Effect of slippage on the thermocapillary migration of a small droplet

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(Received 7 July 2011; accepted 4 September 2011; published online 15 March 2012)

We conduct a numerical investigation and analytical analysis of the effect of slippage on the thermocapillary migration of a small liquid droplet on a horizontal solid surface. The finite element method is employed to solve the Navier-Stokes equations coupled with the energy equation. The effect of the slip behavior on the droplet migration is determined by using the Navier slip condition at the solid-liquid boundary. The results indicate that the dynamic contact angles and the contact angle hysteresis of the droplet are strictly correlated to the slip coefficient. The enhancement of the slip length leads to an increase in the droplet migration velocity due to the enhancement of the net momentum of thermocapillary convection vortices inside the droplet. A larger contact angle leads to an increase in the migration velocity which in turn enlarges the rate of the droplet migration velocity to the slip length. There is good agreement between the analytical and the numerical results when the dynamic contact angle utilizes in the analytical approach obtained from the results of the numerical computation, and the static contact angle is smaller than 50° . © 2011 American Institute of Physics. [doi:10.1063/1.3644382]

I. INTRODUCTION

The movement of a small liquid droplet actuated by the thermal gradient on a horizontal solid surface has received a lot of attention because of potential applications in droplet-based devices.^{1–3} The molecular interaction between a liquid and a solid that occurs as a liquid droplet moves on a horizontal solid surface is very complicated and its mechanism needs to be thoroughly understood. From the macroscopic point of view, there is a common principle in continuum fluid dynamics, the no slip boundary condition, which is assumed to apply, wherein it is assumed that fluid molecules in the immediate vicinity of the solid surface move at exactly the same velocity as the surface. Hence, the relative fluid-solid velocity would be equal to zero. However, this hypothesis is in contrast with the movement of the contact lines of liquid droplets on a solid surface.^{4,5} This assumption is also not strongly supported by molecular simulations and experimental investigations on the microscopic scale.^{6–13} Recently, the idea of “slip length” or “slip coefficient,” first proposed by Navier,¹⁴ has been recognized as valid for determining the slip behavior of a liquid on a solid surface. This idea states that at the solid boundary, due to kinematic reasons, the normal component of the fluid velocity should vanish at an impermeable solid wall. Furthermore, the tangential velocity u is proportional to the shear rate according to the expression $u_\tau = b(\partial u / \partial z)$, where the constant of proportionality b is called the slip length. This parameter is defined as the distance beyond the liquid/solid wall interface where the liquid velocity extrapolates to zero. The magnitude of the slip length depends strongly on the surface wettability and the roughness.¹⁵ Moreover, it has been found that the

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shear rate normal to the surface, and the presence of gaseous layers at the liquid/solid wall interface will also affect the slip behavior.^{9,16} Voronov *et al.*¹⁷ indicated that this coefficient is also functionally dependent on the strength of the affinity of the interfacial energy and the relative diameter of the molecular wall-fluid and fluid-fluid collision. Generally, this value is enhanced for a weak interaction between liquid and solid.¹⁸ For instance, when the liquid sits on a nonwetting surface, and its pressure is lower than the capillary pressure, the liquid will be restricted to the top of the micro-protrusions and the voids in the micro-feature, which are occupied by a gas phase.¹⁹ It is found that the magnitude of the slip could normally be less than a few nanometers for flows over flat hydrophilic surfaces, while it could be as large as tens of micrometers for superhydrophobic surfaces.^{12,20–23} Therefore, the slippage apparently rises from complex small scale liquid/solid boundary conditions. The structural and dynamical properties of the liquid/solid wall interface can be represented by the magnitude of the slip coefficient.

Several studies have investigated the thermocapillary migration of a liquid droplet on a solid surface.^{24–33} The main objective of the theoretical studies has been to predict the steady migration velocity of the droplet. Lubrication approximation has been employed for this purpose in several works.^{24,26–29} Brochard²⁴ assumed the droplet to be approximately wedge-shaped, the wedge angle to be equal to the static contact angle (SCA) and to be sufficiently small. She neglected deformation of the free surface. However, the droplet could become asymmetric as it moves leading to variation in the contact angles, the so-called dynamic contact angles (DCA). The difference in the DCA between the advancing and receding side is called the contact angle hysteresis (CAH). Ford and Nadim²⁶ assumed the droplet to be shaped like a two-dimensional, long, thin ridge, pinned in a solid wall under the Navier slip condition. Their results indicate that the slip length strongly affects the steady migration velocity of the droplet. In an extension of the method developed by Ford and Nadim,²⁶ Chen *et al.*²⁸ used the height profile of a droplet, which they obtained from their experimental work, to calculate the droplet velocity with a fixed CAH for different slip lengths. The results show the speed of the droplet to be more significantly influenced by the CAH than by the slip length. Pratap *et al.*²⁹ carried out a study where the slip length was fixed at a constant value. The results show that the effect of CAH on the critical droplet size (below this size, the droplet does not move) is minimal, and this size is independent of the imposed temperature gradient. It has been demonstrated experimentally^{25,28–31} that the final speed of the droplet is proportional to the footprint radius L and that the critical droplet size depends on the temperature gradient. Tseng *et al.*³¹ found that the droplet shape would change during motion. Recently, Song *et al.*³⁰ indicated that capillary flow plays an important role in the thermocapillary migration of a droplet. Unfortunately, experimental works have only been conducted for droplet movement on a specific solid surface. The effects of the interaction characteristics between the liquid and solid surface on the droplet motion behavior have yet to be clearly presented. Nguyen and Chen^{32,33} did develop a numerical model to study the thermocapillary migration of a droplet on a horizontal solid surface. In these studies, the Navier slip boundary condition is used to overcome the contact line problems. The chosen slip length is of a nanometer order, and a partial wetting surface is assumed.

It is clear that the droplet migration behavior could be affected by the slippage. Furthermore, the shape parameters such as the droplet's DCA, CAH, footprint radius L , and height profiles $h(x)$ vary during motion^{28,31} and could, therefore, be strongly correlated with the slip behavior. Unfortunately, it is impossible to develop an analytical model in which these parameters can be shown as a function of the slip coefficient. It is usually assumed that these factors and the slip coefficient can be considered independently, meaning that the droplet speed is obtained as the effect of slippage or of other parameters separately.²⁸ In addition, in the theoretical analysis, the predicted speed of a droplet with a large SCA, and the speed predicted for different slip lengths might also be uncounted.³² It should also be noted that it is somewhat difficult to carry out precise experimental investigations such as the measurement on the millimeter scale of temperature or flow velocity during the migration of a droplet, the fabrication of solid surfaces with different slip lengths on the nanometer scale, or the establishment of an imposed stable, high temperature gradient on a millimeter length. In spite of the long-standing interest in the slip condition at a liquid/solid boundary, there has been very few theoretical, numerical,

and experimental studies focused on understand how the migration of a small liquid droplet on a solid surface with a uniform temperature gradient is affected by the slippage. Our goal in the present study is to develop a proper numerical model for investigation of the effects of the slip behavior on the thermocapillary migration of a small liquid droplet on a horizontal solid surface. The numerically calculated droplet migration velocities are compared with those predicted from an analytical model, which is a modified version of the technique proposed by Ford and Nadim²⁶ and Chen *et al.*²⁸

II. THEORETICAL DESCRIPTION

A small squalane droplet surrounded by air is placed on a solid substrate then subjected to a uniform horizontal temperature gradient G . The properties of squalane and air are listed in Table I. For the small size of droplet considered here, the density of the liquid within it can be assumed to be a constant value and the effect from the body force can be neglected. The two dimensional equations of the conservation of mass, momentum, and energy for incompressible and Newtonian fluids are

$$\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right]_i = 0, \quad (1)$$

$$\rho_i \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right]_i = -\frac{\partial p}{\partial x} + \left[\frac{\partial}{\partial x} \mu \frac{\partial u}{\partial x} + \frac{\partial}{\partial z} \mu \frac{\partial u}{\partial z} \right]_i + F_x^{SV}, \quad (2)$$

$$\rho_i \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right]_i = -\frac{\partial p}{\partial z} + \left[\frac{\partial}{\partial x} \mu \frac{\partial v}{\partial x} + \frac{\partial}{\partial z} \mu \frac{\partial v}{\partial z} \right]_i + F_z^{SV}, \quad (3)$$

$$\rho_i C_{Pi} \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial z} \right]_i = k_i \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right]_i, \quad (4)$$

where u_i and v_i are the velocity components in the x - and z -directions, respectively; p is the pressure and ρ_i is the fluid density; g is the acceleration constant; μ_i is the dynamic viscosity; C_{Pi} is the specific heat; k_i is the thermal conductivity; and T is the temperature. The subscripts $i = "l"$ and $i = "a"$ denote liquid and air, respectively.

From the numerical point of view, the simulation of the interfacial flow could be very complex due to the existence of the force of surface tension, which is strongly dependent on local variation in the droplet/air interface temperature. Recently, the continuum surface force method, first developed by Brackbill *et al.*,³⁴ has become popular for use in modeling surface tension effects on fluid motion, and this could alleviate the interface topology constraints. It has been employed successfully for modeling incompressible fluid flow, capillarity, and droplet

TABLE I. Physical properties of the fluids (at 25 °C).

Parameters	Air	Squalane
ρ (kg/m ³)	1.1614	809
σ (mN/m)		30.7
γ_T (mN/m K)		0.05
μ (mPa s)	0.0174	29.51
α (m ² /s)	22.5×10^{-6}	1.15×10^{-7}
k (W/m K)	26.3×10^{-3}	140×10^{-3}
C_P (J/kg K)	1007	1500
β (K ⁻¹)	$1/T_{\text{mean}}$	0.996×10^{-3}
κ^{-1} (mm)		2

dynamics.³⁴ In this method, the surface tension, as represented by a body force will only act on an infinitesimal thickness of the element at the interface, so that the interfacial boundary conditions for the normal and tangential stresses are automatically satisfied. The surface tension force acting at the interface can be described by

$$\mathbf{F}^{\text{SV}} = \sigma \kappa \delta \mathbf{n}, \quad (5)$$

where σ is the surface tension; δ is the Dirac delta function that takes a nonzero value at the droplet/air interface only; $\mathbf{n}$ is the unit normal vector to the interface; and κ is the local interfacial curvature. The surface tension force $\mathbf{F}^{\text{SV}}$ varies from point to point at the droplet/air interface as a function of temperature. The surface tension σ can be assumed to be a linear function of temperature³⁵

$$\sigma = \sigma_{\text{ref}} - \gamma_T (T - T_{\text{ref}}), \quad (6)$$

where σ_{ref} is the surface tension at the reference temperature T_{ref} ; and $\gamma_T = -\partial\sigma/\partial T$ is the coefficient of the surface-tension temperature, which in the present study has a positive value for the liquid in the droplet.

Squalane is selected as the liquid for the droplet. Its dynamic viscosity varies with the temperature³⁶ and can be described as

$$\mu_l = 8373.47 \exp(-T/23.51) + 0.00326, \quad (7)$$

where μ and T are inserted in (Pa s) and (K), respectively. The other physical properties of the fluids are assumed to be independent of the temperature.

The appropriate boundary conditions for the flow and temperature field are given by

$$u = v = 0; \quad \frac{\partial T}{\partial x} = 0 \quad \text{at} \quad x = 0; \quad x = W; \quad \text{and} \quad H \geq z \geq 0, \quad (8)$$

$$u = v = 0; \quad T = T_{\text{ref}} \quad \text{at} \quad W \geq x \geq 0, \quad z = H, \quad (9)$$

$$u = v = 0 \quad \text{at} \quad 0 \leq x < x_1 \quad \text{and} \quad x_2 \geq W, \quad (10)$$

$$T = T_H - G \times x \quad \text{at} \quad W \geq x \geq 0, \quad z = 0, \quad (11)$$

and

$$T = T_H \quad \text{at} \quad 0 \quad \text{and} \quad T = T_C \quad \text{at} \quad x = W, \quad (12)$$

where x_1 and x_2 are positions of the droplet's two contact points. The liquid-solid boundary condition is applied. The Navier slip condition is

$$u_\tau = b \frac{\partial u}{\partial z}, \quad (13)$$

where b is the slip length. The liquid/air interface $S(x)$ is set to ensure the continuum of flow and temperature as well as the level set function value

$$\mathbf{V}_l \cdot \nabla S = \mathbf{V}_a \cdot \nabla S, \quad T_a = T_l, \quad \text{and} \quad \phi = 0.5, \quad (14)$$

where $\mathbf{V} = u\mathbf{i} + v\mathbf{j}$.

Before a thermal gradient is imposed on the substrate, the droplet is placed on the substrate at the ambient temperature. Thus, the initial conditions are

$$\mathbf{V}_l(\mathbf{X}, 0) = \mathbf{V}_a(\mathbf{X}, 0) = 0, \quad (15)$$

$$T_{\text{sub}}(x, 0, 0) = T_{\text{ref}}, \quad (16)$$

$$T_l(\mathbf{X}, 0) = T_a(\mathbf{X}, 0) = T_{\text{ref}}, \quad (17)$$

where $\mathbf{X} = x\mathbf{i} + z\mathbf{j}$.

III. ANALYTICAL AND NUMERICAL SOLUTIONS

A. Analytical solution

It is assumed with the droplet considered, that the maximum height h_m is much smaller than the radius of the footprint L ($h_m/L \ll 1$) (Fig. 1), and the dynamic Bond number ($\text{Bo}_D = \rho_l g \beta_1 h_m^2 / \gamma_T$) is no larger than 0.21 ($\text{Bo}_D \leq 0.21$) so that the change in the droplet shape due to the gravitational force and the effect of buoyancy convection can be neglected.³³ Here, the viscosity is assumed to be a constant. Following Ford and Nadim²⁶ and Chen *et al.*,²⁸ the migration speed of a droplet can be expressed as

$$U = \frac{1}{6\mu J} \left[(1 + 2 \cos \varphi_A) |\gamma_T| G - \frac{\sigma_R (\cos \varphi_R - \cos \varphi_A)}{L} \right], \quad (18)$$

where subscripts R and A denote receding and advancing sides, respectively; φ is the dynamic contact angle; and parameter J is described as

$$J = \frac{1}{2L} \int_{-L}^L \frac{dx}{h(x) + 3b}. \quad (19)$$

Clearly, Eq. (19) indicates that any change of the droplet shape and the slip length would contribute to the value of parameter J. Therefore, these factors could influence to the velocity of the droplet, as shown in Eq. (18). If the droplet forms a cylindrical cap and its static contact angle φ_C is less than 90° (Fig. 1), $h(x)$ can be expressed as

FIG. 1. Schematic cross-section of a spherical-cap droplet.

$$h(x) = \sqrt{\frac{L^2}{\sin^2 \varphi_C} - x^2} - L \cot \varphi_C. \quad (20)$$

When the droplet moves forward, the air-liquid interface deforms. Hence, the DCA between the advancing and receding edges is different and L also varies. This leads to a change of $h(x)$. When the DCA between two sides differs, the mid plane of the droplet and the z axis is not coincident. However, these lines can be assumed to be at the same position because the size of the droplet is very small and the difference in the DCA between the two sides is also tiny. Hence, it is reasonable to assume that the front half of the droplet has a DCA of φ_A and the rear side a DCA of φ_R . Parameter J now can be expressed as

$$J = \frac{1}{2L} \left[\int_{-L}^0 \frac{dx}{h_R(x) + 3b} + \int_0^L \frac{dx}{h_A(x) + 3b} \right], \quad (21)$$

in which

$$h_R(x) = \sqrt{\frac{L^2}{\sin^2 \varphi_R} - x^2} - L \cot \varphi_R, \quad (22)$$

and

$$h_A(x) = \sqrt{\frac{L^2}{\sin^2 \varphi_A} - x^2} - L \cot \varphi_A. \quad (23)$$

B. Numerical solution

The conservative level set method is employed in the numerical simulation, because it can deal with the deformation of the interface, while keeping the mass conservation during the motion of the droplet.^{37,38} It can be seen in the schematic diagram in Fig. 2 that the air subdomain Ω_1 and the droplet subdomain Ω_2 are separated by the interface $S(x)$ with the level set function $\Phi = 0.5$. The value of Φ goes smoothly from 0 to 1 with $1 \geq \Phi > 0.5$ in the air subdomain Ω_1 and $0.5 > \Phi \geq 0$ in the droplet subdomain Ω_2 . The reinitialized convection of the interface can be written as

$$\frac{\partial \Phi}{\partial t} + \mathbf{V}_i \cdot \nabla \Phi = \lambda \nabla \cdot \left[\varepsilon \nabla \Phi - \Phi(1 - \Phi) \frac{\nabla \Phi}{|\nabla \Phi|} \right], \quad (24)$$

where λ is the re-initialization parameter; ε is the thickness of the interface; and $\mathbf{V}_i$ is the velocity vector. Generally, the suitable value for λ is the magnitude of the maximum velocity occurring in the problem.³⁹ According to the COMSOL user guide, the selected value of ε is $h_c/2$, where h_c is the characteristic mesh size in the interface region.

FIG. 2. Schematic representation used for computation.

FIG. 3. Typical mesh used in the computational domain ($W=25$ mm and $H=4$ mm) for a droplet with $\varphi=60^\circ$, $L=2.2$ mm, and $h_m=1.29$ mm.

Since the physical variables change significantly near the free surface, the dense mesh near the free interface should be maintained during droplet motion to insure the accuracy of the numerical simulation. Therefore, the arbitrary Lagrangian Eulerian technique is used in the present study to ensure that the fine mesh moves simultaneously with the interface. In this method, the reference coordinate (X, Z) moves with the droplet on the spatial coordinate (x, z) , which is a fixed coordinate with $x=x(X, Z, t)$ and $z=z(X, Z, t)$. The relationship of the two coordinates can be shown as follows:

$$\left. \frac{\partial \Psi}{\partial t} \right|_{x,z} = \left. \frac{\partial \Psi}{\partial t} \right|_{X,Z} - \left. \frac{\partial \Psi}{\partial x} \right|_{X,Z} \dot{x}_{\text{mesh}} - \left. \frac{\partial \Psi}{\partial z} \right|_{X,Z} \dot{z}_{\text{mesh}}, \quad (25)$$

where $(\dot{x}_{\text{mesh}}, \dot{z}_{\text{mesh}})$ is the mesh velocity and Ψ is a dependent variable.

The governing equations with the correlative boundary and initial conditions are solved utilizing the finite element method developed by COMSOL MULTIPHYSICS. The second-order Lagrange triangular elements are employed. The error is controlled by adjusting the relative tolerance A_r and the absolute tolerance A_a so that the iteration step is only accepted when

$$\left(\frac{1}{N} \sum_j \left(\frac{|E_j|}{A_{aj} + A_r |S_j|} \right)^2 \right)^{1/2} < 1, \quad (26)$$

where $\mathbf{S}$ is the solution vector corresponding to the solution at a certain time step; E is the error estimate of the solver in $\mathbf{S}$; and N is the number of degrees of freedom. A higher density of element is chosen in the regions inside and around the droplet to guarantee the correction of this problem. The dependency of the element number on the simulation result is confirmed to ensure the accuracy of the solution. A typical mesh for the case of a droplet with a static contact angle of $\varphi=60^\circ$, footprint radius of $L=2.2$ mm, and maximum height of $h_m=1.29$ mm, is presented in Fig. 3.

IV. RESULTS AND DISCUSSION

In the present study, the influences of the slippage on the thermocapillary migration of a liquid droplet on a horizontal solid surface are investigated by using the Navier slip condition on the liquid/solid interface. The temperature gradient of the solid surface is fixed to be $G=3.12$ K/mm. The transient velocity of the droplet during the migration motion can be determined from the variation of displacement of the droplet center with time. Figure 4 shows the transient velocity of a droplet with $L=2.6$ mm and $\text{SCA}=41^\circ$ for different slip lengths. It can be seen that at the beginning, the droplet accelerates, then, approaches a constant velocity. When the slip length increases, the droplet reaches a quasisteady state earlier with a higher velocity. In our previous work,³² we found that two vortices of unequal size, induced by thermocapillary convection would appear inside the droplet. The net momentum of the thermocapillary convection vortices inside the droplet drives the droplet motion. The position of the stagnation point at the free surface represents the asymmetry of the vortices inside the droplet and hence, the migration behavior of the droplet is significantly dependent on the location of this point. It

FIG. 4. Migration velocity of a droplet with a footprint radius $L = 2.6$ mm, static contact angle $\text{SCA} = 41^\circ$, and temperature gradient $G = 3.12$ K/mm for different slip lengths.

can be seen in Fig. 5 that with a larger slip length, the stagnation point moves closer to the cold side and the asymmetry of the two vortices inside droplet increases. Therefore, the net momentum of the two vortices would be higher for the larger slip length. This results in a higher displacement [Fig. 5(b)] and faster migration speed of the droplet (Fig. 4). In addition, the smaller the value of the slip coefficient, the longer the time the droplet needs to reach a quasi steady state (Fig. 4). It is clear that the transport of the liquid droplet on the solid surface is easier for the larger slip length.

Figure 5(b) shows that the displacement of the advancing edge is always smaller than that of the receding edge. This implies that in the accelerating stage, the velocity of movement of the front edge is always less than at the rear one. As a consequence, the DCAs are always

FIG. 5. (a) Flow field and (b) isotherms inside the squalane droplet with a footprint radius $L = 2.6$ mm, static contact angle $\text{SCA} = 41^\circ$, and temperature gradient $G = 3.12$ K/mm for different slip lengths. The red arrows denote the stagnation point at the droplet-air interface. The black and red curves denote the droplet-air interface at $t = 0$ and $t = 2.5$ s, respectively.

FIG. 6. CAH of the squalane droplet with $L = 2.6$ mm, $SCA = 41^\circ$, and $G = 3.12$ K/mm for different slip lengths.

greater at the front edge than at the rear. In the quasisteady state, the entire droplet moves with the same velocity. In this stage, the DCAs maintain constant values at both edges with the value at the front edge being greater than at the rear. This trend agrees well with the experimental results.³¹ The difference between the two edges also varies for different slip lengths. Figure 6 displays the variation in CAH with the slip coefficient in the quasisteady state. The result shows that the CAH increases as the slip length increases.

Figure 7 illustrates the flow field and isotherms inside the droplets for different SCA with $G = 3.12$ K/mm and $b = 5$ nm, but the same volume. When the SCA is larger, the stagnation point is closer to the advancing edge. This means that the net thermocapillary moment induced by the asymmetry of the two vortices inside the droplet increases for a higher SCA. Figure 8 shows the effect of slip length on the quasi steady velocity for droplets, which have the same volume but different SCAs. It can be seen that for the same slip length, a larger SCA leads to higher velocity. This means that the droplet would move faster on a surface with less wettability. When the SCA is less than 90° , the capillary pressure pushes the fluid flow from the advancing side to the receding one, which creates a force of resistance against the movement of the droplet.³² The resistance from the capillary force diminishes for higher SCA. The other force resisting the forward motion of the droplet is the friction force between the droplet and the substrate. The force of friction on the larger SCA droplet is smaller, due to the smaller contact surface area between the droplet and the substrate. This can explain why the quasisteady velocity of the higher SCA droplet is higher. The rate of increase in the quasisteady velocity in relation to the slip length is higher for a higher SCA.

The effect of slip behavior on the droplet speed is also examined theoretically. Figure 9 presents the variation of droplet speed with of the slip length for different SCAs as calculated from Eqs. (18)–(23), presented in Sec. III A. It can be seen that the droplet speed is enhanced with increase of SCA or slip length. This tendency agrees well with the numerical results. Figure 10 presents the migration velocity of the droplet with $SCA = 41^\circ$ and $L = 2.6$ mm for different slip coefficients, obtained by the analytical method presented in Sec. III A for two cases: (1) CAH is fixed at 1° (called ANA-1), which is similar to the previous work,²⁸ and (2) the DCA and CAH obtained from the present numerical simulations are used (called ANA-2). These analytical results are also compared to those from the present numerical simulations (labeled numerical in Fig. 10). From this figure, we can see that the trend of the variation in migration velocity with the slip length predicted by ANA-2 is different from that predicted by ANA-1, while it is similar to the numerical one. The difference between ANA-1 and ANA-2 is

FIG. 7. (a) Flow field and (b) isotherms inside squalane droplets with $G = 3.12$ K/mm and $b = 5$ nm for different SCAs but all have the same volume. The red arrows denote the stagnation point at the droplet-air interface.

FIG. 8. Quasisteady migration velocity of droplets with $G = 3.12$ K/mm for different SCAs and slip lengths, but the same volume.

FIG. 9. Migration velocity of the squalane droplet with the same volume, $G = 3.6$ K/mm, and $CAH = 0.8^\circ$ versus the SCA for different slip lengths.

evidence that if the droplet velocity is calculated without considering the dependence of the slip length on the CAH and the DCA (case ANA-1), which might lead to inappropriate results. In addition, there might other factors that lead to a slight difference between the ANA-2 results and the numerical investigation results. Unlike the numerical computations, the effect of the nonlinear terms in the Navier-Stokes equations and energy equation is neglected in the analytical approach. Furthermore, it is assumed that the ratio between the height profiles and footprint

FIG. 10. Migration velocity of the squalane droplet with $L = 2.6$ mm, $SCA = 41^\circ$, and $G = 3.12$ K/mm for different slip lengths: comparison between analytical and numerical methods.

FIG. 11. Migration velocity of the squalane droplet with $L = 2.6$ mm, $G = 3.6$ K/mm, and $b = 3$ nm for different static contact angles in both ANA-2 and numerical solutions.

of the droplet is very small and the temperature at the air-droplet interface is the same as that imposed on the solid wall. Based on this assumption, only one vortex will appear inside the droplet. This could be the reason for the faster migration velocity predicted by ANA-2 than that obtained with the numerical investigation, since the net momentum inside the droplet might be higher. Figure 11 shows the difference in droplet migration velocity between the ANA-2 and the numerical investigation for different SCAs. It can be seen that the difference is more significant when the order of SCA is larger. This indicates that the velocity predicted by the analytical model might be closer to the numerical results when the droplet has a small SCA ($< 50^\circ$). It is noted that the migration velocity for ANA-2 is calculated with the DCA and CAH values obtained from numerical solutions. The analytical solution is based on the assumption that the ratio between the height profiles and the footprint of the droplet is very small. Therefore, when the SCA is larger, the ratio is higher and the errors in the analytical solutions might be larger.

V. CONCLUSIONS

It is difficult to determine the thermocapillary migration of a small droplet with a different slip length due to difficulties such as fabricating nanometer high solid walls with different slip lengths, imposing a sufficiently high temperature gradient on the sub-millimeter scale, measuring the temperature and velocity distribution in a sub-millimeter system, and so on. However, such investigations are easier with a proper numerical simulation. In the present study, the effects of the slip behavior on the thermocapillary migration of a small liquid droplet are investigated theoretically and numerically. The numerical results clearly indicate that the slip coefficient, the dynamic contact angles, the contact angle hysteresis, and the migration velocity of the droplet are strictly correlated. The enhancement of the slip length leads to an increase in the droplet speed due to the enhancement of the net momentum of the thermocapillary convection vortices inside the droplet. When the slip coefficient is smaller, it takes a longer time for the droplet to reach a quasi steady state. The larger contact angle leads the higher migration velocity. In addition, the increase in the rate of migration velocity to the slip length is higher for a higher contact angle. In the analytical approach developed based on the lubrication approximation, the velocity is obtained from the known static and dynamic contact angles. This might lead to an inappropriate migration velocity because the dynamic contact angle and slip

condition cannot be considered independently. When the dynamic contact angle obtained from the numerical solution is used in the analytical approach, there is a good agreement between the analytical and numerical results when the static contact angle is less than 50° .

ACKNOWLEDGMENTS

The authors appreciate the financial support received from the National Science Council, Taiwan (R.O.C) under Contract Nos. NSC 95-2221-E-008-138-MY3 and NSC-100-2922-I-008-004.

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