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Stability of an Aqueous Quadrupole Micro-Trap

Jae Hyun Park and Predrag S. Krstić*

Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831

Abstract

Recently demonstrated functionality of an aqueous quadrupole micro- or nano-trap opens a new avenue for applications of the Paul traps, like is confinement of a charged biomolecule which requires water environment for its chemical stability. Besides strong viscosity forces, motion of a charged particle in the aqueous trap is subject to dielectrophoretic and electrophoretic forces. In this study, we describe the general conditions for stability of a charged particle in an aqueous quadrupole trap. We find that for the typical micro-trap parameters, effects of both dielectrophoresis and electrophoresis significantly influence the trap stability. In particular, the aqueous quadrupole trap could play a role of a synthetic virtual nanopore for the 3rd generation of DNA sequencing technology.

1. Introduction

Recently, Guan et al. [1, 2] have showed the feasibility of an *aqueous*, radio-frequency (rf, AC) quadrupole micro-trap for localization and control of charged micro-particles. This is a remarkable extension of the applicability of the quadrupole Paul micro-trap [3, 4] in addition to its conventional applications in mass spectrometry [5], quantum information processing [6, 7], microdynamical sensors [8], etc. Considering that typical biomolecules (e.g. proteins) are stable in water and have a considerable amount of charge [9, 10], this novel aqueous quadrupole micro-trap is an excellent candidate for localization and manipulation of their motion in a confinement region of space [1, 2, 11, 12].

The aqueous quadrupole trap is particularly interesting for the recent large scale scientific endeavor of the 3rd generation methods for the DNA sequencing. Namely, direct DNA sequencing using nanopore [13 – 15] and tunnelling recognition [16 – 18] is potentially orders of magnitude faster and proportionally less costly than the existing Sanger sequencing [19] and the Next-Generation-Sequencing (NGS) [20]. One of the key issues in the direct sequencing technologies is isolation, trapping, localization and control of the DNA motion. The aqueous quadrupole trap is expected to provide the ultra-precise control and localization of biomolecules, the DNA segments in particular, with relaxed fabrication demands to tens of nanometers.

The key property of a Paul trap is the region of its stability, defined by the set of the system parameters for which a charged particle can stay confined for a sufficiently long time to perform a desired trapping function. The stability condition for a spherical homogenous charged particle in a conventional Paul trap in vacuum or in a buffer gas at a low pressure have been studied in details [21] and are determined by the dimensionless “ a ” and “ q ”

*To whom any correspondence should be addressed krsticp@ornl.gov .

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parameters representing quadrupole electrophoretic (EP) forces, $a = \frac{2QU_{dc}}{M\Omega^2 r_0^2}$, $q = \frac{2QU_{ac}}{M\Omega^2 r_0^2}$, where M and Q are the mass and charge of the particle, U_{dc} and U_{ac} are the DC and AC voltage amplitudes, respectively, Ω is the driving frequency of AC input ($U_{ac} \cos(\Omega t)$), and r_0 is the half of cross-size of the trap (see figure 1). In a viscous environment, the stability of a Paul trap is also significantly influenced by the viscous drag of the surrounding medium

expressed by “ b ”- parameter ($b = \frac{2\zeta}{M\Omega}$) where ζ is the friction coefficient of a non-slip spherical particle in Stokes’ drag, $\zeta = 6\pi\eta r_p$, η is the viscosity of medium, and r_p is the

radius of the particle. However, if the complex permittivities $\tilde{\epsilon} = \epsilon_0\epsilon - j\frac{\sigma}{\Omega}$ (ϵ is relative dielectric constant and σ is electric conductivity) of the medium and the particle material are very different, as is a case of a particle in a water environment, the dielectrophoretic forces, although independent on the particle charge, may significantly influence the motion and stability of the particle in the trap due to their polarization nature. The influence of this additional feature of the aqueous Paul trap to the trap stability is the main subject of the present study.

The motion of a “dynamically stable” charged particle in quadrupole trap can be described by the time-dependent amplitude of secular motion on which are superimposed micro-oscillations. The amplitude of the secular motion does not increase in the long-time limit for the dynamically stable particle. Furthermore, in the strongly viscous environment the amplitude of the stable secular motion decreases exponentially with time. This was extensively discussed by Hasegawa and Uehara [22], who derived the closed-form analytical expressions for the stability in a viscous Paul trap, connecting only electrophoretic a and q and the viscous b parameter, without inclusion of the effects of dielectrophoretic forces. They started from the general solution w of damped Mathieu equation in form as $w = A \exp(\mu t) \phi(t) + B \exp(-\mu t) \phi(-t)$ where t is time, A and B are arbitrary constant while the exponent μ is a function of $a - b^2/4$ and q . They assumed $\phi(t)$ is a periodic function with period of π . At the stability border the real part of the exponent μ is zero, and Hasegawa and Uehara have derived the closed form analytical expressions which can be solved numerically with a relatively simple code to establish the stability border.

The dielectrophoretic (DEP) forces on a particle in a non-uniform electric field can be described by

$$\mathbf{F}_{DEP}(t) = (\mathbf{m}(t) \cdot \nabla) \mathbf{E}(t) \quad (1)$$

where $\mathbf{E}(t)$ and $\mathbf{m}(t)$ are AC electric field and (induced) dipole moment of the particle, respectively [23 – 25]. Even for an electrically neutral particle, the induced dipole moment can appear as a result of the difference between polarizability of the medium and of the particle in the AC electric field. The above formula for $\mathbf{F}(t)$ is derived from the Maxwell stress tensor (MST) with condition that the spatial change of electric field inside the particle can be presented by a relation linear with radial position [26]. Although the general formulation of DEP force emerges from the MST approach [27], the above simple dipole approximation is valid when the particle size is small enough in comparison to the characteristic length of change of the applied AC electric field amplitude [28].

With this condition for the dipole approximation fulfilled, the motion of the particle is determined by the real part of the frequency-dependent Clausius-Mossotti (CM) factor,

$$f_{CM} = \frac{\frac{\epsilon_p}{\epsilon_m} - \frac{\epsilon_m}{\epsilon_m}}{\frac{\epsilon_p}{\epsilon_m} + 2\frac{\epsilon_m}{\epsilon_m}} \quad (\text{figure S1 in supplementary material}).$$

The subscripts p and m are for particle

and medium, respectively. If the medium polarizes less than the particle, then $\text{Re}(f_{\text{CM}})$ becomes positive and the particle experiences so-called positive dielectrophoresis (pDEP) and moves toward the high electric field region (i.e. electrodes). However, if the medium polarizes more than the particle, i.e. when $\text{Re}(f_{\text{CM}})$ is negative, the particle experiences negative dielectrophoresis (nDEP) and moves toward the low electric field region (i.e. away from the electrode, toward the trap confinement axis) [29].

In this paper, we determine the stability of a charged particle in an aqueous quadrupole Paul trap by influence of simultaneous presence of both DEP and EP forces. In Section 2, we examine the Langevin-type governing equation for the particle in the trap. In Section 3.1 we focus on the EP-only case, and illustrate the corresponding a -q and b -q stability diagrams. In Section 3.2 we investigate the influence of the DEP on the particle trajectory, and construct the modified a -q and b -q stability diagrams in presence of both EP and nDEP or pDEP forces. Finally, we summarize our study and give our conclusions in Section 4.

2. Equation of Motion: Electrophoretic and Dielectrophoretic Forces

The Langevin-type equation of motion (EOM) of a spherical charged particle in a two-dimensional (linear) aqueous quadrupole trap (parameters defined in the Introduction, and in figure 1), in presence of the DEP force, $\mathbf{F}_{\text{DEP}}$, has the form

$$M \frac{d^2 \mathbf{r}}{dt^2} = -\xi \frac{d\mathbf{r}}{dt} + Q\mathbf{E} + \mathbf{F}_{\text{DEP}} \quad (2)$$

where t is time and $\mathbf{r}$ is the displacement vector of the particle center of mass, $\mathbf{r} = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y$. The first and the second term on the right-hand-side (RHS) of equation (2) are the damping and the EP forces. The EP force, $Q\mathbf{E}$, is due to the gradient of the two-dimensional

quadrupole electric potential, $\Phi = (U_{dc} - U_{ac} \cos \Omega t) \frac{x^2 - y^2}{2r_0^2}$:

$$\mathbf{E} = -\vec{\nabla} \Phi = -\frac{U_{dc} - U_{ac} \cos(\Omega t)}{r_0^2} (x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y). \quad (3)$$

Throughout this study, we assume that the trap size is sufficiently larger than the thermal positional fluctuations of a trapped particle in the long-time limit, and therefore the stability of the particle is determined by its mean, secular motion [30]. Thus, equation (2) describes instantaneous motion of the particle without taking into account random fluctuations. In aqueous quadrupole trap, however, the random motion could be significant because the high viscosity of medium produces the large fluctuations according to the fluctuation-dissipation theorem [31]. If the random fluctuations are larger than the trap size then the particle will not be stable even though the mean motion itself converges. The issues of random fluctuations in an aqueous trap are studied elsewhere (Park et al, submitted to *Small*).

The rigorous expression for the DEP forces acting on a particle contains integration of the MST over the particle surface [26], and is discussed in detail in the supplementary material (SI 2). By assuming that the non-uniformity of electric field inside the particle is small, $\mathbf{F}_{\text{DEP}}$ is reduced to equation (1). For the quadrupole electric field $\mathbf{E}$ of equation (3), the induced dipole moment of the particle, $\mathbf{m}(t)$, is obtained as [32, 33]

$$\mathbf{m}(t) = -\frac{4\pi\epsilon_m\epsilon_0 r_p^3}{r_0^2} [\text{Re}(f_{\text{CM}})(U_{dc} - U_{ac} \cos \Omega t) - \text{Im}(f_{\text{CM}}) U_{ac} \sin \Omega t] (x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y). \quad (4)$$

The instantaneous dielectrophoretic force $\mathbf{F}_{DEP}(t)$ then follows the form

$$\begin{aligned}\mathbf{F}_{DEP} = & \alpha_m U_{ac}^2 \left[\frac{\text{Re}(f_{CM})}{2^p} (1 + \cos 2\Omega t) - \frac{\text{Im}(f_{CM})}{2} \sin 2\Omega t \right] (x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y) \\ & + \alpha_m U_{dc}^2 [\text{Re}(f_{CM})] (x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y) \\ & + \alpha_m U_{dc} U_{ac} [-2\text{Re}(f_{CM}) \cos \Omega t + \text{Im}(f_{CM}) \sin \Omega t] (x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y)\end{aligned}\quad (5)$$

with

$$\alpha_m = \frac{4\pi\epsilon_m\epsilon_0 r_p^3}{r_0^4} \quad (6)$$

The detailed discussion about the point-dipole approximation and the higher-order multipole expression of the DEP force is presented in the supplementary material (SI 2). The DEP force for an ellipsoidal particle in dipole approximation is shown in the supplementary material (SI 3). By inserting equations (3) and (5) into equation (2), we obtain the EOMs, which takes the non-dimensional form

$$\frac{d\mathbf{r}}{d\tau^2} + b \frac{d\mathbf{r}}{d\tau} + \left\{ A_0 + \sum_{i=1}^2 [A_{2i} \cos(2i\tau) + B_{2i} \sin(2i\tau)] \right\} (x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y) = 0 \quad (7)$$

where

$$\begin{aligned}A_0 &= a + \text{Re}(f_{CM}) \left(r_m^2 + \frac{s_m^2}{2} \right), \\ A_2 &= -2[q + r_m s_m \text{Re}(f_{CM})], \quad B_2 = r_m s_m \text{Im}(f_{CM}), \\ A_4 &= \frac{s_m^2}{2} \text{Re}(f_{CM}), \quad B_4 = -\frac{s_m^2}{2} \text{Im}(f_{CM}), \\ \tau &= \frac{\Omega}{2} t, \quad r_m = \frac{U_{dc}}{r_0^2 \Omega} \sqrt{\frac{12\epsilon_m \epsilon_0}{\rho_p}}, \quad s_m = \frac{U_{ac}}{r_0^2 \Omega} \sqrt{\frac{12\epsilon_m \epsilon_0}{\rho_p}}.\end{aligned}\quad (8)$$

With inclusion of the DEP forces two new dimensionless parameters, r_m and s_m , emerge in addition to the EP parameters a and q , and the viscosity b . The magnitudes of DEP force due to DC and AC inputs, are proportional to r_m^2 and s_m^2 , respectively. The product $r_m s_m$ characterizes the “mixed” magnitude of DEP force due to combination of DC and AC inputs. The subscript “m” indicates that r_m and s_m are related with the dielectric property of

medium (ϵ_m), ρ_p is the particle density, $\rho_p = \frac{M}{V_p}$, where V_p is the particle volume, $V_p = \frac{4}{3}\pi r_p^3$, for a spherical particle. The equation (7) consists of non-oscillating leading term A_0 and the

oscillating terms $\sum_{i=1}^2 [A_{2i} \cos(2i\tau) + B_{2i} \sin(2i\tau)]$. The real part of Clausius-Mossotti factor appears only in cosine terms while the imaginary part is present in sine terms.

3. Results and Discussion

The stability of a charged spherical particle in an aqueous quadrupole micro-trap is described by plotting the stability diagrams for trap parameters. For example, the a - q stability diagram illustrates the border between the stable and unstable regions for a given b parameter, while b - q diagram depicts the stability borders for a given parameter a . In this section, we review the stability diagrams with EP-only case (Section 3.1) and then, examine how those diagrams are modified by inclusion of DEP (Section 3.2).

3.1 Stability by Electrophoresis

The a - q stability diagrams for various b -values are shown in figure 2. The a - q diagram for a two-dimensional quadrupole trap in vacuum ($b = 0$) is presented in figure 2(a). The motion in each dimension, x and y , has different stability regions. The solid curves are the stability borders for the x -motion while the dashed lines are the borders for the y -motion. The stable region in each direction is the region between borders. Then, the final stable island in the two-dimensional motion is determined as the intersection of two stable regions in x - and y -directions. Figure 2(b) shows the growth of stable region (surrounded by the lines) with increase of the viscosity parameter b . Among the four lines bordering the stable island, the upper left (B_1) and lower right (B_3) boundaries are due to y -motion while the upper right (B_2) and lower left (B_4) ones are due to the x -motion. For $b = 4$, B_2 and B_4 are out of the considered domain in figure 2(b) ($-2 < a < 2$, $0 < q < 4$). Further details of stability diagrams with various b 's are shown in the supplementary material (SI 4).

Figure 3 shows the b - q stability diagram with zero a -value (no DC field). Unlike the a - q stability diagram in figure 2, the b - q diagrams from both x - and y - motions are identical, because the $\pm$ sign in q introduces only the phase difference in AC input. Obviously, even without a DC field, a considerable stable region in viscous medium can be achieved.

3.2 Particle Trajectories and Stability in Aqueous Quadrupole Trap with Dielectrophoresis

We illustrate changes of a particle trajectory when both EP and DEP are present, for a representative case $b = 4.0$, which is obtained by solving the y -component of equation (7) numerically using ODE solver (ode45) built in MATLAB [34]. Recalling the EP a - q stability diagram with $b = 4.0$ in figure 2(b), the stability of particle in $0 < a < 2$ and $0 < q < 4$ is determined by the y -motion (boundary B_1). Namely, since the a - q stability diagrams are symmetric with respect to $a = 0$ (see figure S2 in supplementary material), the y -stability border for $a > 0$ is enough to determine the dynamics stability of particle in two-dimensional trap. Following the experiments by Guan et al. [1, 2], the considered spherical particle is a polystyrene bead (density is 1050 kg/m^3) of radius $r_p = 490 \text{ nm}$. The trap size is $r_0 = 4 \text{ }\mu\text{m}$ and both DC and AC voltages are 1.2 V . Considering that the cross-over frequency, f_{co} , at which the Clausius-Motssoti factor becomes zero (pDEP for $f < f_{co}$ and nDEP for $f > f_{co}$), is 1.63 MHz (figure S1 in supplementary material), two frequencies of 1.0 and 2.0 MHz were selected to represent pDEP and nDEP, respectively. Under these conditions, $r_m = s_m = 0.034$ for pDEP and $r_m = s_m = 0.017$ for nDEP. The trajectory was obtained by solving the equation of motion for y -direction in equation (7), using the fourth-order Runge-Kutta

method in MATLAB [34] with initial conditions of $(y/y_0)(\tau=0) = 1$ and $\left. \frac{d(y/y_0)}{d\tau} \right|_{\tau=0} = 0$. As is well known for presence of only EP forces (equation (2) without F_{DEP}) the charged particle in the stable regime of a vacuum Paul trap ($\zeta=0$) is performing secular oscillatory motion on which a fast micro-motion is superimposed. In that case the initial conditions above define a particle which is initially (at $\tau=0$) approximately in the amplitude of the secular oscillation. In presence of the viscosity term in equation (7), the oscillations are damped to $y=0$ in the long-time limit, and the above initial conditions define the initial amplitude of the damped oscillations. Finally, in presence of dielectrophoretic forces in equation (7), the above initial conditions define (in analogy to the EP) the approximate amplitude of the secular motion at the beginning of the time counting. Of course, y_0 (and similarly x_0), though arbitrary, is bounded by the maximum distance from the trap center in which both EP and DEP forces can be described by the terms in equation (7) (at most the size of the trap, r_0 , but in practice smaller because of less-than-ideal geometry, influence of the vicinity of the electrode surfaces, finite size of the particle, etc.). Consistently with the above discussion, the initial conditions for y_0 and v_0 can be varied, defining various initial phases of the particle secular

motion. Of course, too large velocity would imply to large amplitude of the secular motion, which might be out of the trap.

Figure 4(a-c) compares three trajectories with EP-only (no DEP), with additional presence of pDEP, and with nDEP, for $b = 4.0$, $q = 0.01$, and $a = 1.0 \times 10^{-5}$ (A). For this particular case of small a and q , and not large b , which belongs to the range of experimental condition [1, 2], the dynamical stability was dominated by the DEP. The EP trajectory in figure 4(a) is neither converging nor diverging, i.e. it is on the stability border. However, with additional inclusion of DEP, if the frequency is in the pDEP region ($f < f_{co}$), the trajectory diverges (figure 4(b)). If it belongs to nDEP region ($f > f_{co}$), it converges (figure 4(c)). For the pDEP case with $f = 1.0$ MHz, the real part of CM factor, $\text{Re}(f_{CM})$, has the positive value of 0.35 while the imaginary part, $\text{Im}(f_{CM})$, has the negative value of -0.73 . Thus, the DEP force has pDEP nature and makes the particle unstable. Under the nDEP condition with $f = 2.0$ MHz, $\text{Re}(f_{CM}) = -0.12$ and $\text{Im}(f_{CM}) = -0.63$, the average DEP force acts as a restoring force. The

constant term from DEP, $a_{DEP} = -\text{Re}(f_{CM}) \left(r_m^2 + \frac{s_m^2}{2} \right)$, is combined with the original a parameter from EP, $a = \frac{2QU}{M\Omega^2 r_0^2}$, defining the effective a parameter,

$a_{eff} = -a - \text{Re}(f_{CM}) \left(r_m^2 + \frac{s_m^2}{2} \right) = -a + a_{DEP}$. Since the EP a is small ($a = 1.0 \times 10^{-5}$), the contribution of DEP to the effective a -value is dominant in determining the stability of the system. The sign of $a_{eff} = -a + a_{DEP}$ is altered according to the sign a_{DEP} .

Figure 4(d-f) compares the trajectories for $b = 4.0$, $q = 1.0$, and $a = 1.0 \times 10^{-2}$ (B). In this combination of (b, a, q) where both q and a are larger than those in case A in figure 4(a-c), the trajectory is always stable because EP confinement is dominating over the de-confinement effects of pDEP. An interesting observation is that the curve from EP+pDEP in figure 4(e) is almost identical to the curve from EP+nDEP in figure 4(f). This indicates that for $a = 1.0 \times 10^{-2}$, the contribution of a_{DEP} on a_{eff} is too small (5 %) for the distinction (made by the different sign of a_{eff}) between nDEP and pDEP to appear.

In the following, we construct the modified a - q and b - q stability diagrams with inclusion of the DEP effects. The stability of an aqueous quadrupole trap is defined by the requirement that a particle confines to the equilibrium position (trap center, i.e. origin) in the long-time limit. When the particle approaches confinement axis with time, the system is considered stable for a given combination of $(b, a, q, r_m, s_m, f_{CM})$ parameters, where r_m , s_m , and f_{CM} are due to DEP. As mentioned in Section 2, r_m^2 , s_m^2 , $r_m s_m$ represent the characteristic magnitudes of DC DEP force, AC DEP force, and mixed AC-DC DEP force, respectively. The real part of Clausius-Mossotti factor, $\text{Re}(f_{CM})$, and its imaginary part, $\text{Im}(f_{CM})$, determine the action of DEP force (see figure S1 in supplementary material). As discussed above the solution y (similarly x) of equation (7) can be written in the Floquet form, as a product of the periodic function and the Floquet exponents μ which is a function of $a - b^2/4$ and q . At the stability border the exponential factor is pure imaginary, and the particle oscillates with constant amplitude. We tested convergence by comparing the amplitudes in two long-time limits, for $\tau=500$ and 1000 . When the difference between the amplitudes of the oscillating trajectory at these two times (for a given set of the trap parameters) is less than a predefined tolerance, here 5×10^{-3} ,

$$\left| \frac{y_{\max}(\tau=1000) - y_{\max}(\tau=500)}{y_0} \right| < 5 \times 10^{-3}, \quad (9)$$

we adopt that the trap is at the stability border. This procedure was repeated for various combinations of the trap parameters to construct the stability border line in the (a, q) (figure 5) and the (b, q) diagrams (figure 6).

Figure 5 presents the modified a - q stability diagrams including DEP forces with two different b -values, $b=1.0$ and 4.0 . The r_m and s_m are fixed to $r_m = s_m = 0.034$ for pDEP and $r_m = s_m = 0.017$ for nDEP. The chosen range of q is $0 < q < 1.0$. For $q > 1.0$ there is almost no difference among EP, EP+pDEP, and EP+nDEP stability borders. As in the previous section, two driving frequencies, 1.0 and 2.0 MHz, were representatively chosen for pDEP and nDEP, respectively. For $b = 1.0$ in figure 5(a), the deviation between EP and EP+pDEP is noticeable for $q < 0.2$ while the deviation between EP and EP+nDEP appears only at $q < 0.02$. The deviations become more visible with decrease of q . For $q < 0.05$, we could not find a value of a for which EP+pDEP reaches a stable condition. However for EP+nDEP, the a -value on border does not change with further decrease of $q < 0.001$. The region of $q < 0.0005$ is, therefore, not shown at the plot. Therefore, the a - q stability diagram is only significantly altered with the presence of DEP for small $q \ll 1$. We note that nDEP enlarges the stable region remarkably, oppositely to pDEP, which suppresses the stable region. For $b = 4.0$ in figure 5(b), all trends are similar to the $b = 1.0$ case in figure 5(a). Thus, the EP+pDEP border curve deviates from the EP border for $q < 0.2$ while the EP+nDEP border deviate from the EP one for $q < 0.05$. The reduction of stable region due to pDEP is more significant with increase of b -values ($b = 4.0$), while the effects of nDEP are barely sensitive to a variation of b . The effects of pDEP to the stability are larger with the variation of b parameter than those of nDEP, because s_m in pDEP regime is larger than s_m in nDEP regime. For example, $r_m \propto 1/\Omega$ and for nDEP $f = \Omega/2\pi > f_{co}$ while for pDEP $f = \Omega/2\pi < f_{co}$. Thus, for $f = 1.0$ MHz for pDEP and $f = 2.0$ MHz for nDEP, r_m of nDEP is four times smaller than r_m of pDEP.

Figure 6 shows comparisons of the b - q stability ($a = 0$) diagrams for EP+DEP and for EP-only. The choice of $0 < b < 4$ reflects the range of b parameters in the experiment [1, 2]. The stability border for EP-only is presented in figure 6(a). With inclusion of pDEP, a new unstable region arises for small q 's (dashed line in figure 6(b)) and the border of this region shifts toward larger q 's with decreasing the driving frequency (figure 6(c)) because the

magnitude of DEP force, proportional to $s_m^2 = \frac{U_{ac}^2}{r_0^4 \Omega^2} \left(\frac{12\epsilon_m \epsilon_0}{\rho_p} \right)$, increases with decreasing the frequency. The inclusion of nDEP does not bring any noticeable change to the EP stability diagram (figure 6(a)). The reason is that inclusion of nDEP only slightly increases the stable region. However, the lower boundary of the stable region is limited by $q = 0$. Moreover, the upper stability border is placed at the large $q \geq 1$, in which the DEP effects are negligible. Thus, the inclusion of nDEP does not affect to the upper stability boundary. Also, the influence of nDEP is inherently smaller because $s_m^2 \propto 1/\Omega^2$ while Ω in nDEP is larger than $f_{co} = 1.63$ MHz. Further details about the stability due to s_m are discussed in the supplementary material (SI 4). In summary, for b - q stability diagram the DEP effects appear only for small $q \ll 1$ due to pDEP.

Throughout this study, we have shown that the effects of DEP on the stability of the charged spherical particles in an aqueous quadrupole micro-trap are considerable only for the small q values. The typical b -values in the recent experiments [1, 2] are of the order of unity while the q -values are of the order 10^{-2} and less, when the effects of DEP are not negligible. Thereby, the conclusions of this study are expected to be useful in designing an aqueous quadrupole micro-trap and defining the regions of its stability for the realistic values of the system parameters.

The DEP effects are not significant for functions of the conventional Paul traps in vacuum or gaseous medium, because the dielectric constant of gaseous medium is small ($\epsilon_m \sim 1$) and possible comparable to the dielectric constant of the particle. In addition, the conventional Paul trap is normally used to manipulate light-mass objects (ion, etc.) for which the corresponding q -value is not small [35], while DEP effects are further suppressed due to the small particle radius.

The qualitative conclusions in this study are not influenced by the shape of the charged particle. However, the quantitative values could vary with the shape because the dielectrophoretic force depends on the shape of object [36, 37]. For a charged particle of a finite size, the electric force exerted on the particle could be calculated by integrating the Maxwell stress tensor over the particle surface [33, 38]. This computation is implemented in the multipole expansion method [27]. The charge of the particle appears only in the zeroth-order term of the expansion and is not function of the particle size. The particle size affects the first-order (force due to dipole) and the higher-order terms (quadrupole, octupole, etc.). Since the electrophoretic (EP) force is related to the particle charge, the zeroth-order term is the EP component in the electric force and it is not a function of particle size. The forces due to dipole and higher multipole moments are the dielectrophoretic (DEP) contribution and these are functions of the particle size.

4. Conclusions

In the present study we discuss the effects of DEP on the stability of a micro-size particle in a linear (two-dimensional) aqueous quadrupole micro-trap. We focus to the trap stability in presence of DEP, and compare it with the standard EP stability conditions. The complete description of DEP in the system necessitates, in addition to the “standard” b , a , and q Paul

trap parameters, three new parameters: $r_m = \frac{U_{dc}}{r_0^2 \Omega} \sqrt{\frac{12\epsilon_m \epsilon_0}{\rho_p}}$, $s_m = \frac{U_{ac}}{r_0^2 \Omega} \sqrt{\frac{12\epsilon_m \epsilon_0}{\rho_p}}$ and

$f_{CM} = \frac{\frac{\epsilon_p}{\rho_p} - \frac{\epsilon_m}{\rho_m}}{\frac{\epsilon_p}{\rho_p} + 2\frac{\epsilon_m}{\rho_m}}$ are introduced. While these parameters carry information on the trap voltages and frequency, as well as on the dimensions and of the dielectric constant of the liquid medium, the only dependence of the DEP on the particle properties is through the particle mass-density, not its charge like in EP. Our results show that the effects of DEP on the stability in the trap become significant only for small q and a parameters. When $q \ll 1$, the stability is governed by DEP rather than EP. nDEP slightly enlarges the stable region while pDEP significantly reduces it. The pDEP effects increase with increase of the medium viscosity and with the decrease of frequency (i.e. with increase of b). The stable region in the b - q diagram is also significantly reduced for small q values due to pDEP

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.

(a) Schematics of a two-dimensional aqueous quadrupole trap. r_0 is the trap size and Ω is the driving angular frequency of AC input. (b) The charged particle is confined within the trap region around the trap axis which is formed by the AC, oscillating, and the DC potentials.

Figure 2.

a - q stability diagrams for EP: (a) a - q stability plot for two-dimensional quadrupole trap for $b = 0.0$. The solid lines are the stability borders from x -motion while the dashed lines are the borders for y -motion. The resultant stable region is filled with the slanted lines; (b) Comparison of a - q stability diagrams with different b -values: $b = 0.0$, 1.0 , and 4.0 .

Figure 3. b-q stability diagram for EP in highly viscous medium, with $a = 0$. (a) The stability diagram is obtained by numerical solution of the conditions of stability [22]. The lower region is the stable region and the upper one represents the unstable region; (b) is the magnified figure for the first stability "island" for $b < 4$.

Figure 4.

(a-c) Trajectories of particle with $b = 4.0$, $q = 0.01$, and $a = 1.0 \times 10^{-5}$. (d-f) Trajectories with $b = 4.0$, $q = 1.0$, and $a = 1.0 \times 10^{-2}$. (e) and (f) are almost identical, as explained in the text. The horizontal axis presents the time in terms of the number of periods $T = 1/\Omega$ of the AC input

Figure 5.

The stability borders in a - q diagram for EP-only, EP+nDEP, and EP+pDEP cases for various values of the b parameter: (a) $b = 1.0$; (b) $b = 4.0$. The frequency for nDEP is 2.0 MHz and that for pDEP is 1.0 MHz. The stable region is lower right region from the stability border.

Figure 6.

The stability b - q diagram for $a = 0$ with various frequencies: (a) EP only; (b) EP+pDEP with $f = 1.0$ MHz; (c) EP+pDEP with $f = 0.1$ MHz. The lines are the stability borders. The solid line is the stability border due to EP and the dashed line is the border due to pDEP. The stable region is determined by the region between the two borders (slanted lines). The region below dashed line is a new unstable region due to pDEP.