Force-dependent mobility and entropic rectification in tubes of periodically varying geometry

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We investigate transport of point Brownian particles in a tube formed by identical periodic compartments of varying diameter, focusing on the effects due to the compartment asymmetry. The paper contains two parts. First, we study the force-dependent mobility of the particle. The mobility is a symmetric non-monotonic function of the driving force, F , when the compartment is symmetric. Compartment asymmetry gives rise to an asymmetric force-dependent mobility, which remains non-monotonic when the compartment asymmetry is not too high. The F -dependence of the mobility becomes monotonic in tubes formed by highly asymmetric compartments. The transition of the F -dependence of the mobility from non-monotonic to monotonic behavior results in important consequences for the particle motion under the action of a time-periodic force with zero mean, which are discussed in the second part of the paper: In a tube formed by moderately asymmetric compartments, the particle under the action of such a force moves with an effective drift velocity that vanishes at small and large values of the force amplitude having a maximum in between. In a tube formed by highly asymmetric compartments, the effective drift velocity monotonically increases with the amplitude of the driving force and becomes unboundedly large as the amplitude tends to infinity.

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I. INTRODUCTION

Transport in systems of varying geometry has become the subject of growing interest among researchers in recent years (see review articles¹ and papers published in the special issue of Chemical Physics² and references therein). The present paper deals with driven Brownian motion in such periodic systems, which is a special case of the general transport problem. Consider a point Brownian particle that moves in a cylindrically symmetric tube of periodically varying diameter under the action of a constant uniform force F directed along the tube axis. We study the particle motion in the high-friction (diffusion) regime. Our analysis focuses on times when the particle displacement significantly exceeds the tube period l . On such time scales, the particle drift velocity $v(F)$ is a nonlinear function of F . Therefore, the particle mobility, defined as the ratio of the drift velocity to the driving force, $\mu(F) = v(F)/F$, depends on F . The mobility is a symmetric function of the driving force, $\mu(F) = \mu(-F)$, if the identical compartments forming the tube are symmetric. When the compartments are of an asymmetric shape, $\mu(F)$ is an asymmetric function of F , $\mu(F) \neq \mu(-F)$. As a consequence, a time-periodic force with zero mean, parallel to the tube axis induces a directed motion of the particle in such a

tube, so that the tube “rectifies” an external periodic signal. This phenomenon is referred to in the literature as “entropic” or “geometric” rectification, since the rectification is achieved due to varying system geometry.^{3–7}

In the present paper, we study variation of the F -dependence of the mobility due to the change in the compartment shape for the tubes schematically shown in Fig. 1. For the tubes shown in panels (a) and (b) of Fig. 1, the F -dependence is qualitatively different from that for the tube shown in panel (c), namely, the former is non-monotonic in F , while the latter is a monotonically increasing function of the driving force. As a consequence, the effective drift velocities of the rectified motion in tubes shown in panels (b) and (c) have qualitatively different dependences on the amplitude of the time-periodic force: For the tube shown in panel (b) the velocity increases with the amplitude, reaches a maximum, and then decreases, whereas for the tube shown in panel (c) the velocity monotonically increases with the amplitude. Thus, the drift velocity has a finite maximum value for the tube shown in panel (b), and can be unboundedly high for the tube shown in panel (c).

The outline of the present paper is as follows. In Sec. II, we discuss variation of the F -dependence of the mobility due to the change in the compartment shape. Then in Sec. III, we

FIG. 1. Schematic representations of cylindrically symmetric tubes formed by identical periodic compartments of different level of asymmetry. The compartment asymmetry increases from panel (a) to panel (c). Tubes formed by symmetric and highly asymmetric compartments are shown in panels (a) and (c), respectively. The case of intermediate level of the compartment asymmetry is shown in panel (b).

discuss the effective drift velocity of the rectified motion induced by a time-periodic force with zero mean that alternates in sign every half period. The velocity depends on the amplitude F_0 and period τ of the external force, $v_{eff}(F_0, \tau)$. We study the drift velocity and the rectification coefficient, $\alpha(F_0, \tau)$, as functions of F_0 in the adiabatic regime ($\tau \rightarrow \infty$), focusing on how these dependences change because of the variation in the compartment shape. In addition, for the tube shown in Fig. 1(c), we study the τ -dependence of the rectification coefficient at large values of F_0 . Using the results obtained in Ref. 8, we develop an analytical theory that shows how $\alpha(F_0, \tau)$ increases with τ from zero to its maximum value in the adiabatic regime as τ increases from zero to infinity. Some concluding remarks are made in Sec. IV.

II. FORCE-DEPENDENT MOBILITY

We study how the F -dependence of the particle mobility changes as we vary the shape of the compartment using the results obtained from Brownian dynamics simulations. The tube-forming compartments are characterized by the maximum and minimum radii of the tube, R and a , respectively, the compartment length l , which is the tube period, and the maximum length l_m between the widest and narrowest cross

sections (Fig. 1). We run simulations for $R = l = 1$, $a = 0.2$, and $l_m = 0.5, 0.7, 0.9, 0.99$, and 1.0 . Varying l_m we convert the compartments from symmetric, at $l_m = 0.5$, to highly asymmetric, at $l_m = 1.0$. Assuming that $k_B T = 1$, where k_B is the Boltzmann constant and T is the absolute temperature, and the particle mobility μ_0 and diffusion coefficient D_0 in a purely cylindrical tube are unity, $\mu_0 = D_0 = 1$, we find $\mu(F)$ for F ranging from -10^5 to 10^5 . The results for positive and negative values of the driving force are shown in panels (a) and (b) of Fig. 2.

Dependences $\mu(F)$ are very similar at positive F : all $\mu(F)$ monotonically increase with F approaching the same upper limit $\mu_0 = 1$ as F becomes sufficiently large (Fig. 2(a)). This is a consequence of the fact that at large positive F the particle spends all the time in the cylinder of radius a passing through the apertures connecting neighboring compartments since collisions with the walls focus the particle into the cylinder. Thus, the compartment asymmetry practically does not manifest itself at positive F .

This is not the case when F is negative (Fig. 2(b)). At $l_m = 0.5$, the compartments are symmetric, therefore, $\mu(F) = \mu(-F)$, and the mobility monotonically decreases from $\mu_0 = 1$ to its minimum value at $F = 0$, as F increases from its large negative values to zero. The difference in the behavior of $\mu(F)$ due to the asymmetry of the compartment shape becomes more and more pronounced as l_m increases. It is most pronounced in the limiting case of $l_m = 1.0$, where the mobility monotonically increases with F from its minimum value $\mu(-\infty) = \mu_0 v^2$, $v = a/R$, at large negative values of F . The transition of the F -dependence of the mobility from non-monotonic to monotonic behavior due to variation of the compartment geometry in two-dimensional periodic channels has been reported earlier,⁴ as well as in a recent publication.⁹

To explain the limiting value of the mobility as $F \rightarrow -\infty$, we note that the pattern of the particle motion at large negative values of F is the same as that in a tube formed by identical cylindrical compartments of radius R and length l separated by infinitely thin periodic partitions with circular openings of radius a in their centers. In such a tube, the particle can be either in the cylinder connecting the openings or in thin layers of thickness $k_B T/|F|$ near the partitions. The marginal distribution of the particle over the tube cross section is uniform and given by $1/(\pi R^2)$ at all F . Therefore, the probability of finding the particle in the cylinder is v^2 . Since the particle moves in the force direction only when it is in the cylinder, the asymptotic value of the mobility in such a tube at large values of $|F|$ is $\mu_0 v^2$.¹⁰

Whereas $\mu(F)$ monotonically decreases with F at negative F when $l_m = 0.5$ and monotonically increases with F when $l_m = 1.0$, at intermediate $l_m = 0.7, 0.9$, and 0.99 , $\mu(F)$ are non-monotonic functions of the driving force (Fig. 2(b)). As $F \rightarrow -\infty$, they approach the limiting value of the mobility, $\mu(-\infty) = \mu_0 = 1$, due to the focusing collisions with the tube walls. The mobilities monotonically decrease as F increases at large negative F , reach their minimum values and then increase as F approaches zero. In contrast to the case of positive F , the details of the behavior of functions $\mu(F)$ at negative F depend on the value of l_m (Fig. 2(b)) and, hence, on the compartment asymmetry.

FIG. 2. The force-dependent mobility at different levels of asymmetry of the tube-forming compartments. Panels (a) and (b) show the mobility for positive and negative values of the driving force, respectively. Panel (c) shows the mobility difference $\Delta\mu(F) = \mu(F) - \mu(-F)$, $F > 0$. Parameters are: $R = l = 1$, $a = 0.2$, numbers in the box are the values of l_m corresponding to different simulations, $D_0 = 1$.

To illustrate the asymmetry of the particle mobility due to the asymmetry of the compartment shape, in Fig. 2(c), we show the difference $\mu(F) - \mu(-F) = \Delta\mu(F)$, $F > 0$, for different value of the parameter l_m that characterizes the compartment asymmetry. Of course, $\Delta\mu(F) \equiv 0$ at $l_m = 0.5$, since the compartments are symmetric (Fig. 1(a)). At highly asymmetric compartments ($l_m = 1.0$) shown in Fig. 1(c), $\Delta\mu(F)$ monotonically increases with F from zero to its maximum value $\Delta\mu(\infty) = \mu_0(1 - v^2)$. At intermediate values of l_m , $\Delta\mu(F)$ are non-monotonic functions of F : they increase with F at small F , reach their maximum values, and then decrease, approaching zero as $F \rightarrow \infty$, since $\mu(F)$ and $\mu(-F)$ have the same asymptotic value $\mu(\pm\infty) = \mu_0$. With the increase in the compartment asymmetry, the maximum value of $\Delta\mu(F)$ increases, and the position of the maximum shifts to larger values of F .

To summarize, in this section we have shown that as l_m increases from 0.5 to 1.0, non-monotonic functions $\mu(F)$ become more and more asymmetric approaching monotonic behavior of $\mu(F)$ at $l_m = 1.0$.

III. ENTROPIC RECTIFICATION

In this section, we consider the particle motion in tubes of different compartment asymmetry, $0.5 \leq l_m \leq 1.0$, under the action of a time-periodic force with zero mean directed along the tube axis. We assume that the force instantly switches between two values $\pm F_0$, where F_0 is the force amplitude, $F_0 > 0$, every half period. When the tube-forming compartments are asymmetric, such a force may generate directed motion of the particle along the tube axis³⁻⁸ with an effective drift velocity $v(F_0, \tau)$, which is a function of amplitude F_0 and period τ of the force. We study this function in the adiabatic regime ($\tau \rightarrow \infty$) in Sec. III A. To characterize the rectification efficiency of the tube, in Sec. III B, we introduce a rectification coefficient $\alpha(F_0, \tau)$ and discuss its behavior as a function of F_0 in the adiabatic regime and as a function of τ at large values of the force amplitude in the special case of highly asymmetric compartments ($l_m = 1.0$).

A. Effective drift velocity

Let $\Delta_{\pm}(F_0, \tau)$ be the mean displacements of the particle during positive and negative half periods of the force, $\Delta_{\pm}(F_0, \tau) > 0$. Then the effective drift velocity is given by

$$v(F_0, \tau) = \frac{\Delta_+(F_0, \tau) - \Delta_-(F_0, \tau)}{\tau} = \frac{1}{2}(v_+(F_0, \tau) - v_-(F_0, \tau)), \quad (1)$$

where $v_{\pm}(F_0, \tau) = \Delta_{\pm}(F_0, \tau)/(\tau/2)$ are the absolute values of the mean velocity of the particle during positive and negative half periods of the force, $v_{\pm}(F_0, \tau) > 0$. In the adiabatic (*ad*) regime,

$$\Delta_{\pm}^{(ad)}(F_0, \tau) = \mu(\pm F_0)F_0\tau/2, \quad (2)$$

and the absolute values of the mean velocities are independent of the force period,

$$v_{\pm}^{(ad)}(F_0) = \mu(\pm F_0)F_0. \quad (3)$$

Substituting this into Eq. (1), we arrive at

$$v_{ad}(F_0) = \frac{1}{2}\Delta\mu(F_0)F_0. \quad (4)$$

This simple formula gives the effective drift velocity of the directed motion of the particle in the adiabatic regime. We discuss its behavior using the results for $\Delta\mu(F_0)$ presented in Sec. II.

Function $v_{ad}(F_0)$ changes its behavior as the compartment shape varies from symmetric ($l_m = 0.5$) to highly asymmetric ($l_m = 1.0$). In the case of symmetric compartments, $\Delta\mu(F_0)|_{l_m=0.5} \equiv 0$, and, hence, $v_{ad}(F_0) \equiv 0$, as it must be. At intermediate values of l_m , $0.5 < l_m < 1.0$, $v_{ad}(F_0)$ is a non-monotonic function of F_0 : it increases with F_0 at small values of the force amplitude, reaches a maximum, and then decreases, approaching zero, as $F_0 \rightarrow \infty$. Finally, at $l_m = 1.0$, $v_{ad}(F_0)$ monotonically increases with F_0 . At large values of the force amplitude, $v_{ad}(F_0)|_{l_m=1.0}$ is linear in F_0 with the slope determined by the ratio of the minimum and maximum tube radii v ,

$$\lim_{A \rightarrow \infty} \frac{dv_{ad}(F_0)|_{l_m=1.0}}{dF_0} = \frac{\Delta\mu(\infty)}{2} = \frac{1 - v^2}{2}. \quad (5)$$

We illustrate the F_0 -dependence of the effective drift velocity in the adiabatic regime in Fig. 3.

To summarize, the effective drift velocity of the particle in the adiabatic regime is a bounded function of the force amplitude at $0.5 < l_m < 1.0$. Its maximum value increases with the increase in the compartment asymmetry. When $l_m = 1.0$, and the compartments are highly asymmetric (Fig. 1(c)), the

FIG. 3. Effective drift velocity as a function of the amplitude of the periodic force in the adiabatic regime at different levels of asymmetry of the tube-forming compartments. The inset illustrates the linear F_0 -dependence of $v_{ad}(F_0)$ at large F_0 when $l_m = 1.0$. (See more details in the Fig. 2 caption.)

velocity can be unboundedly high, since it linearly increases with F_0 at large values of the amplitude.

B. Rectification coefficient

Following Schmid *et al.*,³ we define the rectification coefficient in the adiabatic regime, $\alpha_{ad}(F_0)$, in terms of the force-dependent mobility as

$$\alpha_{ad}(F_0) = \frac{\mu(F_0) - \mu(-F_0)}{\mu(F_0) + \mu(-F_0)} = \frac{\Delta\mu(F_0)}{\mu(F_0) + \mu(-F_0)}. \quad (6)$$

According to this definition, $\alpha_{ad}(F_0)$ can be interpreted as the ratio of $v_{ad}(F_0)$, Eq. (4), to the mean absolute value of the velocity given by $(v_+^{(ad)}(F_0) + v_-^{(ad)}(F_0))/2$, i.e.,

$$\alpha_{ad}(F_0) = \frac{2v_{ad}(F_0)}{v_+^{(ad)}(F_0) + v_-^{(ad)}(F_0)} = \frac{v_+^{(ad)}(F_0) - v_-^{(ad)}(F_0)}{v_+^{(ad)}(F_0) + v_-^{(ad)}(F_0)}. \quad (7)$$

Naturally, the rectification coefficient vanishes for symmetric compartments, $\alpha_{ad}(F_0)|_{l_m=0.5} \equiv 0$. When the compartments are moderately asymmetric, $0.5 < l_m < 1.0$, the rectification coefficient is a non-monotonic function of A that vanishes at small and large values of the force amplitude, having a maximum in between. The maximum value of $\alpha_{ad}(F_0)$ increases with the increasing compartment asymmetry and shifts to larger values of the amplitude.

The F_0 -dependence of the rectification coefficient changes its behavior from non-monotonic to monotonic at $l_m = 1.0$. Function $\alpha_{ad}(F_0)|_{l_m=1.0}$ monotonically increases with the force amplitude, approaching its plateau value $\alpha_{ad}(\infty)|_{l_m=1.0}$, as $F_0 \rightarrow \infty$. The plateau value is independent of the amplitude and determined by the ratio of the minimum and maximum tube radii v ,

$$\alpha_{ad}(\infty)|_{l_m=1.0} = \frac{\mu_+(\infty) - \mu_-(\infty)}{\mu_+(\infty) + \mu_-(\infty)} = \frac{1 - v^2}{1 + v^2}. \quad (8)$$

This formula shows that $\alpha_{ad}(\infty)|_{l_m=1.0}$ approaches unity as the ratio of the tube radii tend to zero. To illustrate the F_0 -dependence of the rectification coefficient in the adiabatic regime, in Fig. 4 we show $\alpha_{ad}(F_0)$ for the compartments of different levels of asymmetry, using our numerical results for the force-dependent mobility presented in Sec. II. The plots of $\alpha_{ad}(F_0)$ shown in Fig. 4 look similar to those of $\Delta\mu(F)$ in Fig. 2(c). This is not surprising since $\alpha_{ad}(F_0)$ is given by the ratio of $\Delta\mu(F_0)$ to the sum $\mu(F_0) + \mu(-F_0)$, which is a weakly varying function of F_0 .

Finally, we discuss the dependence of the rectification coefficient on the force period using the following definition of $\alpha(F_0, \tau)$:

$$\alpha(F_0, \tau) = \frac{v_+(F_0, \tau) - v_-(F_0, \tau)}{v_+(F_0, \tau) + v_-(F_0, \tau)}, \quad (9)$$

which is a generalization of the definition of $\alpha_{ad}(F_0)$ given in Eq. (7). Based on general arguments, one might expect that $\alpha(F_0, \tau)$ monotonically increases with τ from zero to its maximum value $\alpha_{ad}(F_0)$, which is a function of the force amplitude discussed above. The τ -dependence of the rectification coefficient, in general, can be studied only numerically. Nevertheless, it is possible to develop an analytical theory that

FIG. 4. Rectification coefficient as a function of the amplitude of the periodic force in the adiabatic regime at different levels of asymmetry of the tube-forming compartments. (See more details in the Fig. 2 caption.)

describes variation of $\alpha(F_0, \tau)$ from zero to $\alpha_{ad}(F_0)$ in the special case of highly asymmetric compartment, $l_m = 1.0$, assuming that F_0 is large enough, so that $\alpha_{ad}(F_0)$ is independent of the force amplitude and given by Eq. (8). In the rest of this section, we develop such a theory using the results obtained in Ref. 8.

In this special case, $l_m = 1.0$ and $F_0 \rightarrow \infty$, it can be shown that $v_-(F_0, \tau)$ is proportional to $v_+(F_0, \tau)$,

$$v_-(F_0, \tau) = [v^2 + (1 - v^2)(1 - f(\tau))]v_+(F_0, \tau). \quad (10)$$

An exact solution for function $f(\tau)$ can be written in terms of the Bessel functions of the first kind $J_0(z)$ and $J_1(z)$,⁸

$$f(\tau) = 1 - \frac{8}{D\tau(R^2 - a^2)} \sum_{n=1}^{\infty} \frac{J_1^2(\lambda_n a)}{\lambda_n^4 J_0^2(\lambda_n R)} (1 - e^{-\lambda_n^2 D\tau/2}), \quad (11)$$

where λ_n are positive roots of the equation $J_1(\lambda_n R) = 0$, $n = 1, 2, \dots$. As τ goes from zero to infinity, $f(\tau)$ monotonically increase from zero to unity. Its asymptotic behavior at small and large τ is given by⁸

$$f(\tau) \approx \begin{cases} \frac{2\sqrt{2D_0\tau/\pi}}{3a(1-v^2)}, & \tau \rightarrow 0 \\ 1 - 2t_{rel}/\tau, & \tau \rightarrow \infty \end{cases}, \quad (12)$$

where the relaxation time t_{rel} is¹¹

$$t_{rel} = \frac{a^2}{4D_0(1-v^2)}(v^2 - 1 - \ln(v^2)). \quad (13)$$

One can use the relaxation time to write a single-exponential approximation for $f(\tau)$,⁸

$$f(\tau) \approx f_{exp}(\tau) = 1 - \frac{2t_{rel}}{\tau}(1 - e^{-\tau/(2t_{rel})}). \quad (14)$$

Both $f(\tau)$ and $f_{exp}(\tau)$ monotonically increase from zero to unity as τ goes from zero to infinity. Function $f_{exp}(\tau)$ correctly

FIG. 5. Dependence of the rectification coefficient on the force period for the tube shown in Fig. 1(c) in the limiting case of very large amplitude of the periodic force for the tube with $R = l = l_m = 1$ and $a = 0.2$, $D_0 = 1$.

predicts the large- τ asymptotic behavior of $f(\tau)$, Eq. (12) but fails to describe the $\sqrt{\tau}$ increase of $f(\tau)$ at small τ , Eq. (12).

Using Eqs. (9) and (10), we can write the rectification coefficient in terms of function $f(\tau)$ as

$$\alpha(\infty, \tau)|_{l_m=1.0} = \frac{(1+v^2)f(\tau)}{2-(1-v^2)f(\tau)} \alpha_{ad}(\infty)|_{l_m=1.0}. \quad (15)$$

This formula describes monotonic increase of $\alpha(\infty, \tau)|_{l_m=1.0}$ with τ from zero at $\tau = 0$ to its maximum value in the adiabatic regime, $\alpha_{ad}(\infty)|_{l_m=1.0}$, given in Eq. (8). Asymptotic behavior of $\alpha(\infty, \tau)|_{l_m=1.0}$ at small and large τ can be obtained using corresponding asymptotic behavior of $f(\tau)$, Eq. (12),

$$\frac{\alpha(\infty, \tau)|_{l_m=1.0}}{\alpha_{ad}(\infty)|_{l_m=1.0}} \approx \begin{cases} \frac{(1+v^2)\sqrt{2D_0\tau/\pi}}{3a(1-v^2)}, & \tau \rightarrow 0 \\ 1 - 4t_{rel}/[(1+v^2)\tau], & \tau \rightarrow \infty \end{cases}. \quad (16)$$

Substituting the single-exponential approximation of $f(\tau)$, Eq. (14) into Eq. (15), one can obtain a corresponding approximation for the ratio $\alpha(\infty, \tau)|_{l_m=1.0}/\alpha_{ad}(\infty)|_{l_m=1.0}$,

$$\frac{\alpha(\infty, \tau)|_{l_m=1.0}}{\alpha_{ad}(\infty)|_{l_m=1.0}} \approx \frac{1 - (2t_{rel}/\tau)(1 - e^{-\tau/(2t_{rel})})}{1 + (2t_{rel}/\tau)(1 - e^{-\tau/(2t_{rel})})} \alpha_{ad}(\infty)|_{l_m=1.0}. \quad (17)$$

We illustrate the τ -dependence of $\alpha(\infty, \tau)|_{l_m=1.0}$ in Fig. 5.

IV. CONCLUDING REMARKS

In this paper, we have studied driven Brownian motion in a cylindrically symmetric, periodic tube formed by identical compartments of the shape shown in Fig. 1. The focus is on how variation of the compartment asymmetry affects (i) the force-dependent mobility $\mu(F)$ of the particle and (ii) the effective drift velocity of the rectified motion in the adiabatic regime $v_{ad}(F_0)$. Another quantity of our interest is

the rectification coefficient $\alpha(F_0, \tau)$ that characterizes the rectification efficiency of the tube as a function of the amplitude F_0 and period τ of the external force. We have studied its F_0 -dependence in the adiabatic regime, $\alpha_{ad}(F_0)$, for tubes formed by compartments of different level of asymmetry. In addition, we analyzed the τ -dependence of the rectification coefficient in the special case of the highly asymmetric compartment (Fig. 1(c)), assuming that $F_0 \rightarrow \infty$.

We have shown that both $\mu(F)$ and $v_{ad}(F_0)$ have qualitatively different behaviors in tubes formed by symmetric and highly asymmetric compartments schematically shown in Figs. 1(a) and 1(c), respectively. In the former limiting case (Fig. 1(a)), (i) $\mu(F)$ is a non-monotonic, symmetric function of F (Fig. 2) and, therefore, (ii) $v_{ad}(F_0) \equiv 0$ (Fig. 3). In the opposite limiting case (Fig. 1(c)), (i) $\mu(F)$ monotonically increases with F between its limiting values $\mu(-\infty)$ and $\mu(\infty)$ (Fig. 2), while (ii) $v_{ad}(F_0)$ monotonically increases with F_0 and becomes unboundedly large as $F_0 \rightarrow \infty$, since $v_{ad}(F_0) \propto F_0$ at large values of the force amplitude (Fig. 3). As the compartment asymmetry varies between the two limits, both $\mu(F)$ and $v_{ad}(F_0)$ change their behavior between those in the two limiting cases. To be more specific, at intermediate compartment asymmetry (Fig. 1(b)) (i) $\mu(F)$ is a non-monotonic but asymmetric function of F (Fig. 2), while (ii) $v_{ad}(F_0)$ is a non-monotonic bounded function of the force amplitude that first increases with F_0 , reaches a maximum and then decreases as $F_0 \rightarrow \infty$ (Fig. 3). The non-monotonic function $\mu(F)$ becomes monotonic, while non-monotonic, bounded function $v_{ad}(F_0)$ becomes monotonic and unbounded as the compartment becomes highly asymmetric.

Change in the compartment asymmetry also leads to variation of the F_0 -dependence of $\alpha_{ad}(F_0)$. As shown in Fig. 4, $\alpha_{ad}(F_0) \equiv 0$ for the tube formed by symmetric compartments (Fig. 1(a)), $\alpha_{ad}(F_0)$ is a non-monotonic function of F_0 for tubes formed by compartments of intermediate level of asymmetry (Fig. 1(b)), and, finally, $\alpha_{ad}(F_0)$ monotonically increases with F_0 for the tube formed by highly asymmetric compartments (Fig. 1(c)). In the latter case, we developed a

theory that describes the τ -dependence of the rectification coefficient, assuming that $F_0 \rightarrow \infty$. As might be expected on the basis of general arguments, $\alpha(\infty, \tau)$ monotonically increases with τ from zero to $\alpha_{ad}(\infty)$, as τ goes from zero to infinity (Fig. 5). Our analytical results establish the relation between $\alpha(\infty, \tau)$ and geometric parameters of the tube.

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