Comparison of Advection–Diffusion Models and Neural Networks for Prediction of Advanced Water Treatment Effluent

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Abstract

An artificial neural network (ANN) can help in the prediction of advanced water treatment effluent and thus facilitate design practices. In this study, sets of 225 experimental data were obtained from a wastewater treatment process for the removal of phosphorus using oven-dried alum residuals in fixed-bed adsorbers. Five input variables (pH, initial phosphorus concentration, wastewater flow rate, porosity, and time) were used to test the efficiency of phosphorus removal at different times, and ANNs were then used to predict the effluent phosphorus concentration. Results of experiments that were conducted for different values of the input parameters made up the data used to train and test a multilayer perceptron using the back-propagation algorithm of the ANN. Values predicted by the ANN and the experimentally measured values were compared, and the accuracy of the ANN was evaluated. When ANN results were compared to the experimental results, it was concluded that the ANN results were accurate, especially during conditions of high phosphorus concentration. While the ANN model was able to predict the breakthrough point with good accuracy, the conventional advection–diffusion equation was not as accurate. A parametric study conducted to examine the effect of the initial pH and initial phosphorus concentration on the effluent phosphorus concentration at different times showed that lower influent pH values are the most suitable for this advanced treatment system.

Key words: advanced treatment; alum residual; artificial neural network; fixed-bed column test; phosphorus pollution; porosity

Introduction

PHOSPHORUS REMOVAL FROM WASTEWATER is often achieved through an advanced treatment process called adsorption. Effective phosphorus adsorption using dried alum sludge, a waste material from drinking water treatment plants, has been studied and evaluated experimentally with fixed-bed column tests (Mortula and Gagnon, 2007). However, predicting the effluent phosphorus concentration (EPC) of such a treatment process is often very difficult, not only because of the presence of too many factors that could affect the process but also because of the heterogeneous properties of the waste materials.

While effluent predictions of phosphorus adsorption on dried alum sludge are difficult, effluent predictions of the adsorption process on other materials have been done, using both deterministic and probabilistic approaches (see Tables 1 and 2). Adsorption on natural materials has been modeled using deterministic models (Smith and Ghiassi, 2006; Singh *et al.*, 2009). A mass transfer model showed a satisfactory

agreement with experimental results for effluent predictions of phosphorus from an adsorption system on saturated slag (Lee *et al.*, 1997). A surface complexation model was used to predict EPCs from an adsorption system using sand columns (Lindstrom *et al.*, 2007). A surface excess model of arsenic adsorption on maple wood ash facilitated a practical design of arsenic column adsorption systems (Rahman *et al.*, 2004). However, deterministic models have difficulty in evaluating heterogeneous materials.

Over the years several soft-computing models, for example, artificial neural network (ANN) models, have been used to predict effluent concentrations of adsorption systems (Carsky and Do, 1999). Because of recent advances in computer software and hardware, ANN models have come to be seen by engineers and scientists as powerful tools for predicting the contamination and concentration of different effluents and chemicals in drinking water, wastewater, and aquifers. Chelani *et al.* (2002) used a three-layer neural network model with a hidden recurrent layer and a multivariate regression model to predict sulphur dioxide concentration. The predicted values were then compared with the measured concentrations at three sites. The authors concluded that the ANN model was more accurate than multivariate regression models. Raduly *et al.* (2007) used an ANN to predict effluent ammonium, biochemical oxygen demand (BOD₅), and total

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TABLE 1. DETERMINISTIC MODELS APPLIED FOR PREDICTION OF COLUMN TEST RESULTS

<i>Modeling method</i>	<i>Adsorbent</i>	<i>Adsorbate</i>	<i>Comments</i>	<i>Reference</i>
Mass transfer	Iron sorbent	Chromate	n.e.	Smith and Ghiassi, 2006
	Saturated slag media (dust and cake)	Phosphorus	<3% error	Lee <i>et al.</i> , 1997
	Granular ferric hydroxide (GFH)	Arsenate, phosphate, salicylic acid, and groundwater DOC	errors up to 60%	Sperlich <i>et al.</i> , 2008
	Tree leaves & activated carbon	Chromium Cr(VI)	R ² ranged 0.7–0.99	Singh <i>et al.</i> , 2009
	Activated carbon	Mercury	<5%	Goyal <i>et al.</i> , 2009
	Pistachio-nut-shell activated carbon	Pb(II), Cu(II), Cr(III) & Co(II)	R ² ≥ 0.97	Sulaymon <i>et al.</i> , 2009
	Natural subsurface media	Sulfur dioxide (SO ₂)	n.e.	Lua and Yang, 2009
Surface complexation	Uranium(VI)		RMSE=0.1	Barnett <i>et al.</i> , 2000
	Porous media (sand)	Phosphorus	n.e.	Lindstrom <i>et al.</i> , 2007
Surface excess	Red pozzolan	Lead	<5% error	Papini <i>et al.</i> , 1999
	Maple wood ash	Arsenic As(III) & As(V)	<7% error	Rahman <i>et al.</i> , 2004
Finite element	Hydrophobic zeolites (MFI and MOR types)	Sulfur dioxide (SO ₂)	n.e.	Mello and Eic, 2002
	Metal-oxide	Oxygen	distribution n.e.	Öchsner <i>et al.</i> , 2006

n.e., not estimated.

suspended solids. Although the ANN prediction of BOD₅ was accurate when compared to mechanistic wastewater treatment plant model predictions, the ANN prediction of effluent chemical oxygen demand (COD) and total nitrogen concentrations was less accurate. Dellana and West (2009) compared the predictive ability of linear autoregressive integrated moving average (ARIMA) models and nonlinear time-delay neural network models in water quality and wastewater applications. After observing that the two models performed differently depending on the nature of the data (either real world or experimental), they suggested that the nature and intended use of water quality data should be taken into consideration when choosing between neural networks and other statistical methods. Mortula and Sadiq (2009) gave an overview of different soft-computing techniques, including ANN, for modelling and optimizing in water and wastewater treatment. Du *et al.* (2007) found the ANN model of Solanesol adsorption on macroporous resins in packed column systems achieved a correlation coefficient of 0.9869 and thus performed better than the mass transfer model. Cuco *et al.* (2009) used a genetic algorithm for the prediction of effluent bio-

molecules in resin beds (fixed-bed adsorption) and observed a good level of accuracy in the prediction. Mortula *et al.* (2010) have also provided preliminary results for effluent predictions of phosphorus removal using ANNs.

This article discusses the use of an ANN for the prediction of EPCs from an advanced treatment process utilizing dried alum sludge as an adsorbent. The results are compared to advection–diffusion (mass transfer) approaches and some experimental data obtained from fixed-bed adsorption columns.

Experimental Protocols

Fixed-bed column tests were conducted to evaluate EPCs for wastewater treated using dried alum sludge. Rapid, small-scale laboratory experiments were designed to simulate the adsorption behavior in large-scale adsorbers. The experiment was conducted with a fixed column 20 cm in length and 2.5 cm in diameter. Wastewater was passed through the column in an upflow mode (Fig. 1). Effluent water was then tested for phosphorus concentration. The adsorbent particle size was

TABLE 2. STOCHASTIC MODELS APPLIED FOR PREDICTING COLUMN TEST RESULTS

<i>Modeling method</i>	<i>Adsorbent</i>	<i>Adsorbate</i>	<i>Comments</i>	<i>Reference</i>
Artificial neural network (ANN)	Macroporous resins	Solanesol	R ² =0.9869	Du <i>et al.</i> , 2007
	Immobilized <i>Pseudomonas aeruginosa</i> cells	Lanthanide ions (La, Eu, Yb)	RMSE=0.1697	Texier <i>et al.</i> , 2002
	Activated carbon	Binary vapor mixtures	RMSE ranged 0.029–0.4; relative error ranged 1.8%–41.1%	Carsky and Do, 1999
Genetic algorithm	Shells of sunflower	Copper	R ² =0.986; RMSE=0.018	Oguz and Ersoy, 2010
	Resin beds	Lisozyme	0%–0.02%	Cuco <i>et al.</i> , 2009
Generalized extremal optimization algorithm	Resin beds	Lisozyme	0.99%–1.31%	Cuco <i>et al.</i> , 2009

FIG. 1. Fixed-bed column.

0.98 mm, and the wastewater flow rate varied from 1.933 mL/min to 2.45 mL/min. The distribution coefficient (K_d) values used were $0.0112 \text{ cm}^3/\text{g}$. The bulk density of the adsorbent was $1.0328 \text{ g}/\text{cm}^3$. An approximate value of $0.036 \text{ cm}^2/\text{hr}$ was used for D_x (the hydrodynamic dispersion in the direction of flow) for these experiments, and pH was varied in the range of 3–7. To achieve a qualitative pattern of phosphorus adsorption that would identify the breakthrough point clearly, experiments were conducted for at least a week.

Two different models were used to predict EPC from alum sludge adsorption systems: an ANN and advection–diffusion.

The ANN model

An ANN is a computational tool that is capable of estimating and predicting engineering properties that are functions of many variables and parameters. It has proven to be very effective in solving problems in which the relationship between physical phenomena and their parameters are complex, highly nonlinear, and possess a large degree of uncertainty. An ANN has the ability to learn from existing data and adapt to map a set of input parameters into a set of output parameters without knowing the intricate relationship between them. There are several ANNs and architectures that have been used in engineering applications to model or approximate properties. Among feed-forward models, the most widely used ANNs are the multi-layer perceptron (MLP) and the radial basis function (RBF) models. In this investigation, a feed-forward MLP with a back-propagation learning algorithm and two hidden layers was used to estimate EPC in mg/L, using the pH of the influent solution, initial phosphorus concentration (IPC) in mg/L, wastewater flow rate in mL/min, time in hours, and porosity as model inputs. The ANN has a hyperbolic tangent (tanh) activation function in the first and second hidden layers. There are four processing elements

FIG. 2. MLP neural network architecture with two hidden layers.

TABLE 3. RANGE OF TRAINING, VALIDATION AND TESTING DATA

Parameter	Maximum	Minimum
pH of the influent solution	7	3
Initial phosphorus concentration (mg/L)	10	2.5
Time (hours)	238.5	0
Flow rate (mL/min)	2.45	1.933
Porosity	0.594	0.515

in each hidden layer. The output layer also has a hyperbolic tangent (tanh) activation function. The role of the activation function is to map the wide range of input parameters into a limited range of the desired output. In other words, the activation function transforms the activation value of a neuron into an output value. Each neuron calculates its activation value and passes it through the activation function to produce the output value of the neuron. The output of the neuron is used as input for the neurons in the next layer. The output of the neurons of the last layer (output layer) is the output of the entire network (Abdalla *et al.* 2011). For this study, the EPC training and testing data was the result of 225 experimental investigations conducted in fixed-bed column tests (Mortula and Gagnon, 2007), using a fixed particle size of 0.98 mm.

Figure 2 shows the MLP ANN model and its two hidden layers used in this study. The total number of data items used was 225. Of this number, 157 data items were used as the training data set, 32 data items formed the validation set, and the remaining 36 data items formed the testing data set. The data items were randomly divided into the sets. The range of training, validation, and testing data is shown in Table 3. The learning rate used for the first hidden layer is 1.0, for the second hidden layer, 0.1 and for the output layer, 0.01. A momentum factor of 0.7 was used for the model throughout. Initial random values were generated, and the ANN was trained and validated using five runs with 1000 epochs in each run from which the average performance of the five runs was taken (NeuroDimensions, 2005). The average minimum normalized mean square error (NMSE) for the training data was 0.01293 with a standard deviation of 0.00372. For the cross validation data, the average minimum NMSE and the standard deviation were 0.01806 and 0.00459, respectively. The trained ANN model was then used on the test data, and its predicted values were compared with the experimental values.

Table 4 shows the performance indicators of the 36 testing samples. As shown there, the NMSE is 0.0683 and the correlation coefficient is 0.9703. This indicates that the measured and the predicted values show a good correlation.

TABLE 4. PERFORMANCE OF THE ARTIFICIAL NEURAL NETWORK ON THE TEST DATA

Performance criterion	Value
Mean square error (MSE)	0.3907
Normalized mean square error (NMSE)	0.0683
Mean absolute error (MAE)	0.4662
Minimum absolute error	0.0329
Maximum absolute error	1.993
Correlation coefficient (r)	0.9703

Conventional modeling approach

The advection–diffusion equation is important in many physical systems, for example, heat transfer in draining films, water transfer in soils, spread of pollutants in rivers and streams, and flow in porous media (Mohebbi and Dehghan, 2010; Dehghan, 2007). The column setup presented in this article is commonly used to simulate adsorption in ground-water. Therefore, this experimental setup can be represented by an advection–diffusion equation with a mass transfer scenario. This can be done by solving the following one-dimensional advection–dispersion partial differential equation:

$$\frac{\partial C}{\partial t} = D'_x \frac{\partial^2 C}{\partial x^2} - v'_x \frac{\partial C}{\partial x} \quad (1)$$

where

C : Material concentration (mg/L);

t : time (hr);

x : distance (cm)

$D'_x = D_x/R$, where

D_x : Hydrodynamic dispersion in the direction of flow (cm^2/hr)

R : Retardation factor $= 1 + \frac{\rho b}{n} \frac{\partial S}{\partial C}$, where ρ is the bulk mass density of the media (g/m^3); n is the porosity of the media; S is the mass of chemical constituent adsorbed on the solid part of the media (g/g); $\frac{\partial S}{\partial C}$ is the distribution coefficient $= K_d$ (estimated from the batch experiments on the reaction kinetic) (m^3/g)

$v'_x = v_x/R$, where

v_x : Average velocity in the direction of flow (cm/hr)

R : Retardation factor $= 1 + \frac{\rho b}{n} \frac{\partial S}{\partial C}$

This equation can theoretically describe the adsorption of any material in one direction, the direction of flow. While there is a two-dimensional form of the equation that can describe adsorption in the perpendicular direction as well, it gives a more complex solution. Since the experiments dealt with small diameter column tests, resulting in approximately zero adsorption in the perpendicular direction, the complexity of the two-dimensional form of this equation could be avoided. All the parameters needed in the equations were estimated through standard experiments.

Equation (1) was solved numerically, using finite difference approximations, specifically, the Crank-Nicolson method. This method is an implicit finite difference method, which is unconditionally stable and has an accuracy of order $O(h^2, k^2)$ (Mohebbi and Dehghan, 2010). The method involves replacing partial derivatives with the following finite difference approximations:

$$\frac{\partial C}{\partial t} = \frac{C_i^{l+1} - C_i^l}{\Delta t} \quad (\text{forward difference}) \quad (2)$$

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{2} \left(\frac{C_{i+1}^l - 2C_i^l + C_{i-1}^l}{\Delta x^2} + \frac{C_{i+1}^{l+1} - 2C_i^{l+1} + C_{i-1}^{l+1}}{\Delta x^2} \right) \quad (\text{central difference}) \quad (3)$$

$$\frac{\partial C}{\partial x} = \frac{1}{2} \left(\frac{C_{i+1}^l - C_{i-1}^l}{2\Delta x} + \frac{C_{i+1}^{l+1} - C_{i-1}^{l+1}}{2\Delta x} \right) \quad (\text{central difference}) \quad (4)$$

where l is the time index, i is the space index, Δt is the time increment, and Δx is the space increment.

The boundary and initial conditions are:

$C_1^0 \rightarrow C_{v-1}^0$ = Initial concentration inside the column $= 0$
(initial condition)

C_0^l = Concentration at the beginning of the column
(boundary condition)

where n is the space count that corresponds to the end of the column.

Unlike other explicit finite difference methods, this one requires solving systems of linear equations. However, as can be observed from the boundary and initial conditions, the concentration at the end of the column is the unknown. Therefore, to solve the problem, the central difference approximations for the spatial partial derivatives had to be converted at the end of the column to the following forward difference approximations:

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{2} \left(\frac{C_{i+2}^l - 2C_{i+1}^l + C_i^l}{\Delta x^2} + \frac{C_{i+2}^{l+1} - 2C_{i+1}^{l+1} + C_i^{l+1}}{\Delta x^2} \right) \quad (\text{forward difference}) \quad (5)$$

$$\frac{\partial C}{\partial x} = \frac{1}{2} \left(\frac{C_{i+1}^l - C_i^l}{\Delta x} + \frac{C_{i+1}^{l+1} - C_i^{l+1}}{\Delta x} \right) \quad (\text{forward difference}) \quad (6)$$

By substituting Equations (2)–(6) into Equation (1) at the appropriate column location, the following two equations are obtained:

$$(2\alpha_1 + 1)C_i^{l+1} + (\alpha_2 - \alpha_1)C_{i+1}^{l+1} - (\alpha_2 + \alpha_1)C_{i-1}^{l+1} = (\alpha_2 + \alpha_1)C_{i-1}^l + (1 - 2\alpha_1)C_i^l + (\alpha_1 - \alpha_2)C_{i+1}^l \quad (\text{For } i = 1 \rightarrow n - 1) \quad (7)$$

$$(1 - \alpha_1 - 2\alpha_2)C_{i-2}^{l+1} - \alpha_1 C_i^{l+1} + 2(\alpha_2 + \alpha_1)C_{i-1}^{l+1} = -2(\alpha_2 + \alpha_1)C_{i-1}^l + (1 + \alpha_1 + 2\alpha_2)C_{i-2}^l + \alpha_1 C_i^l \quad (\text{For } i = n) \quad (8)$$

where

$$\alpha_1 = \frac{(\Delta t)D'_x}{2(\Delta x)^2}$$

$$\alpha_2 = \frac{(\Delta t)v'_x}{4(\Delta x)}$$

To obtain solutions for as many practical time and space increments as possible, Equations (7) and (8) need to be solved, using a programming language. To do so, the coefficient matrix was first established and then decomposed into lower and upper matrices, using Crout Decomposition. The equations of Crout Decomposition were then implemented to solve the system, using Excel 2007 Visual Basic Application (VBA). The VBA code was divided into two steps. The first step solved the system for the first time step, and the second, using a looping procedure, solved the system for the rest of the time steps. The code was split into two because the first time step required the use of the initial condition, whereas the rest of the steps did not.

Results and Discussion

ANN

An MLP ANN was trained, using the training data. The trained ANN was then tested using the test data to predict EPC (mg/L) from the advanced treatment system. Figure 3 compares the prediction of the trained ANN with the actual measured values that were randomly selected from various experimental sets at different timings and shows the accuracy of these predictions, almost all of which are within ± 1 mg/L. It was observed that while the ANN gave accurate predictions for large EPC values, the predictions were not as accurate for small EPC values. As indicated, the NMSE of the performance of the ANN on the normalized test data was 0.0683 and the correlation coefficient was 0.9703. Figure 4 compares the ANN predicted values of EPC with the experimentally measured values for a single experimental set. The model represented the pattern of the breakthrough curve quite reasonably. The phosphorus concentration at the effluent increased with the passage of time, which indicates the ability of the model to capture the pattern of phosphorus adsorption on alum sludge. The experiments were not conducted until the alum sludge (adsorbent) reached exhaustion. Thus, both the experimental data and the ANN modeling results in Fig. 4 do not show exhaustion. For the early hours of the prediction, however, the model shows negative values, which is unrealistic because the concentration can never be negative. Since ANN is not based on a theoretical analysis of the problem, these negative values may arise. For practical applications of this model, negative values can be assumed to be zero.

Conventional modeling

Comparisons of the experimental data and the results obtained from the advection–diffusion model were established by plotting their EPCs versus time. Figure 4, which compared the ANN predicted EPC values with the experimental data for experimental set, also demonstrates the differences between the conventional model and the experimental data for one experimental set. Even though the conventional model predicted a breakthrough curve that is commonly seen in ad-

FIG. 3. Accuracy of the prediction of effluent phosphorus concentration (mg/L) for test data ($r=0.97$).

FIG. 4. Comparison of an ANN model, an advection–diffusion model, and an experimental data set with the following parameters: influent pH=7; particle size=0.98 mm; initial phosphorus concentration=2.5 mg/L; flow rate=2.133 mL/min; porosity=0.594; $K_d=0.0112$ cm³/g; bulk mass density=1.0328 g/cm³; and $D_x=0.036$ cm²/hr.

sorption processes, the effluent breaks through much faster than the actual results indicate. For this reason, the conventional model was not capable of predicting the process well. The poor performance of the conventional model is mainly due to the fact that alum sludge is a heterogeneous material and a conventional advection–diffusion model with mass transfer processes is not capable of predicting the process dynamics. Since the superiority of the ANN model over the advection–diffusion model is obvious from Fig. 4, statistical analyses are not needed for further justification.

It can be observed from Fig. 4 that the advection–diffusion equation failed to predict EPCs, which leads to the conclusion that the one-dimensional advection–diffusion equation is not suitable for this kind of application. It is not, because it does not take into account the many parameters, such as the pH, involved in the process.

Since the D_x value is an approximation, it is believed that a set of model results similar to experimental data can be achieved if, to slow down the convergence toward the maximum concentration, the D_x is reduced to very small values. However, to avoid oscillation problems, such a modification requires large reductions in the space and time increments. Consequently, unless a very powerful computer is used, a great deal of computational time is required to run the model.

Assuming the reasonable accuracy of D_x , the deterministic model failed to simulate mathematically the adsorption/desorption behavior of phosphorus on dried alum sludge. This may be because the alum sludge, having heterogeneous properties, failed to provide the consistent material properties needed to effectively simulate the behavior using the advection–diffusion model. Although Lee *et al.* (1997) observed reasonable results for blast furnace slag media, it did not happen in this study.

Parametric studies and sensitivity analysis

The parametric study was conducted using the tested ANN model to study the effect of the initial pH and IPC on EPC at different times.

FIG. 5. ANN prediction of the variation of phosphorus concentration with time for different initial phosphorus concentrations with the following parameters: influent pH=6; particle size=0.98 mm; flow rate=2.0 mL/min; and porosity=0.55.

Effect of IPC on EPC. To study the effect of IPC on EPC, an average value of discharge, $Q=2.0$ mL/min, and an average value of porosity, $P=0.55$, were used as constant input parameters. Figure 5 shows the ANN prediction of EPC variation with time for different IPC values (2.5, 5, 7.5, and 10) with the following parameters: influent pH, 6; particle size, 0.98 mm; wastewater flow rate, 2.0 mL/min; porosity, 0.55. As Fig. 5 illustrates, for low values of IPC (e.g., IPC=2.5), the variation of EPC with time is almost linear, that is, almost a

constant rate. For high values of IPC (e.g., IPC=10), the variation is nonlinear. This may be because the available phosphorus for the adsorption sites is low for low phosphorus concentration, and there is low interference from the existing adsorbed phosphorus. A high phosphorus concentration, however, can have potential competition for the adsorption sites, forcing the curve to be nonlinear. For such a case, the increase in EPC was slow initially (the first 40 hours), then increased drastically during the period of 40–140 hours, and then slowed again after 140 hours. Figure 5 shows that the phosphorus concentration breaks through at around 40 hours, and after 140 hours, the adsorbents start to reach exhaustion. Typically, phosphorus treatment in a wastewater treatment plant is done in the advanced treatment stage. For the last wastewater treatment plants, initial phosphorus concentrations for advanced treatment ranges of 2–5 mg/L. Therefore, a linear pattern is more expected. The change from linearity to nonlinearity as IPC increases can be seen even with varying influent pH values, as shown in Fig. 6.

Effect of pH on EPC. A similar pattern of linearity and nonlinearity can be seen in the effect of pH on EPC, which is shown in Fig. 7. An examination of Figures 6a and 7a show that linearity is almost preserved at low influent pH values even with high IPCs. This is due to the reaction of phosphorus with alum sludge, which releases protons, thus reducing the pH levels. When nonlinearity is present, the intervals of slow increase, followed by rapid increase, and then slow increase again are similar to those obtained when IPC varies and the influent pH value is fixed. It indicates the ANN model's deficiency in predicting behavior in the micro-process. Figure 7 shows that exhaustion is reached much sooner at high influent pH values, which implies that acidity is better suited for these

FIG. 6. Variation of effluent phosphorus concentration with pH at different times for initial phosphorus concentrations (IPCs) of (a) 2.5, (b) 5, (c) 7.5, and (d) 10. Flow, $Q=2.0$ L/s; porosity, $P=0.55$.

FIG. 7. Variation of effluent phosphorus concentration with time for pH of (a) 3, (b) 4, (c) 5, and (d) 7. Flow, $Q=2.0$ L/s; porosity, $P=0.55$.

adsorbers. Furthermore, unlike at low IPCs, where exhaustion can be observed at almost the same time for all influent pH values, exhaustion is hardly reached at high IPCs. This may be because a high IPC is associated with the high adsorption density of the alum sludge.

Effect of time on EPC. Figures 8 and 9 reveal that EPC is almost constant during the early stages of the advanced treatment system (26 hours), regardless of the influent pH and IPC values. This is because the breakthrough point has not yet been reached. As time progresses (once phosphorus breaks

FIG. 8. Variation of effluent phosphorus concentration with pH at (a) 26 hours, (b) 70 hours, (c) 142 hours, and (d) 238 hours, with different initial phosphorus concentrations. Particle size, $PS=0.98$ mm; flow, $Q=2.0$ L/s; porosity, $P=0.55$.

FIG. 9. Variation of effluent phosphorus concentration with IPC at (a) 26 hours, (b) 70 hours, (c) 142 hours, and (d) 238 hours, with different pH. Particle size, $PS=0.98$ mm; flow, $Q=2.0$ L/s; porosity $P=0.55$.

through), the impact of influent pH and IPC values becomes more apparent, and it becomes clear that lower influent pH values result in lower EPCs. Figure 8 shows that the linear relation between EPC and influent pH values is almost preserved as time progresses, unlike the relation between EPC and IPC (Fig. 9), which seems much more nonlinear. After 238 hours, the effects of pH value are negligible because the system reaches exhaustion.

Conclusions

In this study an ANN was used to predict EPC. Because input and output data from experiments was used to calibrate the model, the ANN approach accurately represents the process of phosphorus removal by alum sludge. The MLP ANN model was able to predict the pattern of the breakthrough curve quite reasonably. Unlike the ANN model, the conventional modeling approach, depending only on theoretical equations, was relatively inaccurate in representing the process, probably because heterogeneous materials are used. Parametric studies revealed that lower influent pH values are most suited for this advanced treatment system.

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Disclosure Statement

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