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T2 Mapping from highly undersampled data by REconstruction of Principal COmponent coefficient Maps (REPCOM) using Compressed Sensing

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Abstract

Recently, there has been an increased interest in quantitative MR parameters to improve diagnosis and treatment. Parameter mapping requires multiple images acquired with different timings usually resulting in long acquisition times. While acquisition time can be reduced by acquiring undersampled data, obtaining accurate estimates of parameters from undersampled data is a challenging problem, in particular for structures with high spatial frequency content. In this work, Principal Component Analysis (PCA) is combined with a model-based algorithm to reconstruct maps of selected principal component coefficients from highly undersampled radial MRI data. This novel approach linearizes the cost function of the optimization problem yielding a more accurate and reliable estimation of MR parameter maps. The proposed algorithm - REconstruction of Principal COmponent coefficient Maps (REPCOM) using Compressed Sensing - is demonstrated in phantoms and in vivo and compared to two other algorithms previously developed for undersampled data.

Keywords

Parameter mapping; T2; radial MRI; FSE; compressed sensing; Principal Component Analysis

Introduction

MR parameter maps (e.g. T1, T2) have proven to be a valuable quantitative tool for assessing a variety of pathologies. Parameter mapping has been used in the study of brain maturation (1), Parkinson's disease (2, 3), intervertebral disk degeneration (4), cardiac necrosis and edema (5), lung function (6), lesion classification (7, 8), hepatic

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encephalopathy (9), liver cirrhosis (10), cartilage damage (11), as well as in automated tissue/lesion segmentation (12).

The lengthy acquisition times imposed by conventional parameter mapping methods, however, limit their use in the clinic. Long acquisition times not only reduce patient throughput (by increasing the total examination time) but may also affect the quality of parameter maps due to patient motion. Moreover, for imaging applications that require breath holding, parameter mapping is limited by the short time available to acquire the data. Fast imaging methods are a possible solution to these problems with the drawback of low spatial resolution parameter maps and/or a low number of time points for parameter fitting. The acquisition of undersampled data is another potential solution for faster parameter mapping. For moderately undersampled data (2–4 times undersampled), parallel imaging has been proposed for parameter mapping without sacrificing spatial resolution (13). For higher degrees of undersampling (8 – 16 times or even higher), other reconstruction techniques have been proposed for the same goal. An echo sharing (ES) technique (14, 15) was used to obtain T2 maps from radial fast spin-echo (FSE) data. In this technique, a series of T2-weighted images are reconstructed from a single k-space data set (typically 256 k-space lines in total from all echo times) by mixing data acquired during different spin-echo periods but weighting the low-frequency region of k-space to specific TE values. Although this technique has proven to be useful for a series of clinical applications (16, 17), the mixing of TE data in the high spatial frequencies introduces errors in the T2 estimation of structures with high spatial frequency content such as small objects and edges (15). Further improvements in T2 estimation using the ES algorithm include a bent trajectory (where two half radial lines are acquired in the same time required to collect a full line) combined with a homodyne reconstruction to effectively double the number of views without increasing scan time (18). This approach has been shown to reduce the errors in T2 estimation in small objects (19).

As an alternative to conventional MR parameter estimation, where multiple images with varying parameters are needed, model-based approaches have been proposed for both T1 and T2 mapping (20, 21, 22). In the case of T2 mapping, the model-based approaches assume that the intensity of every pixel follows a mono-exponential decay and images with various TE weightings can be synthesized from the initial signal intensity (signal intensity at TE=0) I_0 and T2 (or $R2=1/T2$) maps. These two maps can be obtained from the acquired data by iterative model-fitting techniques. Various constraints or penalties that modify the model-based fitting algorithm have been proposed. These penalties are based on a priori knowledge of the system and are included to improve parameter estimation (20, 22). Recent work utilizes penalty terms based on sparsifying transforms, thus introducing the advantages of the new field of Compressed Sensing (CS) (23) into parameter estimation from undersampled data. Spatial sparsity has been used in the image domain to provide better TE images in a T2 estimation context (24). The idea of using temporal sparsifying penalties in the context of CS has also been reported. Doneva et al. (24, 25) recently showed that a T2 decay can be sparsified through Principal Component Analysis (PCA) or by using an overcomplete dictionary to improve T2 estimation. Using multi-echo Cartesian data, they show that T2 maps obtained from a reduced data set (5 times undersampling) were comparable to the fully sampled gold standard.

A common problem of model based approaches that involve solving a non-linear problem is the scale mismatch between the variables in the model when gradient-based optimization algorithms are used. In the case of T2 mapping for example parameter estimation is sensitive to the scaling factors of T2 and I_0 (20) and scaling mismatches will lead to poorly balanced optimization affecting the accuracy of the reconstructed T2 map. In this work, our approach is to linearize the problem before T2 estimation. We first show that accurate T2 estimation

can be obtained with a small number of Principal Component Coefficient (PCC) maps and use PCA to linearize the signal model. After linearization, the proposed approach falls into the framework of the CS theory, which is well developed for linear systems. Using this linearized model, PCC maps can be estimated from the undersampled k-space data. This technique, REconstruction of Principal Component coefficient Maps (REPCOM), is demonstrated in simulated data, phantoms, and in vivo (brain, liver, and cartilage) imaging.

Theory

The following theoretical considerations are derived for the task of T2 estimation. The methodology, however, can be adapted to the estimation of other MR parameters.

In the model-based algorithm for T2 estimation, assuming a mono-exponential decay model, the main goal is to maximize the data consistency. The data consistency term can be expressed as (20, 21):

$$\sum_j \left\| FT_j(\mathbf{I}_0 \cdot e^{-TE_j/T_2}) - \vec{\mathbf{K}}_j \right\|^2, \quad [1]$$

where TE_j denotes the j^{th} echo time, FT_j is the forward Fourier transform for all k-space locations acquired during the j^{th} spin-echo period, $\mathbf{I}_0$ and $\mathbf{T}_2$ are the initial signal intensity and T2 maps, respectively. $\vec{\mathbf{K}}_j$ is the measured k-space data at echo time TE_j . The data consistency term indicates how different the acquired data are from the signal model. In addition to the data consistency term, one can add constraints (penalties) using a priori knowledge of the object. If P_i represents the penalty functions and λ_i the corresponding regularization parameters which control the relative importance of the penalty functions, an equation for reconstructing T2 maps can be formulated as follows:

$$\hat{\mathbf{I}}_0, \hat{\mathbf{T}}_2 = \arg \min_{\mathbf{I}_0, \mathbf{T}_2} \left\{ \sum_j \left\| FT_j(\mathbf{I}_0 \cdot e^{-TE_j/T_2}) - \vec{\mathbf{K}}_j \right\|^2 + \sum_i \lambda_i P_i(\mathbf{I}_0, \mathbf{T}_2) \right\}. \quad [2]$$

This equation represents the model-based algorithm described in (20). Herein, we refer to it as the MB algorithm.

Due to the non-linearity of the exponential term in Eq. [2], the minimization of the data consistency term using conventional algorithms (e.g. conjugate gradient) suffers from scale mismatches between $\mathbf{I}_0$ and $1/T_2$ (20). One way to overcome this problem is to rescale the TE time points by a *scaling factor* (sf); thus, Eq. [2] becomes:

$$\hat{\mathbf{I}}_0, \hat{\mathbf{T}}_2 = \arg \min_{\mathbf{I}_0, \mathbf{T}_2} \left\{ \sum_j \left\| FT_j(\mathbf{I}_0 \cdot e^{-sf \cdot TE_j/T_2}) - \vec{\mathbf{K}}_j \right\|^2 + \sum_i \lambda_i P_i(\mathbf{I}_0, \mathbf{T}_2) \right\}. \quad [3]$$

As will be shown in this work, the choice of an optimal scaling factor may be challenging. Our approach is to remove the non-linearity related to T2 estimation, which in turn eliminates the scale mismatch. Linearization is achieved by using PCA where the exponential decay can be accurately approximated using a linear combination of a few pre-calculated principal components (PCs). The PCs are calculated from a training set of exponential decay curves covering a given range of T2 and TE times using Singular Value Decomposition (SVD) (26). A detailed description of the generation of the PCs is given in the Appendix.

Let $\vec{v}$ be a real vector representing a noiseless mono-exponential T2 decay curve for specific T2 and a real I_0 values, $v_j = I_0 \cdot e^{-TE_j/T2}$, the elements of $\vec{v}$ are the signal intensity at each TE for this decay. Let L be the number of PCs to be used in the approximation and b_i be the i^{th} PC vector. The T2 decay can then be approximated by the linear combination:

$$\vec{v} = \sum_{i=1}^L m_i \vec{b}_i + \vec{\varepsilon}, \quad [4]$$

where the PC coefficient m_i is the weight of the i^{th} PC for the decay $\vec{v}$, and $\vec{\varepsilon}$ is the approximation error.

Since the PCs are orthonormal, the i^{th} PC coefficient can be computed as follows:

$$m_i = \vec{v} \cdot \vec{b}_i. \quad [5]$$

Let $\mathbf{B}$ denote a matrix whose columns are the N principal components ordered from most to least significant (i.e., largest to smallest corresponding singular value). The dimension of $\mathbf{B}$ is $N \times N$ where N is the number of TE points (i.e. echo train length or ETL). A truncated $N \times L$ matrix $\hat{\mathbf{B}} = (\vec{b}_1, \vec{b}_2, \dots, \vec{b}_L)$ can be formed by selecting the first L columns of $\mathbf{B}$. Let $\mathbf{M}$ denote the $(N_x \times N_y) \times L$ matrix of PC coefficients (N_x, N_y are the dimensions of the TE images); $\mathbf{M} = (M_1, M_2, \dots, M_L)$ where M_i is the vector of the i^{th} PC coefficient of all the pixels in the images. Let $\hat{\mathbf{B}}_j$ denote the j^{th} row of the matrix $\hat{\mathbf{B}}$. Note that $\mathbf{M}\hat{\mathbf{B}}_j^T$ yields the image at echo time TE_j computed from the truncated PC coefficients.

Thus, the data consistency term can be approximated as:

$$\sum_{j=1}^N \left\| FT_j(\mathbf{M}\hat{\mathbf{B}}_j^T) - \vec{\mathbf{K}}_j \right\|^2. \quad [6]$$

Note that in Eq. [6] the non-linearity in the signal model is removed and that the data consistency term is only dependent on the matrix $\mathbf{M}$ since all other variables are known.

Sparsifying penalties ($P_i(\cdot)$ with corresponding weights λ_i) can be incorporated into the model to exploit the sparsity of $\mathbf{I}_0$ and $\mathbf{T}_2$ maps in a transform domain (hereinafter called “spatial sparsity”). Because the PCC maps can be derived from the given $\mathbf{I}_0$ and $\mathbf{T}_2$ maps, PCC maps will inherit this spatial sparsity which can be exploited using the coefficient maps M_i . Altogether the combination of model linearization by PCA and enforcing spatial sparsity leads to the proposed REPCOM algorithm:

$$\hat{\mathbf{M}} = \arg \min_{\mathbf{M}} \left\{ \sum_{j=1}^N \left\| FT_j(\mathbf{M}\hat{\mathbf{B}}_j^T) - \vec{\mathbf{K}}_j \right\|^2 + \sum_i \lambda_i P_i(\mathbf{M}) \right\}. \quad [7]$$

Thus REPCOM incorporates both temporal and spatial sparsity, which can be used according to the CS theory to improve the reconstruction of undersampled data. The temporal sparsity is exploited by the limited number of PC coefficients L (2 – 4). Spatial sparsity is exploited by penalties such as the total variation and the l_1 norm of the wavelet coefficients of the PC maps.

For multi-channel coil data, complex coil sensitivity maps $\mathbf{S}_l$ can be incorporated into the algorithm:

$$\widehat{\mathbf{M}} = \arg \min_{\mathbf{M}} \left\{ \sum_{l=1}^{\#coils} \sum_{j=1}^N \left\| FT_j(\mathbf{S}_l \mathbf{M} \widehat{\mathbf{B}}_j^T) - \vec{\mathbf{K}}_{l,j} \right\|^2 + \sum_i \lambda_i P_i(\mathbf{M}) \right\}, \quad [8]$$

where $\vec{\mathbf{K}}_{l,j}$ is the measured k-space data from coil l at echo time TE_j . Since the phase of the TE images is not expected to change due to the nature of the spin echo acquisition, the phase is incorporated into $\mathbf{S}_l$. Equations [7] or [8] are solved iteratively to reconstruct the PCC maps from which the TE images are obtained. T2 maps are estimated from the TE images by Eq. [4] using a conventional single exponential least squares fitting.

Methods

In vivo data were acquired on a 1.5T MRI scanner (GE Signa NV-CV/I, Milwaukee, WI, USA). Written consent was obtained from subjects prior to imaging. All algorithms were implemented in Matlab (The Mathworks, Natick, MA, USA).

The k-space data for the numerical phantoms used in this work was generated from the analytical Fourier transform of circles. Unless specified, independent identically distributed Gaussian noise was added to the real and imaginary components of k-space to yield a signal-to-noise ratio (SNR, defined as signal mean/noise standard deviation) comparable to in vivo data. In all simulations with noise 100 realizations were used. Each noise realization was generated from a different randomization seed with the same statistical parameters.

Data acquisition

All radial data were acquired with a previously developed radial FSE pulse sequence (16). The angular ordering referred in (27) as “bit-reversed” was used in this work for radial FSE in order to minimize artifacts in the images due to T2 decay and motion (in vivo).

Two physical phantoms both containing 10 mm and 5 mm inner diameter glass tubes filled with MnCl_2 solutions of different concentrations (50 μM , 75 μM , and 170 μM) were prepared. A solution with MnCl_2 concentration of 330 μM was used as background in one of the two phantoms. Data were acquired using the radial FSE sequence with a single channel transmit/receive coil, ETL = 16, echo spacing (i.e. time between 180° pulses) = 8.29 ms, TR = 1 s, slice thickness = 5 mm, receiver bandwidth = ± 31.25 kHz. The FOV was set to 20 cm to yield roughly a 6-pixel diameter object and a 12-pixel diameter object for the 5 mm and 10 mm inner diameter tubes, respectively. The acquisition matrix of 256×256 yielded a total of 16 radial k-space lines for each of the 16 TE data sets. Gold standard data were acquired using the radial FSE method with 256 k-space lines per TE.

For proof-of-concept, Cartesian spin-echo data of a normal volunteer's brain was obtained with a single channel transmit/receive coil, ETL = 16, echo spacing = 8.26 ms, 256×256 matrix for each echo, TR = 1 s, slice thickness = 6 mm, receiver bandwidth = ± 31.25 kHz, FOV = 22 cm. Sixteen TE images were reconstructed from fully-sampled k-space data. From these TE images, three PCC maps were derived and a T2 map was then reconstructed using these PCC maps. In addition, conventional T2 estimation using the 16 TE images was performed.

In vivo brain data were also acquired using the radial FSE sequence. The data sets were acquired with an 8-channel head coil, ETL = 16, echo spacing = 8.80 ms or 9.07 ms, slice thickness = 5 mm, receiver bandwidth = ± 31.25 kHz, TR = 4 s, FOV = 24 cm or 22 cm (specific parameters for each brain image are listed with each figure). The acquisition matrix of 256×256 yielded a total of 16 k-space lines per TE. Gold standard data were acquired using the radial FSE method with 256 k-space lines per TE.

In vivo abdominal data were acquired within a breath hold using the radial FSE sequence. Data were acquired with a 4-channel torso coil, ETL = 16, slice thickness = 8 mm, receiver bandwidth = ± 31.25 kHz, acquisition matrix of 256×256 or 256×192 , with echo spacing varying between 8.76 ms and 9.07 ms, and TR = 1.5 s or 1.8 s (specific parameters for each abdominal image are listed with each figure). The FOV was adjusted according to the body habitus of each subject. Flow from large vessels was suppressed via saturation bands applied above and below the imaging plane. The abdominal scans obtained with the goal of characterizing lesions were acquired with a preparatory time of 35.4 ms to yield anatomical images with the desired visual T2-weighted contrast for radiologists to perform lesion detection. During the preparatory time, a small amount of diffusion ($b = 1.2 \text{ s}^2/\text{mm}$) was applied to suppress the bright signal from small blood vessels that may mimic the signal from lesions.

In vivo data for cartilage were acquired with the radial FSE sequence. Data were acquired with a transmit/receive single channel knee coil, ETL = 8, echo spacing = 8.38 ms, TR = 2 s, slice thickness = 5 mm, FOV = 16 cm, receiver bandwidth = ± 31.25 kHz, NEX = 2, acquisition matrix of 256×256 . Flow from large vessels was suppressed via saturation bands. Gold standard data were acquired using the radial FSE method with 256 k-space lines per TE.

Reconstruction

All anatomical images reconstructed from radial FSE data were obtained by combining data from all TEs (for each RF receiver coil) followed by filtered back projection (FBP). Images from each coil were combined using sum of squares.

For the REPCOM reconstruction the TE values used in the training sets for the generation of PCs were individually generated for each imaging experiment using specific pulse sequence parameters. Unless specified the T2 range used in the generation of PC's was 45 ms to 500 ms. The minimum number of PCs (L) needed for accurate T2 characterization was selected to ensure that the estimates of all the T2 curves in the training set have less than 1% error. In this work L was between 2 to 4.

In the REPCOM reconstruction for radial data we used the non-uniform Fourier transform (NUFFT) toolbox developed by Fessler and Sutton (28) for the forward and inverse Fourier transforms. For data acquired with multiple receivers the complex coil sensitivity maps S_l were obtained prior to T2 mapping from images reconstructed by combining data from all TE's. The sensitivity maps were calculated by dividing the FBP-reconstructed complex images for each coil by the sum of squares. Smoothing was used to reduce noise in the sensitivity maps.

The nonlinear Polak-Ribiere Conjugate Gradient algorithm (29) with line search initialized by steepest descent was used to search for the minimum of Eqs. [7] and [8]. A fixed number of 50 iterations were used in this study based on the initial observation that parameter estimates converged after 15–25 iterations. Zero maps were used as the initial guess for the PCC maps. The l_1 norm of the wavelet coefficients (Daubechies 4, code obtained from <http://www-stat.stanford.edu/~wavelab>) and total variation were used as spatial sparsity penalties on all PCC maps. The weighting parameters for the penalty terms were chosen empirically. In this work, weights λ_i were all set to 0.01 (for the abdominal datasets and one of the brain datasets due to low SNR) or 0.003 (for all other) multiplied by the product of the number of k-space lines and the mean signal intensity of the object pixels in the sum of squares image.

For comparison purposes radial data were reconstructed with the ES algorithm described in (15) and the MB algorithm described in (20). Each TE image reconstructed with the ES algorithm consisted of a central k-space region with only radial lines acquired at a specific TE (TE_j) up to a radius determined by the Nyquist condition (radius at which the distance between k-space samples in the radial direction equals the distance between two k-space samples in the azimuthal direction). In the second k-space tier radial lines acquired at $TE_{j\pm1}$ were added up to the next Nyquist radius. This process was continued until all tiers (number of tiers = ETL) were progressively populated with TE data. The last tier contained data from all TEs. The missing data points on each tier were estimated by linear interpolation. TE images were reconstructed using FBP and T2 maps were generated from them using a single exponential decay model.

The MB algorithm was implemented as described in (20). NUFFT was used for the forward and inverse Fourier transforms. In order to scale I_0 and $1/T_2$, TE's were rescaled by the scaling factor sf as shown in Eq [3]. The nonlinear Polak-Ribiere Conjugate Gradient algorithm was used to search the minimum. The penalty terms in Eq. [3] were the L2 norms of the finite differences of the I_0 and $1/T_2$ maps' Fourier transforms as suggested in (20). To reduce the dependence in T2 estimation on I_0 (see below), the Fourier data were normalized by the mean object I_0 which was obtained from the ES reconstruction. The same sensitivity map generation as in REPCOM was used.

Results

Figure 1 shows the results of the simulations to determine the number of PCCs needed for accurate characterization of T2 decay curves. In this figure, $L = 2, 3, 4$ were used and the error between the true T2 and the T2 estimated from the corresponding PCCs are shown for the given T2 range (45–500 ms). The estimated T2 is calculated by using conventional least squares fitting of the decay curve reconstructed from 2, 3 and 4 PCCs. In this case, three PCCs ($L = 3$) are enough for accurate T2 estimation. The largest 5 singular values (obtained from Eq. [A.3]) for the given parameters are 8.07×10^2 , 1.43×10^1 , 8.47×10^{-2} , 2.07×10^{-4} and 2.72×10^{-7} , which indicates that the contribution of PCCs greater than 4 to the exponential curves is small.

Figure 2 shows brain PCC Maps of the first 3 PCs derived from images fully sampled at all 16 TEs using a Cartesian acquisition. Note that these maps are all spatially correlated and can be well-represented by a small number of coefficients in a transform domain (such as total variation and wavelet transform).

Figure 3b is the T2 map derived from the 3 PCC maps shown in Figure 2. Figure 3a is the T2 map reconstructed from all 16 TE images using conventional least squares fitting. Figure 3c quantifies the difference between the two T2 maps. The difference is below 2% almost everywhere in the brain. This is further evidence that accurate T2 mapping can be obtained with a small number of PCC maps.

To evaluate the performance of the REPCOM algorithm we compare results to the ES and MB algorithms, both developed for undersampled radial data. Figure 4 shows brain T2 maps reconstructed by these algorithms using the same undersampled radial FSE data (16 radial k-space lines per TE). The figure also includes the gold standard T2 map generated from 16 TE images each reconstructed from 256 radial lines per TE. The gold standard took 17 minutes and 20 seconds to acquire, while the acquisition of undersampled data took 1 minute and 20 seconds. The mean T2 values of the Region Of Interest (ROI) illustrated in Figure 4a are: 89.7 ms (gold standard), 89.7 ms (REPCOM), 89.6 ms (MB) and 91.2 ms (ES), indicating that for this large ROI the estimated T2's by these three algorithms are

comparable and close to the gold standard. However, the fine structures are blurred in the ES T2 map; these are better preserved in the T2 maps obtained with the MB and REPCOM algorithms.

Simulations were performed to further compare these algorithms. Figure 5 shows a comparison between REPCOM and the ES algorithms using a numerical phantom consisting of a single circular object with diameter varying from 4 to 8 pixels embedded in a larger circular background (112 pixels in diameter) with varying T2. The k-space data for the phantom was generated assuming a radial FSE acquisition with 16 k-space lines per TE, echo spacing = 9.00 ms and 16 TE points. Figure 5 shows the % error of the mean T2 from 100 noise realizations. Note that for the ES algorithm the estimation is more accurate when the T2 of the background is closer to the T2 of the object (100 ms) and that the error increases as the background T2 differs more from it, being maximum when there is no background (represented by $T_{2\text{background}} = 0$ ms). As previously reported (15), the error depends on the diameter of the object; the smaller the object the larger the error. T2 estimates obtained from REPCOM are less affected by the T2 of the background and by the size of the object. As shown in Figure 5, the % error produced by REPCOM for 4–8 pixel diameter objects ranges from –2% to 2% for the various backgrounds studied. These errors are significantly lower than those of the ES algorithm.

As shown in Figure 4 the MB algorithm yielded a T2 map comparable to the gold standard and T2 map by REPCOM. However, the performance of the MB algorithm is dependent on the scaling factor (20). This is illustrated in Figure 6 for a simulated object consisting of three small circular objects (6 pixels in diameter) with $T_2 = 80$ ms, 150 ms, and 230 ms with a $T_{2\text{background}}$ of 50 ms or 100 ms, chosen to represent T2 for liver and brain parenchyma (1, 7). In the simulations the object and the background have the same uniform I_0 value. The values of I_0 represent the variations seen in the same in vivo image due to coil sensitivities. Results are shown for noiseless data and 200 iterations. From the figure we can see that T2 estimates depend on the scaling factor and that the optimal scaling factor varies with the I_0 and T2 values of the object being imaged. These results indicate that without prior information on T2 and I_0 , it is not possible to choose an optimal scaling factor which in turn will compromise the accuracy of T2 estimation. As noted previously (20), the dependence of T2 estimation on the scaling factor can be reduced with a large number (> 1000) of iterations. For comparison, Table 1 shows the % error in T2 estimation with REPCOM for the same objects used in Figure 6 and for the same reconstruction parameters with 50 iterations. Note that the REPCOM T2 estimates for the three objects are not sensitive to the change of I_0 and $T_{2\text{background}}$.

The three algorithms were also tested on physical phantoms. Figure 7 shows the T2 maps of the phantoms reconstructed from radial FSE data with REPCOM. The T2 for the objects (vials) estimated by the ES, MB and REPCOM algorithms are also shown. The undersampled radial FSE data set used for T2 estimation consisted of 16 TE data sets with 16 k-space radial lines per TE. The gold standard was obtained from TE images reconstructed from 256 k-space lines per TE. For the phantom without background the % error in the T2 estimates obtained from the REPCOM and MB algorithms are in the range of –2.33% to 2.41% and –2.71% to 4.92%, respectively. The % T2 error yielded by the ES algorithm ranges from 1.64% to 18.97%. For the phantom with a background, however, only REPCOM yields good T2 estimates (% T2 error ranges from –0.48 – 1.60%). The corresponding % error range yielded by the MB and ES algorithms is –13.03% to 13.45% and –19.34% to 8.05%, respectively. For the MB algorithm, the best scaling factor was chosen from sf values ranging from 1 to 120. Thus, even for the optimal scaling factor (possible to determine in the phantom since the “truth” for T2 and I_0 is known), the % error in the T2 mean is still object dependent.

Among the properties of the REPCOM algorithm, we investigated the benefit of using spatial sparsity constraints in addition to the temporal constraints imposed by the PCA reconstruction. Figure 8 shows brain T2 maps reconstructed from data acquired with radial FSE where a preparatory time was used between the 90° excitation pulse and the train of 180° refocusing pulses to vary the SNR. The images in Figure 8a–c were obtained from data acquired with no preparatory time between the 90° excitation pulse and the train of 180° refocusing pulses (higher SNR) whereas the images in Figure 8d–f were obtained from data acquired with a preparatory time = 35.20 ms (lower SNR). When no spatial constraints are incorporated into the REPCOM algorithm, the reconstructed T2 map from high SNR data (Figure 8a) is similar to the corresponding gold standard (Figure 8c). There are some noise-like artifacts in Figure 8a which are reduced when spatial penalties are used in the reconstruction (Figure 8b). For low SNR data, the noise is more evident in the T2 map reconstructed without penalties (Figure 8d). The improvement introduced by the spatial penalty is more significant and the corresponding T2 map (Figure 8e) matches better the gold standard (Figure 8f).

The performance of the REPCOM algorithm under different levels of noise was also investigated using the numerical phantom used in Figure 6 (for $T2_{\text{background}}=50$ ms, $I_0=1$). Simulations were conducted for two object diameters (6 and 12 pixels) for different levels of noise. The results are summarized in Figure 9 where SNR_{60} is the SNR at TE = 60 ms (the approximate TE contrast of the images reconstructed from all radial FSE lines). As reference, the noise level $\text{SNR}_{60} = 50$ –100 is comparable to the SNR of white and grey matter in the brain images shown in Figure 8. The noise level $\text{SNR}_{60} = 13$ –25 is comparable to the SNR of abdominal lesions in the radial FSE images (16). For the 6- and 12-pixel diameter objects, the biases of T2 estimates are all below 3% for $\text{SNR}_{60} \geq 13$. The standard deviation increases with decreasing SNR with the standard deviations for the 12-pixel objects being smaller due to the larger number of pixels in the ROI.

Table 2 was obtained using radial FSE data with $\text{SNR}_{60} = 25$ for the numerical phantom used in Figure 9 (the phantom with 6-pixel diameter objects). The number of TE points (ETL) was varied from 4 to 16, while the same TE coverage was enforced by adjusting the echo spacing. The number of total radial k-space lines was varied from 128 to 1024 yielding different levels of undersampling for each TE time point depending on the ETL. As expected, for all three objects, the accuracy of T2 estimates increases with increasing number of k-space lines per TE. When the number of k-space lines is fixed, very little difference is observed among the various echo train length for objects with longer T2 (230 ms, 150 ms). For the object with T2 = 80 ms, the standard deviation was found to improve as the echo train increases. This is because there is less signal at later TE points for short T2s, and T2 estimation is more accurate when there are more data from early TE points.

We investigated the utility of T2 mapping using REPCOM in other applications besides the brain. Figure 10 shows the T2 maps of cartilage (color map) overlaid onto the corresponding anatomical knee image of an asymptomatic young adult male. The gold standard T2 map was reconstructed from 8 TE images with 256 k-space radial lines each. The REPCOM T2 map was reconstructed from 8 TE images with 32 k-space lines per TE. As reported previously, there is a progressive increase in the T2 (30 ms – 70 ms) from the radial to the superficial zone of the cartilage (30). This is seen in both the gold standard and REPCOM T2 maps. Although the T2 map by REPCOM algorithm has a blockier appearance compared to the gold standard, the focal changes in the cartilage are still well preserved in the REPCOM T2 map which is reconstructed from 8 times undersampled data. This blockier appearance is mainly due to undersampling and lower SNR. Because of the spatial sparsity penalties, a noisier dataset will lead to more “regulated” PCC maps, which in turns leads to a blockier T2 map. For quantitative comparison, we calculated the mean T2 and standard

deviations of the entire cartilage. The mean T2 and standard deviation obtained from the T2 REPCOM map of 43.0 ms and 7.1 ms are comparable to the mean T2 and standard deviation of 42.7 ms and 7.2 ms obtained from the gold standard. The acquisition time for the gold standard and REPCOM data sets are significantly different: 17 minutes and 8 seconds for the gold standard and only 2 minutes and 12 seconds for the data used in the REPCOM reconstruction.

Figure 11 shows the anatomical images (Figure 11a–d) and corresponding T2 maps (Figure 11e–h) for a subject with a healthy liver and three subjects with different types of focal liver lesions. The lesions represent the most common type of focal liver lesions: metastasis, hemangiomas, and cysts; the last two being benign lesions. Both the anatomical images and the T2 maps were obtained from the same radial FSE data set acquired in a single breath hold. The mean T2 values of two of the metastatic lesions (arrows in Figure 11b and 11f) are 88 ms and 89 ms. The mean T2 value of the hemangioma (arrow in Figure 11c and 10g) is 170 ms. The mean T2 value of the cyst (arrow in Figure 11d and 10h) is 599 ms. The difference in T2 values among them is consistent with lesion type as previously reported (16, 31, 32). Also note that the signal intensities of the two metastatic lesions in Figure 11b are different due to the coil sensitivity modulation, which may confound diagnosis. Since the T2 maps are not affected by the sensitivity profile of the receiver RF coil, both lesions have similar T2 values. It should be pointed out that the T2 maps shown in Figure 11f–h have some degree of smoothness compared to the one in Figure 11e. The data for the maps in Figure 11f–h were acquired with a preparatory time (a 35.40 ms delay at the beginning of the echo train was used to yield anatomical images with the desired T2-weighted contrast for radiologists to perform lesion detection) which reduced the SNR, particularly in the liver parenchyma which has a shorter T2. A stronger spatial penalty was enforced on the images shown in Figure 11f–h resulting in smoother maps. Should the same spatial penalty used in Figure 11e was enforced on the data of Figure 11f–h, the reconstructed T2 map would become grainy due to the lower SNR.

Discussion

The results presented in this work demonstrate that data acquisition for parametric MR imaging can be substantially accelerated using the proposed REPCOM algorithm. REPCOM is capable of exploiting both spatial and temporal redundancies in data. Temporal sparsity is exploited by using a small number of principal components across the temporal dimension. Spatial sparsity is exploited by constraining the total variation and the l_1 norm of the wavelet coefficients of the PCC maps. The linearization of the signal model in REPCOM results in avoidance of the scale mismatch problem associated with gradient-based minimization of non-linear signal models (20, 21). As shown in Figure 6 the scaling factor required for the minimization of the data consistency in Eq. [3] is object dependent. Thus, objects with different T2 and I_0 (due to coil sensitivities, changes in T1 relaxation, etc) may require a different scaling factor precluding accurate T2 estimation for all objects being imaged (as shown in Figure 7). Although this problem could be reduced by using large number of iterations (>1000) (20), this would require very long reconstruction times. Since REPCOM is based on the linearization of the data consistency term, T2 estimation is not dependent on the scaling factor and its performance is significantly less dependent on the object characteristics. Moreover, accurate T2 estimates are obtained with only 50 iterations. Thus, REPCOM is superior to the previously developed ES and MB algorithms.

The ability of the proposed algorithm to enable reconstruction of accurate parameter maps in greatly reduced scanning time can be utilized in many clinical applications. Parameter maps obtained from highly undersampled in vivo data are presented in this study for brain,

cartilage, and abdominal imaging applications. In the case of abdominal scans, the proposed technique can be especially useful because imaging time is limited to the breath-hold period.

While this study demonstrates that the proposed method is a promising technique for rapid parametric MR imaging, further investigation is needed in several areas: The weights that are used to trade-off between the spatial and temporal sparsity constraints and data consistency were empirically determined in this study. Optimal determination of these weights remains a topic for future research. Similarly, the orthonormal wavelet transform, together with total variation, was used as the spatial sparsifying transform similar to previous CS methods. The optimal spatial transform for a particular application remains to be determined (33). There are recent studies (34) illustrating that the conjugate gradient algorithm used in this work may not yield optimal performance when it comes to solving the l_1 minimization problem. Utilization of more sophisticated solvers can further improve the performance of the proposed technique. There are conventional methods such as parallel imaging and partial k-space sampling used in accelerated imaging. While REPCOM incorporates coil sensitivity information into the reconstruction process, the incorporation of partial k-space sampling techniques as part of the reconstruction was not investigated in this study. Additional acceleration can be expected from incorporation of these techniques and further investigation is needed in this area.

The proposed method uses PCA to approximate exponential decays using a linear combination of principal components. Any approximation will only work within a certain range. In this method, the accuracy of the approximation is enforced by training the principal components for the specific T2 range and by having sufficient number of principal components (L) to ensure less than 1% error for all values in the given T2 range. The T2 range can be either estimated based on the T2 map obtained by the ES algorithm or by using prior knowledge based on the anatomy being imaged. Although larger values of L will lead to less error, if perfectly reconstructed, increasing L will increase the dimensionality of the optimization problem at the expense of the temporal sparsity constraint. Given the T2 range and TE coverage, the optimal L and optimal design of the training set remain open problems.

Because the PCA approach is an approximation there is a residual error compared to an exponential decay. This error introduces some extra noise which is T2 dependent and thus not white. As a result, there could be interference between the residual error and the underlying white noise assumption in the data consistency term. This effect may partly contribute to the noise dependent bias shown in Figure 9. However, as shown in the figure, the bias is small.

One significant advantage of the proposed method not investigated in this study is its ability to allow parameter mapping using multi-exponential signal models. Although the PCs are generated from training data of mono-exponential decay curves in this work, the algorithm can be used to reconstruct multi-exponential decays for each pixel. Suppose that the signal

originates from n distinct species: $\vec{v} = \sum_{i=1}^n \vec{v}_i$. If the signal from each species follows an exponential decay, it can be expressed as a linear combination of PCs as previously

illustrated in this study: $\vec{v}_i = \sum_{j=1}^L m_{i,j} \vec{b}_j$. It is easy to demonstrate that the combined signal from all species can also be expressed as a linear combination of PCs:

$$\vec{v} = \sum_{i=1}^n \sum_{j=1}^L m_{i,j} \vec{b}_j = \sum_{j=1}^L \left(\sum_{i=1}^n m_{i,j} \right) \vec{b}_j. \quad [10]$$

Thus, the proposed algorithm can be used to reconstruct the coefficients $\hat{m}_j = \sum_{i=1}^n m_{i,j}$, which can then be used to obtain decay rates using conventional least squares fitting. The investigation of the performance of the algorithm for multi-exponential decay models remains an important area for future research.

For the non-optimized Matlab code used here, with 8 coils, 256 k-space lines, 50 iterations, the particular penalties and their weightings, the reconstruction took 35 minutes on a desktop computer with an Intel Core 2 Quad CPU at 2.4 GHz using a single core. However, in this algorithm, the most computationally heavy step, forward and inverse NUFFT, is quite parallelizable. Thus, an order of magnitude or more reduction in reconstruction time can be expected from a parallelized implementation running on a Graphics Processing Unit (GPU).

The experimental results in this work concentrated on radial trajectories and T2 estimation. However, the algorithm is general enough to allow reconstruction of maps in other parametric imaging applications such as T1 mapping and diffusion imaging, and using other trajectories. Further studies are needed to evaluate the performance of the algorithm under these conditions.

Conclusions

In this work, it is shown that the non-linearity in the mono-exponential decay model can be removed by Principal Component Decomposition. By applying this linearization to a model-based reconstruction, the REPCOM algorithm provides a numerically well-defined method for accurately estimating non-linear MR parameters from highly undersampled data. The algorithm exploits both the temporal and spatial sparsity of the underlying data and yields accurate T2 estimates. Compared to the echo sharing and model-based algorithms previously published, we have shown that REPCOM provides better T2 estimates and has little dependence on the object. The performance of the proposed REPCOM algorithm was demonstrated in a variety of in vivo imaging applications.

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Appendix. The Generation of Principal Components

To determine the Principal Components, the following information is used: TE points, a set of training T2 values (the T2 range of interest, dependent on the application).

A T2 decay curve vector $\vec{D}_i$ of value T_2 is modeled by a mono-exponential decay with proton density (the signal intensity when TE = 0ms) normalized to 1:

$$\vec{D}_i = (e^{-TE_1/T_2}, e^{-TE_2/T_2}, \dots, e^{-TE_n/T_2})^T. \quad [A.1]$$

A training matrix **D** can be formed as

$$\mathbf{D} = (\vec{D}_1, \vec{D}_2, \dots), \quad [A.2]$$

where the number of columns is the number of training T2 values.

The Principal Components are then computed by the eigenvalue decomposition of the correlation matrix $\mathbf{D}\mathbf{D}^T$ (or use the SVD of **D**), which is:

$$\mathbf{D}\mathbf{D}^T = \mathbf{B} \mathbf{\Sigma} \mathbf{B}^T, \quad [A.3]$$

where **B** is the matrix of the PC vectors:

$$\mathbf{B} = (\vec{b}_1, \vec{b}_2, \dots, \vec{b}_n), \quad [A.4]$$

Given L , the number of PCs desired, the truncated matrix of the PC vectors $\hat{\mathbf{B}}$ is then determined:

$$\hat{\mathbf{B}} = (\vec{b}_1, \vec{b}_2, \dots, \vec{b}_L) \quad [\text{A.5}]$$

Figure 1.

Percent error between the true T2 and the T2 estimated from the characterization of the first 2–4 PCCs. Simulations were performed on noiseless data using 16 TE time points ranging from 9 ms to 144 ms in steps of 9 ms. The training set consists of T2 curves with T2 values from 45 ms to 500 ms.

Figure 2.
PCC Maps of the first 3 PCs derived from Cartesian SE images fully sampled at all TE time points.

Figure 3.

(a) T2 map calculated from 16 fully sampled TE images using conventional least squares fitting. (b) T2 map recovered from the three PCC maps shown in Figure 2. (c) Percent error of T2 map in (b) taking (a) as the gold standard.

Figure 4. (a) Reference anatomical brain image, (b) gold standard brain T2 map reconstructed from radial FSE data with 256 radial k-space lines per TE. T2 maps reconstructed with (c) REPCOM, (d) MB, and (e) the ES algorithms using the same undersampled radial FSE data set (16 k-space lines per TE). The data were acquired with an 8-channel head coil, ETL=16, echo spacing= 9.07 ms, slice thickness = 5 mm, receiver bandwidth = ± 31.25 kHz, TR = 4 s, FOV = 22 cm. Data for the gold standard were obtained from a different scan than the undersampled data set. The scaling factor used for the MB algorithm is 90.

Figure 5.

Percent error of mean T2 estimates for simulated circles of varying diameters (4, 6 and 8 pixel diameter) embedded in a background (112 pixels in diameter) with varying T2's. The true T2 of the circles is 100 ms. The mean T2 estimates were calculated from 100 noise realizations. Simulations were performed for 16 TE points and 16 k-space lines per TE (256 data points per radial line). The echo spacing was 9.00 ms and the SNR corresponding to TE = 60 ms was set to 25. For convenience and consistency of notation, $T2_{\text{background}} = 0$ ms represents a phantom with no background. In the simulations it was ensured that the T2 range for the training set covered object T2 and $T2_{\text{background}}$.

Figure 6.

Percent error of the mean T2 estimates calculated by the MB algorithm vs. scaling factor (sf in Eq. 3) for various I_0 (i.e., signal intensity at $TE = 0$) and T2 values. The object and the background have the same uniform I_0 . The simulations are for three 6-pixel diameter circular objects with $T_2 = 230$ ms, 150 ms and 80 ms embedded in a background of 112 pixels in diameter. Noiseless data were used in the simulations with 16 k-space lines per TE and 16 TE points.

Figure 7.

T2 maps of the two physical phantoms and T2 estimates obtained by REPCOM, MB and ES algorithms compared to the gold standard. For the MB algorithm the best scaling factor were selected from a range of 1 to 120. Data were acquired with radial FSE with ETL = 16 and acquisition matrix = 256×256 to yield 16 k-space lines per TE for the undersampled data and 256 k-space lines per TE for the gold standard. The echo spacing was 8.29 ms. The number of iterations used in REPCOM and MB was 50 and 200, respectively.

Figure 8.

Effect of spatial sparsities in the T2 maps reconstructed with REPCOM. (a, b, d, e) T2 maps reconstructed with REPCOM from radial FSE data acquired with ETL=16 and 16 radial lines per TE. (c, f) Corresponding gold standard T2 maps reconstructed from radial FSE data acquired in a separate scan with ETL=16 and 256 radial lines per TE. Data in (d–f) were acquired with a preparatory time = 35.20 ms between the 90° excitation pulse and the train of 180° refocusing pulses to yield a lower SNR data set. The T2 maps in (a, d) were reconstructed without spatial penalties. The T2 maps in (b, e) were reconstructed with spatial penalties. Other imaging parameters are echo spacing = (a–c) 9.07 ms or (d–f) 8.80 ms, FOV = 22 cm, slice thickness = 5 mm, number of readout points = 256.

Figure 9.

(a, b) The error of REPCOM T2 estimates for various SNR levels. The simulations are for three 6-pixel (or 12-pixel) diameter circular objects with $T_2 = 230$ ms, 150 ms and 80 ms embedded in a background of 112 pixels in diameter ($T_{2\text{background}} = 50$ ms). I_0 was set to be 1 for all pixels in the phantom. Data were generated in radial FSE acquisition style with 16 TE points, 16 radial k-space lines per TE, and echo spacing = 9 ms. SNR_{60} represents the SNR at TE = 60 ms which is the approximate TE contrast of anatomical images reconstructed from all radial FSE k-space lines. One hundred noise realizations were used in the simulations.

Figure 10.

Color T2 map of cartilage (overlaid onto its corresponding anatomical knee image) from an asymptomatic young male adult. The mean T2 and standard deviation of the cartilage ROI obtained from the REPCOM T2 map are 43.0 ms and 7.1 ms, respectively. These values are comparable to the mean T2 (42.7 ms) and standard deviations (7.2 ms) of the gold standard. Data were acquired with radial FSE with ETL = 8, echo spacing = 8.38 ms, TR = 2 s, slice thickness = 5 mm, FOV = 16 cm, receiver bandwidth = ± 31.25 kHz, NEX = 2, acquisition matrix of 256×2048 (yields 256 k-space lines per TE, for gold standard), 256×256 (yields 32 k-space lines per TE, for REPCOM). The range of the T2 values of the training set for the REPCOM reconstruction was 20 – 100 ms. REPCOM data were acquired in only 2 minutes and 12 seconds compared to the 17 minutes and 8 seconds acquisition needed for the gold standard.

Figure 11.

(a–d) Anatomical images and (e–h) corresponding T2 maps obtained by REPCOM for abdominal images representative of (a) a healthy liver and three most common focal liver lesions: (b) metastasis, (e) hemangiomas and (d) cysts. The mean T2 values of the two metastatic lesions (arrows in b and f) are 88 ms and 89 ms. The mean T2 value of the hemangioma (arrow in c and g) is 170 ms. The mean T2 value of the cyst (arrow in d and h) is 599 ms. Data were acquired with radial FSE with ETL=16 and echo spacings: 8.76 ms (a), 8.80 ms (b), 9.07 ms (c), and 8.84 ms (d). Other acquisition parameters were TR=1.8 s (a, c, d) and 1.5 s (b), slice thickness = 8 mm, receiver bandwidth = ± 31.25 kHz, and acquisition matrices of 256×256 (a, b) and 256×192 (c, d). Data used for (b–d) were acquired with a preparatory time to yield higher T2-weighting for better lesion detection and are thus noisier compared to (a).

Table 1

The percent error of the mean T2 values calculated with REPCOM for three 6-pixel diameter objects with T2= 80 ms, 150 ms, and 230 ms with various background conditions. Simulations were performed with the same noiseless data used in Figure 6.

T2 of the object	230 ms	150 ms	80 ms
$I_0 = 1$, No background	3.67%	-0.76%	1.73%
$I_0 = 1$, $T_{2\text{background}} = 50$ ms	0.20%	0.21%	0.52%
$I_0 = 1.5$, $T_{2\text{background}} = 50$ ms	0.20%	0.22%	0.54%
$I_0 = 1$, $T_{2\text{background}} = 100$ ms	-0.67%	0.55%	0.95%

T2 estimates (mean $\pm$ standard deviation) obtained with REPCOM for three circular objects with various numbers of TE points and total number of k-space lines. The data were obtained using a numerical phantom. The diameter of the objects is 6 pixels, the T2's are (a) 230 ms, (b) 150 ms and (c) 80 ms. These objects were embedded into a 112-pixel diameter circular background. For the various numbers of TE, the same TE coverage was enforced. TE points are equally spaced. Complex Gaussian noise was added to the k-space such that the SNR at TE = 60 ms is about 25 for the object with T2 = 80 ms.

Table 2

	# lines	128	256	512	1024
T2 truth = 230 ms	ETL = 4	238.14 $\pm$ 8.12	234.14 $\pm$ 5.39	231.72 $\pm$ 3.98	234.04 $\pm$ 2.55
	ETL = 8	237.80 $\pm$ 7.10	233.90 $\pm$ 5.30	231.82 $\pm$ 4.24	232.15 $\pm$ 3.62
	ETL = 16	233.64 $\pm$ 7.04	233.85 $\pm$ 5.13	233.76 $\pm$ 3.44	232.64 $\pm$ 2.99
T2 truth = 150 ms	ETL = 4	154.63 $\pm$ 3.35	151.70 $\pm$ 2.71	152.27 $\pm$ 1.93	150.89 $\pm$ 1.38
	ETL = 8	154.45 $\pm$ 3.76	151.44 $\pm$ 2.81	151.70 $\pm$ 1.67	150.11 $\pm$ 1.66
	ETL = 16	152.62 $\pm$ 3.27	151.52 $\pm$ 2.30	152.45 $\pm$ 1.73	150.37 $\pm$ 1.85
T2 truth = 80 ms	ETL = 4	81.25 $\pm$ 2.50	80.57 $\pm$ 1.79	80.27 $\pm$ 1.25	80.27 $\pm$ 0.88
	ETL = 8	81.18 $\pm$ 2.01	80.30 $\pm$ 1.43	80.37 $\pm$ 0.88	80.15 $\pm$ 0.69
	ETL = 16	80.87 $\pm$ 1.68	80.35 $\pm$ 1.17	80.64 $\pm$ 0.78	80.28 $\pm$ 0.68