A Fast Region-Based Active Contour Model for Boundary Detection of Echocardiographic Images

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Abstract This paper presents the boundary detection of atrium and ventricle in echocardiographic images. In case of mitral regurgitation, atrium and ventricle may get dilated. To examine this, doctors draw the boundary manually. Here the aim of this paper is to evolve the automatic boundary detection for carrying out segmentation of echocardiography images. Active contour method is selected for this purpose. There is an enhancement of Chan–Vese paper on active contours without edges. Our algorithm is based on Chan–Vese paper active contours without edges, but it is much faster than Chan–Vese model. Here we have developed a method by which it is possible to detect much faster the echocardiographic boundaries. The method is based on the region information of an image. The region-based force provides a global segmentation with variational flow robust to noise. Implementation is based on level set theory so it easy to deal with topological changes. In this paper, Newton–Raphson method is used which makes possible the fast boundary detection.

Keywords Echocardiographic images · Mitral regurgitation · Atrium · Ventricle · Active contours · Level set theory · Topological

Introduction

Leakage of blood from the left ventricle into the left atrium during systole is called mitral regurgitation. It is caused by various mechanisms related to structural or functional abnormalities of the mitral apparatus, adjacent myocardium, or both. The most common causes of mitral regurgitation are rheumatic heart disease, myxomatous degeneration, chordal rupture, infective endocarditis, coronary artery disease, and cardiomyopathy. About 2% of the population with a similar prevalence in males and females have significant mitral valve regurgitation [1].

In case of significant MR, left ventricle has to accommodate both the stroke volume and regurgitant volume with each heart beat so it leads to volume overload of the left ventricle. The left ventricle dilates and becomes hyperdynamic for compensation. The left atrial and pulmonary venous pressures increase quickly in case of acute severe MR, leading to pulmonary congestion and pulmonary edema. A gradual increase in left atrial size and compliance compensates in chronic MR, so that left atrial and pulmonary venous pressures do not increase until late in the course of the disease. An increase in after load, contractile dysfunction, and heart failure occur in case of progressive left ventricular dilation. Left atrial enlargement leads the patient to atrial fibrillation and arterial thromboembolism. Patients may develop pulmonary hypertension and right-sided heart failure in long-standing MR. A study has been done in [2] for finding accuracy of transesophageal echocardiography in functional mitral regurgitation estimation. Quantification of mitral regurgitation, hemodynamic parameters, and statistical analysis is included in the study. Transthoracic echocardiography and transesophageal echocardiography echo Doppler data have been compared with hemodynamic, angiographic, and surgical data for

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patients with acute severe MR [3]. Accuracy of the proximal isovelocity surface area method for quantitative assessment of the severity of mitral regurgitation in children has been investigated in [4]. Progression of mitral regurgitation has been studied in [5] with Doppler echocardiography.

Two-dimensional echocardiography is a good method for finding out left atrial and ventricle dilation in case of mitral regurgitation. There are many papers in the literature which discuss the different measurements related to left atrium and left ventricle [6–10].

There are different techniques for left atrium and ventricle boundary detection. In [11], the LV center and LV edge points were detected using neural network classifiers with new input feature vectors. These neural classifiers combined with knowledge-based techniques in the center detection stage refine the center estimate. In [12], double thresholding method is used for left ventricle endocardium detection. First step of this method was to suppress the noise by taking mean of two adjacent frames four times. Next step was ternary thresholding in which two thresholds divide the image into three regions. Final step was second thresholding, in which a binarized image is obtained. In [13], snake-based algorithm is used for endocardial boundary detection. A correlation between geometrically normalized images from the patient and from the database is also proposed.

Here the objective is to detect the left atrium and left ventricle boundaries using active contour method. Active contour model is firstly proposed in [14]. There are many research papers which are based on deformable models or active contour models [15–21]. An energy minimizing deformable spline influenced by constraint and image forces that pull it toward object contours is called active contour. Snakes are largely used in applications like boundary detection object tracking, shape recognition, segmentation, edge detection, stereo matching [14], etc.

Energy Minimization Formulation and Level Set Method

The basic objective of the energy minimizing formulation of deformable contours is to find a parameterized curve that minimizes the weighted sum of internal energy and external energy. The internal energy specifies the tension or the smoothness of the contour. The external energy is defined over the image domain and typically possesses local minima at the image intensity edges occurring at object boundaries. Minimizing the total energy yields internal forces and external forces. Internal forces have two parts: Elasticity forces hold the curve together and bending energy keeps it from bending too much. External forces attract the curve toward the desired object boundaries.

Deformable contour is a curve $X(s)=(X(s), Y(s)), s \in [0, 1]$ which moves through the spatial domain of an image to minimize the following energy functional:

$$E(X) = E_{\text{int}}(X) + E_{\text{ext}}(X) \quad (1)$$

Internal energy of snake is the summation of elastic energy and bending energy and is given as:

$$E_{\text{int}} = \frac{\partial}{\partial S} \left(\alpha \frac{\partial X}{\partial S} \right) - \frac{\partial^2}{\partial S^2} \left(\beta \frac{\partial^2 X}{\partial S^2} \right) \quad (2)$$

The first term discourages stretching and makes the model behave like an elastic string. The second term discourages bending and makes the model behave like a rigid rod. The external energy is given as the potential energy:

$$E_{\text{ext}} = -\nabla P(X) \quad (3)$$

External energy attracts the curve toward the desired object boundaries. To find the object boundary, parametric curves are initialized within the image domain and are forced to move toward the potential energy minima under the influence of both the forces.

The level set method has the property of automatic topology adaptation. In the level set method, the curve is represented implicitly as a level set of a 2-D scalar function referred to as the level set function which is usually defined on the same domain as the image. The level set is defined as the set of points that have the same function value. Instead of tracking a curve through time, the level set method evolves a curve by updating the level set function at fixed coordinates through time. This perspective is similar to that of a Eulerian formulation of motion. An advantage of level set function is that it remains a valid function while the embedded curve can change its topology.

Given level set function $\phi[x, y, t]$ with contour $X(s, t)$, as its zero level set, we have

$$\phi[X(s, t), t] = 0 \quad (4)$$

Differentiating Eq. 4 with respect to t and using the chain rule, we obtain:

$$\frac{\partial \phi}{\partial t} + \nabla \phi \cdot \frac{\partial X}{\partial t} = 0 \quad (5)$$

where $\nabla \phi$ denotes the gradient of ϕ .

Assume that ϕ is negative inside the zero level set and positive outside. Accordingly, the inward unit normal to the level set curve is given by:

$$N = -\frac{\nabla \phi}{|\nabla \phi|} \quad (6)$$

Using the curve evolution theory, we can rewrite Eq. 5 as

$$\frac{\partial \phi}{\partial t} = V(k)|\nabla \phi| \quad (7)$$

where k , the zero level set, is given by

$$k = \nabla \frac{\nabla \phi}{|\nabla \phi|} = \frac{\phi_{xx}\phi_y^2 - 2\phi_x\phi_y\phi_{xy} + \phi_{yy}\phi_x^2}{\phi_x^2 + \phi_y^2} \quad (8)$$

Chan–Vese Model for Boundary Detection

Chan–Vese proposed a model for boundary detection in which stopping term did not depend on the gradient of the image, as in conventional active contours [22]. An image u_0 is usually considered a real-valued bounded function defined on $\overline{\Omega}$, where Ω is a bounded and open subset of R^2 with $\partial\Omega$ as its boundary.

According to level set theory originally proposed by Osher and Sethian in [23], a geometric active contour can be represented by zero level set of real-valued function $\Phi: \Omega \subset R^2 \rightarrow R$ which evolves in an image u_0 according to variational flow in order to segment the object from the image background.

Chan–Vese active contour algorithm comes from segmentation problem formulated by Mumford and Shah. Given an observed image u_0 , find decomposition Ω_i of Ω and an optimal piecewise smooth approximation u of u_0 , such that u varies smoothly within each Ω_i and rapidly or discontinuously across the boundaries of Ω_i .

To solve this problem, Mumford and Shah [24] proposed the following minimization problem:

$$\inf \left\{ F^{MS}(u, C) = \int_{\Omega} (u - u_0)^2 dx dy + \mu \int_{\Omega \setminus C} |\nabla u|^2 dx dy + \nu |C| \right\} \quad (9)$$

A reduced case of the model is obtained by restricting the segmented image u to piecewise constant function, i.e., $u = \text{constant } c_i$ inside each connected component Ω_i . Then this problem is called “minimal partition problem” and its functional is:

$$E^{MS}(u, c) = \sum_i \int_{\Omega} (u - c_i)^2 dx dy + \nu |C| \quad (10)$$

It is easy to see that, for fixed C , the energy from Eq. 10 is minimized in the variable c_i by setting

$$c_i = \text{mean}(u_0) \text{ in } \Omega_i \quad (11)$$

By the functional for “minimal partition problem” and given the curve $C = \partial\omega$, with $\omega \subset \Omega$ an open subset and two unknown constants c_1 and c_2 denoting $\Omega_1 = \omega$, $\Omega_2 = \Omega - \omega$. They have proposed to minimize the following energy with respect to c_1, c_2 , and C .

$$\begin{aligned} F(c_1, c_2, C) = & \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(\text{inside}(C)) \\ & + \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy \\ & + \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy \end{aligned} \quad (12)$$

With level set formulation

$$\begin{aligned} C = \{(x, y) | \phi(x, y) = 0\} F(c_1, c_2, \phi) = & \mu \int_{\Omega} \delta(\phi(x, y)) |\nabla \phi(x, y)| dx dy \\ & + \nu \int_{\Omega} H(\phi(x, y)) dx dy + \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 H(\phi(x, y)) dx dy \\ & + \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) dx dy \end{aligned} \quad (13)$$

where $H(\cdot)$ is Heaviside function and $u_0(x, y)$ is input image.

c_1 and c_2 are the mean intensities inside and outside the active contour defined as follows:

$$c_1 = \frac{\int_{\Omega} u_0(x, y) H(\phi(x, y)) dx dy}{\int_{\Omega} H(\phi(x, y)) dx dy} \quad (14)$$

$$c_2 = \frac{\int_{\Omega} u_0(x, y) (1 - H(\phi(x, y))) dx dy}{\int_{\Omega} (1 - H(\phi(x, y))) dx dy} \quad (15)$$

Chan–Vese used the gradient decent equation to find the evolution equation as follows:

$$\frac{\partial \phi}{\partial t} = \delta_{\epsilon}(\phi) \left[\mu \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] \quad (16)$$

Proposed Method

Chan–Vese used gradient descent method for finding evolution. Here in this paper the aim is to find the evolution equation using Newton–Raphson method. The method differs from the steepest descent in that the information of the second derivative is used to locate the minimum of the function. This results in faster conver-

Fig. 1 Images for boundary detection in different views of heart. **a** Apical four-chamber view of heart. **b** Parasternal long axis view of heart. **c** Parasternal long axis view of heart. **d** Parasternal long axis view of heart. **e** Apical four-chamber view of heart. **f** Parasternal long axis view of heart. **g** Parasternal long axis view of heart. **h** Parasternal long axis view of heart. **i** Parasternal long axis view of heart. **j** Apical two-chamber view of heart

gence. This iterative process follows a set guideline to approximate one root, considering the function, its derivative, and an initial x -value. The Newton–Raphson method uses an iterative process to approach one root of a

Fig. 2 Boundary detection of left ventricle and left atrium with Chan–Vese method in different view of heart. **a** Left ventricle boundary detection in apical four-chamber view of heart. **b** Left ventricle boundary detection in parasternal long axis view of heart. **c** Left atrium boundary detection in parasternal long axis view of heart. **d** Left atrium boundary detection in parasternal long axis view of heart. **e** Left ventricle boundary detection in apical four-chamber view of heart. **f** Left atrium boundary detection in parasternal long axis view of heart. **g** Left atrium boundary detection in parasternal long axis view of heart. **h** Left atrium boundary detection in parasternal long axis view of heart. **i** Left atrium boundary detection in parasternal long axis view of heart. **j** Left atrium boundary detection in apical two-chamber view of heart

Fig. 3 Boundary detection of left ventricle and left atrium with proposed method in different view of heart. **a** Left ventricle boundary detection in apical four-chamber view of heart. **b** Left ventricle boundary detection in parasternal long axis view of heart. **c** Left atrium boundary detection in parasternal long axis view of heart. **d** Left atrium boundary detection in parasternal long axis view of heart. **e** Left ventricle boundary detection in apical four-chamber view of heart. **f** Left atrium boundary detection in parasternal long axis view of heart. **g** Left atrium boundary detection in parasternal long axis view of heart. **h** Left atrium boundary detection in parasternal long axis view of heart. **i** Left atrium boundary detection in parasternal long axis view of heart. **j** Left atrium boundary detection in apical two-chamber view of heart

Table 1 Comparison between Chan–Vese and proposed algorithm based on time and iterations taken by two methods to complete the boundary detection

Images	Mask	Chan–Vese method		Proposed method	
		Iterations	Time (s)	Iterations	Time (s)
1	50×50	400	20.83	300	15.61
2	60×50	400	22.33	300	16.78
3	55×50	300	16.73	250	14.09
4	122×70	700	38.53	500	27.42
5	100×100	350	19.46	250	13.54
6	100×60	230	14.08	200	12.29
7	50×80	230	13.97	200	12.14
8	60×40	350	20.96	300	17.99
9	80×80	250	15.21	200	12.05
10	100×100	250	15.16	200	12.18

function. The specific root that the process locates depends on the initial arbitrarily chosen x -value.

$$X_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (17)$$

Now applying the Newton–Raphson method on Eq. 13 for finding out evolution equation:

$$\frac{\partial \phi}{\partial t} = \left\{ \delta_\epsilon(\phi) \left[\mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - v - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] \right\}^{-1} \bullet F_\epsilon(c_1, c_2, \phi) \quad (18)$$

Fig. 4 Comparison between Chan–Vese and proposed method based on time taken to complete the boundary detection

Fig. 5 Comparison between Chan–Vese and proposed method based on iterations taken to complete the boundary detection

Fig. 6 Comparison between Chan–Vese and proposed method based on mean

Let h be the space step and Δt the time step and then the discretization of Eq. 18 is as follows:

$$\frac{\phi_{i,j}^{N+1} - \phi_{i,j}^N}{\Delta T} = \left\{ \delta_h(\phi_{i,j}^n) \left[\frac{\mu}{h^2} \Delta^x \left(\frac{\Delta_+^x \phi_{i,j}^n}{\sqrt{(\Delta^x + \phi_{i,j}^n)^2 / (h)^2 + (\phi_{i,j+1}^n - \phi_{i,j-1}^n)^2 / (2h)^2}} \right) \right. \right. \\ \left. \left. + \frac{\mu}{h^2} \Delta^y \left(\frac{\Delta_+^y \phi_{i,j}^{n+1}}{\sqrt{(\phi_{i+1,j}^n - \phi_{i-1,j}^n)^2 / (2h)^2 + (\Delta_+^y \phi_{i,j}^n)^2 / h^2}} \right) - v - \lambda_1 (u_{o,i,j} - c_1(\phi^n)^2) + \lambda_2 (u_{o,i,j} - c_2(\phi^n)^2) \right] \right\}^{-1} \bullet F_h(c_1, c_2, \phi_{i,j}^n) \quad (19)$$

Experimental Results

In this section, some results have been shown which prove that the proposed algorithm gives the response faster than that given by the method proposed by Chan–Vese. Figure 1 shows the

input images. These images are original images taken from Cardiology Department in Postgraduate Institute of Medical and Education Research (PGIMER) Chandigarh, India.

Patients suffering from mitral regurgitation are taken for the experiment. Here 10 such images are presented as

Table 2 Comparison between Chan–Vese and proposed algorithm-based statistical analysis

Images	Chan–Vese method				Proposed method			
	Mean	Median	Standard deviation	Variance	Mean	Median	Standard deviation	Variance
1	7.72	7.77	4.35	7.76	6.24	6.27	3.50	12.25
2	11.34	11.32	6.42	41.23	8.53	8.54	4.82	23.22
3	8.52	8.55	4.82	23.21	7.15	7.18	4.034	16.31
4	19.41	19.39	11.08	122.83	13.86	13.89	7.88	62.09
5	9.84	9.83	5.58	31.15	6.89	6.94	3.87	15.00
6	7.17	7.16	4.03	16.25	6.29	6.32	3.53	12.53
7	7.12	7.11	4.00	16.00	6.23	6.26	3.48	12.14
8	10.62	10.63	6.02	36.20	9.17	9.21	5.17	26.69
9	7.73	7.76	4.36	19.014	6.17	6.19	3.46	11.98
10	7.73	7.77	4.35	7.76	6.24	6.27	3.50	12.25

Fig. 7 Comparison between Chan–Vese and proposed method based on median

example. The images would be obtained by a trained echocardiographer using standard parasternal and apical views of heart. In these cases, it is easy to see atrium and ventricle.

Figure 2 shows the boundaries detected by Chan–Vese method, the boundary detected images corresponding to the images which are in Fig. 1. Images 1 and 5 are apical four-chamber view of heart. In these images, the boundary of left ventricle has been detected. In image 2, also left ventricle has been detected but this is the case of parasternal long axis view. In images 3, 4, 6, 7, 8, 9, and 10, the boundary of left atrium is detected. These are all parasternal long axis view of heart.

Figure 3 shows the boundaries detected by proposed method. It can easily be seen that boundaries are approximately same; however, proposed method takes less time and iteration for detecting same boundary. First and most

Fig. 8 Comparison between Chan–Vese and proposed method based on standard deviation

Fig. 9 Comparison between Chan–Vese and proposed method based on variance

important step is to decide the position of the mask from where the contour is to start to converge on the boundaries. This becomes possible with the MATLAB tool. Once the position of contour is decided, the boundary of the atrium and ventricle becomes automatic with the help of the present algorithm.

Table 1 shows the comparison between the Chan–Vese’s and the proposed algorithm boundary detection techniques. From this table, we can see that according to image, masks are of different size. Figures 4 and 5 show the chart corresponding the data in the table. Figure 4 shows the comparison of Chan–Vese method and proposed method based on time. Bars related to proposed method cover greater value of time. Figure 5 shows the comparison based on iterations. In this case, also bars related to proposed method cover greater value of iterations than that of Chan–Vese.

Proposed algorithm presents an automatic boundary detection technique. It has also improved the speed of the contour to converge on the boundaries. Here the no. of iterations and the time taken to converge are considered for comparison. In case of the no. of iterations, it is noticed that there is a difference of 100 iterations in case of images 1, 2, and 5. Proposed algorithm requires 100 iterations fewer than the Chan–Vese algorithm takes. Similarly, in case of images 3, 8, 9, and 10, proposed algorithm requires 50 fewer iterations. In case of images 5 and 7, this gap is of 30 iterations, and in case of image 4, it has reached to 200 fewer iterations.

Time difference is also there. Proposed algorithm takes less time to find desired boundary. According to the observations, images 1 through 10 take time less by 5.22, 5.55, 2.64, 11.11, 5.92, 1.79, 1.83, 2.97, 3.16, and 2.98 in seconds, respectively. Time taken per iterations is also less than that taken by Chan–Vese method.

Table 2 represents the statistical analysis of the method. Mean, median, standard deviation, and variance have been taken for comparison. These four parameters have been calculated for all ten images based on Chan–Vese method as well as proposed method. There graphical representation can be shown in Figs. 6, 7, 8, and 9, respectively.

Conclusions and Discussions

The proposed algorithm presented in this paper gives a way to automatically find out the boundaries of left ventricle and left atrium which are dilated due to mitral regurgitation. All echocardiographic images are original images taken from PGIMER, Chandigarh, India. Boundary detection is required for finding out the area which is a measure of severity of mitral regurgitation. Active contour method is best suitable for this work. Chan–Vese algorithm has been enhanced in this paper. Results show that the proposed algorithm exhibits a faster response. This algorithm takes less no. of iterations and less time to get desired boundaries.

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