Published in final edited form as:

J Biomech. 2011 November 10; 44(16): 2802–2808. doi:10.1016/j.jbiomech.2011.08.007.

Assessment of the Applicability of Hertzian Contact Theory to Edge-Loaded Prosthetic Hip Bearings

Anthony P. Sanders, M.S. and

Dept. of Mechanical Engineering, University of Utah, Salt Lake City, Utah and Ortho Development Corp., Draper, Utah, 936 Diestel Rd., Salt Lake City, UT 84105, Phone: 801-619-3436, Fax: 801-619-8936, ap.sanders@utah.edu

Rebecca M. Brannon, Ph.D.

Dept. of Mechanical Engineering, University of Utah, Salt Lake City, Utah

Abstract

The components of prosthetic hip bearings may experience *in-vivo* subluxation and edge loading on the acetabular socket as a result of joint laxity, causing abnormally high, damaging contact stresses. In this research, edge-loaded contact of prosthetic hips is examined analytically and experimentally in the most commonly used categories of material pairs. In edge-loaded ceramic-on-ceramic hips, Hertzian contact theory yields accurate (conservatively, <10% error) predictions of the contact dimensions. Moreover, Hertzian theory successfully captures slope and curvature trends in the dependence of contact patch geometry on the applied load. In an edge-loaded ceramic-on-metal pair, a similar degree of accuracy is observed in the contact patch length; however, the contact width is less accurately predicted due to the onset of subsurface plasticity, which is predicted for loads >400 N. Hertzian contact theory is shown to be ill-suited to edge-loaded ceramic-on-polyethylene pairs due to polyethylene's nonlinear material behavior. This work elucidates the methods and the accuracy of applying classical contact theory to edge-loaded hip bearings. The results help to define the applicability of Hertzian theory to the design of new components and materials to better resist severe edge loading contact stresses.

Keywords

Prosthetic hip bearings; Hertzian contact; edge loading; microseparation; ceramic-on-ceramic

1. Introduction

The femoral head and acetabular liner of a prosthetic hip joint may not always function as the ideal ball-and-socket joint they are designed to be, which may be a root cause for clinically observed wear modes. Hip replacement surgery may alter the structural dimensions and the tissue constraints of a hip joint such that the ball and socket are held together more loosely than planned for in design. Thus, when examining the potential wear

© 2011 Elsevier Ltd. All rights reserved.

Publisher's Disclaimer: This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Conflict of interest statement

The authors declare that they have no conflicts of interest in publishing this article.

modes of prosthetic hips, it is important to consider that the joint has the potential to be damaged from adverse behaviors such as disassociation and eccentric contact.

Previous investigations have described various abnormal behaviors in prosthetic hip joints. Dislocation, with the ball (or *head*) fully exiting the socket, has been widely reported and is the extreme example (Lewinnek et al., 1978). Fluoroscopic studies have revealed smaller ball-socket separations (subluxation) during various hip motions (Lombardi et al., 2000; Dennis et al., 2001; Komistek et al., 2002). Such subluxation has been called *microseparation* (Nevelos et al., 2000) and *micro-lateralization* (Sariali et al., 2010). Implant malalignment and small femoral heads may contribute to femoral neck impingement on the liner's rim (Nadzadi et al., 2002; Crowninshield et al., 2004). Impingement may lever the head out of the socket, causing subluxation where the head bears upon the socket's edge (*edge loading*) (Scifert et al., 2001; Kluess et al., 2007). A relatively vertical cup orientation may cause edge loading (Mellon et al., 2010), and a recent clinical study found 34% of 1884 acetabular cups to be abducted above the ideal 45° maximum (Callanan et al., 2010). Thus, systematic laboratory investigations are needed to quantify the wear characteristics or other consequences of these ways that a prosthetic hip might deviate from ideal behavior.

Dislocation can severely scratch the femoral head (Bourne et al., 2005). Microseparation can cause ceramic component stripe wear (Walter et al., 2004; Yamamoto et al., 2005), and in laboratory tests, stripe wear has contributed to squeaking in ceramic hips (Taylor et al., 2007). Finite element analyses (FEA) of edge loading contact stresses from microseparation (Mak et al., 2002) and impingement (Kluess et al., 2007) have revealed markedly elevated contact stresses. Hip simulator wear tests imparting microseparation and edge loading have elicited clinically relevant stripe wear on ceramic prostheses (Nevelos et al., 2000; Manaka et al., 2004). Each of these effects is a potential failure mode in the sense that the bearings accrue potentially harmful damage.

In ideal concentric loading, the ball and socket are in conforming contact, meaning that their contacting surface radii are closely matched (e.g. <100 μm difference); thus, loads produce large contact areas and low contact stresses. In contrast, adverse loading induces high contact stress because the ball and socket come into non-conforming contact, meaning that the contacting surfaces have radii that differ greatly. Under such conditions, loads generate smaller contact areas and higher contact stresses.

Although Hertzian contact theory has been applied in the case of concentric ceramic-on-ceramic hip contact (Mak and Jin, 2002), its merits and shortcomings have not been assessed in abnormal states and multiple material couples. For analyses of adverse loading, Hertzian theory may be useful and applicable provided that the following key assumptions of the theory are approximately satisfied: 1) The materials are homogeneous, linear elastic, and isotropic; 2) The surfaces are perfectly smooth and frictionless; 3) The surfaces are non-conforming; and 4) The contact dimensions are much smaller than the surface radii at the contact point (Hertz, 1882; Johnson, 1985). Considering these in the case of edge-loaded ceramic and metal hip prostheses, three preliminary observations are made:

- Items 1 and 2 are as closely approximated as they are in the case of concentric contact.
- For Item 3, the surfaces are less conforming than in the case of concentric contact.
- Thus, Item 4 is more likely to be valid than in the case of concentric contact.

Based on these observations, this study examines the hypothesis that the contact dimensions of edge-loaded ceramic and metal hip bearings can be accurately estimated using Hertzian

theory. For contrast, the study also examines the hypothesis that a plastic bearing material with nonlinear constitutive behavior will not be accurately modeled by the theory.

2. Materials and Methods

2.1. Hertzian contact analysis

Contact between a femoral head and a liner's radiused edge was modeled as a sphere-torus contact (Fig. 1). Predictions of the contact dimensions were computed using Hertzian contact theory. For a contact point on the torus surface, the principal curvatures are given by (Irons, 2005):

$$\kappa_1 = H - \sqrt{H^2 - K} \quad \kappa_2 = H + \sqrt{H^2 - K} \quad (1)$$

Here, K is the Gaussian curvature and H is the mean curvature, which are:

$$K = \frac{\cos \phi}{r(R+r \cos \phi)} \quad H = \frac{R+2r \cos \phi}{2r(R+r \cos \phi)} \quad (2)$$

Here, R and r are the torus' major and minor radii, respectively, and ϕ is the angle about the torus' minor diameter. Given the liner inclination θ in Fig. 1, ϕ was related to θ as: $\phi = \pi - \theta$. The dimensions of the contact ellipse relate to the contacting surfaces' configuration as (Nikas, 2002):

$$\frac{(1-e^2)^{-1} \mathbf{E}(e) - \mathbf{K}(e)}{\mathbf{K}(e) - \mathbf{E}(e)} = \frac{\Sigma + \Delta}{\Sigma - \Delta} \quad (3)$$

where (Hertz, 1882; Harris, 1991):

$$\Sigma = \kappa_1 + \kappa_2 + \rho_1 + \rho_2 \quad \text{and} \quad \Delta = \left[(\kappa_1 - \kappa_2)^2 + (\rho_1 - \rho_2)^2 + 2(\kappa_1 - \kappa_2)(\rho_1 - \rho_2) \cos(2\alpha) \right]^{1/2} \quad (4)$$

Here, $\rho_1 (= \rho_2)$ was the curvature of the sphere, and $\alpha = 0$ since one surface was spherical. Further, $e = \sqrt{1 - b^2/a^2}$, where a is the ellipse's major semi-axis and b its minor semi-axis. $\mathbf{K}(e)$ and $\mathbf{E}(e)$ are the complete elliptic integrals of the first and second kind. Equation (3) was solved numerically for e .

Starting from Hertz's integral formulas (Hertz, 1882), we derived formulas for the contact ellipse dimensions:

$$a = \left\{ \frac{6Q}{\pi e^2 E^* (\Sigma - \Delta)} [\mathbf{K}(e) - \mathbf{E}(e)] \right\}^{1/3} \quad b = \left\{ \frac{6Q e' [\mathbf{E}(e) - (e')^2 \mathbf{K}(e)]}{\pi e^2 E^* (\Sigma + \Delta)} \right\}^{1/3} \quad (5)$$

where $e' = \sqrt{1 - e^2}$ (Erdelyi, 1953). The contacting bodies' Young's moduli (E_1, E_2) and Poisson's ratios (ν_1, ν_2) appear within E^* : $1/E^* = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$. The maximum contact pressure is $P_{\max} = 3Q/2\pi ab$, and the mean pressure is $\bar{P} = Q/\pi ab$ (Johnson, 1985).

2.2. Test specimens

To validate this study's hypotheses, three different material pairs were tested: ceramic-on-ceramic (*CoC*), ceramic-on-metal (*CoM*), and ceramic-on-plastic (*CoP*). The ceramic components were Al_2O_3 products (BioloX Forte, Ceramtec, Germany). Two *CoC* pairs, nominally Ø28 mm and Ø36 mm, were examined. The *CoCr* (ASTM F 1537) liner was a custom-made facsimile; to reduce cost, its concave ID was simplified from a sphere to a cylinder, though without affecting the desired form of the edge radius. This liner had a nominal 36 mm ID and a 4.0 mm edge radius polished to $0.05 \mu\text{m R}_a$. There were also two plastic liners, nominally Ø28 and Ø36 mm. They were custom-made from crosslinked ultra-high molecular weight polyethylene (*XLPE*), and their 2.54 mm edge radii were polished to $0.2 \mu\text{m R}_a$. All components are pictured in the on-line material. The relevant material properties are given in Table 1.

In the ceramic liners, the edge radius was observed to consist of multiple segments. The edge was measured by three means: 1) a touch-probe coordinate-measuring-machine (*CMM*) with resolution $<5 \mu\text{m}$ (Contura, Zeiss, Germany), 2) an arc-encoder stylus with resolution $<0.5 \mu\text{m}$ (Contracer, Mitutoyo, Japan), and 3) a laser sensor with resolution $<0.1 \mu\text{m}$ (Nexiv, Nikon, Japan). The measurements revealed that the edge's profile was best represented as a series of arc segments (Fig. 2, dimensions in Table 2). Each segment, being a surface of revolution, was regarded as a partial torus surface. Both ceramic liners exhibited a distinct crest where the edge met the spherical surface; otherwise, the junctions between surface segments were tangent. In the *CoCr* and *XLPE* liners, the edge radii were closely controlled during manufacture to yield single arc segments. These specimens' key dimensions are in Table 3.

2.3. Contact tests

The liner was secured via a Morse taper with a titanium acetabular shell. The shell was securely assembled within a hemispherical cavity in a 481 kg/m^3 polyurethane (ASTM F-1839) test block. As an exception, the *CoCr* liner was clamped directly in the test fixtures.

The tests created quasi-static contact between the femoral head and specified points on the liner's edge. Figure 3 shows the typical set-up; the on-line material provides additional photos. The head was attached to the uniaxial test frame's actuator. A sine plate was attached to the test frame's load cell, and the plate oriented the liner (angle θ , Fig. 1) such that the specified contact point had a vertical normal vector. The liner assembly was placed on the sine plate underneath the head. To align the head's pole to the nadir of the liner's edge, we used a repetitive process of making a contact mark between the head and edge, followed by shimming the test block to adjust the alignment. Alignment accurate to $20 \mu\text{m}$ was confirmed using an optical *CMM* (Nexiv, Nikon, Japan). The test procedure consisted of applying a vertical force to press the head into the liner's edge while creating a record of the contact patch using a so-called fingerprinting technique (Sanders and Brannon, 2011) illustrated in the on-line material. The contact patch was measured at $37\text{--}143\times$ magnification using the optical *CMM*. Tests were conducted on multiple liner edge points; the test matrix is in Table 4. For the *CoC* pairs, *Configuration 1* gave contact on *R Minor 1* (Fig. 2) and *Configuration 2* gave contact on *R Minor 2*; another created contact directly upon the crest (Fig. 2).

3. Results

Figure 4 shows the recorded contact patches with one example from each material pair. The on-line material provides additional examples, with a series of images for each material pair. The dimensional results are in Figure 5, and Table 5 summarizes the results considering the

complete load ranges. Table 6 gives statistics for several tests involving multiple trials. Graphs for the remaining contact pairs are in the on-line material.

4. Discussion

The fingerprinting contact measurement technique was inspired by Hertz, who used “the thinnest possible layer of lampblack” to measure small contacts in glass (Hertz, 1882). Figure 6 graphs some of Hertz’ experimental results. Like our results, Hertz’ measurements were generally greater than his predictions, perhaps caused by squeezing of the lampblack outside of the true contact patch. In our technique, it is likely that contact would squeeze some of the grease slightly beyond the contact border; furthermore, powder particles may adhere to the transfer film’s edge. Both effects could exaggerate contact dimensions, resulting in increasing relative error with decreasing contact size. In contrast, with Hertzian theory’s assumption that the contact patch is small, its predictions of the contact dimensions should increase in accuracy with decreasing contact size, and vice versa. In Fig. 6, Hertz’ data for Load <15 kg shows an erratic error (percent difference), indicating that the measurement technique loses relative precision for small contact patches. The data for Load >15 kg trends toward decreased error; this may indicate that with increasing contact size, decreasing measurement error overrides the diminution of the contact theory’s accuracy. As described below, our results exhibited trends consistent with the sources of potential measurement error. Regarding repeatability of the results, the data ranges were larger for the CoP pairs than the CoC pairs (Table 6), probably because the rougher surface of the plastic parts made the contact patch borders less distinct. Based on the small ranges in the CoC tests, the CoM tests were performed using single trials.

Hertzian predictions were most accurate in the CoC pairs. Researchers have reported that Al_2O_3 maintains an elastic Hertzian response up to a mean contact pressure of ~5 GPa (maximum pressure ~7.5 GPa) (Guiberteau et al., 1994). In the present tests of the Ø36 pair, the maximum predicted $\bar{P}$ was 3.8 GPa, with $P_{\max}=5.7$ GPa ($\theta=25^\circ$, $Q=2500$ N); thus, based on the prior research, all the present CoC trials were within the elastic range. The predictions were more accurate at $\theta=25^\circ$ than at $\theta=16.1^\circ$, which is expected because $\theta=25^\circ$ gives reduced surface conformity and therefore better satisfaction of the theory’s assumptions. The error in semi-minor b (roughly, 10–15%) was greater than that in semi-major a (roughly, 1–8%). This is explained by the tendency of the measurement technique to exaggerate smaller contact patch dimensions, which was also manifested by increasing error with decreasing a . The lowest-error results in a (<2% Dif., Fig. 5a, $\theta=25^\circ$) compare favorably with Hertz’ results (<4% Dif., Fig. 6), in spite of our greater measurement complexity (opaque materials and a grease transfer film). An assessment of the theory’s accuracy for CoC edge loading is difficult due to measurement error and the lack of an independent measurement standard. Yet, after slightly discounting the size-dependent error in b , Hertzian predictions conservatively have <10% error for CoC edge loading.

In the CoM pair, maintenance of elastic conditions may be judged approximately using a criterion for avoiding subsurface yield in circular contacts, $\bar{P} < 1.1S_y$ (Fischer-Cripps, 2000), where S_y is the metal’s tensile yield strength (965 MPa, certification data). For example, at $\theta=15^\circ$ and 400 N, the predicted $\bar{P}$ was 1134 MPa; this fails the criterion, so subsurface yield likely occurred in the CoCr for $Q \geq 400$ N. Each trial was conducted at a fresh edge point, and no permanent indentations were observed. In spite of predicted plasticity, the CoM contact patches were elliptical, and both a and b followed the trends predicted by Hertzian theory. The error in a was consistently around –5% for $Q \geq 300$ N, indicating that a was large with respect to a subsurface plastic zone. In contrast, the error in b trended greater with $Q > 400$ N, indicating that plasticity had a noticeable effect on this smaller dimension. In both a and b , the error clearly trended greater for $Q < 300$ N, probably due to the size-dependent

measurement errors. To consider metal-on-metal (*MoM*) contact, we analyzed a Ø36 mm CoCr head-liner contact at $\theta=15^\circ$. This virtual pair gave $\bar{P}>1.1S_y$ at $Q=490$ N; thus, in a *MoM* pair, a higher load is needed to induce subsurface plasticity than in a *CoM* pair. The propensity for subsurface yield means that Hertzian predictions have a limited range of accuracy, roughly <400 N in this study, in edge-loaded metal bearings. Measurement errors limited our ability to demonstrate the theory's applicability throughout this load range; yet, in a , the error was <10% for $Q>100$ N.

The error was greatest in the XLPE liners, which is attributed to the material's nonlinear elastic properties. As an example, the Ø36 pair at $\theta=15^\circ$ and 10 N had a predicted $P_{\max}$ of 25.3 MPa, a value that is beyond XLPE's small linear range (<10 MPa, manufacturer's data). The *CoP* pairs are an example for which edge-load contact cannot be accurately analyzed using Hertzian theory. This conclusion may be extrapolated to metal-on-plastic (*MoP*) pairs, because *CoP* and *MoP* each act essentially as a rigid compliant pair (Bartel et al., 1985).

The contact pressure caused by edge loading is much greater than that caused by concentric contact. For example, with the Ø36 Al_2O_3 pair edge loaded to 1000 N ($\theta=16.1^\circ$), $P_{\max}$ is 1950 MPa, but for concentric contact, it is 45 MPa. Similar dramatic differences were reported in a FEA of microseparation (Mak et al., 2002). Researchers reported contact pressures >1 GPa for head-liner contact in FEA of edge-loaded hard bearings (Elkins et al., 2011). Small elements are needed to yield highly accurate estimates of edge-loaded contact stresses by FEA. Elkins et al. reported that ~ 20 elements/ mm^3 were needed to obtain convergence to the "gold standard" Hertzian analysis. Yet, this volumetric density implies an element edge length of 0.36 mm, which would be insufficient to accurately model the small contact patch minor axes (e.g. <0.3 mm) seen in the present study. Submodelling techniques have been used for FEA of small stress concentrations and contacts (Cormier et al., 1999), and these techniques may enable more accurate FEA of edge-loaded hip bearings. The present study's results and techniques may serve to experimentally validate such analyses.

The separation δ (Fig. 1) was 3.0–3.5 mm for angles $\theta\approx 16^\circ$, which is near the upper end of fluoroscopically measured values (Lombardi et al., 2000; Dennis et al., 2001; Komistek et al., 2002). Separation of ~ 6 mm for $\theta=25^\circ$ is beyond values measured *in-vivo*. The δ values were chosen partly to reflect *in-vivo* measurements (thus: $\delta=2, 3.0, 3.5$ mm), and partly to broaden the range of conditions (thus: $\delta=6, 6.5$ mm) and better elucidate the applicability of Hertzian theory. Little is currently known about the *in-vivo* contact loads experienced during subluxation; therefore, this study examined widely varying loads to reflect the likelihood that *in-vivo* contact loads span a broad range.

Researchers have subjected fine-grained Al_2O_3 to reciprocated sliding under a mean contact pressure of 950 MPa, which elicited severe intergranular fracture and grain pullout in 12,000 cycles (Cho et al., 1992); *CoC* hip stripe wear is characterized by a similar microstructural appearance (Walter et al., 2004). For further comparison, analysis of the Ø36 *CoC* pair ($\theta=16.1^\circ$) shows that $\bar{P}=950$ MPa is attained at $Q\approx 400$ N. From these data, it appears that relatively low edge-load contact forces, occurring repeatedly during *in-vivo* subluxation, could be sufficient to cause *CoC* stripe wear. Notably, a 400 N spring force was employed to instigate edge loading in microseparation wear tests that produced *CoC* stripe wear (Nevelos et al., 2000).

The ceramic liners' crests (Figure 2) have been observed previously (Walter et al., 2004). In this study, contact upon this crest occurred for a separation, $\delta=2.0$ mm, equal to the average observed in a group of treadmill-walking hip replacement patients (Komistek et al., 2002).

The observed contact patch (Fig. 4d) had 15% less area than the patch at $\delta=3.0$ mm ($\theta=16.1^\circ$). This implies that crest loading induces particularly elevated contact stresses, which exposes yet another potential cause of stripe wear.

This study has provided extensive evidence that the accuracy of Hertzian contact theory for predicting contact dimensions in edge loading depends on the bearing materials and the contact position. The theory is most applicable in CoC pairs (with error <10%) because these maintain elastic response to high contact pressures. This finding is valuable because it implies that Hertzian theory may be used to predict edge-loading contact stresses (see, e.g., Fabrikant, 2005), thereby expediting development of materials and components that are more durable under adverse loads. Hertzian theory should also be useful for edge-loaded CoM (and MoM) bearings, though for loads <400 N since, above that, subsurface plasticity is predicted. The theory is ill-suited to CoP (and MoP) bearings due to nonlinear material behavior.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

Nomenclature

E, E^*	Young's modulus, apparent elastic modulus
$K(e), E(e)$	complete elliptic integrals of first, second kind, for modulus e
H, K	mean, Gaussian curvatures
ID, OD	inner diameter, outer diameter
Q	normal contact force
$P_{\max}$	maximum contact pressure
$\bar{P}$	mean contact pressure
R, r	major, minor radii of a torus
a, b	major, minor semi-axes of a contact patch
e	ellipse eccentricity
e'	complementary eccentricity
α	angle between planes of minimum absolute principal curvature
θ	orientation angle between a contacting torus and nested sphere
δ	polar separation between femoral head and acetabular liner
Σ, Δ	functions of the principal curvatures and orientation of contacting surfaces
κ_1, κ_2	principal curvatures at the contact point of a first body in a contact couple
ρ_1, ρ_2	principal curvatures at the contact point of a second body in a contact couple
ν	Poisson's ratio
ϕ	angle about a minor diameter in parametric equations of a torus

Acknowledgments

This work was supported by Award #R21AR056374 from the NIAMS/NIH.

References

- Bartel DL, Burstein AH, Toda MD, Edwards DL. The effect of conformity and plastic thickness on contact stresses in metal-backed plastic implants. *Journal of Biomechanical Engineering*. 1985; 107:193–199. [PubMed: 4046559]
- Bourne RB, Barrack R, Rorabeck CH, Salehi A, Good V. Arthroplasty options for the young patient: Oxinium on cross-linked polyethylene. *Clinical Orthopaedics and Related Research*. 2005; 441:159–167. [PubMed: 16330999]
- Callanan MC, Jarrett B, Bragdon CR, Zurakowski D, Rubash HE, Freiberg AA, Malchau H. The John Charnley Award: Risk factors for cup malpositioning: Quality improvement through a joint registry at a tertiary hospital. *Clinical Orthopaedics and Related Research*. 2010; 18:18–28.
- Cho SJ, Moon H, Hockey BJ, Hsu SM. The transition from mild to severe wear in alumina during sliding. *Acta Metallurgica et Materialia*. 1992; 40:185–192.
- Cormier NG, Smallwood BS, Sinclair GB, Meda G. Aggressive submodelling of stress concentrations. *International Journal for Numerical Methods in Engineering*. 1999; 46:889–909.
- Crowninshield RD, Maloney WJ, Wentz DH, Humphrey SM, Blanchard CR. Biomechanics of large femoral heads: What they do and don't do. *Clinical Orthopaedics and Related Research*. 2004; 429:102–107. [PubMed: 15577473]
- Dennis D, Komistek R, Northcut E, Ochoa J, Ritchie A. In-vivo determination of the hip joint separation and the forces generated due to impact loading conditions. *Journal of Biomechanics*. 2001; 34:623–629. [PubMed: 11311703]
- Elkins JM, O'Brien MK, Stroud NJ, Pedersen DR, Callaghan JJ, Brown TD. Hard-on-hard total hip impingement causes extreme contact stress concentrations. *Clinical Orthopaedics and Related Research*. 2011; 469:454–463. [PubMed: 20953853]
- Erdelyi, A. Higher transcendental functions. New York: McGraw-Hill; 1953.
- Fabrikant VI. A new symbolism for solving the Hertz contact problem. *Quarterly Journal of Mechanics and Applied Mathematics*. 2005; 58:367–381.
- Fischer-Cripps, AC. Introduction to contact mechanics. New York: Springer; 2000.
- Glaser D, Dennis DA, Komistek RD, Miner TM. In vivo comparison of hip mechanics for minimally invasive versus traditional total hip arthroplasty. *Clinical Biomechanics*. 2008; 23:127–134. [PubMed: 18022739]
- Guiberteau F, Padture NP, Lawn BR. Effect of grain size on Hertzian contact damage in alumina. *Journal of the American Ceramic Society*. 1994; 77:1825–1831.
- Harris, TA. Rolling bearing analysis. New York: John Wiley & Sons; 1991.
- Hertz, H. On the contact of rigid elastic solids and on hardness. Hertz, H.; Schott, JA., editors. London: MacMillan; 1882. p. 164–183. Miscellaneous papers by
- Irons ML. The curvature and geodesics of the torus. 2005 from <http://www.rdrop.com/~half/math/torus/index.xhtml>.
- Johnson, KL. Contact mechanics. Cambridge: Cambridge University Press; 1985.
- Klues D, Martin H, Mittelmeier W, Schmitz KP, Bader R. Influence of femoral head size on impingement, dislocation and stress distribution in total hip replacement. *Medical Engineering and Physics*. 2007; 29:465–471. [PubMed: 16901743]
- Komistek RD, Dennis DA, Ochoa JA, Haas BD, Hammill C. In vivo comparison of hip separation after metal-on-metal or metal-on-polyethylene total hip arthroplasty. *Journal of Bone and Joint Surgery American*. 2002; 84:1836–1841.
- Lawn BR, Padture NP, Guiberteau F, Cai H. Model for microcrack initiation and propagation beneath hertzian contacts in polycrystalline ceramics. *Acta Metallurgica et Materialia*. 1994; 42:1683–1693.
- Lewinnek GE, Lewis JL, Tarr R, Compere CL, Zimmerman JR. Dislocations after total hip-replacement arthroplasties. *Journal of Bone and Joint Surgery American*. 1978; 60:217–220.
- Lombardi AV, Mallory TH, Dennis DA, Komistek RD, Fada RA, Northcut EJ. An in vivo determination of total hip arthroplasty pistoning during activity. *Journal of Arthroplasty*. 2000; 15:702–709. [PubMed: 11021445]

- Mak MM, Besong AA, Jin ZM, Fisher J. Effect of microseparation on contact mechanics in ceramic-on-ceramic hip joint replacements. *Proceedings of the Institution of Mechanical Engineers. Part H: Journal of Engineering in Medicine*. 2002; 216:403–408.
- Mak MM, Jin ZM. Analysis of contact mechanics in ceramic-on-ceramic hip joint replacements. *Proceedings of the Institution of Mechanical Engineers. Part H: Journal of Engineering in Medicine*. 2002; 216:231–236.
- Manaka M, Clarke IC, Yamamoto K, Shishido T, Gustafson A, Imakiire A. Stripe wear rates in alumina THR - comparison of microseparation simulator study with retrieved implants. *Journal of Biomedical Materials Research - Part B Applied Biomaterials*. 2004; 69:149–157.
- Mellon, SJ.; Simpson, DJ.; Kwon, YM.; Murray, DW.; Gill, HS. The influence of acetabular component orientation on stress during gait in metal-on-metal hip resurfacing. 56th Annual Meeting of the Orthopaedic Research Society; New Orleans. 2010.
- Nadzadi ME, Pedersen DR, Callaghan JJ, Brown TD. Effects of acetabular component orientation on dislocation propensity for small-head-size total hip arthroplasty. *Clinical Biomechanics*. 2002; 17:32–40. [PubMed: 11779644]
- Nevelos J, Ingham E, Doyle C, Streicher R, Nevelos A, Walter W, Fisher J. Microseparation of the centers of alumina-alumina artificial hip joints during simulator testing produces clinically relevant wear rates and patterns. *Journal of Arthroplasty*. 2000; 15:793–795. [PubMed: 11021457]
- Nikas GK. Fatigue life and traction modeling of continuously variable transmissions. *Journal of Tribology*. 2002; 124:689–698.
- Sanders AP, Brannon RM. Determining a surrogate contact pair in a Hertzian contact problem. *Journal of Tribology*. 2011; 133:024502–024506.
- Sariali E, Stewart T, Jin Z, Fisher J. Three-dimensional modeling of in vitro hip kinematics under micro-separation regime for ceramic on ceramic total hip prosthesis: An analysis of vibration and noise. *Journal of Biomechanics*. 2010; 43:326–333. [PubMed: 19892358]
- Scifert CF, Noble PC, Brown TD, Bartz RL, Kadakia N, Sugano N, Johnston RC, Pedersen DR, Callaghan JJ. Experimental and computational simulation of total hip arthroplasty dislocation. *Orthopedic Clinics of North America*. 2001; 32:553–567. [PubMed: 11689369]
- Taylor S, Manley MT, Sutton K. The role of stripe wear in causing acoustic emissions from alumina ceramic-on-ceramic bearings. *Journal of Arthroplasty*. 2007; 22:47–51. [PubMed: 17919593]
- Walter WL, Insley GM, Walter WK, Tuke MA. Edge loading in third generation alumina ceramic-on-ceramic bearings: Stripe wear. *Journal of Arthroplasty*. 2004; 19:402–413. [PubMed: 15188097]
- Yamamoto T, Saito M, Ueno M, Hananouchi T, Tokugawa Y, Yonenobu K. Wear analysis of retrieved ceramic-on-ceramic articulations in total hip arthroplasty: Femoral head makes contact with the rim of the socket outside of the bearing surface. *Journal of Biomedical Materials Research - Part B Applied Biomaterials*. 2005; 73:301–307.

Figure 1. Model of edge-loaded contact, with socket shown in cross section. View gives an anterior-posterior perspective with cup oriented at 50° abduction. In the experiments, the entire assembly was posed with the load vector Q vertical and downward; otherwise, the relative positions were as shown.

Figure 2.

Measurements revealed that the edge profile between the face and the ID of the Al_2O_3 liners was best described as a series of arc segments. Detail views illustrate the cross-sectional profiles of the Ø28 and Ø36 liners.

Figure 3.
Configuration of edge-load test on ceramic prostheses.

Figure 4.

Examples of recorded contact patches. Superimposed dashed red ellipses show Hertzian predictions. a) CoC, Ø28, 1500 N, $\theta=16.7^\circ$. b) CoM, Ø36, 400 N, $\theta=15^\circ$. c) CoP, Ø28, 20 N, $\theta=20^\circ$. d) CoC, Ø36, 1000 N, contact on crest illustrated in Fig. 2.

Figure 5. Contact patch dimensions and % difference of Hertzian predictions compared to measurements. a) and b) CoC, $\varnothing 36$. c) and d) MoM. e) and f) CoP, $\varnothing 36$, $\theta = 15^\circ$.

Figure 6.
Hertz' original experimental contact data, graphed from data in (Hertz, 1882).

Table 1

Material properties for Hertzian contact analysis

Material	Al ₂ O ₃ *	Co-Cr **	XLPE [#]
Young's modulus (MPa)	380,000	241,000	1040
Poisson's ratio	0.26	0.30	0.41

* (Mak and Jin, 2002)

** Manufacturer's data. BioDur CCM Plus, Carpenter, Wyomissing, PA

[#] Manufacturer's data: Ortho Development, Draper, UT

Table 2

Key edge dimensions (in mm) of Al₂O₃ liner specimens (*R Minor*/*Major* headings per Figure 2).

nom. ID	<i>R Minor 1</i>	<i>R Major 1</i>	<i>R Minor 2</i>	<i>R Major 2</i>	<i>R Minor 3</i>	<i>R Major 3</i>
Ø28	17.364	30.665	7.715	21.346	2.290	16.416
Ø36	11.822	29.442	2.260	20.174	n.a.	n.a.

Table 3

Key edge dimensions (in mm) of CoCr and XLPE liner specimens.

Material	nom. ID	R Major	R Minor
CoCr	Ø36	21.920	4.013
XLPE	Ø28	16.807	2.54
	Ø36	20.811	2.54

Table 4

Matrix of prosthesis edge-loading tests. One specimen per pair. θ and δ per Figure 1, n = number of trials performed.

Pair	Size	Configuration 1			Configuration 2		
		θ	δ (mm)	n	θ	δ (mm)	n
CoC	28	16.7°	3.0	2	25°	5.7	2
CoC	36	16.1°	3.5	2	25°	6.3	3
CoM	36	15°	n.a.	1	30°	n.a.	1
CoP	28	20°	5.1	1-2	—	—	—
CoP	36	15°	4.8	1-3	—	—	—

Edge-loaded prostheses; difference between predicted and measured contact dimensions, summarized over the entire load range. (RMS = root-mean-square)

Table 5

Pair	θ	RMS difference		mean % difference	
		<i>a</i> (μm)	<i>b</i> (μm)	<i>a</i>	<i>b</i>
CoC Ø28	25.0°	11.5	24.7	0.9%	-14.2%
	16.7°	15.1	28.4	-0.3%	-15.7%
CoC Ø36	25.0°	9.4	9.1	-0.6%	-9.3%
	16.1°	124.7	18.7	-6.3%	-11.4%
CoM Ø36	30°	55.1	27.7	-7.4%	-31.8%
	15°	105.6	22.5	-7.7%	-33.1%
CoP Ø28	20°	524.4	113.3	-21.3%	-47.4%
CoP Ø36	15°	1178.8	83.5	-31.5%	-42.9%

Table 6

Statistics from several tests with multiple trials (semi-axis dimensions in μm).

Pair	Ø28 CoC	Ø36 CoC	Ø28 CoP	Ø36 CoP
Orientation	16.7°	25°	20°	15°
Load	1000 N	1500 N	10 N	10 N
No. of trials	2	3	2	3
Semi-axis	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>
	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>
Mean	1577	1604	2255	3175
	180	102	220	178
Range	6	9	36	63
	1	4	15	29