On the role of glottis-interior sources in the production of voiced sound

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The voice source is dominated by aeroacoustic sources downstream of the glottis. In this paper an investigation is made of the contribution to voiced speech of secondary sources within the glottis. The acoustic waveform is ultimately determined by the volume velocity of air at the glottis, which is controlled by vocal fold vibration, pressure forcing from the lungs, and unsteady backreactions from the sound and from the supraglottal air jet. The theory of aerodynamic sound is applied to study the influence on the fine details of the acoustic waveform of “potential flow” added-mass-type glottal sources, glottis friction, and vorticity either in the glottis-wall boundary layer or in the portion of the free jet shear layer within the glottis. These sources govern predominantly the high frequency content of the sound when the glottis is near closure. A detailed analysis performed for a canonical, cylindrical glottis of rectangular cross section indicates that glottis-interior boundary/shear layer vortex sources and the surface frictional source are of comparable importance; the influence of the potential flow source is about an order of magnitude smaller.

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I. INTRODUCTION

The “source-filter” theory of voicing identifies the glottis as a monopole source of volume velocity $Q(t)$, say, t being time, that radiates into the supraglottal vocal tract (Fant, 1960; Flanagan, 1972; Stevens, 1998). Quasi-steady flow through the glottis produced by contraction of the lung cavity is modulated during “voiced speech” by oscillations of the glottis cross section caused by flow-excited vibrations of the vocal folds. This generates periodic fluctuations in $Q(t)$, and source-filter theory determines the acoustic pressure as the solution of an “analogy” problem in which the glottis is replaced by a monopole of strength $Q(t)$ at the closed, lower end of the supraglottal tract. The sound pressure $p(\mathbf{x}, t)$ heard by a listener at $\mathbf{x} = (x_1, x_2, x_3)$ in free space is then given by (Rothenberg, 1981; Titze, 2008; Howe and McGowan, 2011b)

$$\frac{p(\mathbf{x}, t)}{\rho_o} = - \int_{-\infty}^{\infty} Q(\tau) \frac{\partial G_o}{\partial \tau}(\mathbf{x}, t - \tau) d\tau, \quad (1)$$

where G_o is the corresponding (source-filter) Green’s function defining the radiation from a source at the lower end of the supraglottal tract when the end is regarded as *closed*, and ρ_o is the mean air density at the listener.

This quite general result is most easily illustrated by the familiar model configuration depicted in Fig. 1(a), where the supraglottal tract is idealized as a uniform, hard-walled duct of length L and cross-sectional area A_{VT} , radiating from the

“lip opening” to the listener at $\mathbf{x}$. The subglottal tract is also rigid and uniform with cross-section area A_L and terminates in the upstream lung complex. Take the coordinate origin on the axis of symmetry of the supraglottal tract in the plane of its lower end, with x_1 increasing toward the lip opening. Green’s function G_o characterizes the sound (i.e., the velocity potential) produced by a unit source when the end $x_1 = 0$ is assumed to be closed [as in Fig. 1(b)]. When dissipation at the walls is ignored and the mean air density is uniform, and for the most important case where acoustic wavelengths are much greater than the duct diameter $\sim \sqrt{A_{VT}}$ (so that only plane waves can propagate), it is known that

$$G_o(\mathbf{x}, t - \tau) \simeq \frac{1}{8\pi^2 R} \int_{-\infty}^{\infty} \frac{e^{-i\omega(t-\tau-R/c_o)} d\omega}{\cos\{k_o(L + \ell_E + ik_o A_{VT}/4\pi)\}}, \quad (2)$$

$k_o = \omega/c_o,$

where $R \gg \sqrt{A_{VT}}$ is the distance to $\mathbf{x}$ from the lip opening at $x_1 = L$, ℓ_E is the end-correction for the area of the lip opening (Howe, 1998; Rayleigh, 1945), and c_o is the speed of sound. The integrand has simple poles at the zeros of the cosine term at values of ω equal approximately to $\pm\omega_1, \pm\omega_2, \pm\omega_3, \dots$ together with small negative imaginary parts that account for damping of supraglottal tract resonances by radiation from the mouth (the only dissipation in this simple model), where $f_n = \omega_n/2\pi$ is the n th formant. The widths of the peaks of the integrand about these formant frequencies govern the “filtering” characteristics of the system.

The voiced sound pressure at $\mathbf{x}$ can also be expressed in terms of aeroacoustic sources within the vocal tract by means of a Green’s function $G(\mathbf{x}, \mathbf{y}, t, \tau)$ valid for arbitrary source points $\mathbf{y} = (y_1, y_2, y_3)$ anywhere within the entire

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FIG. 1. (a) Basic model of the subglottal and supraglottal vocal tracts. (b) Reduced geometry used in the source-filter representation of the sound at the free space point $\mathbf{x}$ generated by a point source of strength $Q(t)$ at the nominal position of the glottis when the glottal end of the supraglottal tract is assumed to be closed.

vocal tract. This is an *advanced potential* propagating as a function of $(\mathbf{y}, \tau)$ that satisfies

$$\left(\frac{1}{c_o^2} \frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial y_j^2} \right) G = \delta(\mathbf{x} - \mathbf{y}) \delta(t - \tau), \quad \text{where} \\ G = 0 \quad \text{for} \quad \tau > t \quad (3)$$

and the condition $\partial G / \partial y_n = 0$ on the moving surface $S(\tau)$, say, of the vocal tract, where y_n is a local normal coordinate directed into the fluid. Then (Howe, 2008; Howe and McGowan, 2007, 2011a)

$$\frac{p(\mathbf{x}, t)}{\rho_o} = \int_{-\infty}^{\infty} \oint_{S(\tau)} \left[G \frac{\partial \mathbf{v}}{\partial \tau} - \nu \frac{\partial G}{\partial \mathbf{y}} \wedge \boldsymbol{\omega} \right] \cdot d\mathbf{S}(\mathbf{y}) d\tau \\ - \int_{-\infty}^{\infty} \int_{V(\tau)} \frac{\partial G}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y} d\tau, \quad (4)$$

where $\mathbf{v}$ denotes velocity, $\boldsymbol{\omega} = \text{curl } \mathbf{v}$ is the vorticity, ν is the kinematic viscosity, the vector surface element $d\mathbf{S}$ on $S(\tau)$ is directed into the fluid, and $V(\tau)$ is the region occupied by the fluid at time τ .

This formula expresses the production of sound in terms of the surface motions and unsteady flow within the vocal tract, and can be used to derive the “Fant” equation governing the volume velocity $Q(t)$. The first term in the square brackets of the surface integral includes the direct contribution from lung contraction and from surface motions within and near the glottis. The latter resembles a surface monopole, but also accounts for the influence of variations in the vocal fold “added mass” (Lighthill, 1978). The viscous source is concentrated principally within the glottis. The vortex source $\boldsymbol{\omega} \wedge \mathbf{v}$ produces sound of dipole type. Because $\partial G / \partial y_n$ vanishes on $S(\tau)$, its main contribution is from the jet just downstream of the glottis, and from surface boundary layers within the glottis, where glottal wall motions cause $\mathbf{v}$ to have a finite component in the y_n direction.

The function G can be interpreted as a velocity potential of flow having vanishing normal velocity on the instantaneous positions of the vocal tract walls. Thus, within and near the glottis, which is small compared to the characteristic acoustic wavelength, we can put

$$G = \alpha(\tau) + \beta(\tau) Y(\mathbf{y}, \tau) \quad (5)$$

where $\alpha(\tau)$, $\beta(\tau)$ do not depend on $\mathbf{y}$, and where the function $Y(\mathbf{y}, \tau)$ (the “Kirchhoff vector”) is a solution of Laplace’s equation that represents the velocity potential of ideal incompressible flow through the glottis [with $\partial Y / \partial y_n = 0$ on $S(\tau)$, Howe, 2008]. It is usual to normalize $Y(\mathbf{y}, \tau)$ such that

$$Y(\mathbf{y}, \tau) \sim \begin{cases} y_1 A_L / A_{VT} & \text{for } |\mathbf{y}| \gg \sqrt{A_g} \text{ in the supraglottal tract} \\ y_1 - \ell(\tau) & \text{for } |\mathbf{y}| \gg \sqrt{A_g} \text{ in the subglottal tract} \end{cases}, \quad (6)$$

where A_g is the minimum cross-sectional area of the glottis, and $\ell(\tau)$ is the effective length of the fluid slug engaged in coherent motion through the glottis, which is given in terms of the Rayleigh conductivity $K_g \equiv K_g(\tau)$ ($\leq 2\sqrt{A_g}/\pi$) of the glottis by (Rayleigh, 1945; Howe, 1998)

$$\ell(\tau) = \frac{A_L}{K_g(\tau)}. \quad (7)$$

Because ℓ depends on time it is not usually possible to obtain a closed analytical formula for $G(\mathbf{x}, \mathbf{y}, t, \tau)$, even for the geometrically simple model of Fig. 1(a). However, Eq. (4) can be manipulated into the form

$$\frac{p(\mathbf{x}, t)}{\rho_o} = \int_{-\infty}^{\infty} \beta(\tau) \mathcal{F}(\tau) d\tau, \quad (8)$$

where $\mathcal{F}(\tau)$ depends on the vocal tract aeroacoustic sources on the right of Eq. (4), and the function $\beta(\tau)$ can be shown to satisfy

$$\mathcal{L}(\tau) \beta(\tau) = \frac{\partial G_o}{\partial \tau}(\mathbf{x}, t - \tau), \quad (9)$$

where $\mathcal{L}(\tau)$ is an integro-differential operator and G_o is the source-filter Green’s function of Eq. (1). The details of these calculations are discussed in Howe and McGowan (2011b). The properties of $\mathcal{L}(\tau)$ depend on the geometrical and mechanical details of the glottis and vocal tract.

The volume velocity $Q(t)$ can be determined from Eqs. (1), (8), and (9) without detailed knowledge of $G(\mathbf{x}, \mathbf{y}, t, \tau)$ or $\beta(\tau)$ by introducing the adjoint operator $\hat{\mathcal{L}}(\tau)$. This satisfies

$$\int_{-\infty}^{\infty} \left(Q(\tau) \mathcal{L}(\tau) \beta(\tau) - \beta(\tau) \hat{\mathcal{L}}(\tau) Q(\tau) \right) d\tau = 0 \quad (10)$$

because the integrand is a perfect differential and $\beta(\tau)$, $Q(\tau)$ respectively vanish at $\tau = \pm\infty$. It follows from Eqs. (1), (8),

and (9) that $Q(t)$ is the solution of *Fant's* equation (Fant, 1960)

$$\hat{L}(t)Q(t) = -\mathcal{F}(t). \quad (11)$$

For complex systems this equation must be solved numerically to determine $Q(t)$ in terms of the aeroacoustic sources $\mathcal{F}(t)$. The solution is used in Eq. (1) to calculate the voiced speech at $\mathbf{x}$.

Equation (11) forms the basis of a “reduced complexity” treatment (Howe and McGowan, 2007, 2011a, b), as opposed to the more involved method of calculation by direct numerical simulation of the entire structural and compressible motion (Zhao *et al.*, 2002; Zhang *et al.*, 2002; Hofmans *et al.*, 2003; Thomson *et al.*, 2005; Duncan *et al.*, 2006). These more general approaches are computationally intensive and often cannot be run in a timely manner for more than one or two voicing cycles. In principle they can account for all possible mechanisms that contribute to the sound, including the backreaction on the glottis of supraglottal resonances, although the latter still awaits to be explored. Such backreactions can easily be included in Eq. (11) (Howe and McGowan, 2011b), as well as in many numerical schemes that use alternative approximations to Eq. (11) (e.g., Flanagan and Landgraf, 1968; Gupta *et al.*, 1973; Zanartu *et al.*, 2007; Titze, 2008).

In this paper the general form of the source function $\mathcal{F}(t)$ is discussed in Sec. II. A simplified glottis of rectangular cross section is used in Sec. III to obtain explicit representations of sources contributing to $\mathcal{F}(t)$ from within the glottis. The corresponding Fant equation is examined numerically in Sec. IV to assess the relative contributions of the glottis sources to voiced speech.

II. THE AEROACOUSTIC SOURCE FUNCTION $\mathcal{F}(t)$

A. General representation of the sound

To fix ideas, our general discussion of the representation (4) of the acoustic pressure will be framed in terms of the simple model of Fig. 1(a). Lung contraction (represented in the figure by peripheral contraction of a section of the subglottal tract centered at $x_1 = -\ell_L$) produces a slowly varying overpressure $p_L(t - x_1/c_o)$, say, traveling toward the glottis (Howe and McGowan, 2007). The resulting glottis volume velocity $Q(t)$ is modulated by vibration of the vocal folds and produces an unsteady jet. In a first approximation it can be assumed that sound radiated from the glottis toward the lungs is absorbed without reflection within the lung complex. This hypothesis simplifies the analysis, but does not affect our present discussion; it has been examined in more detail by Howe and McGowan (2009).

B. Surface normal motions

Consider first the component $\int_{-\infty}^{\infty} \oint_{S(\tau)} G(\partial \mathbf{v} / \partial \tau) \cdot d\mathbf{S}(\mathbf{y}) d\tau$ of the first term on the right-hand side of Eq. (4). The normal component of velocity on $S(\tau)$ can be assumed to vanish everywhere except within and in the immediate

neighborhood of the glottis, and at the lung contraction source. In either of these regions the local motion may be regarded as incompressible ($\text{div } \mathbf{v} \simeq 0$), and a simple transformation shows that

$$\int_{-\infty}^{\infty} \oint_{S(\tau)} G \frac{\partial \mathbf{v}}{\partial \tau} \cdot d\mathbf{S}(\mathbf{y}) d\tau = - \int_{-\infty}^{\infty} \oint_{S(\tau)} \left\{ \frac{\partial G}{\partial \tau} + \mathbf{v} \cdot \frac{\partial G}{\partial \mathbf{y}} \right\} \times \mathbf{v} \cdot d\mathbf{S}(\mathbf{y}) d\tau \quad (12a)$$

$$\simeq - \int_{-\infty}^{\infty} \oint_{S(\tau)} \frac{\partial G}{\partial \tau} \mathbf{v} \cdot d\mathbf{S}(\mathbf{y}) d\tau. \quad (12b)$$

The simplification (12b) is applicable because the no-slip condition implies that the tangential component of velocity on $S(\tau)$ (which is approximately parallel to $\partial G / \partial \mathbf{y}$) is negligible.

The direct contribution to Eq. (12b) from the lung contraction source [at $x_1 \sim -\ell_L$ in Fig. 1(a)] can easily be evaluated (as described by Howe and McGowan, 2009; 2011a, b) in the form

$$\frac{2A_L}{\rho_o} \int_{-\infty}^{\infty} \beta(\tau) p_L(\tau) d\tau, \quad (13)$$

where $\beta(\tau)$ is the coefficient in definition (5) of G , and $p_L(\tau)$ is the pressure incident from the lung at the glottis at time τ .

To evaluate the corresponding contribution to Eq. (12b) from glottis surface motions we shall introduce the simplification $\oint_{S(\tau)} \mathbf{v} \cdot d\mathbf{S} = 0$, where the integration is over that section of $S(\tau)$ within and close to the glottis. In other words, it is assumed that no net change in tissue volume occurs during vocal fold oscillations. This is not an essential approximation, but simplifies the discussion without affecting the main thrust of the paper. Then the contribution to Eq. (12b) from the glottal surface sources is just

$$- \int_{-\infty}^{\infty} \oint_{S(\tau)} \frac{\partial}{\partial \tau} (\beta Y) \mathbf{v} \cdot d\mathbf{S} d\tau = - \int_{-\infty}^{\infty} \beta(\tau) \left[\oint_{S(\tau)} \frac{\partial Y}{\partial \tau} \mathbf{v} \cdot d\mathbf{S} - \frac{\partial}{\partial \tau} \oint_{S(\tau)} Y \mathbf{v} \cdot d\mathbf{S} \right] d\tau,$$

where the right-hand side is obtained after integration by parts subject to the causality conditions that $\beta(\tau)$ vanishes at $\tau = +\infty$ and source terms vanish at $\tau = -\infty$. Combining this formula with Eq. (13), we therefore conclude that

$$\int_{-\infty}^{\infty} \oint_{S(\tau)} G \frac{\partial \mathbf{v}}{\partial \tau} \cdot d\mathbf{S}(\mathbf{y}) d\tau \simeq - \int_{-\infty}^{\infty} \beta(\tau) \left[- \frac{2A_L}{\rho_o} p_L(\tau) + \oint_{S(\tau)} \frac{\partial Y}{\partial \tau} \times \mathbf{v} \cdot d\mathbf{S} - \frac{\partial}{\partial \tau} \oint_{S(\tau)} Y \mathbf{v} \cdot d\mathbf{S} \right] d\tau. \quad (14)$$

The surface integrals in this formula can be evaluated when the normal velocity of motion of the glottis is known or prescribed.

C. Surface friction

Surface frictional effects as a source of sound are important only within the glottis, and then only when the glottis cross section is very small ($A_g \sim 0$). Therefore, we can put

$$-\int_{-\infty}^{\infty} \oint_{S(\tau)} \nu \frac{\partial G}{\partial \mathbf{y}} \wedge \boldsymbol{\omega} \cdot d\mathbf{S} d\tau = -\int_{-\infty}^{\infty} \beta(\tau) \oint_{S(\tau)} \times \nu \frac{\partial Y}{\partial \mathbf{y}} \wedge \boldsymbol{\omega} \cdot d\mathbf{S} d\tau. \quad (15)$$

On the glottis wall the vorticity $\boldsymbol{\omega}$ and $\partial G/\partial \mathbf{y} = \beta(\tau) \partial Y/\partial \mathbf{y}$ are tangential to $S(\tau)$, and are mutually perpendicular in a first approximation, because $\partial Y/\partial \mathbf{y}$ is parallel to the mean flow just outside the boundary layer.

D. Fluid volume sources

The vortex source $\boldsymbol{\omega} \wedge \mathbf{v}$ is significant (i.e., of *dipole* order) only in the vicinity of the glottis or in other regions where it can interact with vocal tract surface inhomogeneities. Elsewhere, at distances from the glottis exceeding $\sim \sqrt{A_{VT}}$ for the model of Fig. 1(a), vorticity produces weaker sound of *quadrupole* order (because at such points $\partial G/\partial \mathbf{y} \sim$ constant over a characteristic vorticity correlation scale, for which $\int \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y} \simeq 0$; see Howe and McGowan, 2007). We can therefore put

$$-\int_{-\infty}^{\infty} \int_{V(\tau)} \frac{\partial G}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y} d\tau = -\int_{-\infty}^{\infty} \int_{V(\tau)} \beta(\tau) \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y} d\tau. \quad (16)$$

The main contribution to the volume integral is from vorticity in the free jet just downstream of the glottis, within a distance less than about $\sqrt{A_{VT}}$, and possibly also from boundary layer vorticity *within* the glottis where the normal component of velocity of $S(\tau)$ is non-zero. On stationary sections of $S(\tau)$ the vectors $\boldsymbol{\omega} \wedge \mathbf{v}$ and $\partial G/\partial \mathbf{y}$ are effectively perpendicular within the boundary layer and the contribution to Eq. (16) is negligible.

E. The source function $\mathcal{F}(t)$

The substitution of the results (14)–(16) into Eq. (4) then supplies the following explicit form of the aeroacoustic source function of Eqs. (8), (11):

$$\begin{aligned} \mathcal{F}(\tau) = & \frac{2A_L}{\rho_o} p_L(\tau) - \oint_{S(\tau)} \frac{\partial Y}{\partial \tau} \mathbf{v} \cdot d\mathbf{S} + \frac{\partial}{\partial \tau} \oint_{S(\tau)} Y \mathbf{v} \cdot d\mathbf{S} \\ & - \oint_{S(\tau)} \nu \frac{\partial Y}{\partial \mathbf{y}} \wedge \boldsymbol{\omega} \cdot d\mathbf{S} - \int_{V(\tau)} \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y}, \end{aligned} \quad (17)$$

where each integrand is assumed to be expressed as a function of $(\mathbf{y}, \tau)$. At low Mach numbers it is usually sufficient to evaluate $\mathcal{F}(\tau)$ from the equations of incompressible flow. Substitution into the Fant equation (11) (the explicit form of which is discussed in Sec. IV A) then determines the volume

velocity $Q(t)$ directly in terms of $\mathcal{F}(\tau)$ and thence the voiced pressure by means of Eq. (1).

Numerical schemes that determine the aeroacoustic sources in Eq. (8) from a direct numerical simulation of the local incompressible flow must at the same time evaluate $\beta(\tau)$. This is avoided by use of the Fant equation (11), for which a separate knowledge of $\beta(\tau)$ is not required. The method is very effective provided the properties of the unsteady glottal flow and motion are sufficiently well understood to permit the source function $\mathcal{F}(\tau)$ to be expressed in terms of the volume velocity Q . The Fant equation must still be solved numerically for $Q(t)$, simultaneously with coupled equations for the motion of the vocal folds, etc., but once this has been done the voiced sound pressure is given by the independent equation (1).

III. ANALYSIS OF THE AEROACOUSTIC SOURCES

The first two surface integrals on the right-hand side of Eq. (17) represent *potential flow sources* (added mass effects) and are formally of dipole type (Howe, 2008). They depend only on the normal component of vocal fold velocity and on the Kirchhoff vector $Y(\mathbf{y}, \tau)$, and are independent of the flow within the glottis except insofar as that flow influences the glottal motion. These integrals can be evaluated as soon as the time-dependent glottal geometry is specified.

The remaining source integrals in Eq. (17) involve the vorticity $\boldsymbol{\omega}$ both within the flow and on the vocal tract walls. In the simplest approximation the volume vorticity source (proportional to the *Lamb vector* $\boldsymbol{\omega} \wedge \mathbf{v}$) can be regarded as confined to thin shear layers at the edge of the jet formed by separation of the mean flow through the glottis, as discussed by Howe and McGowan (2007, 2011a, b). However, there are also important contributions to this term from the boundary layers within the glottis, which we shall attempt to elucidate in this section. The frictional vortex source (involving the kinematic viscosity ν) is important when the glottis is near closure (Fant, 1960; Flanagan, 1972).

The relative contributions of these different sources are easily assessed by consideration of the simplified glottal geometry and motion illustrated in Fig. 2. The glottis is taken to be cylindrical of rectangular cross section and length ℓ_g in the mean flow direction, with constant spanwise dimension d (out of the plane of the paper in Fig. 2), and time-dependent width $w = w(t)$, so that the glottal cross-sectional area $A_g(t) = w(t)d$. This is similar to the original “single-mass” model of Fant (1960); Flanagan (1972); and others.

Flow separation is assumed to occur at the fold “leading” edge (A in Fig. 2) when $w(t)$ is increasing, and from the “trailing” edge B during contraction of the glottal cross section, in accordance with the analytical model of Howe and McGowan (2010). When the coordinate origin O is taken at the supraglottal end of the glottis the Kirchhoff vector $Y(\mathbf{y}, \tau)$ can be approximated within the glottis by

$$Y(\mathbf{y}, \tau) \simeq \frac{A_L y_1}{A_g(\tau)}, \quad -\ell_g < y_1 < 0 \quad \text{so that} \quad \ell(\tau) \simeq \frac{A_L \ell_g}{A_g(\tau)}. \quad (18)$$

FIG. 2. Simplified vocal fold geometry used to evaluate glottal source contributions to $\mathcal{F}(\tau)$. The glottis has streamwise length L and rectangular cross section of spanwise length d (out of the plane of the paper) and variable width $w(t)$.

A. The potential flow (“added mass”) sources

This approximation supplies the following representation of the potential flow source terms of equation (17):

$$-\oint_{S(\tau)} \nu \frac{\partial Y}{\partial \tau} \mathbf{v} \cdot d\mathbf{S} + \frac{\partial}{\partial \tau} \oint_{S(\tau)} Y \mathbf{v} \cdot d\mathbf{S} = \frac{\ell_g^2 A_L}{2A_g} \frac{\partial^2 A_g}{\partial \tau^2}, \quad (19)$$

where we have used

$$\oint_{S(\tau)} f(y_1) \mathbf{v} \cdot d\mathbf{S} = -\dot{A}_g(\tau) \int_{-\ell_g}^0 f(y_1) dy_1, \quad \dot{A}_g(\tau) = \partial A_g / \partial \tau, \quad (20)$$

in which contributions from vocal fold normal motions outside the glottis are neglected

B. The frictional drag source

The third surface integral on the right-hand side of Eq. (17) is the frictional drag dipole, and is important only when $w(t) \rightarrow 0$, when the glottis Reynolds number is small enough that the flow may be regarded as laminar-viscous with the conventional parabolic velocity profile (Fant, 1960; Flanagan, 1972; Stevens, 1998). Then

$$-\oint_{S(\tau)} \nu \frac{\partial Y}{\partial \tau} \wedge \boldsymbol{\omega} \cdot d\mathbf{S} \simeq -\nu \oint_{S(\tau)} \frac{\partial Y}{\partial y_1} \frac{\partial v_1}{\partial y_n} dS, \quad (21)$$

$$v_1 \simeq \frac{6U}{w^2} y_n (w - y_n),$$

where U is the mean axial velocity within the glottis and y_n is measured from the wall into the fluid. Hence, using Eq. (18) and the identity $Q = A_g U$, we obtain the familiar approximation

$$-\oint_{S(\tau)} \nu \frac{\partial Y}{\partial \tau} \wedge \boldsymbol{\omega} \cdot d\mathbf{S} \simeq -\frac{12\nu d^2 \ell_g^3 Q}{A_L^2 \ell_g^2} \equiv -\frac{12\nu d^2 A_L \ell_g Q}{A_g^3}. \quad (22)$$

C. The Lamb vector source, $\boldsymbol{\omega} \wedge \mathbf{v}$

In the first order of approximation there are two principal contributors to the vortex source $-\int_{V(\tau)} (\partial Y / \partial \mathbf{y}) \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y}$. The dominant one is furnished by the free vorticity in the shear layers of the jet emerging from the supraglottal end of the glottis. This can be estimated by assuming the vorticity to be confined to a free stream surface at the outer edge of the jet, such that $\boldsymbol{\omega} \wedge \mathbf{v} \simeq (1/2) U_j^2 \delta(s_\perp) \mathbf{n}$, where U_j is the fully contracted, free jet speed downstream of the glottis, and $s_\perp$ is distance measured in the direction of the outward unit normal $\mathbf{n}$ from the jet.

The main contribution to the integral from this term is from the section of the jet very close to the glottis, where the streamlines of the velocity potential $Y(\mathbf{y}, \tau)$ cut across the shear layer (see Howe and McGowan, 2007, 2011a for details):

$$-\int_{V(\tau)} \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y} = -\frac{1}{2} U_j^2 \oint_{S_j} \frac{\partial Y}{\partial s_\perp} dS \approx -\frac{1}{2} A_L U_j^2, \quad (23)$$

where the second integral is over the curved vortex sheet approximation $S_j(\tau)$ to the shear layer. The approximation is valid provided $A_g \ll A_{VT}$ and U_j is effectively constant on $S_j(\tau)$ over a short distance $\sim O(A_g^{1/2})$ downstream of the glottis. This is the case if temporal changes in the shape of the jet can be regarded as quasi-static, i.e., provided the Strouhal number $f_o \sqrt{A_g} / U_j$ of the vocal fold oscillations at frequency f_o is small. It is typically less than about 0.1 for a female adult during voiced speech, and still smaller for males.

The second, smaller contribution from the Lamb vector is a direct consequence of the normal component of vocal fold surface motion. Consider first the case where the flow is fully attached to the glottis wall, which is contracting at velocity $-(1/2)(dw/d\tau)$ and equal to the normal component $v_n(\tau)$ of fluid velocity at the wall. Then within the thin boundary layer on the wall of the glottis

$$-\int \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v}_n d^3 \mathbf{y} \approx \frac{A_L}{A_g} \int \frac{\partial v_1}{\partial y_n} v_n d^3 \mathbf{y} \approx \frac{A_L}{A_g} \oint_{S(\tau)} U(\tau) \mathbf{v} \cdot d\mathbf{S}, \quad \dot{A}_g(\tau) < 0. \quad (24)$$

The final surface integral is obtained by integration across the boundary layer, neglecting the variation of v_n , where $U(\tau)$ is the uniform mean glottal velocity, so that $A_g U = Q$.

Hence, using the relations (18), (20) we find

$$-\int \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v}_n d^3 \mathbf{y} \approx \frac{Q A_L \ell_g \dot{A}_g}{A_g^2} = -Q \frac{\partial \ell}{\partial \tau}, \quad \dot{A}_g(\tau) < 0. \quad (25)$$

This formula is applicable provided the flow is attached to the glottis wall, which for the single mass model implies that $\dot{A}_g < 0$ (closing phase). When $\dot{A}_g > 0$ (opening phase,

Fig. 2) flow separation and jet formation occur at the subglottal end A. In this case $-\int (\partial Y/\partial \mathbf{y}) \cdot \boldsymbol{\omega} \wedge \mathbf{v}_n d^3 \mathbf{y}$ acquires a non-zero contribution only from that section of the jet lying *within* the glottis, because as the glottis expands at a rate $\dot{A}_g$, the cross section of the fully contracted jet must similarly expand at the rate $\sigma \dot{A}_g$, where σ is the jet contraction ratio. The integration (24) is now confined to the jet shear layer within the glottis, with U replaced by U_j , leading to

$$-\int \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v}_n d^3 \mathbf{y} \approx \frac{A_L}{A_g} \oint_{S_j(\tau)} U_j(\tau) v_n dS \approx \frac{U_j A_L \ell_g \sigma \dot{A}_g}{A_g} \\ = \frac{Q A_L \ell_g \dot{A}_g}{A_g^2} = -Q \frac{\partial \ell}{\partial \tau}, \quad \dot{A}_g(\tau) > 0, \quad (26)$$

because $Q = U_j \sigma A_g$.

Combining the results (23), (25), (26), the overall contribution from the Lamb vector source is finally

$$-\int \frac{\partial Y}{\partial \mathbf{y}} \cdot \boldsymbol{\omega} \wedge \mathbf{v} d^3 \mathbf{y} \approx -\frac{A_L}{2A_g^2} \frac{Q^2}{\sigma^2} - Q \frac{\partial \ell}{\partial \tau}, \quad (27)$$

where the relation $Q = U_j \sigma A_g$ has been used.

D. The source function $\mathcal{F}(\tau)$

Substituting the results (19), (22), (27) into the definition (17), the source function $\mathcal{F}(\tau)$ is found to be

$$\mathcal{F}(\tau) \approx \frac{2A_L}{\rho_o} p_L(\tau) + \frac{\ell_g^2 A_L}{2A_g} \frac{\partial^2 A_g}{\partial \tau^2} - \frac{12\nu d^2 A_L \ell_g Q}{A_g^3} \\ - \frac{A_L}{2A_g^2} \frac{Q^2}{\sigma^2} - Q \frac{\partial \ell}{\partial \tau}. \quad (28)$$

The terms on the right-hand side represent, respectively, the contributions from the lung pressure, glottis potential flow, glottis viscous drag, the free jet Lamb vector, and glottis vorticity (see Table I).

E. The limit of ideal flow

For ideal, irrotational flow ($\boldsymbol{\omega} \equiv 0$ everywhere) arriving from the lungs there can be no jet formation at the glottis and no viscous drag, so that Eq. (28) reduces to

$$\mathcal{F}(\tau) \approx \frac{2A_L}{\rho_o} p_L(\tau) + \frac{\ell_g^2 A_L}{2A_g} \frac{\partial^2 A_g}{\partial \tau^2} - Q \frac{\partial \ell}{\partial \tau}. \quad (29)$$

According to Sec. III C, the term $-Q \partial \ell / \partial \tau$ in this formula is produced by vorticity within the glottis. It occurs also in this irrotational limit because the absence of the no-slip condition

when there is ideal flow past the wall now requires the retention of the second (previously discarded) term in the integrand on the right-hand side of Eq. (12a). The irrotational form of Eq. (17) is therefore

$$\mathcal{F}(\tau) = \frac{2A_L}{\rho_o} p_L(\tau) - \oint_{S(\tau)} \frac{\partial Y}{\partial \tau} \mathbf{v} \cdot d\mathbf{S} + \frac{\partial}{\partial \tau} \oint_{S(\tau)} Y \mathbf{v} \cdot d\mathbf{S} \\ - \oint_{S(\tau)} \mathbf{U} \cdot \frac{\partial Y}{\partial \mathbf{y}} \mathbf{v} \cdot d\mathbf{S}. \quad (30)$$

where $\mathbf{U} = (U, 0, 0)$ is the uniform mean flow velocity in the glottis.

The final integral in Eq. (30) is the inviscid limit of the boundary layer integral of Eqs. (27), (28), because [by Eqs. (18), (20)]

$$-\oint_{S(\tau)} \mathbf{U} \cdot \frac{\partial Y}{\partial \mathbf{y}} \mathbf{v} \cdot d\mathbf{S} = -\frac{U A_L}{A_g} \oint_{S(\tau)} \mathbf{v} \cdot d\mathbf{S} = \frac{U A_L \ell_g}{A_g} \dot{A}_g(\tau) \\ \equiv -Q \frac{\partial \ell}{\partial \tau}, \quad (31)$$

wherein $Q = A_g U$.

IV. THE SIGNIFICANCE OF THE GLOTTIS-INTERIOR VOICE SOURCES

The unsteady volume source strength $Q(t)$ for the simple glottis model of Fig. 2 satisfies the Fant equation (11) with $\mathcal{F}(\tau)$ given by Eq. (28) (with τ replaced by t). The principal flow sources are summarized in Table I.

A. Simplified Fant equation

The general Fant equation accordingly takes the form

$$\hat{\mathcal{L}}(t) Q(t) = -\frac{2A_L}{\rho_o} p_L(t) + \frac{A_L}{2A_g^2} \frac{Q^2}{\sigma^2} + Q \frac{\partial \ell}{\partial t} + \frac{12\nu d^2 A_L \ell_g Q}{A_g^3} \\ - \frac{\ell_g^2 A_L}{2A_g} \frac{\partial^2 A_g}{\partial t^2}. \quad (32)$$

To examine the relative contributions of the sources of Table I it will be sufficient to consider the case where the reflection of sound from the lip opening is ignored. This should be an adequate approximation provided f_0 is not close to an upper tract resonance frequency (cf. Zhang *et al.*, 2006; Howe and McGowan, 2009). Then $\hat{\mathcal{L}}(t)Q = -\ell(\partial Q/\partial t) - c_o(1 + (A_L/A_{VT}))Q$ (Howe and McGowan, 2011b), and Eq. (32) becomes

$$\ell \frac{\partial Q}{\partial t} + c_o \left(1 + \frac{A_L}{A_{VT}} \right) Q = \frac{2A_L}{\rho_o} p_L(t) - \frac{A_L}{2A_g^2} \frac{Q^2}{\sigma^2} - Q \frac{\partial \ell}{\partial t} \\ - \frac{12\nu d^2 A_L \ell_g Q}{A_g^3} + \frac{\ell_g^2 A_L}{2A_g} \frac{\partial^2 A_g}{\partial t^2}. \quad (33)$$

To simplify the comparison we shall introduce the further approximation $A_L = A_{VT}$. Equation (33) can then be recast in the form

TABLE I. Principal Fant equation source terms.

(a) Free jet Lamb vector ($\boldsymbol{\omega} \wedge \mathbf{v}$)	$-A_L Q^2 / 2A_g^2 \sigma^2$
(b) Glottis jet/boundary layer Lamb vector	$-Q \partial \ell / \partial t$
(c) Glottis viscous drag	$-12\nu d^2 A_L \ell_g Q / A_g^3$
(d) Glottis potential flow (added mass) source	$(\ell_g^2 A_L / 2A_g) \partial^2 A_g / \partial t^2$

$$\rho_o \frac{\partial}{\partial t}(\ell Q) + 12\rho_o \frac{\nu d^2 A_{VT} \ell_g Q}{A_g^3} = 2A_{VT} p_L(t) - 2\rho_o c_o Q - \rho_o \frac{A_{VT}}{2A_g^2} \times \frac{Q^2}{\sigma^2} + \frac{\rho_o \ell_g^2 A_{VT}}{2A_g} \frac{\partial^2 A_g}{\partial t^2}. \quad (34)$$

Substitution of the solution $Q(t)$ into Eq. (1) and use of the corresponding source-filter Green's function $G_o = (c_o/A_{VT}) H([t] - \tau)$, where H is the Heaviside step function and $[t] = t - x_1/c_o$ is the retarded time, yields the acoustic pressure $p = \rho_o c_o Q(t - x_1/c_o)/A_{VT}$ that radiates toward the lip opening. It can be deduced similarly that the pressure wave radiated back into the lungs in the subglottal tract is $p = p_L(t + x_1/c_o) - \rho_o c_o Q(t + x_1/c_o)/A_{VT}$. Our model assumes that this wave is absorbed within the lungs without reflection to the glottis.

The first term on the left-hand side of Eq. (34) represents the rate of change of x_1 -momentum of the effective mass of fluid involved in accelerated motion through the glottis. The second term is the frictional resistance to this motion produced by viscous action within the glottis (important when $A_g \rightarrow 0$). The term in Q^2 determines the dissipative action of the jet, which converts potential energy of pressure into rotational fluid motions. The driving pressure $2p_L(t) - 2\rho_o c_o Q(t)/A_{VT}$ can be decomposed into contributions from the limiting values as $x_1/c_o \rightarrow 0$ of the three sound waves $p_L(t - x_1/c_o)$, $\{p_L(t + x_1/c_o) - \rho_o c_o Q(t + x_1/c_o)/A_{VT}\}$ and $\rho_o c_o Q(t - x_1/c_o)/A_{VT}$, which correspond, respectively, to the incident overpressure, the pressure wave radiated back into the lungs, and the pressure radiated toward the lip opening. If Eq. (34) is recast in the more conventional form

$$\rho_o \frac{\partial}{\partial t}(\ell Q) + 2\rho_o \left(c_o + \frac{6\nu d^2 A_{VT} \ell_g}{A_g^3} \right) Q + \rho_o \frac{A_{VT}}{2A_g^2} \frac{Q^2}{\sigma^2} = 2A_{VT} p_L(t) + \frac{\rho_o \ell_g^2 A_{VT}}{2A_g} \frac{\partial^2 A_g}{\partial t^2}, \quad (35)$$

it is evident that energy radiation via the waves $\pm Q(t \mp x_1/c_o)$ is manifested as an additional damping of motion within the glottis. This damping is always large relative to the viscous dissipation except when the glottis is very nearly closed, i.e., except when

$$\frac{A_g}{A_{VT}} < \left[6 \left(\frac{\nu}{c_o \ell_g} \right) \left(\frac{d \ell_g}{A_{VT}} \right)^2 \right]^{1/3}. \quad (36)$$

B. NUMERICAL RESULTS

Consider a steplike incident pressure $p_L(t) = p_I H(t)$, and revert to the following form of Eq. (35):

$$\ell \frac{\partial Q}{\partial t} + 2c_o Q + \frac{A_{VT}}{2A_g^2} \frac{Q^2}{\sigma^2} = \frac{2A_{VT} p_I}{\rho_o} H(t) - Q \frac{\partial \ell}{\partial t} - \frac{12\nu d^2 A_{VT} \ell_g Q}{A_g^3} + \frac{\ell_g^2 A_{VT}}{2A_g} \frac{\partial^2 A_g}{\partial t^2}. \quad (37)$$

The last three terms on the right-hand side correspond to the glottis-interior sources (b)–(d) of Table I.

A complete analysis involves the simultaneous solution of this equation with equations that govern the unsteady pressure, friction, and muscular restoring forces responsible for the self-sustaining oscillations of the vocal folds. But, to compare the relative contributions to the sound from the individual sources of Table I it is permissible to specify the vocal fold motions in what Titze (2008) has called a “Level I” model (cf. Zhao *et al.*, 2002; Howe and McGowan, 2007, 2011b). We do this by assuming that the glottis cross section oscillates at a given frequency f_0 , with $A_g \equiv w(t)d$ given by

$$\frac{A_g}{A_{VT}} = a_0 + \frac{a_1}{2} \{1 - \cos(2\pi f_0 t)\}, \quad (38)$$

where a_0 and $a_0 + a_1$ are, respectively the minimum and maximum values of the open area ratio A_g/A_{VT} . Complete closure occurs if $a_0 = 0$, but because Eq. (37) must be solved numerically it is necessary to assign a very small, non-zero value to a_0 . For ideal, inviscid flow a small, finite value of a_0 would permit “leakage” through the glottis. However, our choice for a_0 will be governed entirely by the need to avoid numerical instabilities; the glottal flow is still blocked in this case because of the very large action of viscosity.

The following fixed parameter values for the glottis and vocal tract will be assumed:

$$\left. \begin{aligned} f_0 &= 125 \text{ Hz}, \quad \ell_g = 0.3 \text{ cm}, \quad d = 1.8 \text{ cm}, \quad A_{VT} = \pi \text{ cm}^2, \\ a_0 &= 0.00025, \quad c_o = 350 \text{ m/s}, \quad \nu = 1.6 \times 10^{-5} \text{ m}^2/\text{s}. \end{aligned} \right\} \quad (39)$$

The value $f_0 = 125$ Hz is typical of an adult male, and other values are taken from Flanagan (1972). In all of the results displayed in the following it is assumed that the lung overpressure $p_I = 5$ cm of water. The maximum subglottal overpressure is then $\sim 2p_I = 10$ cm of water (~ 1 kPa), which corresponds approximately to the typical measured maximum (Fant, 1960; Flanagan, 1972; Stevens, 1998). The jet contraction ratio σ varies with time during the opening and closing phases of the glottis (Pelorson *et al.*, 1994; Park and Mongeau, 2007; Zanartu *et al.*, 2007; Howe and McGowan, 2010). For the simple model of Fig. 2 the jet separation point jumps discontinuously between the ends of the glottis, and there is a corresponding jump in the value of σ . This occurs when the glottis is fully open and produces a rapid variation in the volume velocity profile, which is often avoided by artificial interpolation across the transition (see, for example, Zanartu *et al.*, 2007). The discontinuity can be avoided simply by using a constant value of σ throughout. This has no practical effect on the relative contributions of the glottis-interior sources because they are important only when the glottis is near closure. We shall therefore ignore changes in σ and take $\sigma = 1$ throughout our calculations.

Equation (37) with $A_g(t)$ determined by the model (38) has been solved by fourth order Runge–Kutta integration (Abramowitz and Stegun, 1970), subject to the initial

FIG. 3. Predictions of the Fant equation (37) for condition (39) when $p_l = 5$ cm of water and $(A_g/A_{VT})_{\max} \simeq a_1 = 0.1$, plotted as functions of the non-dimensional retarded time $f_0[t]$. (a) Overall pressure $p/p_l = \rho_o c_o Q(t)/p_l A_{VT}$ radiated toward the lip opening. (b)–(d) Separate contributions to p/p_l from the corresponding glottal sources of Table I. (e) Net contribution from all glottal sources. (f) Glottis area ratio A_g/A_{VT} .

condition $Q = 0$ at $t = 0$, the motion being initiated by the arrival of the incident step rise in pressure p_l . The non-dimensional profile of the pressure wave $p/p_l \equiv \rho_o c_o Q(t - x_1/c_o)/p_l A_{VT}$ radiated toward the lip opening is plotted in Fig. 3 [curve (a)] as a function of $f_0[t]$ ($[t] = t - x_1/c_o$ being the retarded time) for three cycles when $(A_g/A_{VT})_{\max} \simeq a_1 = 0.1$. The motion becomes essentially periodic within one period $1/f_0$ after the arrival at the glottis of the step rise in pressure, and the plot-range $0 < f_0[t] < 3$ may therefore be assumed to correspond to any three-cycle interval after this time. The lower curve (f) in Fig. 3 shows the corresponding (retarded) variation of the glottal cross-sectional area. The curves labeled (b)–(d) in Fig. 3 depict the respective contributions to the overall radiated pressure p/p_l from the glottis sources (b)–(d) of Table I. Each one is evaluated by calculating the acoustic pressure with and without the corresponding source term. Evidently the influence of the potential flow/added mass source is always negligible. The viscous frictional source (c) is important only when $A_g/A_{VT} \rightarrow 0$, when it strongly opposes the glottis volume flux. According to Eq. (36) this is the case when A_g/A_{VT} is smaller than about 0.01 for conditions (39). The “ $\partial \ell / \partial t$ ” component (b), from the boundary layer Lamb vector source $\omega \wedge \mathbf{v}_n$ within the glottis, is comparable to the viscous induced pressure wave, but makes a positive/negative contribution according to whether A_g/A_{VT} is increasing or decreasing (when, respectively, $\partial \ell / \partial t \lesseqgtr 0$). The plot labeled (e) in Fig. 3

FIG. 4. Predictions of the Fant equation (37) for condition (39) when $p_l = 5$ cm of water and $(A_g/A_{VT})_{\max} \simeq a_1 = 0.2$, plotted as functions of the non-dimensional retarded time $f_0[t]$. (a) Overall pressure $p/p_l = \rho_o c_o Q(t)/p_l A_{VT}$ radiated toward the lip opening. (b)–(d) Separate contributions to p/p_l from the corresponding glottal sources of Table I. (e) Net contribution from all glottal sources. (f) Glottis area ratio A_g/A_{VT} .

illustrates the net contribution of all of the glottis-interior sources to the overall pressure. This is significant near closure and is generally negative except for a short intermediate interval of the cycle after the glottis opens which terminates when the glottis is fully open; its effect is to skew the main pressure pulse, raising and lowering the fore and aft sections of the pulse profile by amounts determined by the positive and negative excursions of curve (e) in Fig. 3. It is more pronounced at larger peak values of A_g/A_{VT} , when non-linearity becomes more important, as illustrated in the corresponding results shown in Fig. 4 for $(A_g/A_{VT})_{\max} \simeq a_1 = 0.2$.

These conclusions are illustrated in alternative ways in Figs. 5 and 6. In Fig. 5 the pressure waveform $p/p_l = \rho_o c_o Q(t - x_1/c_o)/p_l A_{VT}$ is plotted with (—) and without (---) account being taken of the glottis-interior source contributions (b)–(d) of Table I. In the absence of the interior sources the waveform is symmetric about the maximum pressure and leakage is evident when A_g assumes its minimum, non-zero value (at $f_0[t] = 0, 1$, and 2 in Fig. 5); the interior sources act to skew the wave profile, principally because of viscous action when the glottis is near closure, which effectively blocks the flow and increases the high frequency content of the waveform. The effect at high frequencies is shown in Fig. 6, where a comparison is made of the magnitudes $|c_n|$ of the Fourier coefficients of the two waveforms with (• • •) and without (○ ○ ○) the interior glottis sources, where

FIG. 5. Predicted pressure $p/p_I = \rho_o c_o Q(t)/p_I A_{VT}$ radiated toward the lip opening with (—) and without (---) the glottis-interior source contributions of Table I, plotted as functions of the non-dimensional retarded time $f_0[t]$. For condition (39) when $p_I = 5$ cm of water: (a) $(A_g/A_{VT})_{\max} \simeq a_1 = 0.1$, (b) 0.2.

$$c_n = f_0 \int_{t_o}^{t_o+1/f_0} \frac{p(t)}{p_I} e^{2\pi n f_0 t} dt \quad (40)$$

and where t_o is large enough for the waveform to be periodic (i.e., $t_o > 1/f_0$). For the two illustrated cases of $(A_g/A_{VT})_{\max} \simeq a_1 = 0.1, 0.2$ it is seen that the leakage implicit in the

FIG. 6. Predicted non-dimensional magnitudes $|c_n|$ of the pressure wave Fourier coefficients defined in Eq. (40) with (●●●) and without (○○○) the glottis-interior source contributions of Table I. For condition (39) when $p_I = 5$ cm of water: (a) $(A_g/A_{VT})_{\max} \simeq a_1 = 0.1$, (b) 0.2.

neglect of the interior glottis sources reduces the higher frequency content of the waveform, by as much as ~ 10 dB when $n \sim 5-10$, although the lower frequency components ($n = 0-4$) are essentially unaffected and therefore independent of the interior glottis sources. In both cases, however, the higher frequency content is relatively weaker by ~ 20 dB or more.

V. CONCLUSION

The shape of the voiced pressure wave radiated toward the lip opening depends on the unsteady volume velocity $Q(t)$ of air through the glottis. $Q(t)$ can in principle be determined from the Fant equation governing motion within the glottis in terms of forcing dominated by the subglottal overpressure, the backreactions from the radiated sound, and vorticity in the jet flow from the glottis. Such forcing can be represented by means of the equations of aerodynamic sound, which reveal, however, the existence of second order “sources” that influence the fine details of the acoustic waveform. These sources are associated with motion of the jet separation point within the glottis, glottis wall “potential flow” effects, surface friction, and vortex sources within the glottis-wall boundary layer and in the transversally oscillating jet shear layers when separation occurs within or near the subglottal end of the glottis.

The second order forcing is found to modify the high frequency content of the volume velocity, especially when the glottis is near closure. It has been examined in this paper in terms of a simplified glottis modeled as a cylindrical duct of rectangular cross section. This indicates that the glottis-interior boundary layer and shear layer vortex sources and the surface friction source are of comparable importance near closure, and dominate the acoustic waveform at such times, whereas the potential flow source is always about an order of magnitude smaller.

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NOMENCLATURE

- A_g = glottis cross-sectional area
- A_L = subglottal tract cross-sectional area
- A_{VT} = supraglottal tract cross-sectional area
- a_0, a_1 = see Eq. (38)
- c_n = Fourier amplitude, Eq. (40)
- c_o = speed of sound
- d = glottis span, anterior-posterior direction
- $\mathcal{F}$ = aeroacoustic source function
- f_0 = vocal fold frequency
- G, G_o = Green's function
- K_g = conductivity of glottis
- k_o = acoustic wavenumber, ω/c_o
- L = length of supraglottal tract
- $\mathcal{L}, \hat{\mathcal{L}}$ = see Eqs. (9), (11)
- ℓ = slug length of glottis, Eq. (6)

ℓ_E = lip opening end correction
 ℓ_g = length of glottis
 P = pressure
 p_L, p_I = lung over-pressure
 Q = volume velocity
 $[t]$ = retarded time, $t - x/c_0$
 U = mean velocity in glottis
 U_j = contracted jet speed
 $\mathbf{v}$ = velocity
 w = glottis width (medial)
 Y = Kirchhoff vector, Eq. (5)
 α, β = Green's function coefficients, Eq. (5)
 ν = kinematic viscosity
 ρ = density
 σ = contraction ratio
 ω = radian frequency
 $\boldsymbol{\omega}$ = vorticity, curl $\mathbf{v}$

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