An analysis of the mechanical parameters used for finite element compression of a high-resolution 3D breast phantom

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(Received 21 October 2010; revised 11 July 2011; accepted for publication 22 August 2011; published 27 September 2011)

Purpose: The authors previously introduced a methodology to generate a realistic three-dimensional (3D), high-resolution, computer-simulated breast phantom based on empirical data. One of the key components of such a phantom is that it provides a means to produce a realistic simulation of clinical breast compression. In the current study, they have evaluated a finite element (FE) model of compression and have demonstrated the effect of a variety of mechanical properties on the model using a dense mesh generated from empirical breast data. While several groups have demonstrated an effective compression simulation with lower density finite element meshes, the presented study offers a mesh density that is able to model the morphology of the inner breast structures more realistically than lower density meshes. This approach may prove beneficial for multimodality breast imaging research, since it provides a high level of anatomical detail throughout the simulation study.

Methods: In this paper, the authors describe methods to improve the high-resolution performance of a FE compression model. In order to create the compressible breast phantom, dedicated breast CT data was segmented and a mesh was generated with 4-noded tetrahedral elements. Using an explicit FE solver to simulate breast compression, several properties were analyzed to evaluate their effect on the compression model including: mesh density, element type, density, and stiffness of various tissue types, friction between the skin and the compression plates, and breast density. Following compression, a simulated projection was generated to demonstrate the ability of the compressible breast phantom to produce realistic simulated mammographic images.

Results: Small alterations in the properties of the breast model can change the final distribution of the tissue under compression by more than 1 cm; which ultimately results in different representations of the breast model in the simulated images. The model properties that impact displacement the most are mesh density, friction between the skin and the plates, and the relative stiffness of the different tissue types.

Conclusions: The authors have developed a 3D, FE breast model that can yield high spatial resolution breast deformations under uniaxial compression for imaging research purposes and demonstrated that small changes in the mechanical properties can affect images generated using the phantom. © 2011 American Association of Physicists in Medicine. [DOI: 10.1118/1.3637500]

Key words: biomechanical, model, phantom, simulation, deformation, breast imaging, finite element

I. INTRODUCTION

There is considerable effort underway to improve the detection of breast cancer, and imaging modalities have played an

important role in that endeavor.^{1–14} The optimization of breast imaging techniques requires a realistic imaging environment that can replicate the clinical imaging process. Due to radiation dose concerns and time constraints, it would be

impractical and unethical to perform many of these studies with human subjects. In addition, it would be prohibitively expensive to develop physical phantoms that are able to simulate the heterogeneity in patient anatomy and pathology. A realistic computerized, compressible, breast phantom is a practical alternative as it can provide a “known-truth” for evaluating new techniques and parameters while requiring only software to run the simulation.

We previously introduced a methodology to develop a 3D computer-simulated breast phantom based on empirical data.¹⁵ One of the major components of the model is to provide a realistic compression simulation; this will allow the breast phantom to be used in the development and improvement of compressed breast imaging modalities as well as other applications such as multimodal image registration, tumor tracking, and surgical planning. In this paper, we describe a finite element (FE) model that is able to simulate the type of compression used in mammography. Section II will review the mechanical models used to simulate compression of the breast tissues. Section III initially describes the segmentation algorithm used to delineate the different materials of the breast (skin, adipose, and glandular tissues) from CT images. It further explains the incorporation of those different material definitions into a 3D, FE model to simulate breast compression, and the process required to simulate a projection of the compressed phantom. Section IV shows a parametric analysis of the breast model properties that affect the compression of the breast, including the relative stiffness of the different tissue types, skin friction, mesh density, element type, breast density, and material/mass density. Finally, Sec. V explores the impact that the model parameters have on the simulated 3D breast deformation, future efforts to improve the accuracy of the model, as well as limitations of the current implementation of breast compression described in this work.

II. BACKGROUND

Breast compression simulation methods have typically used finite element techniques because of their ability to solve for the large (finite strain) deformation of complex 3D structures. Mammographic compression plates induce a large-body strain, which can be greater than 50%, on the breast in the direction of compression. During simulation, FE methods evaluate the strain and displacement induced in the other dimensions in response to a prescribed uniaxial compression. For linear, isotropic, elastic materials under such uniaxial compression, strain energy, and stress are related by Hooke's law such that¹⁶

$$\sigma_{ij} = \frac{E_Y}{1 + \nu} \left(\epsilon_{ij} + \frac{\nu}{1 - 2\nu} \delta_{ij} \epsilon_{kk} \right), \quad (1)$$

where σ_{ij} is the stress tensor, E_Y is the material's Young's modulus, ν is the Poisson's ratio, ϵ_{ij} is the strain tensor, δ_{ij} is the Kronecker delta, and ϵ_{kk} is the first scalar invariant of strain. Strain is related to displacement, u , as a symmetric tensor using the following equation:

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (2)$$

Because a dynamic solution was obtained through the use of an explicit FE solver, mass effects were included in this model as described by

$$\rho_o \frac{\partial^2 u_i}{\partial t^2} = \rho_o B_i + \frac{\partial \sigma_{ij}}{\partial x_j}, \quad (3)$$

where ρ_o is the materials mass density, and gravity was neglected in this analysis such that external body forces, B_i , are zero. For the models described herein, a prescribed, uniaxial displacement (strain) was applied to the breast through a contact problem with rigid plates, and the resultant 3D stress/strain data were solved for throughout the volume of the simulated breast.

The level of accuracy of a biomechanical simulation is largely dependent on how physically realistic and detailed the substructures of the object are represented in the model. Several patient-specific biomechanical breast models^{17–29} have been developed to simulate compression and are able to predict the compressed location of registration markers to less than 5 mm. Most of these models have been optimized to yield results, while the patient is still at the treatment center (e.g., in a clinically relevant timeframe of <30 min). Therefore, they tend to utilize relatively coarse meshes (node spacing on the order of centimeters) to decrease computational time at the expense of spatial resolution.

Although successful for their clinical end points, the patient-specific models are not intended to provide high spatial resolution or be used as a tool for evaluating imaging techniques. An imaging phantom should be able to be used to optimize and develop new devices and imaging processing techniques that may require different levels of anatomical detail and varying levels or positions of mechanical strain. Therefore, it would be beneficial to have a multimodality phantom with a detailed representation of the breast anatomy that is maintained throughout the simulated imaging study. This phantom could realistically model the interaction of inner breast structures under different imaging approaches, or alternately, be simplified as necessary while offering a consistent structural basis. Our method simulates 3D compression of a high-resolution breast phantom using finite element methods that will ultimately be used for imaging research purposes. In addition to generating high-resolution images, a suitable breast phantom should also have the ability to simulate changes in the breast over time due to age and hormonal variations as well as accommodate user-defined mechanical properties to encompass the wide variability in breast composition (e.g., breast glandular density) that exists among female patients; therefore, the effect of changing these parameters was analyzed in this study.

III. METHODS

High-resolution, volumetric images of 17 pendant breasts were acquired with a prototype dedicated breast CT scanner at UC Davis.^{5,6} A custom denoising algorithm was used on the projection images to suppress noise resulting from the low-dose acquisition.³⁰ The denoised datasets were

reconstructed using a custom filtered back projection algorithm to generate 300 768² coronal images with an in-plane resolution of 250 μm and a slice thickness of 500 μm . A postreconstruction scatter correction method was used to correct for the cupping artifact and improve uniformity.³¹

III.A. Segmentation

The methods used to segment the dedicated breast CT images have been described in detail previously¹⁵ but our current work incorporated a few changes that are summarized here. The denoised and scatter-corrected datasets were segmented into three components: adipose, glandular, and skin tissues using a semiautomated segmentation algorithm developed specifically for these dedicated breast CT datasets.¹⁵ The first coronal slice was defined for each dataset by visual inspection as the location where the denoised data appeared to be fully within the breast volume and without substantial scatter artifacts. The last coronal slice was defined for each dataset by finding the last slice that appeared to have breast data. All pixels before the first slice and after the last slice were assigned to zero. A breast-to-air threshold was defined by using a certain percentage, determined by trial and error for each breast volume, ranging from 25% to 55% of the maximum value in each breast slice. Each breast was masked such that all air pixels were set to zero. An initial segmentation on the breast tissue was performed using an iterative histogram classification technique that separated glandular and skin tissue from adipose. For each slice, the left and right bounds of the histogram were used to calculate the midpoint of the breast values. Next, the average of each half of the histogram was used to redefine the bounds and iteratively recalculate the midpoint until it converged to a single value. A second order polynomial fit to the calculated midpoints across all of the slices was used to make a smoothly varying segmentation threshold. The threshold was applied and all values below the slice specific threshold were assigned to zero.

Next, the breast mask and segmented glandular and skin data were used to determine the skin thickness. For each coronal slice, the mask was eroded by a single pixel and then the eroded mask was subtracted from the breast mask to get a single-pixel-thick mask. The sum of the segmented breast

values located within the thin mask was found. This process was repeated until the mask hit the skin-fat barrier, which was determined as the point when the sum of the values dropped by greater than 40%. An average thickness of the skin was determined from all of the slices and used to define the skin for the breast. Typical breast skin ranges from 1 to 3 mm in thickness;³² the determined skin thickness for the different datasets ranged from 1.5 to 2.5 mm.

The segmented breast values located within the skin were removed and subsequent operations were performed solely on the segmented glandular tissue. A series of morphological operations were performed in the coronal, sagittal, and axial planes using the MATLAB R2007a (The MathWorks, Inc., Natick, MA) *bwmorph* “bridge” and “diag” operations to fill small holes between segmented glandular areas. This was repeated three times and then *bwmorph* “close” and “majority” operations as well as the *bwareaopen* function were used in the coronal slices to remove isolated islands of glandular tissue that were smaller than 2 mm in diameter. This step was important for the mesh generation since glandular segments smaller than 2 mm in diameter do not need to be defined by the generated mesh. As described in the phantom creation methodology,¹⁵ the segmented glandular tissue was further classified into three different types of glandular tissue. This information was used for the simulated image projection but was not used for the described compression analysis. The previously defined skin mask was added to the segmented glandular image to complete the segmented breast volume shown in Fig. 1.

In order to categorize the 17 breast datasets considered in this study, the breasts’ glandular density percentage was calculated as the ratio of the total number of voxels assigned to glandular tissue to the total number of voxels in the breast volume. The breast densities were calculated for all of the datasets and resulted in an average density of $25\% \pm 16\%$ (Table I). The breasts were categorized into three groups based on glandular density (Table I). Breast density categories were chosen to represent an evenly spaced grouping of densities that covered the range of glandular densities in the available breast datasets. The average breast volume for each density category was calculated and demonstrates that the average breast volume decreased with increasing glandular density. Parametric FEM analysis was performed on the

FIG. 1. Columns show breasts in different density categories from left to right: 14% dense, 28% dense, and 40% dense: scatter-corrected breast data is in the top row, and segmented data are in the bottom row.

TABLE I. Breast CT data categorized by glandular density and size.

%Density		0%–15%		15%–30%		30+%	
Number of datasets		7		4		6	
Average volume	Volume range (cm ³)	890	477–1324	732	513–952	467	241–767

28% dense breast and two additional subjects were selected in different density ranges for analysis on the effect of glandular density on simulated compression (Fig. 1).

III.B. Mesh generation and boundary condition assignment

The segmented volume was resized with bilinear interpolation to $384 \times 384 \times 300$ resulting in isotropic $500 \mu\text{m}$ resolution. An isosurface that encapsulated the resized segmented breast volume was generated using MATLAB to create a shell structure. The shell was imported into Hypermesh (Hypermesh 10, Altair Engineering, Inc., Troy, MI),³³ which was used to produce the mesh basis for the FE model. Hypermesh's *shrinkwrap* function was applied to achieve a spatial low-pass filter on the imported outer shell of the 3D breast volume to facilitate automatic meshing with 4-noded, solid tetrahedral elements. Mesh-generation penalties were imposed for elements with high aspect ratios (>25) or extreme element angles ($<5^\circ$) to avoid numerical artifacts due to malformed elements undergoing finite deformations.

Mesh refinement was studied using average element edge lengths of 1.5 mm, 2.5 mm, 3.75 mm, 5 mm, and 1 cm to ensure that element density was not a first-order determinant of the deformation data. Figure 2 shows how the higher mesh density exponentially increases the number of elements, and Fig. 3 demonstrates how the breast is represented with the varying mesh densities. An average element edge length of 2.5 mm was chosen for most simulations. Using an average element edge length of 2.5 mm, the total number of

elements across the 17 different breasts ranged from 131 026 to 718 928.

Material properties for the skin, glandular, and adipose components of the breast were assigned based on the segmented data. The following criteria were used to assign material properties to elements that were ambiguously located in the segmented image: (1) elements that had vertices in multiple materials were defined based on the breast material corresponding to the location of the element's centroid, (2) if the centroid was outside of the defined breast volume or close to air (within $500 \mu\text{m}$ in the coronal plane or 1 mm in the anterior direction), it was assigned skin material properties using the assumption that all elements adjacent to air were skin, ensuring a continuous layer of skin around the breast. The meshes were continuous solid elements without contact interfaces between the different materials. All elements located near or on the first coronal slice were assumed to be attached to the chest wall, with restricted motion in the anterior–posterior direction but permitted degrees of freedom in the superior–inferior and medial–lateral directions.

III.C. Finite element analysis

LS-DYNA (Livermore Software Technology Corp., Livermore, CA),³⁴ an explicit, time-domain finite element package was used to analyze the breast models. We chose an explicit over an implicit solver³⁵ to reduce the RAM requirement for the simulation, and while the full transient deformation of the breast was solved for using this approach, only the final steady-state compression of the breast was used for our analysis. These models were run on Intel Xeon 5140 processors operating at 2.33 GHz in an SMP parallel environment over 4 CPU cores using <1 GB of RAM; typical runtimes ranged from 3 to 4 h.

Typically, the 4-noded tetrahedral elements utilized for these models are not ideal for modeling incompressible soft tissues because they generate numerical artifacts from their innate stiffness. To investigate if it was necessary to capture second order behavior, the type of elements used to define

FIG. 2. Demonstrates how the element count exponentially increases as the edge length decreases.

FIG. 3. Graphical representation of different mesh densities generated using different average element edge lengths. A magnified view of the mesh density is shown to the right of example: (a) = 1.5 mm, (b) = 2.5 mm, (c) = 3.75 mm, (d) = 5 mm, and (e) = 10 mm. The axis shows the orientation of the breast and planes used throughout this work. Notice how the qualitative curvatures of the breast are poorly represented with the more coarse meshes.

the mesh was investigated to compare how 4-noded tetrahedral elements differed from simulations using 10-noded tetrahedral elements and 1-point tetrahedral elements. The difference between these types of elements was the number of integration points internal to the tetrahedron where the strain and displacement were calculated. The 4-noded element had 4 integration points internal to the element from which the calculated strain and displacement was interpolated to the 4 nodes. While it still had 4 nodes, the 1-point element had a single integration point at the centroid of the tetrahedron. The 10-noded element had nodal points located at the four vertices and the six midside nodes and ten integration points located internal to the tetrahedron. The 10-noded element allowed for second order behavior, which essentially means that the sides of the element could bend.

Although breast tissue is typically defined as a hyperelastic material, a linear elastic definition has been shown to sufficiently approximate its behavior.^{17,20,28,29} Therefore, the breast materials (skin, glandular, and adipose) were modeled as linear elastic, isotropic solids. Unfortunately, there are difficulties measuring the stress-strain relationship of breast tissues accurately because the mechanical properties are dependent on the *in situ* environment of the tissue and the mechanical measurement technique (i.e., static vs dynamic). Consequently, there are a wide range of values for the elastic modulus of different breast tissues used in current biomechanical models:^{17–29,36} adipose tissue ranges from 0.5 to 25 kPa, glandular tissue from 0.08 to 272 kPa, and skin from 0.088 to 3 MPa. For our analysis, changing the separate tissues' Young's moduli relative to one another was parametrically

studied to determine the impact on the simulated tissue compression (Table II). We chose a range of stiffness values similar to those previously presented by other researchers.^{17,28}

In addition to varying the stiffness of the tissue, the impact of friction between the compression plates and the skin was parametrically evaluated since factors such as patient age, skin moisture, and sample location can affect this mechanical response. Coefficients of kinetic friction (μ) ranging from 0 to 1.0 were studied in these simulations.^{37,38} The level of material incompressibility was also initially evaluated in this study. However, as described in Sec. V, the Poisson's ratio was fixed at 0.49 for all models in order to achieve a level of incompressibility that did not make the

TABLE II. Mechanical properties investigated.

	Fat	Glandular	Skin
Mass density (Refs. 45, 46) ($\text{g}\cdot\text{cm}^{-3}$)	0.928	1.035	1.1
Poisson's ratio	0.49	0.49	0.49
Coefficient of friction (skin-plates) (μ)			0, 0.1, 0.46, 1.0
Young's Modulus (kPa)			
Scenario 1	1	1	1
Scenario 2	1	5	1
Scenario 3	1	10	1
Scenario 4	1	1	10
Scenario 5	1	1	88
Scenario 6	1	2.5	10
Scenario 7	1	5	10
Scenario 8	1	10	88
Scenario 9	10	50	100

FIG. 4. The 28% dense breast model. (a) precompression; (b) postcompression, (c) off-axis view to get a 3D view of the compression.

model numerically unstable. All material properties used for analysis are shown in Table II.

The compression was modeled using two rigid plates that were infinite in the axial plane at predefined sagittal locations set with one located superior and the other inferior to the breast without initial contact. In order to simulate typical compression levels of mammography, the plates were moved at a constant velocity of 6.25 mm/s to achieve a 5 cm compression thickness in 10 s; this compression was held 50 additional seconds in order for inertial transients to be damped and achieve a steady-state compression. The pre-compression and the postcompression profiles of the breast are shown in Fig. 4.

Since a purely elastic material without material viscosity was modeled, oscillations from the dynamic compression resonate through the material unless numerically damped to achieve the steady-state response. A critical damping coefficient (D_{cr}) was calculated by first solving the undamped model to find the resonant frequency ω_r of the system.³⁹

$$D_{cr} = 2\omega_r, \quad (4)$$

The power spectrum of the displacement signal in the direction of plate movement, of a surface node that did not contact the compression plates and was located near the nipple, was used to determine the resonant frequency. The average damping coefficient for all the different breast models was calculated to be 0.686 ± 0.08 ; $D_{cr} = 0.686$ rad/s was

used for all models presented herein. The effect of damping on the model is illustrated in Fig. 5.

For analysis, the compressed nodal information generated for the finite element tetrahedral mesh was linearly interpolated in three dimensions to provide a new location for each uncompressed image voxel. The relative overall displacement between compressed breasts was calculated in traditional imaging planes and evaluated as a function of the different mechanical parameters in the model to determine their impact on the simulated deformation of the breast phantoms.

III.D. Simulated image projection

Following the finite element compression, a simulated projection of the model was generated. The projection code requires a subdivision surface definition of the phantom; therefore, MATLAB's *isosurface* function was used to generate triangular surfaces from the segmented breast data for each of the different breast materials. The new compressed location for each node in the triangular surface definition of the original uncompressed breast was given by the aforementioned 3D linear interpolation. However, due to the spatial low-pass filtering effect of the *shrinkwrap* function used during mesh generation, not all of the triangulated surface nodes from the segmented data could be defined with the interpolation from the tetrahedral mesh. The new location for

FIG. 5. Damping coefficient effect. The undamped solution was used to calculate the resonant frequency that was suppressed in order to attain a critically damped solution; note the convergence to the steady-state behavior occurs much earlier in time after cessation of the plate compression (10 s).

FIG. 6. Effect of changing mesh density on final displacement. (28% dense, 4-noded tetrahedral elements, $E(\text{fat:glandular:skin}) = 1:5:10$ kPa, $\nu = 0.49$, $\rho = \text{real}$, and $\mu = 0.46$)

undefined nodes was determined by linearly interpolating in 3D between the two nearest defined nodes. The FE compressed triangular surface definition was then used to generate a simulated projection of the breast phantom using the method described in Li *et al.*¹⁵ and Segars *et al.*⁴⁰

IV. RESULTS

IV.A. Categorization of segmented breast models

The breasts from the three different density categories used in this study are shown in Fig. 1. The top row shows slices from the scatter-corrected CT datasets, and the bottom row shows the corresponding slices from the resulting segmented datasets. The least dense breast used was for analysis was 14% dense, the midrange breast was 28% dense, and the densest breast was 40% dense. The average breast volume for each of the breast density categories is shown in Table I.

IV.B. Finite element results

The majority of the parametric analysis was performed using the same breast model. Figure 4 shows the representative breast (28% dense) precompression and postcompression, as well as an off-axis view for 3D display. The initial chest wall diameter was 12.8 cm and 5 cm in thickness post-compression (61% strain).

Figure 5 shows the effect that the imposed critical damping has on the displacement of the analyzed node near the nipple; note the convergence to the steady-state behavior occurs much earlier after cessation of the plate compression (10 s).

Several different analyses were run to demonstrate the difference between using different mesh densities as well as

different breast tissues with varying mechanical properties. The majority of analyses were performed on a 28% dense breast; however, comparisons were also done with a 14% dense and 40% dense breast to show the effect of modulus choice on breasts with different glandular densities. Figures 6–9 show box plots representing the distribution of overall displacements of all the nodes in the breast model to provide a graphical representation of the relative global breast distortion between different material parameters. In these box plots, the solid line is the median; the box is the interquartile range, and the box plots whiskers range from 2.7σ above and below the mean of the displacement data as shown in the legend of Fig. 6. Figure 6 demonstrates how the final compressed locations of each voxel in the original breast data changes as a function of average overall displacement between the different models, from the 2.5 mm mesh breast to the other mesh densities. Figure 7 shows the overall displacement changes when changing different mechanical properties of the model: (a) using real mass density values as defined in Table II for each material versus a uniform mass density of $1 \text{ g}\cdot\text{cm}^{-3}$ for all materials; (b) changing the type of the element used to define the mesh (10-noded and 1-point tetrahedral elements) versus the 4-noded tetrahedral element; and (c) including a coefficient of friction between the skin and the compression plates. Figure 8 shows how changing the ratio of Young's modulus for the different breast tissues affects the final overall displacement of the breast compared to a mesh where all the tissues have the same modulus. Figure 9 illustrates how breast density changes the overall displacement between breasts using different modulus ratios.

A mesh-refinement study was performed to evaluate the impact of element density on the final deformations

FIG. 7. Changing different parameters effect on displacement. (a) average displacement between using the real density for each material and uniform density values; (b) average displacement from the 4-noded tetrahedron mesh when 10-noded and 1 point tetrahedrons are used to define the mesh (note the very small displacement scale); (c) average displacement between a coefficient of friction $\mu = 0.1$ and 0.46 versus no friction. [28% dense, 4-noded tetrahedral elements, $E(\text{fat:glandular:skin}) = 1:5:10 \text{ kPa}$ for all but the mass density comparison, which used $E(\text{fat:glandular:skin}) = 1:1:1 \text{ kPa}$].

simulated by the breast phantom. Figure 10 shows an axial slice through the representative breast (28% dense, $E(\text{fat:glandular:skin}) = 1:5:10 \text{ kPa}$, $\nu = 0.49$, $\rho = \text{real}$ (Table 2), and $\mu = 0.46$) at full compression using the five different element sizes ranging from 1.5 to 10 mm; the different colors represent the different materials. Note how the distribution of the materials converges for the finer meshes, with greater structural detail being preserved.

Figure 11 shows the postcompression morphology of the breast tissue and the total-displacement profiles in the same central axial slice using three different combinations of modulus values for fat:glandular:skin tissue (1:1:1, 1:5:10, 1:10:88 kPa with $\nu = 0.49$, $\rho = \text{real}$ (Table 2), and $\mu = 0.46$). In order to demonstrate the spatial distribution of displacement throughout the breast due to the effect of different mechanical properties, a sagittal slice through the center of the breast is shown in Figs. 12–14. For Figs. 12–14, the two left images show the interpolated nodal displacement of the two datasets under analysis (in the direction of interest), and the right image shows the difference between the two datasets to highlight where they differ the most. Because the compressed profiles may change between differently generated datasets, the images display the nodal-displacements in their original uncompressed location. Figure 12 illustrates how utilizing real mass density values changes the distribution of displacements throughout the breast; Fig. 13 shows how including a model for friction between the skin and the compression plates affects tissue displacement; and Fig. 14 demonstrates the distribution in displacements that could occur when the stiffness ratios of the various tissues are altered.

IV.C. Simulated image projection

A cranial-caudal projection and a medial–lateral projection were acquired using the projection code. To demonstrate the effect of varying the mechanical properties of the different breast tissues, projections of two modulus ratios (1:5:10 and 1:10:88) are illustrated in Fig. 15. For comparison to the FEM compression presented here, ML and CC projections of the breast phantom using the non FE simplistic compression method described previously,¹⁵ which did not take into consideration varying mechanical properties, are also shown.

V. DISCUSSION

It would be beneficial for a breast phantom designed for multimodality imaging research purposes to have high spatial resolution information about the breast tissue, including a realistic estimation of the redistribution of breast tissues under different levels of mechanical strain. This would allow the researcher to define assumptions and level of complexity for a specific project. Our investigation into the effect of mesh density and different material properties in a breast phantom show that slight alterations in the models properties can change the final distribution of the tissue under compression. The effects due to small changes in the model could potentially be of interest for the development of new imaging tools and techniques for breast cancer research, although they may not be significant for the purposes of tumor

FIG. 8. Effect of changing moduli on final displacement from a model with uniform moduli. (28% dense, 4-noded tetrahedral elements, $\nu = 0.49$, $\rho = \text{real}$, and $\mu = 0.46$)

tracking for biopsy or postsurgical outcomes that require a model within a relatively short time frame.

V.A. Relative displacement analysis

A higher mesh density with smaller elements is necessary to capture small structures and maintain the mechanical integrity of the model. Figure 6 shows that changing the mesh

density can alter the overall average deformation and demonstrates that lower density meshes can result in larger differences from the 2.5 mm mesh. The differences in tissue distributions shown in Fig. 10 can affect the overall deformation of the breast that occurs due to the stiffness of different breast materials. The overall effect of mesh density on the average total displacement from the 2.5 mm element edge length mesh was less than 5 mm for meshes utilizing edge

FIG. 9. Effect due to breast density. How displacement changes using different moduli ratios from the uniform model in breasts of different densities. (28% dense, 4-noded tetrahedral elements, $\nu = 0.49$, $\rho = \text{real}$, and $\mu = 0.46$)

FIG. 10. Effect of mesh density on tissue distribution under compression. Using a breast, that is, 28% dense, 4-noded tetrahedral elements, $E(\text{fat:glandular:skin}) = 1:5:10$ kPa, $\nu = 0.49$, $\rho = \text{real}$, and $\mu = 0.46$. Fat, glandular, and skin tissue are displayed. (a) 1.5 mm, (b) 2.5 mm, (c) 3.75 mm, (d) 5 mm, and (e) 10 mm.

lengths smaller than 5 mm; however, subtle movements of small glandular structures may not be captured with coarser meshes (Fig. 6). Although the overall shape of the glandular tissue is captured, small and/or thin glandular regions are not represented in detail using the larger mesh. In addition, structures smaller than 2 mm in diameter in the coronal plane were removed in the segmentation step and would not be represented in the generated mesh; hence, a 1.5 mm edge length mesh may not be necessary for the current FE model. However, future iterations of the breast phantom may include additional models for the suspensory Cooper's ligaments, which

will further affect the distribution of tissue through the compressed breast. The addition of smaller structures in the breast will require even further mesh refinement, which will subsequently increase runtime. The 1.5 mm elements took over seven times longer to run than the 2.5 mm elements used for the results in this manuscript. Smaller elements would impose even greater memory penalties if formulated in an implicit solver, requiring computers with great degrees of parallelization and RAM to run in practical amounts of time.

A large difference in displacement was not apparent when using real mass density values or different types of

FIG. 11. An axial slice through the same breast at full compression showing different tissue locations on the left and total-displacement values on the right. Moduli ratios from top to bottom: 1:1:1, 1:5:10, 1:10:88 kPa (28% dense, 4-noded tetrahedral elements, $\nu = 0.49$, $\rho = \text{real}$, and $\mu = 0.46$). A zoom view of the boxed section is shown in the middle to illustrate the different breast morphology using different moduli ratios. The color map for the displacement images range from 15 to 35 mm. Notice how the overall shape of the breast and the tissue deformation deep in the breast varies depending on the relative stiffness ratio between the different tissue types.

FIG. 12. Difference using real mass density values: left is the total displacement with uniform mass density values, middle is total displacement with real density values as defined in Table II, and right is the total difference between the models. Note the small scale of the difference image compared to the original models.

elements. An average displacement of 0.4 mm indicates that utilizing real values for the density of the tissues did not greatly affect the overall displacement from using uniform values for the different tissues [Fig. 7(a)]. However, in contrast with modulus ratios, the density values for the breast tissues are well studied and can be defined without questioning their actual value, therefore, the real density level should be included. Altering the type of the element used in the model to 10-noded tetrahedral elements and 1-point tetrahedral element did not greatly alter the overall relative displacement from the 4-noded tetrahedral element with the average overall displacement between models of $<25 \mu\text{m}$ [Fig. 7(b)]. This is most likely because the 2.5 mm element edge length mesh was refined enough that the second order behavior captured by a 10-noded tetrahedral was unnecessary for this analysis. Although not part of this analysis, potentially using a 10-noded element mesh with fewer elements may provide similar displacements as the high density mesh used in this study.

Modeling the friction of the skin against the compression plates had the most effect on the skin's deformation. Figure 13 demonstrates that using a nonzero coefficient of friction mostly affects deformation in the edges of the breast, but not as much in the center. Medial-lateral displacements in a sagittal plane centered in the breast, and superior-inferior displacement in an axial plane centered in the breast, were not affected by the presence or absence of plate friction. This is most likely because simulated friction restricts the breast from sliding across the compression plates, while no friction allows the skin to slide, affecting the breast tissue near the edges, but not the deformation nearer the middle of the breast. Increasing the coefficient of friction to 1.0 did not

have as substantial an impact as simply including a coefficient of friction of 0.1, which indicates that some model of friction is necessary but the precise value has a secondary effect. Computationally, sliding contact interfaces can increase run times over 25%, therefore, depending on the area of interest, this may not be a viable parameter to include. Studies have investigated the effect of modeling the breast with a different material definition of skin. However, the skin is 1–3 mm thick and a coarse mesh on the order of centimeters will not be able to characterize or model skin effectively. We can define a thin skin that realistically encapsulates the breast fully and remains $<3 \text{ mm}$ thick. Figure 8 shows that with increasing skin modulus, the overall displacement increases, which demonstrates that skin does affect breast deformation. This demonstrates the need for a high density mesh with a skin definition that is able to describe the skin with realistic mesh elements.

Altering the relative stiffness ratio had a great effect on the displacement distributions. Figure 8 shows that the absolute value of the moduli has less effect on the relative deformation than changing the ratio of the moduli. This is likely due to the fact that the breast tissue was similarly displaced ($<0.5 \text{ mm}$) with the 1:5:10 ratio as with the 10:50:100 ratio. In addition, it demonstrates that each change of the modulus has an effect on the overall deformation from a breast with uniform modulus; the larger the ratio change, the larger the change in the nodal displacement. Figure 14 shows that the overall deformation differences with higher modulus ratios are dispersed throughout the breast, which shifts the tissues around in a nonuniform manner. This effect is further demonstrated in Fig. 11, which shows how changing the modulus alters the deformation of different tissues and the shape of

FIG. 13. Difference due to friction: top without friction; middle with friction; and bottom is the difference between the models. It is clearly demonstrated that the change due to adding friction between the skin and the compression plates is concentrated primarily on the edges of the breast.

FIG. 14. Change in moduli: left is uniform ratio of 1:1:1, middle is 1:10:88, and right is total change between displacements. The figure shows that the difference between models using varying moduli ratios is distributed throughout the breast.

the overall breast under compression. The magnified region shows how some tissues come into plane differently with different moduli, and the displacement image shows how the tissues move differently with varying the defined modulus ratios. The effect of changing modulus ratios is further compounded when the breast glandular density is altered. Figure 9 illustrates that changing the ratios of Young's modulus has the greatest overall effect on the least dense breast where the more compliant adipose tissue is more dependent on the smaller volume percentage of stiffer glandular tissue that can concentrate the stress from the plate compression. The greatest differences in the simulated deformations occurred in the medial-lateral direction, orthogonal to the compression direction, which is where the breast redistributes across the compression plates to maintain the incompressibility constraint according to Eq. (1).

Simulated mammographic projection images shown in Fig. 15 demonstrate the effect of using a high-resolution FEM compression algorithm on the breast phantom. The shape and overall morphology of the phantom are very different in the resulting simulated projections generated using the FEM compression algorithm from the original simplistic compression algorithm that did not consider the properties and interactions of the different tissues. Figure 15 also demonstrates how different mechanical parameters (i.e., varying the modulus ratios of the different tissues) can affect the

distribution of the tissue in the resulting projection. This figure demonstrates that images simulated using the phantom can be influenced from the simulated compression model implemented and that the ultimate representation of the breast changes, which may influence studies that use the breast phantom.

V.B. Limitations and future directions

Both linear and hyperelastic material models have been used in the past to model breast compression. However, hyperelastic materials introduce even more degrees of freedom in the parametric analysis that would further complicate the first-order material dependencies that we studied with this model and may also increase the computational overhead. The results presented under the assumptions of purely elastic materials can be treated as a foundation on which the higher-order effects such as hyperelasticity can be added. In addition, another group has demonstrated that the breast exhibits anisotropic deformation and can be modeled using transverse isotropic materials;⁴¹ tissue anisotropy can also be added to these models in the future, though information about tissue orientation, etc., must be provided from the segmented image data.

The interleaved tissues within the modeled breasts were connected to one another without the ability to slide past one

FIG. 15. Simulated image projection of the 28% dense phantom (using 4-noded tetrahedral elements, $\nu = 0.49$, $\rho = \text{real}$, and $\mu = 0.46$). (a) is an ML projection with moduli ratio = 1:5:10; (b) is a CC projection with moduli ratio = 1:5:10; (c) is an ML projection with moduli ratio = 1:10:88; (d) is a CC projection with moduli ratio = 1:10:88; (e) is an ML projection using a simplistic compression algorithm; (f) is a CC projection using a simplistic compression algorithm. Note how the projection of the breast is different using different moduli ratios. Additionally, the new refined FEM compression method distributes the tissue differently than the old simplistic compression, which did not take into account the varying mechanical properties of the different breast tissues.

another. While this is a good approximation for most of the breast where fibrous interconnects tether tissues to one another, it would be interesting to investigate the effect on displacement from imposing slip boundary conditions between disparate tissue types. Initial investigation into generating a high-resolution compression model with compartmentalized tissues was attempted; however, automatic mesh generation of these complex 3D entities yielded malformed elements that prevented analysis. More highly refined segmented entities with smoother transitions between different material types may facilitate such an effort.

Other groups have used registration markers and the outline of the breast in a real mammogram to evaluate the accuracy of the simulated compression.^{17,19–27} Although evaluating the accuracy of the simulated compression was unnecessary for the purpose of the presented breast phantom package, it would be interesting to quantitatively evaluate the realism of simulated compression in the future.

Changing the level of material incompressibility also changes the ultimate deformation of the breast. However, because we chose to use an explicit solver and an elastic material with large deformation, the elements moved into a numerically unstable state as a Poisson's ratio of 0.5 was approached from 0.45 to 0.49999. For analysis, the models were run using $\nu = 0.49$, to approximate a level of incompressibility that did not produce artifacts in the overall deformation and did not make the materials unrealistically compressible. If an alternate FE tool or Poisson's ratio is used for compression analysis, the results should be verified that they do not contain artifacts from numerically unstable elements.

The 4-noded tetrahedral elements utilized for these models are not ideal for modeling incompressible soft tissues; however, our mesh was refined enough that we did not need the second order behavior captured by higher-order (10-noded) tetrahedral elements, which may also have prohibitively long runtimes and greater memory requirements. More ideal hexahedral elements would require extreme manual intervention to make well-conditioned numerical ele-

ments that did not have extreme aspect ratios and also possessed proper node connectivity between structures with nonregular geometries during the mesh-generation process, which would be dependent on each image that was segmented, and would, therefore, not be amenable to a generalized breast phantom model. Numerical artifacts associated with the 4-noded tetrahedral elements were minimized with the prescribed mesh-generation penalties, though true accuracy in the compression can only be evaluated in a study with precompression and postcompression image comparison that was not available for these studies.

The compression plates were modeled as infinite rigid walls in the axial plane. Although this is not realistic as the plates normally contact and terminate at the chest wall, we assumed that the breast data we had available contained limited chest wall information and would be fully compressed during mammogram acquisition. Thus, the breast data under compression was attached to an imaginary chest wall and allowed to fully compress. In the future, we can model a generic chest wall and pectoral muscle to attach the breast to, and then use a compression plate definition that is finite in the axial plane and contacts and terminates at the chest wall.

In our simulation, we did not account for the initial position of the breast during mammography, where the breast is rested on the inferior compression plate, while the superior plate is moved down. In addition, we did not remove the effects of gravity imposed on the pendant breast during image acquisition from the phantom generated from the dedicated breast CT images. In an elastic simulation, such as those presented, the effects of gravity on the breast would be superimposed on the compressed solutions and would likely be relatively small, though this approach would not be appropriate when utilizing hyperelastic material definitions where the initial strain-state of the tissues is important.

The compression algorithm generated in this work utilized proprietary mesh generation and FEM solvers that require specific parameters. Although the methods can be translated to other tools, the compression may entail some

FIG. 16. Images from FEBio simulation—left: uncompressed phantom; right: compressed phantom.

slight modifications to the procedure used herein. A demonstration of the compression algorithm translated to nonproprietary tools is given in the Appendix.

VI. CONCLUSION

We have developed a 3D, finite element breast phantom model that can yield high spatial resolution breast deformations under uniaxial compression for imaging research purposes. Skin, adipose, and glandular tissue were successfully segmented from breast CT images and mapped onto a 3D tetrahedral finite element mesh. This breast phantom allows for user-defined mechanical properties of the breast tissue and the analysis demonstrates potential differences due to the chosen assignment. Varying the relative Young's modulus of the constituent tissues of the breast can have a significant impact of the 3D deformation of the breast, especially in less dense breasts. The presence of skin and its assigned modulus can affect the overall deformation. In addition, incorporating friction between the skin and the compression plates affects deformations near the edges of the breast, but not as much in the center. Future iterations of this model will incorporate connectivity with adjacent chest wall tissues, include smaller and more refined structures such as Cooper's ligaments, and be extended to more complicated material models. This work demonstrates the effect of several parameters on tissue compression. It can be a basis for researchers choosing material properties and mechanical parameters in breast phantoms or in simulation studies for other applications such as tumor tracking or predicting surgical outcomes.

ACKNOWLEDGMENTS

The authors would like to thank Dr. John Boone and his lab at UC Davis Medical Center for providing the dedicated breast CT data that was used for this work. In addition, the authors are grateful to Dr. Joseph Lo and Dr. Jay Baker from Duke University Medical Center for their assistance with image analysis and interpretation. Finally, they would like to thank Ned Daniely at Duke University for the extensive system support he provided throughout this project. This work has been supported by the Department of Defense Breast Cancer Research Program (W81XWH-06-1-0732) and National Institutes of Health NIH/NCI (R01CA134658), NIH/NCI (R01EB001838), NIH/NCI (R01CA112437), and NIH/NCI (R01CA94236).

APPENDIX

1. Using open-source FEM software

In order to demonstrate that the breast phantom can be used by researchers without access to proprietary mesh-generation tools and FEM solvers such as HYPERMESH and LS-DYNA, open-source FEM software was used to simulate compression on the phantom. The segmented breast data were used to generate an STL file of the breast surface using a function called "SURF2STL," available from the MATLAB Central website.⁴² The STL file was input into LS-PrePost

(Livermore Software Technology Corp., Livermore, CA) (Ref. 43) to generate a 4-noded tetrahedral mesh with elements of edge length ~ 5 mm. Materials were assigned to the mesh elements in the same way as described for the LS-DYNA method. The model was run using FEBio (Musculoskeletal Research Laboratories, University of Utah, Salt Lake City, UT) (Ref. 44) with slightly different material definitions in order for the model to run to completion: Neo-Hookean materials, no coefficient of friction, restricted movement in the coronal and axial directions at the chest wall, and a slower compression (30 s to 5 cm compression thickness). In addition, it was necessary to define contact surfaces between the skin and the rigid wall compression plates. We selected the surface elements located superior or inferior of the axial midplane of the breast to contact their respective rigid walls. Figure 16 illustrates the breast phantom precompression and postcompression using FEBio. This small foray into open-source FEM packages demonstrates that it is possible to use the phantom with alternative tools. Therefore, if a research group is more familiar with another FEM package, they can still use the breast phantom for their studies with compressed simulation.

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