changes in R refine the ET predictions about this curvature.

Table 1

Optimal fatigue, F, and recovery, R, parameters by joint region.

Joint Region	F	R	Within 95% PI	F:R Ratio	Observed Asymptote (%MVC)	Predicted Asymptote* (%MVC)
Ankle	0.00589	0.00058	8/9	10.2	8.99	8.96
Knee	0.01500	0.00149	8/9	10.1	9.05	9.04
Trunk	0.00755	0.00075	8/9	10.1	9.05	9.04
Shoulder	0.01820	0.00168	8/9	10.8	8.47	8.45
Elbow	0.00912	0.00094	8/9	9.7	9.36	9.34
Hand/Grip	0.00980	0.00064	7/9	15.3	6.14	6.13
General	0.00970	0.00091	8/9	10.7	8.59	8.57

* Predicted Asymptote = 1/(F:R+1)*100%

Table 2

Model accuracy for ET predictions using optimal F and R parameters for optimization and test task intensities.

Joint Region	Optimization Intensities [†]		Test Intensities [‡]	
	Mean RMS Error (s)	Mean Relative Error (%)	Mean RMS Error (s)	Mean Relative Error (%)
Ankle	11.2	0.6	28.2	-4.0
Knee	6.7	14.0	15.4	-17.0
Trunk	9.3	2.2	23.1	-1.9
Shoulder	2.7	-6.9	6.1	-10.0
Elbow	9.9	12.0	25.5	7.9
Hand/Grip	5.6	16.2	6.0	-20.0
General	7.3	2.4	11.8	-2.0

[†]Optimization task intensities: 10% – 90% of maximum, in 10% increments (total of 9 simulations).

[‡]Test task intensities: 15% – 95% of maximum, in 10% increments (total of 9 simulations).

Note: Errors calculated between predicted and expected ET based on statistical intensity-ET models previously reported (Frey Law and Avin, 2010).