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Image reconstruction in photoacoustic tomography involving layered acoustic media

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Abstract

Photoacoustic tomography (PAT), also known as thermoacoustic or optoacoustic tomography, is a rapidly emerging biomedical imaging technique that combines optical image contrast with ultrasound detection principles. Most existing reconstruction algorithms for PAT assume the object of interest possesses homogeneous acoustic properties. The images produced by such algorithms can contain significant distortions and artifacts when the object's acoustic properties are spatially variant. In this work, we establish an image reconstruction formula for PAT applications in which a planar detection surface is employed and the to-be-imaged optical absorber is embedded in a known planar layered acoustic medium. The reconstruction formula is exact in a mathematical sense and accounts for multiple acoustic reflections between the layers of the medium. Computer-simulation studies are conducted to demonstrate and investigate the proposed method.

1. INTRODUCTION

Photoacoustic tomography (PAT), also known as thermoacoustic or optoacoustic tomography, is a rapidly emerging imaging technique that holds great promise for biomedical applications [1–4]. PAT is a hybrid technique that exploits the high optical contrast of tissue with the high spatial resolution of ultrasonic methods. The goal of PAT is to determine an estimate of an object's spatially variant absorbed optical energy density from measurements of pressure wave fields that are induced via the thermoacoustic effect [2]. Because the optical absorption characteristics of tissue vary strongly with hemoglobin content, knowledge of the absorbed optical energy distribution can yield both structural and functional information. Accordingly, PAT may facilitate a variety of important human and animal imaging studies and is being developed actively by numerous research groups worldwide [2,3,5–10].

When a short laser pulse is used to irradiate biological tissue, the thermoacoustic effect results in the emission of acoustic pressure signals that can be measured outside the object by use of wideband ultrasonic transducers. The PAT image reconstruction problem can be interpreted as an acoustic inverse source problem [11], and a variety of analytic image reconstruction algorithms have been developed for three-dimensional (3D) PAT assuming canonical measurement apertures [2,12–18]. Most existing PAT image reconstruction

algorithms assume the object of interest possesses homogeneous acoustic properties. However, the images produced by such algorithms can contain significant distortions and artifacts when the object's speed-of-sound and density distributions are spatially variant [19,20]. The development of PAT image reconstruction algorithms that can compensate for variations in the acoustical properties of an object remains an important area of current research [21–26].

In this work, we develop a PAT reconstruction formula for applications in which a planar detection surface is employed, and the to-be-imaged optical absorber is embedded in a planar-layered acoustic medium. The speed of sound, density, thickness of each layer, and number of layers are assumed to be known, and the illuminated optical absorber is assumed to be confined to only one layer. The reconstruction formula is exact in a mathematical sense and accounts for multiple reflections of the induced photoacoustic wave field between the layers of the medium. The formula establishes a mapping that relates the 3D Fourier components of the sought-after optical energy density distribution to those of the measured pressure data that correspond to propagating wave modes. In the special case of an object with homogeneous speed-of-sound and density distributions, the reconstruction formula reduces to a previously published one [16] when the number of layers is set to one. Computer-simulation studies are conducted to demonstrate and investigate the proposed method.

2. BACKGROUND: STANDARD FORMULATION OF PAT WITH A PLANAR MEASUREMENT APERTURE

The standard imaging model for PAT is derived from the acoustic wave equation in either the space–time or space–frequency domain. The space–frequency domain representation $\tilde{p}(\mathbf{r}, \omega)$ of the acoustic field is related to the space–time representation $p(\mathbf{r}, t)$ by a Fourier transform, viz.

$$\tilde{p}(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} dt p(\mathbf{r}, t) e^{i\omega t}, \quad (1)$$

where ω is the temporal frequency coordinate and $\mathbf{r} = (x, y, z)$.

In PAT applications, the acoustic field obeys an inhomogeneous Helmholtz equation:

$$[\nabla^2 + k^2(\mathbf{r})]\tilde{p}(\mathbf{r}, \omega) = \frac{i\omega\beta}{C_p} A(\mathbf{r})H(\omega), \quad (2)$$

where $k(\mathbf{r}) = \omega/c(\mathbf{r})$ is the spatially varying wavenumber, $c(\mathbf{r})$ is the local speed of sound, β is the thermal expansion coefficient, C_p is the specific heat capacity (at constant pressure), $A(\mathbf{r})$ is the absorbed optical energy density, and $H(\omega)$ describes the spectral content of the exciting optical or microwave pulse. We will let $A(x, y, z)$ denote $A(\mathbf{r})$ expressed explicitly in Cartesian coordinates. The pressure field away from the acoustic source can be expressed as

$$\tilde{p}(\mathbf{r}, \omega) = \frac{i\omega\beta H(\omega)}{C_p} \iiint_V d^3\mathbf{r}' G(\mathbf{r}, \mathbf{r}', \omega) A(\mathbf{r}'), \quad (3)$$

where $G(\mathbf{r}, \mathbf{r}', \omega)$ is an appropriate Green function [27] and V denotes the support volume of $A(\mathbf{r})$. Equation (3) represents an imaging model for PAT expressed in the temporal

frequency domain. For homogenous acoustic media, $G(\mathbf{r}, \mathbf{r}', \omega) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$. In general, the Green function can only be found analytically when the speed-of-sound map possesses certain symmetries. Otherwise, numerical methods must be employed to approximate the Green function needed to specify the imaging model in Eq. (3).

The solution to the inverse problem for PAT, i.e., the estimation of $A(\mathbf{r})$ based on knowledge of $\tilde{p}(\mathbf{r}, \omega)$ and $H(\omega)$, is based on Eq. (3) but also incorporates information about the measurement geometry. In the case where the medium is acoustically homogeneous with the speed of sound c and the measurement aperture corresponds to a plane, taken to be $z = 0$ without loss of generality, a Fourier-transform-based solution to the inverse problem has been established [16,17]. Let $\tilde{p}(k_x, k_y, \omega)$ denote the two-dimensional (2D) spatial Fourier transform of the pressure data $\tilde{p}(x, y, z, \omega)$ evaluated on the measurement plane $z = 0$:

$$\tilde{p}(k_x, k_y, \omega) = \iint_{-\infty}^{\infty} dx dy \tilde{p}(x, y, z=0, \omega) e^{-i(k_x x + k_y y)}. \quad (4)$$

Similarly, let $\mathcal{A}(k_x, k_y, k_z)$ denote the 3D Fourier transform of $A(x, y, z)$:

$$\mathcal{A}(k_x, k_y, k_z) = \iiint_{-\infty}^{\infty} dx dy dz A(x, y, z) e^{-i(k_x x + k_y y + k_z z)}. \quad (5)$$

It has been demonstrated [16] that the values of $\mathcal{A}(k_x, k_y, k_z)$ that reside within a sphere of radius $\frac{\omega}{c}$ centered at the origin of 3D Fourier space can be determined from the measured pressure data as

$$\mathcal{A}(k_x, k_y, \sqrt{\omega^2/c^2 - k_x^2 - k_y^2}) = \frac{-2C_p \tilde{p}(k_x, k_y, \omega)}{\omega \beta H(\omega)} \times \sqrt{\omega^2/c^2 - k_x^2 - k_y^2}, \quad (6)$$

where effects related to the transducer response have been suppressed. One notes that the k_z coordinate of $\mathcal{A}(k_x, k_y, k_z)$ is found via a nonlinear mapping of k_x, k_y , and ω . From knowledge of the estimated Fourier components, a low-pass-filtered estimate of $A(\mathbf{r})$ can be determined by use of the 3D inverse Fourier transform [28]. In Section 3 a generalization of Eq. (6) is established for the case where the optical absorber described by $A(\mathbf{r})$ is embedded in a stratified planar acoustic medium. The effects of the finite size of the detector and the finite length of the excitation pulse can be readily included in the reconstruction algorithm as described in [16].

3. PAT RECONSTRUCTION FORMULA FOR LAYERED ACOUSTIC MEDIA

Many PAT reconstruction methods developed to date are predicated on the assumption that the medium surrounding the source is acoustically homogenous. Below we develop a reconstruction formula for PAT employing a planar measurement aperture for the case of an inhomogeneous layered acoustic medium. The optical absorber described by $A(\mathbf{r})$ is assumed to be confined to only one layer. The speed-of-sound and density values of the medium within each layer are assumed to be constant but can be different in each layer. The planar boundaries of the layers are assumed to be parallel to the measurement plane $z = 0$. The solution strategy comprises two steps: (1) determine the Green function for the stated layered medium, thereby specifying the imaging model in Eq. (3), and (2) invert the resulting imaging model to obtain a Fourier-based inversion formula.

A. Green Function for Layered Acoustic Media

Consider a medium whose local speed of sound and density are functions of only one spatial variable, z , and the variation takes the form of discontinuous jumps at specific values of z . Specifically, as depicted in Fig. 1, the speed of sound $c(\mathbf{r})$ and density $\rho(\mathbf{r})$ distributions are defined as

$$\rho(\mathbf{r}) = \rho_m \quad \text{for } z_m \leq z < z_{m+1}, \quad (7a)$$

$$c(\mathbf{r}) = c_m \quad \text{for } z_m \leq z < z_{m+1}, \quad (7b)$$

where the index $m = 0, 1, \dots, N$ indicates the layer in the medium, ρ_m and c_m denote the values of $\rho(\mathbf{r})$ and $c(\mathbf{r})$ within the m th layer, and the planes $z = z_m$ and $z = z_{m+1}$ bound the m th layer.

Time domain solutions for the photoacoustic field induced in such a medium are complicated, and solutions are more tractable in the frequency domain. In the frequency domain, the pressure field in the layered medium satisfies the Helmholtz equation:

$$(\nabla^2 + k_m^2)\tilde{p}(\mathbf{r}, \omega) = 0 \quad \text{for } z_m \leq z < z_{m+1}, \quad (8)$$

where $k_m = \omega/c_m$. Note that the analysis that follows can be extended to compensate for acoustic wave field dispersion and frequency-dependent absorption [29] by consideration of an appropriately defined complex-valued k_m . The acoustic field must also obey the following boundary conditions:

$$\tilde{p}(\mathbf{r}, \omega)|_{z_m^-} = \tilde{p}(\mathbf{r}, \omega)|_{z_m^+} \quad \forall m, \quad (9)$$

$$\frac{1}{\rho_{m+1}} \frac{\partial \tilde{p}}{\partial z} \Big|_{z_m^-} = \frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial z} \Big|_{z_m^+} \quad \forall m. \quad (10)$$

The Green function for a homogeneous medium, where $c(\mathbf{r}) = c$ and $\rho(\mathbf{r}) = \rho$, satisfies the equation

$$(\nabla^2 + k^2)G_0(\mathbf{r}, \mathbf{r}'; \omega) = -\delta(\mathbf{r} - \mathbf{r}'), \quad (11)$$

where $\delta(\mathbf{r})$ is the 3D Dirac delta function and has a well-known angular spectrum representation [30]:

$$G_0(\mathbf{r}, \mathbf{r}'; \omega) = \frac{\exp(ik|\mathbf{r} - \mathbf{r}'|)}{4\pi|\mathbf{r} - \mathbf{r}'|} = \iint_{\infty} \frac{d^2k_{\parallel}}{(2\pi)^2} \frac{i}{2k_z} e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{r} - \mathbf{r}')} e^{ik_z|z - z'|}. \quad (12)$$

Here $\mathbf{k}_{\parallel}$ is a 3D vector with units of inverse length and of the form $[k_x, k_y, 0]^T$, $d^2k_{\parallel} = dk_x dk_y$, $k_z(\mathbf{k}_{\parallel}) = \sqrt{k^2 - k_{\parallel}^2}$, and $k = \omega/c$ is the wavenumber for the medium.

The Green function for the layered medium defined by Eq. (7) may be found through a number of methods, including modal analysis [27]. In this method, the Green function is

generally expressed as a summation over expansion functions. For the case of the layered medium, the expansion functions in each layer will be the plane-wave functions.

Consider that the photoacoustic source described by $A(\mathbf{r})$ is confined to layer $m = P$ in the medium. The associated Green function will be denoted as $G_P(\mathbf{r}, \mathbf{r}'; \omega)$ and by definition satisfies

$$(\nabla^2 + k_p^2)G_P(\mathbf{r}, \mathbf{r}'; \omega) = -\delta(\mathbf{r} - \mathbf{r}') \quad \text{for } z_p \leq z < z_{p-1}, \quad (13)$$

$$(\nabla^2 + k_m^2)G_P(\mathbf{r}, \mathbf{r}'; \omega) = 0 \quad \text{for } z_m \leq z < z_{m-1}, \quad m \neq P. \quad (14)$$

The angular spectrum representation of $G_P(\mathbf{r}, \mathbf{r}'; \omega)$ takes the form

$$G_P(\mathbf{r}, \mathbf{r}'; \omega) = \iint_{-\infty}^{\infty} \frac{d^2 k_{\parallel}}{(2\pi)^2} \left\{ e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} [a_m(\mathbf{k}_{\parallel}) e^{ik_z^{(m)} z} + b_m(\mathbf{k}_{\parallel}) e^{-ik_z^{(m)} z}] + \frac{i}{2k_z^{(m)}} e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{r} - \mathbf{r}') e^{ik_z^{(m)} |z - z'|}} \delta_{mP} \right\} \quad (15)$$

for $z_m \leq z < z_{m-1}$,

where δ_{mn} is the Kronecker delta function and $k_z^{(m)} \equiv \sqrt{k_m^2 - k_{\parallel}^2}$. The quantities $a_m(\mathbf{k}_{\parallel})$ and $b_m(\mathbf{k}_{\parallel})$ are layer-dependent coefficients that can be interpreted as plane-wave amplitudes associated with the forward and backward travelling field components in each layer. Note that the third quantity in the integrand of Eq. (15) describes the unscattered field within layer P produced by a point source at location $\mathbf{r}'$ within that layer [e.g., see Eq. (12)].

In order to specify $G_P(\mathbf{r}, \mathbf{r}'; \omega)$ the unknown quantities $a_m(\mathbf{k}_{\parallel})$ and $b_m(\mathbf{k}_{\parallel})$ must be determined. This can be accomplished by forcing $G_P(\mathbf{r}, \mathbf{r}'; \omega)$ to satisfy the boundary conditions described by Eqs. (9) and (10) at each interface $z = z_m$. By use of Eqs. (9), (10), and (15), a system of $2N - 2$ linear equations containing the $2N - 2$ unknowns $\{a_1, a_2, \dots, a_{N-1}\}$ and $\{b_2, b_3, \dots, b_N\}$ can be established. These equations are of the form

$$a_{m+1} + b_{m+1} - a_m \exp(-ik_z^{(m)} d_m) - b_m \exp(ik_z^{(m)} d_m) = -S_1 \delta_{P,m+1} + S_2 \delta_{P,m}, \quad (16)$$

$$\frac{k_z^{(m+1)}}{\rho_{m+1}} [a_{m+1} - b_{m+1}] - \frac{k_z^{(m)}}{\rho_m} [a_m \exp(-ik_z^{(m)} d_m) - b_m \exp(ik_z^{(m)} d_m)] = -\frac{k_z^{(m+1)}}{\rho_{m+1}} S_1 \delta_{P,m+1} - \frac{k_z^{(m)}}{\rho_m} S_2 \delta_{P,m}, \quad (17)$$

where the dependence on $\mathbf{k}_{\parallel}$ has been suppressed in the functions $a_m(\mathbf{k}_{\parallel})$, $b_m(\mathbf{k}_{\parallel})$, and $k_z(\mathbf{k}_{\parallel})$,

$$S_1 \equiv \frac{i}{2k_z^{(m+1)}} \exp(-i\mathbf{k}_{\parallel} \cdot \mathbf{r}') \exp[ik_z^{(m+1)}(z_m - z')], \quad (18)$$

$$S_2 \equiv \frac{i}{2k_z^{(m)}} \exp(-i\mathbf{k}_{\parallel} \cdot \mathbf{r}') \exp[ik_z^{(m)}(z' - z_m)], \quad (19)$$

and d_m are the distances between successive layers ($d_m = z_{m-1} - z_m$). Note that the above linear system must be solved for every temporal frequency, ω , and every spatial frequency, $\mathbf{k}_{\parallel}$, of interest.

The system of equations described above can be represented as $\underline{M} \cdot \vec{g} = \vec{s}$, where $\underline{M}$ is a $(2N - 2) \times (2N - 2)$ matrix whose elements are functions of $\mathbf{k}_{\parallel}$, ω , and the acoustical properties of the layers; $\vec{g}$ is a vector of length $2N - 2$ of plane-wave amplitudes associated with the specified $\mathbf{k}_{\parallel}$ and ω with elements $\vec{g} = [a_1, b_2, a_2, b_3, \dots, a_{N-1}, b_N]$; and $\vec{s}$ is a vector with four nonzero elements, two of which are proportional to $\exp(-ik_z^{(P)} z')$ and two of which are proportional to $\exp(ik_z^{(P)} z')$. The placement of these four nonzero elements in $\vec{s}$ depends on the layer in which the optical absorber resides. For example, in the case where the optical absorber resides in layer P , the four nonzero elements are given by

$$[\vec{s}]_{2P-3} = -\frac{i}{2k_z^{(P)}} \exp(-i\mathbf{k}_{\parallel} \cdot \mathbf{r}') \exp[ik_z^{(P)}(z_{P-1} - z')], \quad (20a)$$

$$[\vec{s}]_{2P-2} = -\frac{i}{2\rho_P} \exp(-i\mathbf{k}_{\parallel} \cdot \mathbf{r}') \exp[ik_z^{(P)}(z_{P-1} - z')], \quad (20b)$$

$$[\vec{s}]_{2P-1} = \frac{i}{2k_z^{(P)}} \exp(-i\mathbf{k}_{\parallel} \cdot \mathbf{r}') \exp[ik_z^{(P)}(z' - z_P)], \quad (20c)$$

$$[\vec{s}]_{2P} = -\frac{i}{2\rho_P} \exp(-i\mathbf{k}_{\parallel} \cdot \mathbf{r}') \exp[ik_z^{(P)}(z' - z_P)], \quad (20d)$$

where $[\vec{s}]_n$ denotes the n th component of the vector $\vec{s}$.

The values of the coefficients $a_m(\mathbf{k}_{\parallel})$ and $b_m(\mathbf{k}_{\parallel})$ are found for one angular frequency, ω , and one spatial frequency, $\mathbf{k}_{\parallel}$, by solving

$$\vec{g} = \underline{M}^{-1} \cdot \vec{s}. \quad (21)$$

By iterating this process through all combinations of angular and spatial frequencies, $G_P(\mathbf{r}, \mathbf{r}'; \omega)$ can be fully determined via Eq. (15).

B. Image Reconstruction Formula

Because of the form of the forcing terms in Eqs. (18) and (19), each plane-wave amplitude for the Green function takes the form

$$[\vec{g}]_n(\mathbf{k}_{\parallel}; \mathbf{r}') = T_n(\mathbf{k}_{\parallel}) e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}'} e^{ik_z^{(P)} z'} + R_n(\mathbf{k}_{\parallel}) e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}'} e^{-ik_z^{(P)} z'}, \quad n=1, 2, 3, \dots, 2N-2, \quad (22)$$

where, again, P denotes the layer that contains the optical absorber and T_n and R_n are found by explicit solution of Eq. (21). The functions T_n and R_n are explicitly functions of $\mathbf{k}_{\parallel}$, but also depend on all of the system parameters: $\{d_m\}$, $\{\rho_m\}$, and $\{c_m\}$.

It is assumed that detection of the acoustic field is performed on the plane $z = 0$. In this region, Eq. (15) takes on the simplified form

$$G_p^{\text{meas}}(\mathbf{r}, \mathbf{r}'; \omega) = \iint_{\infty} \frac{d^2 k_{\parallel}}{(2\pi)^2} e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} a_1(\mathbf{k}_{\parallel}; \mathbf{r}'), \quad (23)$$

where $a_1(\mathbf{k}_{\parallel}; \mathbf{r}') = [\vec{g}]_1$ is the outgoing plane-wave amplitude in the final (detection) layer. The acoustic pressure, $p(\mathbf{r}, \omega)$, evaluated on the detection plane $z = 0$, is given by the equation

$$p(\mathbf{r}, \omega) = \frac{i\omega\beta H(\omega)}{C_p} \iiint_V d^3 r' G_p^{\text{meas}}(\mathbf{r}, \mathbf{r}'; \omega) A(\mathbf{r}'). \quad (24)$$

On substitution from Eq. (23) into Eq. (24), and using Eq. (22) and the 2D Fourier transform, it can be verified that

$$\tilde{p}(\mathbf{k}_{\parallel}, \omega) = \frac{i\omega\beta H(\omega)}{C_p} [T_1(\mathbf{k}_{\parallel}) \mathcal{A}(\mathbf{k}_{\parallel}; k_z^{(P)}) + R_1(\mathbf{k}_{\parallel}) \mathcal{A}(\mathbf{k}_{\parallel}; -k_z^{(P)})], \quad (25)$$

where $\mathcal{A}(\mathbf{k}_{\parallel}; k_z^{(P)}) \equiv \mathcal{A}(k_x, k_y, \sqrt{k_p^2 - k_x^2 - k_y^2})$ and $\tilde{p}(\mathbf{k}_{\parallel}, \omega)$ is used to denote $\tilde{p}(k_x, k_y, \omega)$. Each plane-wave component of the detected field is made up of two components: one is related to $\mathcal{A}(\mathbf{k}_{\parallel}; k_z^{(P)})$, and the other is related to $\mathcal{A}(\mathbf{k}_{\parallel}; -k_z^{(P)})$. Note that, when the illuminated object is in the leftmost layer (layer N), all of the $R_n(\mathbf{k}_{\parallel})$ are identically zero.

Using the fact that $A(\mathbf{r})$ is real, and that both R and T are functions only of the magnitude of $\mathbf{k}_{\parallel}$, the measured pressure on a plane parallel to the interface is sufficient to estimate the spatial Fourier components of $A(\mathbf{r})$ as

$$\mathcal{A}(\mathbf{k}_{\parallel}; k_z^{(P)}) = \frac{C_p}{i\omega\beta H(\omega)} \frac{[T_1^*(\mathbf{k}_{\parallel}) \tilde{p}(\mathbf{k}_{\parallel}, \omega) - R_1(\mathbf{k}_{\parallel}) \tilde{p}^*(-\mathbf{k}_{\parallel}, \omega)]}{|T_1|^2 - |R_1|^2}. \quad (26)$$

Equation (26) is the main result of this paper. It shows that the Fourier components of the object are simply related to the Fourier components of the acoustic signal measured on a plane. It can be verified that, in the special case of an object with homogeneous speed-of-sound and density distributions, i.e., a medium described by a single layer, Eq. (26) reduces to Eq. (6).

4. COMPUTER SIMULATIONS

Computer simulations were performed to verify the proposed reconstruction method. A numerical phantom was considered that contained two small spheres located within the first of three layers. The spheres were centered at positions (1, -1.34, -1.5) cm and (0, 1.4, -1.4) cm, had radii of 1 cm, and were band limited by convolving each with a 3D Gaussian function of width 0.5 mm. The value of $A(\mathbf{r})$ was assigned to be one for each sphere. Spheres were used in the simulations because they have a known Fourier representation, eliminating machine error in calculating the Fourier transform of the object. The first layer of the background was assumed to be unbounded in the $-z$ direction and have a speed of sound of 1550 m/s (consistent with muscle tissue). The second layer had a thickness 3 mm and has a speed of sound of 1476 m/s (consistent with fat tissue). The third layer had a speed of sound of 1498 m/s (consistent with skin). The densities of each layer were considered to

be equal. The pressure wave field was assumed to be recorded in the a layer that is matched to the skin layer a distance of $d = 1$ mm away from the skin edge.

Samples of $\tilde{p}(\mathbf{r}, \omega)$ were generated according to Eq. (24), where the integral in Eq. (23) was evaluated numerically for $|\mathbf{k}_{\parallel}| < 3k_0$. Temporal frequency domain data were generated at a sampling rate of 1.59 MHz on a rectangular grid of 128×128 idealized, pointlike, transducers, with lateral spacing $\Delta x = 0.5$ mm. The effects of an imperfect transducer response on the reconstructed images are expected to be the same as described elsewhere [11,31].

Sampled values of $p(\mathbf{r}, t)$ were obtained by application of the one-dimensional inverse fast Fourier transform (FFT) to the computed values of $\tilde{p}(\mathbf{r}, \omega)$. Noisy versions of $p(\mathbf{r}, t)$ were created by addition of identically distributed white Gaussian noise at each transducer location. The noise level was referenced to the average value of $p(\mathbf{r}, t)$ computed over all values of t and transducer locations $\mathbf{r}$.

For the three layer object described above, Eq. (26) can be expressed explicitly as

$$\mathcal{A}(\mathbf{k}_{\parallel}; k_z^{(m)}) = \frac{2C_p k_z^{(m)}}{i\omega\beta H(\omega)} \frac{1 + e^{2ik_z^{(f)}d_f} r_{mf}(\mathbf{k}_{\parallel}, \omega) r_{fs}(\mathbf{k}_{\parallel}, \omega)}{e^{ik_z^{(s)}d} e^{ik_z^{(f)}d_f} t_{mf}(\mathbf{k}_{\parallel}, \omega) t_{fs}(\mathbf{k}_{\parallel}, \omega)} \tilde{p}(\mathbf{k}_{\parallel}, \omega), \quad (27)$$

where $k_z^{(l)}$ is the z component of the wave vector for muscle ($l = m$), fat ($l = f$) or skin ($l = s$), d_f is the thickness of the fat layer (3 mm in these simulations), and t and r are the standard amplitude transmission and reflection coefficients for acoustic fields [32]:

$$r_{ij}(\mathbf{k}_{\parallel}, \omega) = \frac{\rho_i k_z^{(j)}(\mathbf{k}_{\parallel}, \omega) - \rho_j k_z^{(i)}(\mathbf{k}_{\parallel}, \omega)}{\rho_i k_z^{(j)}(\mathbf{k}_{\parallel}, \omega) + \rho_j k_z^{(i)}(\mathbf{k}_{\parallel}, \omega)}, \quad (28a)$$

$$t_{ij}(\mathbf{k}_{\parallel}, \omega) = \frac{2\rho_i k_z^{(j)}(\mathbf{k}_{\parallel}, \omega)}{\rho_i k_z^{(j)}(\mathbf{k}_{\parallel}, \omega) + \rho_j k_z^{(i)}(\mathbf{k}_{\parallel}, \omega)}. \quad (28b)$$

The reconstruction algorithm was implemented as follows. The 3D FFT algorithm was employed to compute sampled values of $p(k_x, k_y, \omega)$ from the noiseless or noisy measurement data $p(\mathbf{r}, t)$. A low-pass filter was applied so that $p(k_x, k_y, \omega) = 0$ for

$k_x^2 + k_y^2 \geq \omega^2/c_p^2$, where c_p is the speed of sound in the layer in which the object resides. The filtering is performed to preserve the relationship expressed in Eq. (26). For

$k_x^2 + k_y^2 \geq \omega^2/c_p^2$, $k_z^{(p)}$ is imaginary, and $\mathcal{A}$ is thus a Fourier–Laplace transform and not a Fourier transform. The resulting data were employed in Eq. (27) to obtain an estimate of the object's Fourier transform on a uniform grid in k_x and k_y and a nonuniform grid in k_z . For a

given k_x and k_y , the sampled values of k_z corresponded to $k_z = \sqrt{\omega^2/c_p^2 - k_x^2 - k_y^2}$, where c_p is the speed of sound in the layer containing the objects. The object estimate was then interpolated onto a uniform grid in k_z for $k_z > 0$ for each k_x and k_y using linear interpolation. Because the object being reconstructed was purely real, the Fourier relation $\mathcal{A}^*(-\mathbf{k}) = \mathcal{A}(\mathbf{k})$ was used to obtain the Fourier components of the object for $k_z < 0$. Finally, the Fourier transform of the object was performed using a 3D FFT to obtain the estimate of the original phantom.

5. RESULTS

The estimate of $A(\mathbf{r})$ that was reconstructed by use of Eq. (27) from the noiseless simulation data is displayed in Fig. 2. Specifically, a 2D slice corresponding to the plane $z = -1.37$ cm is shown in the top-left panel. The other three panels contain line plots through the reconstructed image (in black) and through the original phantom along the lines $x = 0.75$ cm and $z = -1.37$ cm (top right), $y = -1.33$ cm and $z = -1.51$ cm (bottom left), and $y = 1.42$ cm and $z = -1.42$ cm (bottom right). Note that the reconstructed and original objects are so similar as to be indistinguishable. This is expected and corroborates the mathematical correctness of the reconstruction formula.

To demonstrate the improvement in reconstruction accuracy that can be obtained by use of the proposed reconstruction algorithm, images were reconstructed from the noiseless simulation data by use of Eq. (6) [see Eq. (17) of [16]], which assumes an acoustically homogeneous background. In Fig. 3, a 2D slice through the reconstruction of the original phantom based on the assumption of a homogeneous background is shown in the plane $z = -1.37$ cm. The assumed speed of sound employed by the reconstruction algorithm was consistent with muscle tissue (i.e., the speed of sound in the layer containing the optical absorbers). One notes that the image reconstructed by use of Eq. (6) is qualitatively similar to the original object but does not accurately estimate the two sphere's radii or the values of $A(\mathbf{r})$. In the line plots (in the upper-right and bottom two panels), one can see more clearly the failure of the homogenous model in quantitatively reconstructing objects in a layered medium. In cases where the acoustic properties of the layers are not so closely matched, e.g., when one layer is bone, the degradation in the accuracy of images produced by use of the homogeneous medium assumption can be expected to be even more dramatic.

As the acoustical properties of the medium layers may not precisely known in practice, robustness tests were performed to determine how well the algorithm performs with respect to imprecise knowledge of various system parameters, e.g., the speed of sound. To this end, a number of simulations were performed in which the speeds of sound assumed in the inversion algorithm were varied from the true values. In Fig. 4, a 2D slice of the reconstruction through the plane $z = -1.37$ cm is shown in which the speed of sound assumed in the inversion algorithm differed from the actual value in the first layer only. Specifically, the speed of sound assumed by the inversion algorithm is 1627 m/s (5% larger than the actual value of 1550 m/s). One can see that the reconstructed spheres have a larger radius than both the original phantoms and the reconstruction based on the assumption of homogeneity. The value for the estimated absorbed energy density is also less accurate than in either of the previous reconstructions. By misestimating the speed of sound c_P in the layer

containing the optical absorbers, the mapping $k_z(\mathbf{k}_{\parallel}) = \sqrt{\frac{\omega^2}{c_P^2} - k_{\parallel}^2}$ employed in Eq. (27) produces an erroneous set of k_z values and therefore error in the estimated Fourier components of $A(\mathbf{r})$. This results in either a compression (when the speed of sound used is smaller than actual value) or expansion (when the speed of sound used is larger than the actual value) of the object in the z direction (the interpolated direction in Fourier space).

As a further test of the robustness of the algorithm, we investigated the case when the reconstruction algorithm employed the correct the speed-of-sound value in the layer containing the optical absorber but employed the wrong values for the other two layers. In Fig. 5, a 2D slice through the plane $z = -1.37$ cm of the reconstruction of the original phantoms is shown in which the speed of sound assumed in the inversion algorithm differs from the actual value in the second and third layer only. In the second layer, the assumed speed of sound (1535 m/s) is 4% larger than the actual speed of sound (1476 m/s), and in the third layer, the assumed speed of sound (1543 m/s) is 3% larger than the actual speed of

sound (1498 m/s). One sees immediately that misestimating the acoustical properties of the layers that do not contain the object has a less deleterious effect on the reconstruction than misestimating the acoustical properties in the object layer.

Finally, a preliminary investigation of the proposed reconstruction algorithm's robustness to stochastic measurement noise was performed. In the top-left panel of Fig. 6, a 2D slice of the image reconstructed by use of Eq. (27) from one realization of the noisy data is displayed for the plane $z = -1.37$ cm. The noise level in the pressure data was 1%, as defined in Section 4. To quantify the noise level, empirical estimates of the image variance were computed [33]. To compare the imaging model presented in this paper to a previous model [16], the image variance over 1000 realizations of the noise process was calculated for two situations. In the first, the phantoms were embedded in a known, layered medium, and Eq. (27) was used to estimate the image. In the second, the phantoms were embedded in a known, homogeneous medium, and Eq. (6) was used to estimate the image. The variance for the two cases is shown in Fig. 6 along the lines $x = 0.75$ cm and $z = -1.37$ cm (top-right panel), $y = -1.33$ cm and $z = -1.51$ cm (bottom-left panel), and $y = 1.42$ cm and $z = -1.42$ cm (bottom-right panel), where the blue, dashed lines denote the variance of the image when the object is embedded in the layered medium and the black, solid lines denote the variance of the image when the object is embedded in a homogeneous medium. The image variance in both cases are reasonably flat. The imaging model presented in this paper has a similar robustness to noise to the previously introduced imaging model [16]. A more detailed investigation of the statistical properties of the reconstruction presented in this paper is the subject of future research.

6. CONCLUSION

In this paper, a reconstruction formula has been introduced for PAT in the case that the optical absorber is embedded in a layered medium and acoustical detection is performed on a planar surface. The reconstruction formula demonstrates a mapping between the 3D Fourier transform of the optical absorber and the 2D spatial Fourier transform of the acoustic pressure data acquired on the detection plane. Through the use of simulations, the reconstruction formula was compared to a previous formula valid only for the case when the optical absorber is embedded in a homogeneous medium and acoustical detection occurs on a planar surface. It was shown that the formula that is valid for a homogeneous medium does not return accurate estimates of object structure when the object is embedded in a layered medium, although there is qualitative similarity between the output of the reconstruction formula and the original phantom. The robustness of the formula has been explored when imprecise knowledge of the layered medium is used as an input to the reconstruction formula. The stability of the inversion formula with respect to additive noise has also been investigated.

Further work is required to extend the layered media formalism for PAT to the case when the acoustic properties of each layer are dispersive and absorptive. Absorptive processes complicate the simple Fourier relationship between acoustic pressure and optical absorption density presented in this paper. Additionally, experimental applications of this work in biological imaging need to be explored.

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Fig. 1.

Example of a layered medium. Each layer, bounded between z_m and z_{m-1} and having length d_m , may have different speed of sound, c_m and particle density ρ_m . The detection plane is assumed to be at $z_0 = 0$, and the object is contained in layer n .

Fig. 2.

(Color online) (top left) Image of the numerical phantom in the plane $z = -1.37$ cm reconstructed by use of Eq. (27) from noiseless data. (top right) Profile through the reconstructed image (black) and original phantom (blue dashed) along the line $x = 0.75$ cm and $z = -1.37$ cm. (bottom left) Corresponding profiles along the line $y = -1.33$ cm and $z = -1.51$ cm. (bottom right) Corresponding profiles along the line $y = 1.42$ cm and $z = -1.42$ cm.

Fig. 3.

(Color online) (top left) Image of the numerical phantom in the plane $z = -1.37$ cm reconstructed by use of Eq. (27) from noiseless data using the assumption of a homogeneous medium with speed of sound consistent with the first layer of the actual layered structure.

(top right) Profile through the reconstructed image (black) and original phantom (blue dashed) along the line $x = 0.75$ cm and $z = -1.37$ cm. (bottom left) Corresponding profiles along the line $y = -1.33$ cm and $z = -1.51$ cm. (bottom right) Corresponding profiles along the line $y = 1.42$ cm and $z = -1.42$ cm.

Fig. 4.

(Color online) (top left) image of the numerical phantom in the plane $z = -1.37$ cm reconstructed by use of Eq. (27) from noiseless data when the speed of sound in the first layer of the structure is incorrectly estimated. (top right) profile through the reconstructed image (black) and original phantom (blue dashed) along the line $x = 0.75$ cm and $z = -1.37$ cm. (bottom left) Corresponding profiles along the line $y = -1.33$ cm and $z = -1.51$ cm. (bottom right) Corresponding profiles along the line $y = 1.42$ cm and $z = -1.42$ cm.

Fig. 5.

(Color online) (top left) Image of the numerical phantom in the plane $z = -1.37$ cm reconstructed by use of Eq. (27) from noiseless data when the speed of sound in the second and third layer of the structure are incorrectly estimated. (top right) Profile through the reconstructed image (black) and original phantom (blue dashed) along the line $x = 0.75$ cm and $z = -1.37$ cm. (bottom left) Corresponding profiles along the line $y = -1.33$ cm and $z = -1.51$ cm. (bottom right) Corresponding profiles along the line $y = 1.42$ cm and $z = -1.42$ cm.

Fig. 6.

(Color online) (top left) Image of the numerical phantom in the plane $z = -1.37$ cm reconstructed by use of Eq. (27) when 1% noise is added to the acoustic signal. (top right) Profile of the variance of the reconstructed object for the case when the object is embedded in a layered medium (blue dashed) and a profile of the variance of the reconstructed object when the object is embedded in a homogeneous medium (black) along the line $x = 0.75$ cm and $z = -1.37$ cm. (bottom left) Corresponding profiles along the line $y = -1.33$ cm and $z = -1.51$ cm. (bottom right) Corresponding profiles along the line $y = 1.42$ cm and $z = -1.42$ cm.