A distance-based dynamical transition analysis of time series signals and application to biological systems

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Abstract This study demonstrates an application of distance-based numerical measures to the phase space of time series signals, in order to obtain a temporal analysis of complex dynamical systems. This method is capable of detecting alterations appearing in the characters of the deterministic dynamical systems and provides a simple tool for the real-time analysis of time series data obtained from a complex dynamical system even with black box functionality. The study presents a possible application of the method in the dynamical transition analysis of real EEG records from epilepsy patients.

Keywords Dynamical system analysis · Chaos · Epilepsy seizure prediction from EEG signal

1 Introduction

The behavioral model of a dynamical system is well characterized by state equations, which define the transitions of the system states in a state space. Therefore, system behavior depends on the state transitions; a transient inspection of the shape of the state transition trajectories yields some useful information about the momentary behavior of a dynamical system. In this way, a prospective irregular dynamical transition or a variation in the system's character can be estimated by means of the transient analysis carried out on state transition trajectories. A real-time analysis of the state trajectories of a deterministic dynamical system might be utilized to detect an early indication of a varying trend in system behavior.

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The most popular methods for the analysis of a dynamical system are Kolmogorov entropy [1], Lyapunov characteristic exponents [2–5], and correlation dimension [6]. The correlation dimension mainly gives an estimate of the complexity of a dynamical system, while the Kolmogorov entropy and the Lyapunov exponents are utilized in the analysis of the degree of chaotic behavior.

Lyapunov exponents, as a measure of the chaoticity of a dynamical system, calculate how distance between two nearby orbits grows exponentially. This measure has shown to be a useful dynamical diagnostic for chaotic systems and its calculation can be described for experimental time series [5]. Lyapunov exponents were also implemented in the signal processing of nonlinear dynamicals and chaos. One of the famous applications of Lyapunov exponents in signal analysis was in epilepsy seizure prediction from analyses of the electroencephalogram (EEG) and electrocorticogram (ECoG) signals [6–9]. Seizure prediction from the Lyapunov exponents of EEGs is based on the assumption that a multi-channel EEG record from a healthy brain shows the outputs of a complex dynamical system in chaos. An epilepsy seizure was reported to decrease the degrees of chaos in the brain. Iasemidis et al. discovered a preictal transition that precedes seizures by periods of minutes to hours [6, 7]. This preictal dynamical transition was reported to be characterized by a progressive convergence of dynamical measures, such as short-term maximum Lyapunov exponents (STLmax), at specific anatomical areas of patients' brains. A long-term, prospective, real-time, seizure prediction algorithm, based on convergence of STLmax measures of EEG channels, was developed, and tested on the long-term and multi-seizure EEG records of two epilepsy patients [8].

Lai et al. tested the performance of the Lyapunov exponents computed from a time series to predict or detect critical events in a dynamical system [10]. In their testing on the Ikeda-Hammel-Jones-Moloney (IHJM) map, which was designed to provide a deterministic chaotic system model with controllable parameter variations to mimic the non-stationary ECoG data, the finite-time statistical fluctuations and noise in the signals hindered the predictive power of the Lyapunov exponents computed from a time series. This sensitivity of Lyapunov exponents to statistical fluctuations and noise mainly comes from the strong dependence of Lyapunov exponent calculations on the initial distance between two nearby neighbor orbits. Lai et al. reported from their controlled tests that Lyapunov exponents from a time series were capable of detecting simple, local bifurcations in the presence of very low noise. However, they pointed out that Lyapunov exponents were arguably disadvantageous for real detection applications due to the inevitable presence of noise, the statistical fluctuations of realistic signals, and the high computational complexity of Lyapunov exponents [11]. It was generally accepted that standalone use of Lyapunov exponent analysis of an EEG channel was not efficient as a reliable epilepsy seizure predictor; however, spatiotemporal evaluation of Lyapunov exponents from multi-channel EEG signals could be useful in a reliable epilepsy seizure prediction application [8, 11].

This paper is devoted to applying the state transition distance metrics to phase space of time series signals, which is obtained from a deterministic, complex, dynamical system. The proposed method computes the average distance of all states to the first state in a windowed orbit of the state space. Referred to as short-time average distance (STAD), STAD is beneficial for assessment of whether the trajectory of the state transition is enwrapping or escaping. An escaping trajectory can be a sign of chaoticity or instability, whereas an enwrapping trajectory can be an indication of the dissipative or oscillatory nature of a deterministic dynamical system. Figure 1a illustrates a representation of the two types of trajectories in a two-dimensional state space. State transition velocity (STV) of a trajectory is calculated by the average distance of consecutive state transitions in a time window. STV

Fig. 1 **a** Illustration of trajectory curvatures in two-dimensional state space $X = [x_1 \ x_2]$. **b** Trajectory 1 is an escaping type trajectory with $D_E = 0.36$, $V_E = 0.0028$ and $D_{EN} = 128.67$. Trajectory 2 is a slightly enwrapping type trajectory with $D_E = 0.31$, $V_E = 0.0028$ and $D_{EN} = 109.84$. Trajectory 3 is an enwrapping type trajectory with $D_E = 0.23$, $V_E = 0.0053$ and $D_{EN} = 43.51$ (length of trajectory and size of time window ($w = 500$) are equal)

of a time window is used to normalize STAD in order to make it distance independent; the normalized STAD is referred to as normalized average distance (NAD) in this paper. The NAD measure is responsive to alterations in the curvature of state transition trajectories.

In the following sections, the STAD, STV, and NAD are arithmetically derived for a multidimensional phase space of a time series signal. What it yields for dynamical systems is discussed in terms of the results obtained for synthetic test data from the logistic equation, which is used to model the growth of an animal population. It is also discussed using results obtained for epileptic EEG records, which are the time series signals from the electrical activities of a real neurobiological system.

2 Methodology

2.1 Mathematical foundation for the normalized average distance calculation

Let $X(t)$ express a state vector in a p -dimensional state space for a given time t as:

$$X(t) = [x_1 \ x_2 \ \dots \ x_p], \quad (1)$$

where, $x_1, x_2, x_3, \dots, x_p$ are components of the state vector. The distance metric between two states in the space is written as the following:

$$d(X(t_1), X(t_2)) = |X(t_1) - X(t_2)| = \left(\sum_{i=1}^p (x_i(t_1) - x_i(t_2))^2 \right)^{\frac{1}{2}}. \quad (2)$$

The short-time average distance D_E for a moving window with size w as in Fig. 2a is expressed as:

$$D_E(t) = \frac{1}{w} \sum_{j=1}^w d(X(t), X(t+j)). \quad (3)$$

When state space has a large number of states or the width of time window (w) is quite large, it is convenient to express D_E in the logarithmic scale as the following:

$$D_{EL}(t) = \log_{10} D_E(t). \quad (4)$$

The average state transition velocity on a trajectory of state space can be defined, depending on the distance between state transitions, as the following:

$$V_E(t) = \frac{1}{w} \cdot \sum_{j=1}^w d(X(t+j), X(t+j-1)). \quad (5)$$

Then, the normalized average distance is expressed as:

$$D_{EN}(t) = \frac{D_E(t)}{V_E(t)}. \quad (6)$$

Larger values of $V_E(t)$ can be a sign of chaotic or unstable behavior in the system; however, this does not imply anything about the geometry of the trajectory. $D_E(t)$ strongly depends on the shape of the trajectory at a constant state transition velocity. In the case of a varying state transition velocity on the trajectory, the proposed normalized average distance $D_{EN}(t)$ can provide an indication about the curvature of the state transition trajectory, and, correspondingly, about the character of the dynamical system. Figure 1b illustrates calculated measures of $D_E(t)$, $V_E(t)$ and $D_{EN}(t)$ for the different trajectory forms.

In practice, a discrete time series signal or a data sequence can be obtained from the observable states of a complex dynamical system, which may be a biological system, a social system, or a physical system on the micro or macro scale. When the state equation of a system is not definite for an observer, this system is supposed to have black box functionality for the observer. In such a dynamical system with black box functionality, any observable parameters of the system can be used to collect data to form a time series signal. Then, a phase space can be built using the delayed samples of the time series signal as represented in Fig. 2b.

Fig. 2 **a** An orbit in the state space and distance vectors for states on a trajectory. **b** A representation of the complex dynamical system and the corresponding phase space $X_n = [x_1 \ x_2 \ x_3 \ x_p]$ from time-delayed samples of the time series signal $u(n)$

Denote this time series signal by $u(n)$. A state vector of the delayed phase space is defined as the following [12, 13]:

$$X_u(n) = [u(n) \quad u(n - \tau) \quad u(n - 2\tau) \quad \dots \quad u(n - (p - 1)\tau)] \quad (7)$$

Here, τ is the time lag, p is the dimension of the phase space, and n is the discrete time index. D_E , given by Eq. (3), can be adopted for the phase space as follows:

$$D_{EU}(n) = \frac{1}{w} \sum_{j=1}^w \left(\sum_{i=0}^{p-1} (u(n - i\tau) - u(n + j - i\tau))^2 \right)^{\frac{1}{2}}. \quad (8)$$

The speed of the state transition for the phase state can be expressed, according to Eq. (5), as:

$$V_{EU}(n) = \frac{1}{w} \cdot \sum_{j=1}^w \left(\sum_{i=0}^{p-1} (u(n + j - i\tau) - u(n + j - 1 - i\tau))^2 \right)^{\frac{1}{2}}. \quad (9)$$

Then, the normalized average distance for the phase space can be written as:

$$D_{ENU}(n) = \frac{D_{EU}(n)}{V_{EU}(n)}. \quad (10)$$

2.2 A discussion on results obtained for biological dynamical systems

The logistic equation is used for approximating the evolution of an animal population over time [14] and it provides controllable biological test data with an adjustable chaoticity degrees. Growth of the population was modeled by the logistic difference equation, which is commonly expressed by a discrete form as:

$$A(n + 1) = r \cdot A(n) \cdot (1 - A(n)). \quad (11)$$

$A(n)$ is the number of members in the population this year, $A(n + 1)$ is an estimation of the member count for next year. The parameter r is defined as the growth rate, and it determines the character of the system. When r is greater than 3.56994..., it describes a chaotic system. Figure 3 illustrates the results obtained for the logistic difference equation with various r values. In the figure, when the logistic equation defines a chaotic system by setting $r > 3.57$, the $D_{ENU}(n)$ responds with a clear and consistent rise. This demonstrates that the method is capable of detecting chaotic behavior in the case of a deterministic dynamical system. In the case of step-wise increase of r as the population is growing, responses of the population, responses of the logarithmic STAD, the logarithmic STV, NAD, and the windowed variance (WV) are illustrated in Fig. 3d. In the non-chaotic case, where $r \leq 3.57$, WV, logarithmic STAD, and logarithmic STV exhibited parallel responses to changes in r . When the system behaves chaotically, $r > 3.57$, discrimination among the responses of the methods becomes more apparent. In the chaotic case, WV and logarithmic STV exhibited similar responses to changes in r , since both measures are highly dependent on instantaneous deviations in the amplitude of the signal. However, when $r = 3.7$, the logarithmic STAD did not decrease as did WV and logarithmic STD, because the STAD and NAD measures predominantly consider the form of the state trajectory. The NAD, in the non-chaotic case, did not give a significant response to the step-rise in r , but in the chaotic case, NAD can give a clear

Fig. 3 Logarithmic STAD in **a**, logarithmic STV in **b** and NAD in **c** for the logistic difference equation with the parameters of $A(0) = 0.1$ and various values of r . Response of methods to stepwise temporal variation in the values of r in **d** (other parameters are $w = 500$, $\tau = 3$, $p = 100$, window sampling period $T_s = 300$)

response to step-wise increases in r . This implies that NAD is largely sensitive to changes in the degrees of chaoticity.

Figures 4 and 5 present the results obtained for the two real EEG records, which contain seizure records of the epilepsy patients (A and B). The EEG records seen in Figs. 4a and 5a are the sum of 19 EEG channels. Analyzing the sum of all the EEG channels implies

Fig. 4 **a** Sum of the single-channel EEG records from patient A. **b** WV of cumulative EEG in **a**. **c** STAD in logarithmic scale. **d** STV in logarithmic scale. **e** NAD (parameter settings: $w = 10000$, $\tau = 1$, $p = 1000$)

consideration of the whole brain as a complete dynamical system. EEG data was collected by a portable device from the patients, who continued their daily activities during recording. In those figures, dip trends occurred in the NAD plots several minutes before the onset of the generalized seizures of patients A and B. Here, the NAD indicates the existence of a strengthening, enwrapping tendency in the trajectory of the phase space at the onset of the seizures and it implies a decrease in the magnitude of chaotic oscillations, which is also

Fig. 5 **a** Sum of the single-channel EEG records from patient B. **b** WV of cumulative EEG in **a**. **c** STAD in logarithmic scale. **d** STV in logarithmic scale. **e** NAD (parameter Settings: $w = 10000$, $\tau = 1$, $p = 1000$)

observed as a decreasing trend in WV and the logarithmic STV plots before the seizures in Figs. 4 and 5. This decreasing trend in preictal dynamical transitions was mainly considered as the reducing of the chaoticity degrees of the neural system before an epilepsy seizure [6–8]. One possible reason for the decrease in the magnitude of chaotic oscillation observed in the preictal state might be degradation in neural electrical activity because of a loss of synchrony among the neurons. In a previous work, the effect of loss of synchrony in a preictal state was explained by Mormann et al. as “the neural system might also provide an

‘idle’ population of neurons, which may be more easily recruited into the epileptic process” [15]. Hence, increasing the portion of the idle population in a neural system easily leads to the decreasing magnitude of the chaotic oscillation observed in EEG signals.

The Lyapunov exponent calculation is carried out using the distances between states of two nearby orbits. The result of the STLmax calculation is highly dependent on selecting an appropriate nearest-neighbor orbit in the state space. It makes the method difficult to implement in real-time applications for two reasons it requires a rigorous backward search to locate the nearest state of neighbor orbits, and noise in time series signals can easily result in failure to select the appropriate nearest neighbors. This makes the Lyapunov exponent calculation vulnerable to noise. In the NAD method, the distance calculations are carried out among the states in a time-windowed fraction of the same orbit; there is no need for backward searching to find an appropriate state of a neighbor orbit, as in the calculation of Lyapunov exponents. This advantageous makes the NAD method easy to implement in real-time applications.

The robustness of the method against noise is a key feature for the applicability of this method to realistic signals [11]. Figure 6 demonstrates the effects of noise on STAD, STV, and NAD at various signal-to-noise ratio (SNR) levels of the logistic difference equation. The SNR is defined as $20 \log_{10} \frac{A}{N_s}$. In order to manage noise insertion into the test system, an additive pseudo-random noise signal with uniform distribution $N_s(n)$ was added to the next state of the logistic difference equation, to give $A(n+1) = r \cdot A(n) \cdot (1 - A(n)) + N_s(n)$. The noise level has an influence on the consistency of the WV, whereas the NAD measure shows slighter deviation against various SNR levels of the noised logistic difference equation (see Table 1 for a quantitative comparison of the largest deviations in measures under varying SNR levels). This advantage makes the proposed method a potential candidate for real-world applications with low levels of noise background. In a recent work, Li et al. reported that a judgment distance, defined as rate of maximum state transition distance to total number of nodes in a multidimensional phase space of a time series signal, can distinguish a chaotic series from a random series for a signal-to-noise ratio (SNR) over 50 ($SNR \geq 50$) [16].

As a marker of seizure onset, consecutive absolute minima and absolute maxima patterns were observed in STAD and STV plots. The probability of an absolute minimum followed by an absolute maximum was about 0.5 and the probability of the consecutive absolute minima-maxima patterns coinciding with ictal events in the plots of patients was 1.0 based on 440 min of EEG data, recorded in three separate sessions. Although these statistics seem to be significant for detection of seizures in EEG records, an advanced algorithm based on the evaluation of several measures will be needed for more reliable seizure prediction.

Fig. 6 Logarithmic STAD, logarithmic STV, NAD, and $10 \times WV$ are plotted versus various SNR levels of noise. STAD, STV, and NAD are more robust against noise than WV

Table 1 The largest deviation of measures in the SNR range of [28,42] dB

	WV	$\log_{10} STAD$	$\log_{10} STV$	$\log_{10} NAD$
Max. deviation (Δx)	0.011	0.032	0.025	0.016
Mean value (x)	0.074	0.585	0.679	0.807
Normalized deviation ($\Delta x/x$)	14.8%	5.5%	3.7%	2.0%

The authors observed two drawbacks of the methods, which should be considered in the application development. One is that the proposed measures may not discriminate instability from the chaotic behavior of the system. The other is that the method does not yield an exact value for the degrees of chaoticity; in fact, it can yield an output that is sensitive to variations in the chaoticity degrees of a stable dynamical system. However, from an application point of view, detecting alterations in the chaoticity degrees is helpful in predicting changes in the system's character.

3 Conclusions

The paper introduced the state transition distance-based measures indicating the wrapping of orbits in a state space and proposed them as a time series signal processing method to be utilized in the analysis of signals coming from biological, dynamical systems. The methods are easy to implement and yet they were effective in detecting alterations in the nature of a complex dynamical system. In the controlled tests on the logistic equation, the proposed measures were sensitive to alterations in the degrees of chaoticity and they were significantly consistent in the case of the existence of a low level of uncertainty in the signal, such as the presence of noise. For a realistic evaluation, the proposed methods were employed in the analysis of epileptic EEG signals; they were shown to be capable of responding to a preictal dynamical transition several minutes earlier than the onset of generalized epilepsy seizures. However, it should be noted that a more advanced prediction algorithm on the bases of the proposed measures should be developed for an accurate and reliable prediction of epilepsy seizures.

The study demonstrated that state transition distance-based analysis can be utilized to detect a change in the character of a dynamical system with black box functionality. Specifically, this study analyzed a single output system by means of constructing a multi-dimensional phase space. However, as a future work, a multi-dimensional state space, constructed by multi-channel time series signals, could be tested in the detection of system anomalies.

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