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The Cognitive Predictors of Computational Skill with Whole versus Rational Numbers: An Exploratory Study

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Abstract

The purpose of the present study was to explore the 3rd-grade cognitive predictors of 5th-grade computational skill with rational numbers and how those are similar to and different from the cognitive predictors of whole-number computational skill. Students ($n = 688$) were assessed on incoming whole-number calculation skill, language, nonverbal reasoning, concept formation, processing speed, and working memory in the fall of 3rd grade. Students were followed longitudinally and assessed on calculation skill with whole numbers and with rational numbers in the spring of 5th grade. The unique predictors of skill with whole-number computation were incoming whole-number calculation skill, nonverbal reasoning, concept formation, and working memory (numerical executive control). In addition to these cognitive abilities, language emerged as a unique predictor of rational-number computational skill.

Keywords

mathematics; elementary; cognitive predictor; rational numbers; computation

The National Mathematics Advisory Panel (NMAP, 2008) concluded that the pervasive difficulty many students experience with rational numbers, including fractions, decimals, and percents, is a key obstacle to progress with algebra. Difficulty with rational numbers may be a result of whole-number bias; that is, students' tendency to misapply whole-number concepts and procedures to fractions (Ni & Zhou, 2005). In fact, the NMAP considers fractions to be one of the "Critical Foundations of Algebra" (p. 17-19) and states that proficiency with fractions should be a major focus at the elementary school level.

Given the important role competence with rational numbers plays in future mathematics learning, it is unfortunate that little is known about the cognitive abilities that contribute to

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its development. In the present study, we examined whether the same set of domain-general abilities (such as working memory) that influence skill with whole numbers affect skill with rational numbers or whether there are qualitative differences in the cognitive abilities that underlie the development of rational-number skill. Such an understanding may highlight areas of weakness in students who are at risk for developing difficulty with rational numbers and help pinpoint strategies for early intervention.

Several studies have investigated relations between cognitive resources and whole-number computational skill. Two predictors that have been replicated across studies are processing speed (Fuchs et al., 2006; Fuchs et al., 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Swanson & Kim, 2007) and working memory (Fuchs et al., 2006; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Swanson, 2006; Swanson & Beebe-Frankenburger, 2004). In focusing on different components of working memory and their relations to arithmetic computation, Swanson and Kim (2007) demonstrated the contribution of the phonological loop and the central executive to the prediction of mathematics performance in school-aged children. Further support for the central executive's role in mathematics development was provided by Meyer et al., who contrasted the relative contribution of the three core components of working memory (i.e., the central executive, phonological loop, and visuo-spatial sketchpad; Baddeley & Logie, 1999) to the development of mathematical skill for second and third graders. The authors found that, for younger students, the central executive and phonological loop uniquely predicted mathematics reasoning; for older students, visuo-spatial working memory predicted skill in both mathematics reasoning and numerical operations. Fuchs et al. (2010b) similarly investigated the role of the central executive, phonological loop, and visuo-spatial sketchpad components of working memory (in addition to five other domain-general abilities and two aspects of basic numerical competency) in predicting mathematics development across first grade. The central executive, in the form of *Counting Recall* (from the Working Memory Test Battery for Children [Pickering & Gathercole, 2001]), was uniquely predictive of procedural calculations development; measures of the phonological loop and visuo-spatial sketchpad were not. Across these studies, the central executive component of working memory seems important for whole-number computation skill, and additional study is warranted to understand if the central executive similarly predicts skill with rational-number calculations.

The role of processing speed as a candidate for predicting computational skill with whole and rational numbers is also a possibility. For example, Bull and Johnston (1997) found processing speed to be the best predictor of whole-number arithmetic ability for 7-year-old children, when simultaneously considering abilities of short-term memory, sequencing, and long-term memory. In addition, Fuchs, Fuchs, Stuebing, et al. (2008) found that processing speed was associated with calculations difficulty but not word-problem difficulty at third grade. By contrast, Seethaler and Fuchs (2006) examined the role of a large set of cognitive and mathematical abilities on the whole-number computational estimation skill for third-grade students, of which processing speed was included. They found arithmetic number combination skill, nonverbal reasoning, concept formation, working memory, and inattentive behavior, but not processing speed, to uniquely predict estimation skill. Perhaps with respect to computational estimation skill, speed is less integral than mastering the procedural steps necessary for accurate problem solution. Additional study is needed to explore the role of processing speed in computational skill with whole and rational numbers.

A different facet of whole-number mathematics, word problems, appears to draw on cognitive resources distinct from those required for computation. Although working memory is related both to computation and word-problem skill (Fuchs et al., 2006; Swanson, 2006; Swanson & Beebe-Frankenburger, 2004), word-problem proficiency, but not whole-number calculation skill, appears to be uniquely predicted by concept formation, nonverbal

reasoning, sight word efficiency, language, and reading (Fuchs et al., 2006, 2008, 2010b; Swanson, 2006). Fuchs and colleagues (2008) suggest that skill with calculations and word problems may be distinct and dissociable aspects of mathematical cognition, while Hart, Petrill, Thompson, and Plomin (2009) provide evidence of different genetic and environmental influences on math problem solving versus calculation skill. A motivating question in the present study was whether a similar distinction is warranted between computation skill in the domain of whole numbers versus rational numbers.

It is possible that processing speed and working memory, shown to influence skill with whole-number computation (e.g., Fuchs et al., 2006; 2010b; Swanson & Beebe-Frankenburger, 2004), also predict computational skill with rational numbers. Automatic and fluent execution of simple tasks (i.e., processing speed) and the use of a specialized cognitive system for storing and manipulating information (i.e., working memory; Baddeley & Hitch, 1974) are likely to be as relevant to the domain of rational-number computation as whole-number computation. Due to the hierarchical nature of mathematics (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004), skill with rational-number computation (a relatively advanced mathematics topic) builds upon skill with foundational topics (such as proficiency with whole-number computation). Thus, one would expect that many of the cognitive correlates of whole-number computational skill would remain important for development of rational-number computational skill. However, because many students who are competent with whole-number calculations struggle in the domain of rational numbers (NMAP, 2008), the predictors of whole-number computational competence may not be sufficient to account for computational skill with rational numbers.

Thus, the question arises, What are additional plausible predictors for rational-number computational competence? Recently, Hecht and Vagi (in press) found that individual differences in two domain-general abilities (i.e., working memory and attentive behavior) and two domain-specific abilities (i.e., arithmetic fluency and conceptual knowledge about fractions) were uniquely predictive in the development of computation, word problem, and estimation skill with common fractions in fourth- and fifth-grade students. In the present study, we expanded on Hecht and Vagi by incorporating additional domain-general variables. First, we included processing speed as a possible predictor of both the domains of whole-number and rational-number computation skill given prior work with whole-number calculations sometimes showing the unique contribution of processing speed but not working memory when both predictors compete for variance in the same model (e.g., Bull & Johnston, 1997; Fuchs, Fuchs, Stuebing, et al., 2008; Fuchs et al., 2010b). Also, we included nonverbal reasoning and concept formation, which although linked to skill with word-problem competencies (e.g., Fuchs et al. 2006), have not been associated with whole-number calculations. The procedural rules for operating with rational numbers are more complex and less intuitive, raising the possibility that nonverbal reasoning and concept formation are involved in rational-number calculations.

In addition, we considered language as a possible predictor for skill with rational-number computation. Language may be related to rational-number competence through conceptual knowledge. As children mature intellectually, they rely increasingly on a symbolic and verbal understanding of number, necessary to develop school-based mathematical competence (Jordan, Glutting, & Ramineni, 2010). Using an inherent “rational number sense” (Sowder, Bezuk & Sowder, 1993 cited in Mazzocco & Devlin, 2008), children develop an informal understanding of rational numbers, which is focused primarily on the part-whole and measurement constructs. Oral language skill may support students’ conceptual understanding of rational numbers, formally in response to school instruction as well as informally in response to everyday activities that reinforce fractional concepts (e.g., understanding the meaning of “half of something”). Students with stronger oral language

skill, who better differentiate the many subconstructs of fractions (e.g., measure, quotient, ratio, multiplicative operator) in response to instruction, may be more likely to choose correct problem solution procedures. This language-mediated hypothesis is supported by Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999). They found that Korean students whose native language embeds the concept of fractional parts in the mathematics terms used for fractions (i.e., $1/3$ is read as, “of three parts, one”) show evidence of greater understanding of pictorial representations of fractions, even without formal instruction in fractions, compared to students whose language does not (e.g., students from Croatia or the United States, where $1/3$ is read as “one-third”). In a similar way, according to Ng and Rao (2010), the transparency of Chinese-based systems of number words and mathematical terms may be partly responsible (along with cultural beliefs) for the superior performance East Asian students demonstrate.

On the other hand, Gelman and Butterworth (2005) argued that numerical competence develops independently of language, with evidence from neuroimaging studies, developmental perspectives, and an investigation of certain Amazonian peoples who develop particular number skills with a limited mathematics vocabulary. Further, Donlan, Cowan, Newton, and Lloyd (2007) examined mathematics skills for students with specific language impairments as contrasted with age- and language-matched controls. They found that, although students with language impairments showed deficits in producing the count-word sequence, basic calculation, and place value understanding, there was no difference between groups with respect to arithmetic principles. Thus, students with language impairments may not be impeded in their understanding of arithmetic concepts, even as they show deficits in other number processing skills. Based on these conflicting accounts, we were interested in exploring whether language may account for unique variance in rational-number computational skill.

In these ways, the purpose of the present study was to explore the cognitive predictors of fifth-grade computational skill with rational numbers and how those are similar to and different from the cognitive predictors of whole-number computational skill. Although some studies have examined the cognitive abilities underlying whole-number computational skill (e.g., Fuchs et al., 2006; Meyer et al. 2010; Wilson & Swanson, 2001) and computational estimation skill (Seethaler & Fuchs, 2006), we identified no previous studies examining differences in cognitive predictors of whole-number versus rational-number computational competence. Specifically, we expanded on Hecht and Vagi (in press) in four ways. First, as already described, we investigated a broader set of cognitive predictors. Second, we lengthened the prediction timeframe. Students in the present study were tested in the fall of third grade, with follow-up testing occurring during spring of fifth grade, allowing for almost three academic years to elapse. In contrast, Hecht and Vagi followed students from the fourth through fifth grade. Third, whereas Hecht and Vagi limited their outcome to common fractions, we considered skill with rational numbers more generally (i.e., common fractions, decimals, decimal percents, and ratios). Fourth, Hecht and Vagi did not contrast predictors of rational numbers against those of whole numbers, as done in the present study. To our knowledge, this is the first empirical look at patterns of cognitive characteristics that may predict calculations performance with whole numbers versus rational numbers. We note that because the present study relies on an extant database, in which the available measures of computational outcomes were restricted, readers should understand the present study as exploratory, providing a basis for generating hypotheses for additional study.

Method

Participants

The data described in this article were collected as part of a larger, prospective 4-year study evaluating the effects of mathematics instruction and the cognitive correlates of mathematical problem solving. In a southeastern metropolitan school district, 120 third-grade classrooms were enrolled in four cohorts, with 30 classrooms entering each year, such that data collection spanned six years to follow each cohort three school years. One teacher left the study during the first month due to personal reasons, thereby withdrawing students in her class. In the remaining 119 classrooms, we screened 2,023 consented students using the Test of Computational Fluency (Fuchs, Hamlett, & Fuchs, 1990). We obtained a representative sample of 1,153 students, blocking within classroom and so that 25% had scores 1 standard deviation (*SD*) below the mean of the distribution, 50% had scores within 1 *SD* of the mean of the distribution, and 25% had scores 1 *SD* above the mean of the distribution. We excluded students with a standard score below 80 on both subtests of the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999). The sample in the present study is the subgroup of 688 students whom we located at the end of fifth grade and on whom we had complete data for the study variables. The 688 students were comparable to the exited students on the majority of the study variables measured at the beginning of third grade (see Measures section), $F(1, 1151) = .02 - 1.5, p = .222-.886$. The exceptions were with two language measures (i.e., WDRB Listening Comprehension and TOLD Grammatical Closure; see Measures section), $F(1, 1151) = 6.60, p = .010$ and $F(1, 1151) = 9.28, p = .002$, and one working memory measure (i.e., WMTB-C Listening Recall; see Measures section), $F(1, 1151) = 6.57, p = .011$. However, the effect sizes contrasting these two groups were small (0.15, 0.18, and 0.15, respectively), with means of the respective measures of 21.57 ($SD = 4.06$), 19.62 ($SD = 6.16$), and 10.26 ($SD = 3.21$) for the exiters and 20.91 ($SD = 4.44$), 18.42 ($SD = 6.79$), and 9.77 ($SD = 3.23$) for the students who remained. These 688 students had participated in 485 classrooms across the three school years. See Table 1 for raw scores (as well as standard scores where applicable) on the variables reported in the present study. On the Wechsler Abbreviated Scale of Intelligence (Wechsler), at the beginning of third grade, the mean IQ of this sample was 97.32 ($SD = 14.19$); on the Wide Range Achievement Test (WRAT; Wilkinson, 1993), the mean mathematics standard score was 97.54 ($SD = 10.19$). Of these 688 students, 362 (52.6%) were female; 380 (55.2%) received subsidized lunch; 245 (35.6%) were White, 309 (44.9%) were African American, 77 (11.2%) were Hispanic, 57 (8.3%) were identified as “other”; 32 (4.7%) were English language learners (ELLs); and 49 (7.1%) received special education services.

Procedure

Students were tested in September and October of third grade in three whole-class testing sessions, each lasting 30 to 60 min, and two 45-min individual testing sessions. In May of fifth grade, students completed two 60- to 90-min testing sessions (i.e., one individual and one whole-class). Research assistants administered the tests after demonstrating 100% accuracy in training. Every individual test administration was audiotaped, with 18.6% of the audiotaped sessions randomly selected for independent rescoring; interscorer agreement was 99.8% across the sessions. In these sessions, students were tested on a broader set of measures, relevant to the larger investigation. For the present study, we report data on the variables relevant to the present research questions.

Measures: Fall of Third-Grade Predictors

Calculation skill—The *Test of Computational Fluency* (Fuchs, Hamlett, & Fuchs, 1990) is a 1-page test displaying 25 items that sample the typical second-grade computation

curriculum, including adding and subtracting number combinations and procedural computation. Students have 3 min to complete as many answers as possible. The score is the number of correct responses. Staff entered responses into a computerized scoring program on an item-by-item basis, with 15% of tests re-entered by an independent scorer. Data-entry agreement was 99.6. On the sample of 1153, coefficient alpha was .94, and criterion validity with the previous spring's TerraNova (CTB/McGraw-Hill, 1997) Total Math score was .60.

Language—Woodcock Diagnostic Reading Battery (WDRB) Listening Comprehension (Woodcock, 1997) tests the ability to understand sentences or brief paragraphs. The examiner reads aloud 38 items, one at a time, pausing at the end of the sentence or paragraph for the student to supply a missing word. The passages become increasingly complex, progressing from simple statements with basic vocabulary, to paragraphs with advanced vocabulary and more complicated sentence structure. Students earn one point for each correct response; testing is discontinued after six consecutive errors. The score is the number of correct responses. Reliability is .80 for ages 5-18.

WASI Vocabulary (Wechsler, 1999) measures expressive vocabulary and verbal knowledge. The examiner presents 42 items, one at a time, and immediately scores the oral response as worth 0, 1, or 2 points, based on quality. For the first four items, the student identifies a color picture with one word; for the remaining items, the student provides a definition for a word. Testing is discontinued after five consecutive responses scored as 0. The score is the total number of points for responses. Zhu (1999) reports split-half reliability at .86-.87 for ages 6-7; the correlation with the Wechsler Intelligence Scale for Children (1999) is .72.

Test of Language Development (TOLD) Grammatical Closure (Newcomer & Hammill, 1988) measures the ability to understand and use varied morphological forms of the English language. The examiner reads 30 sentences, one at a time, and pauses for the student to supply a missing word from the sentence. Each correct response earns one point; testing is discontinued after six consecutive errors. The score is the total number of correct responses. As reported by the test developer, reliability for 8 year olds is .88.

Nonverbal reasoning—*WASI Matrix Reasoning* (Wechsler, 1999) measures nonverbal reasoning with four tasks: pattern completion, classification, analogy, and serial reasoning. The tester presents the student with a color picture of a matrix with a missing piece; the student then chooses the correct piece to complete the matrix from five options displayed at the bottom of the page. Each correct response earns one point; testing is discontinued after four errors on five consecutive items. As reported by the test developer, reliability for 8 year olds is .94.

Concept formation—*Woodcock-Johnson III (WJ-III) Concept Formation* (Woodcock, McGrew, & Mather, 2001) measures students' ability to state the rule(s) for concepts when presented with icons that do and do not illustrate the concept. Each correct response earns one point; testing is discontinued according to cutoff points. The score is the total number of correct responses. As reported by the test developer, reliability for 8 year olds is .93.

Processing speed—*WJ-III Visual Matching* (Woodcock et al., 2001) provides students 3 min to find and circle the only two identical numbers from a row of six numbers, in each of 60 rows. Students earn one point for each correctly circled pair of numbers; the score is the total number of points. As reported by the test developer, reliability is .91.

Working memory-central executive sentences—With Working Memory Test Battery for Children (WMTB-C) Listening Recall (Pickering & Gathercole, 2001), the examiner reads a series of sentences (beginning with one sentence and progressing to six

sentences), some of which are nonsensical. Upon hearing each sentence, the student states whether each sentence is true or false. After all sentences in a series are determined to be true or false, the student states the last word from each sentence, preserving the order in which the sentences were presented. The student earns one point for each series of sentences for which all words were recalled in the correct order (regardless of accuracy in true/false determination). Testing is discontinued once the student makes three errors in one series of sentences; the score is the total number of points earned. As reported by the test developer, reliability is .93.

Working memory-central executive numerical—With *WJ-III Numbers Reversed* (Woodcock et al., 2001), students repeat a string of numbers in reverse order. The string of numbers becomes progressively longer. Testing is discontinued when the student makes three errors in a block of items. The score is the total number of items repeated correctly in reverse order. As reported by the test developer, reliability for 8 year olds is .86.

Measures: Spring of Fifth-Grade Outcomes

Fifth-grade computational skill—With *WRAT-3 Arithmetic* (Wilkinson, 1993), students have 15 min to write answers to 40 computation items of increasing difficulty, including single- and multi-digit addition, subtraction, multiplication, and division computation with whole numbers, fractions, decimals, percents/ratios, and algebraic equations. To index whole-number versus rational-number computational skill, one author independently coded each item as a whole-number item or a rational-number item. A second author independently re-coded the items with 100% agreement. Twenty-one (52.5%) were whole-number items; 13 (32.5%), rational-number items. (Six or 15% involved algebraic equations and were excluded from the present study.) The 21 whole-number items included 6 addition, 5 subtraction, 5 multiplication, and 5 division problems; 10 were single-digit and 11 were multi-digit. The 13 rational-number items included 7 items of addition, subtraction, multiplication, or division of fractions and mixed numeral fractions; 2 multiplication items with decimal numbers; 2 problems that require translation of a decimal number to a percentage or a fraction; and 2 items that require computing the percentage or subset of a given number. The scores were the number of whole-number items correct and the number of rational-number items correct. Alpha coefficient on this sample was .78 for whole numbers; .53 for rational numbers. We note that although .78 is deemed adequate, .53 suffices only for exploratory purposes (Nunnally, 1967), as in the present study.

Data Analysis and Results

For language, where we had more than one measure, we created a weighted composite variable using a principal components factor analysis across TOLD Grammatical Closure, WDRB Listening Comprehension, and WASI Vocabulary. (Because the principal components factor analysis yielded only one factor, no rotation was necessary.) For other constructs, only one measure was available: incoming calculation skill (Test of Computation Fluency), nonverbal reasoning (WASI Matrix Reasoning), concept formation (WJ-III Concept Formation), processing speed (WJ-III Visual Matching), working memory-sentences (WJ-III Listening Recall), and working memory-numerical (WJ-III Numbers Reversed). See Table 1 for raw score means and *SDs*, standard score means and *SDs* as available, and correlations of the seven predictor variables and the two outcome variables.

We entered the set of seven beginning-of-third-grade predictors into a simultaneous regression analyses to predict end-of-fifth-grade whole-number computational skill. The significant predictors were incoming calculation skill, nonverbal reasoning, concept formation, and working memory-numerical. Together, these accounted for 31.2% of the variance in whole-number computation scores, $R^2 = .312$, $F(7, 678) = 43.88$, $p < .001$. See

Table 2 for the unstandardized and standardized beta coefficients, standard errors, and R^2 change for each predictor variable, with all others controlled.

Then we conducted parallel analyses using end-of-fifth-grade rational-number computational scores as the outcome. As with the whole-number computational skill, the simultaneous regression analysis showed that incoming calculation skill, nonverbal reasoning, concept formation, and working memory-numerical accounted for unique variance. Additionally, however, language emerged as a significant predictor. Together, these five abilities accounted for 29.6% of the variance in rational-number computation scores, $R^2 = .296$, $F(7, 678) = 40.75$, $p < .001$. See Table 2 for the unstandardized and standardized beta coefficients, standard errors, and R^2 change for each predictor variable, after all others had been entered.

Discussion

The purpose of this study was to explore the cognitive factors affecting whole-number versus rational-number computational skill. We investigated which cognitive abilities uniquely account for development of computational skill with rational numbers but not for whole numbers, while controlling for incoming calculation skill. The cognitive abilities, assessed in the fall of third grade, were language, nonverbal reasoning, concept formation, processing speed, working memory-sentences, and working memory-numerical. Calculation skill with whole numbers and with rational numbers (i.e., common fractions, decimals, and percentages) was assessed three academic years later, at the end of fifth grade. Results indicated a common set of cognitive predictors for the two types of calculation skill as well as one ability that was uniquely predictive of rational-number calculation skill.

Four of the seven predictors (i.e., incoming calculation skill, nonverbal reasoning, concept formation, and working memory-numerical) uniquely accounted for individual differences in the development of whole-number and rational-number computation skill. Together, the predictor variables accounted for 31.2% of the variance in whole-number skill, with incoming calculation skill accounting for the greatest percentage of variance (i.e., 10.0). Research has shown that elementary school students' difficulty persists across grades (e.g., Fletcher, Lyon, Fuchs, & Barnes, 2007; Shalev, Manor, Auerbach, & Gross-Tsur, 1998). Thus, one would expect arithmetic calculation skill at third grade to predict competence with whole-number computation in the fifth grade, even as the calculation tasks increase in complexity and difficulty. In a similar way, incoming calculation skill also accounted for the greatest percentage of variance in end-of-fifth-grade rational-number computational skill (i.e., 4.8). Hecht and Vagi (in press) also identified arithmetic fluency as a unique predictor of growth in common fraction skills from fourth to fifth grade. Our findings extend those results to include a composite measure of skill with a broader set of rational numbers: decimal fractions, percents, and ratios.

In addition, nonverbal reasoning and concept formation were unique predictors of skill with whole as well as rational-number computation, each accounting for approximately 1% of variance. These domain-general abilities have not, to our knowledge, been linked to skill with whole-number or rational-number computation in previous research, although they have been associated with word-problem skill (e.g., Fuchs et al., 2006; Fuchs et al., 2010). One exception is Cowan, Donlan, Newton, and Lloyd (2005), who investigated relations between number skills and nonverbal reasoning (among other cognitive variables) for 7-9-year-old children with specific language impairments and control groups matched for age and language skill. Cowan et al. found nonverbal reasoning to uniquely account for variance in several number skill tasks, but suggested that nonverbal reasoning may have captured aspects of working memory as well. In the present study, we controlled for working memory

by including it as a competing cognitive dimension within analyses. Our nonverbal reasoning measure assessed pattern completion, classification, analogy, and serial reasoning tasks, whereas the concept formation measure measured the ability to identify, categorize, and determine rules to classify objects. The substeps and decisions embedded in the computation items of the fifth-grade calculations test, decisions that involve the complex and not particularly intuitive procedural rules for operating with multi-digit whole numbers and rational numbers, appear to draw on the same cognitive abilities measured by these nonverbal reasoning and concept formation measures.

The fourth cognitive ability predicting individual differences in the development of computation competence was the central executive component of working memory (numerical, specifically), which uniquely accounted for .5% of variance in whole-number and 1.2% of variance in rational-number computational skill. Note that, although the central executive component of working memory as indexed with numbers was uniquely predictive of individual differences in the development of both forms of calculation skill (whole-number and rational-number), listening recall was not. This indicates that competence with calculations derives strength from the specific ability to handle numbers within working memory and suggests individual differences in working memory for numbers versus words, as others have previously documented (Dark & Benbow, 1991; Siegel & Ryan, 1989). On the other hand, as Fuchs et al. (2010) demonstrated, two central executive tasks involving numerals (backward digit span and counting recall) correlated with each other less well than the listening recall and counting recall tasks correlated with each other (perhaps due to parallel assessment methods). This suggests, in contradiction to our present findings, a common working memory mechanism that is not distinct for numbers and words. It is nevertheless possible that some individuals' word or number representations are more highly active in working memory. Strong activation of Arabic numerals and corresponding magnitudes in working memory may facilitate execution of procedural calculations (whereas strong activation of verbal information may aid in one or several component processes, such as building problem models involved in solving word problems, as shown by Fuchs et al.). Future work examining these possibilities is warranted.

Although computational skill with whole as well as rational numbers appears to draw on a shared set of cognitive resources, namely incoming calculation skill, nonverbal reasoning, concept formation, and central executive working memory, individual differences in the development of computational skill with rational numbers does appear to draw upon one resource that was not associated with whole-number performance: language ability. Our analyses showed that language uniquely accounted for 1.1% of the variance in rational-number skill. In considering why language uniquely supports the development of calculation skill with rational but not whole numbers, it may be helpful to consider the role oral language plays in students' early number development. Research suggests that some non-human primates (e.g., Hauser, MacNeilage, & Ware, 1996) and preverbal infants (e.g., Wynn, 1992) demonstrate an innate capacity for numerosity independent of language ability. However, as infants grow into toddlers and preschoolers, the development of early number constructs appears to vary according to specific language patterns (see Fletcher et al., 2007). For example, Hodent, Bryant, and Houde (2005) conducted an experiment with French- and English-speaking toddlers, evaluating their ability to judge the accuracy of simple computations such as $1 + 1 = 3$. They found that interference between the toddlers' early arithmetic abilities and their burgeoning language skills was apparent only for the French-speaking children, whose language uses the same word, *un*, to denote cardinality and singularity; the English language does not. A further example of how language supports the early development of children's numeracy concepts, foundational to later computational skill, is demonstrated with the Chinese-based system of number words and mathematical terms, which may help explain East Asian students' superior performance over their

English-speaking counterparts (Ng & Rao, 2010; Miura et al., 1999). For example, a literal translation of the number 12 from Chinese into English is *ten-two* or *ten and two ones*, transparently connecting the language of the number words to the base-ten system. By contrast, English-speaking children learn the word, *twelve*, which provides no intuitive or instructional semantic advantage for the young learner. Finally, Zuber, Pixner, Moeller, and Nuerk (2009) examined the difficulty transcoding causes in languages with inversion (i.e., saying “hundred three and twenty” to name $100 + 23 = 123$), such as German. They found that German-speaking children struggle with this basic number-processing task.

Oral language appears to support early whole-number development; it is possible that language supports students’ early development of rational-number skill, as well, by promoting conceptual understanding of calculations with fractions, decimals, and percentages. Research has shown that the influence of conceptual and procedural knowledge is iterative and bi-directional (Rittle-Johnson & Siegler, 1998), with advances in conceptual understanding leading to more accurate choice of procedures (and vice versa). When exposed to a new mathematical concept (as the rational-number items likely are for many of students), they are tasked to simultaneously construct meanings for the written symbols and become familiar with the procedures used to calculate with those symbols (Wearne & Hiebert, 1989). In fact, Hecht and Vagi (in press) found that students’ fourth-grade conceptual knowledge concerning fractions uniquely contributed to their growth in fraction computation skill over the course of the next year. Our results suggest that students with stronger language skills may derive greater benefit from classroom instruction concerning the many different subconstructs of rational numbers and thus be better positioned to choose problem solution strategies more efficiently than peers with weaker oral language skills. Students in the fourth and fifth grade, who have only recently been exposed to computation with fractions, decimals, percentages, and ratios, may rely on oral language cues to develop a deeper conceptual understanding of the symbols of rational numbers (Johanning, 2008). Future research should examine what unique aspects of spoken and written language might facilitate this deeper conceptual understanding and allow for alternative interpretation of our results.

At this point, we remind readers of the exploratory nature of the present study due to two important study limitations. The first concerns our outcome measure of computation skill. Competence with whole numbers and with rational numbers was indexed via subtest scores from WRAT Arithmetic (Wilkinson, 1993), which mixes whole-number items with rational-number items to proceed from easiest to hardest. Thus, students were presented with more whole-number items earlier in the test. In addition, although alpha was .78 for whole-number items, it fell to .53 for rational-number items (probably due to a smaller number of rational-number items). It is nevertheless important for readers to note that all students completed the assessment within the time limit and that .53 is considered adequate within exploratory studies (Nunnally, 1967). Clearly, however, additional research is warranted using separate measures of each domain of computational skill. A second limitation is that over the three school years, from the beginning of the study in third grade to the end of fifth grade, 465 students became unavailable for outcome testing. In the school district from which our sample was drawn, students frequently transfer to schools within and outside of the district, sometimes creating difficulties in locating students for follow-up testing. Although the exiters appeared similar to study participants on the predictor variables, we cannot know for sure if the results would replicate had the exiters remained.

With these limitations in mind, findings provide the basis for generating hypotheses for additional study even as they suggest preliminary implications for instruction. First, it is important to note that the greatest amount of explained variance for both whole- and rational-number computation skill was accounted for by incoming calculation skill. That is,

students' beginning-of-third-grade computational fluency was the greatest predictor of computational skill three academic grades later. This finding underscores the importance of intervening early to address students' deficits with foundational mathematics skills to offset future and more pervasive difficulty. Further, specific domain-general abilities (i.e., nonverbal reasoning, concept formation, and working memory related to numbers) may influence computational skill with both whole and rational numbers. Thus, instruction that engages these specific cognitive abilities, to emphasize the decision-making inherent in selecting and in monitoring the completion of accurate procedural strategies for both types of computation, may produce stronger learning. Finally, language may be an important facilitator of competence with rational-number computation. This may be important for teachers to consider so they can accommodate students' oral language strengths and weaknesses during instruction, particularly with respect to initial instruction of rational-number constructs. Of course, these hypotheses about the factors that contribute to stronger rational-number development require additional empirical testing, including the use of randomized control studies.

We also note that processing speed and the central executive component of working memory in the form of sentences did not account for individual differences in the development of whole-number or rational-number computation skill. We expected processing speed to account for individual differences on the calculation outcome measures, given previous work showing that association with lower-level skill with number combinations (Fuchs et al., 2006) as well as with performance on a broad measure of mathematics achievement (Geary, 2010). Findings on this association are, however, mixed (see, for example, Seethaler & Fuchs, 2006; Fuchs et al., 2010), and methodological features of the studies may explain inconsistency in findings. For example, in the present study, variance associated with processing speed may have been captured by our measure of incoming calculation skill, which incorporated a stringent time limit, thereby requiring fluency. In any case, present findings emphasize the need for further study about the predictive role of processing speed in the development of calculation skill.

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References

- Aunola K, Leskinen E, Lerkkanen M, Nurmi J. Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*. 2004; 96:699–713.
- Baddeley, AD.; Hitch, GJ. Working memory.. In: Bower, GH., editor. *The psychology of learning and motivation*. Academic Press; New York: 1974. p. 47-90.
- Baddeley, AD.; Logie, RH. Working memory: The multi-component model.. In: Miyake, A.; Shah, P., editors. *Models of working memory: Mechanisms of active maintenance and executive control*. Cambridge University Press; Cambridge: 1999. p. 28-61.
- Bull R, Johnston RS. Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Psychology*. 1997; 65:1–24.
- Cowan R, Donlan C, Newton EJ, Lloyd D. Number skills and knowledge in children with specific language impairment. *Journal of Educational Psychology*. 2005; 97:732–744.
- CTB/McGrawHill. *TerraNova Technical Manual*. Author; Monterey, CA: 1997.

- Dark VJ, Benbow CP. Differentiated enhancement of working memory with mathematical versus verbal precocity. *Journal of Educational Psychology*. 1991; 83:48–60.
- Donlan C, Cowan R, Newton EJ, Lloyd D. The role of language in mathematical development: Evidence from children with specific language impairments. *Cognition*. 2007; 103:23–33. [PubMed: 16581052]
- Fletcher, JM.; Lyon, GR.; Fuchs, LS.; Barnes, MA. *Learning disabilities: From identification to intervention*. Guilford Press; New York: 2007.
- Fuchs LS, Fuchs D, Compton DL, Powell SR, Seethaler PM, Capizzi AM, et al. The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*. 2006; 98:29–43.
- Fuchs LF, Fuchs D, Steubing K, Fletcher JM, Hamlett CL, Lambert W. Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology*. 2008; 100:30–47. [PubMed: 20057912]
- Fuchs LS, Geary DC, Compton DL, Fuchs D, Hamlett CL, Bryant JD. The contributions of numerosity and domain-general abilities to school readiness. *Child Development*. 2010; 81:1520–1533. [PubMed: 20840238]
- Fuchs LS, Geary DC, Compton DL, Fuchs D, Hamlett CL, Seethaler PM, et al. Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology*. 2010b; 46:1731–1746. [PubMed: 20822213]
- Fuchs, LS.; Hamlett, CL.; Fuchs, DS. *Test of Computational Fluency*. Vanderbilt University; Nashville: 1990.
- Geary DC. Cognitive predictors of achievement growth in mathematics and reading: A five year longitudinal study. 2010 Manuscript submitted for publication.
- Geary DC, Hoard MK, Byrd-Craven J, Nugent L, Numtee C. Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*. 2007; 2007:1343–1359. [PubMed: 17650142]
- Gelman R, Butterworth B. Number and language: How are they related? *Trends in Cognitive Science*. 2005; 9:6–10.
- Hart SA, Petrill SA, Thompson LA, Plomin R. The ABCs of math: A genetic analysis of mathematics and its links with reading ability and general cognitive ability. *Journal of Educational Psychology*. 2009; 101:388–402. [PubMed: 20157630]
- Hauser MD, MacNeilage P, Ware M. Numerical representations in primates. *Proceedings of the National Academy of Sciences, USA*. 1996; 93:1514–1517.
- Hecht SA, Vagi KJ. Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*. (in press).
- Hodent C, Bryant P, Houde O. Language-specific effects on number computation in toddlers. *Developmental Science*. 2005; 8:420–423. [PubMed: 16048514]
- Johanning DI. Learning to use fractions: Examining middle school students' emerging fraction literacy. *Journal for Research in Mathematics Education*. 2008; 39:281–310.
- Jordan NC, Glutting J, Ramineni C. The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*. 2010; 20:82–88. [PubMed: 20401327]
- Mazzocco MMM, Devlin KT. Parts and holes: Gaps in rational number sense in children with vs. without mathematical learning disability. *Developmental Science*. 2008; 11:681–691. [PubMed: 18801123]
- Meyer ML, Salimpoor VN, Wu SS, Geary DC, Menon V. Differential contribution of specific working memory components to mathematics achievement in 2nd and 3rd graders. *Learning and Individual Differences*. 2010; 20:101–109. [PubMed: 21660238]
- Miura IT, Okamoto Y, Vlahovic-Stetic V, Kim CC, Han JH. Language supports for children's understanding of numerical fractions: Cross-national comparisons. *Journal of Experimental Child Psychology*. 1999; 74:356–365. [PubMed: 10552923]
- National Mathematics Advisory Panel. *The Final Report of the National Mathematics Advisory Panel*. U.S. Department of Education; Washington DC: 2008.
- Newcomer, PL.; Hammill, DD. *Test of Language Development*. Rev. ed.. Pro-Ed; Austin, TX: 1998.

- Ngan Ng SS, Rao N. Chinese number words, culture, and mathematics learning. *Review of Educational Research*. 2010; 80:180–206.
- Ni Y, Zhou Y. Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*. 2005; 40:27–52.
- Nunnally, JC. *Psychometric theory*. McGraw-Hill; New York: 1967.
- Nunnally, JC. *Psychometric theory*. 2nd ed.. McGraw-Hill; New York: 1978.
- Opfer JE, Siegler RS. Representational change and children's numerical estimation. *Cognitive Psychology*. 2007; 55:169–195. [PubMed: 17462620]
- Pickering, S.; Gathercole, S. *Working Memory Test Battery for Children*. The Psychological Corporation; London: 2001.
- Rittle-Johnson, B.; Siegler, RS. The relation between conceptual and procedural knowledge in learning mathematics: A review.. In: Donlan, C., editor. *The development of mathematical skills*. Psychology Press; Hove, United Kingdom: 1998. p. 75-110.
- Seethaler PM, Fuchs LS. The cognitive correlates of computational estimation skill among third-grade students. *Learning Disabilities Research & Practice*. 2006; 21:233–243.
- Shalev RS, Manor O, Auerbach J, Gross-Tsur V. Persistence of developmental dyscalculia: What counts? Results from a 3-year prospective follow-up study. *Journal of Pediatrics*. 1998; 133:358–362. [PubMed: 9738716]
- Siegel LS, Ryan EB. The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development*. 1989; 60:973–980. [PubMed: 2758890]
- Sowder, JT.; Bezuk, N.; Sowder, LK. Using principles from cognitive psychology to guide rational number instruction for prospective teachers.. In: Carpenter, TP.; Fennema, E.; Romberg, TA., editors. *Rational numbers: An integration of research*. Lawrence Erlbaum Associates; Hillsdale, NJ: 1993. p. 239-259.
- Swanson HL, Beebe-Frankenburger M. The relationship between working memory and mathematical problem-solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology*. 2004; 96:471–491.
- Swanson HL. Cross-sectional and incremental changes in working memory and mathematical problem-solving. *Journal of Educational Psychology*. 2006; 98:265–281.
- Swanson HL, Kim K. Working memory, short-term memory, and naming speed as predictors of children's mathematical performance. *Intelligence*. 2007; 35:151–168.
- Wearne D, Hiebert J. Cognitive changes during conceptually based instruction on decimal fractions. *Journal of Educational Psychology*. 1989; 81:507–513.
- Wechsler, D. *Wechsler Abbreviated Scale of Intelligence*. The Psychological Corporation; San Antonio, TX: 1999.
- Wilkinson, GS. *Wide Range Achievement Test 3*. Wide Range; Wilmington: 1993.
- Wilson KM, Swanson HL. Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? *Journal of Learning Disabilities*. 2001; 34:237–248. [PubMed: 15499878]
- Woodcock, RW. *Woodcock Diagnostic Reading Battery*. Riverside; Itasca, IL: 1997.
- Woodcock, RW.; McGrew, KS.; Mather, N. *Woodcock-Johnson III*. Riverside; Itasca, IL: 2001.
- Wynn K. Addition and subtraction by human infants. *Nature*. 1992; 358:749–750. [PubMed: 1508269]
- Zhu, J. *WASI Manual*. The Psychological Corporation; San Antonio: 1999.
- Zuber J, Pixner S, Moeller K, Nuerk HC. On the language specificity of basic number processing: Transcoding in a language with inversion and its relation to working memory capacity. *Journal of Experimental Child Psychology*. 2009; 102:60–77. [PubMed: 18499120]

Research Highlights

- We explore the cognitive predictors of whole- versus rational-number computational skill.
- We assess 3rd-grade students on calculation skill, language, nonverbal reasoning, concept formation, processing speed, and working memory.
- We assess whole-number and rational-number computational skill outcomes in the spring of 5th grade.
- Unique predictors of whole-number computation skill include calculation skill, nonverbal reasoning, concept formation, and working memory-numerical.
- In addition to these abilities, language emerges as a unique predictor of rational-number computation skill.

Table 1

Means, Standard Deviations, and Correlations^a Among Predictors and Computation Outcomes (n = 688)

	Raw Score		Standard Score ^b		CBM	LC	Voc.	GC	LF	MR	CF	VM	LR	NR	W	R
	X	(SD)	X	(SD)												
Predictors																
Incoming calculation skill: CBM Computation (CBM)	12.23	(6.10)			--											
Language: WDRB listening comp. (LC)	20.91	(4.44)	95.90	(18.64)	.20	--										
WASI vocabulary (Voc.)	27.66	(6.60)	47.00	(10.11)	.33	.52	--									
TOLD grammatic closure (GC)	18.42	(6.79)	85.07	(11.05)	.26	.50	.52	--								
Language factor (LF)	-0.13	(2.55)			.32	.82	.83	.82	--							
Nonverbal reasoning: WASI matrix reasoning (MR)	15.73	(6.38)	49.04	(11.03)	.26	.28	.32	.30	.37	--						
Concept formation: WJ concept formation (CF)	15.78	(7.22)	92.84	(13.63)	.37	.43	.46	.41	.52	.39	--					
Processing speed: WJ visual matching (VM)	475.13	(19.46)	98.11	(15.49)	.44	.19	.19	.21	.24	.25	.30	--				
Working memory-sentences: WMTB-C listening recall (LR)	9.77	(3.23)	92.16	(15.94)	.20	.38	.38	.42	.48	.27	.41	.18	--			
Working memory-numerical: WJ numbers reversed (NR)	9.31	(2.76)	95.55	(13.78)	.21	.18	.24	.24	.27	.27	.29	.26	.35	--		
Outcomes																
Whole number computation (W)	15.63	(3.10)			.49	.20	.28	.23	.29	.30	.37	.29	.26	.26	--	
Rational number computation (R)	1.82	(1.50)			.42	.27	.36	.29	.37	.32	.39	.29	.27	.29	.52	--

Note.

CBM = Curriculum-based measurement; WDRB = Woodcock Diagnostic Reading Battery; comp. = Comprehension; WASI = Weschler Abbreviated Scale of Intelligence; TOLD = Test of Language Development-Primary; WJ = Woodcock-Johnson III; WMTB-C = Working Memory Test Battery-Children.

^a All correlations significant, $p < .01$.

^b Means and standard deviations for WASI vocabulary and WASI matrix reasoning are reported as T scores.

Table 2
Multiple Regression Results in Predicting Whole-number and Rational-number Computational Skill

<i>Predictors</i>	Whole-number Skill Outcome ^a				Rational-number Skill Outcome ^b			
	<i>B</i>	<i>SE</i>	<i>Beta</i>	<i>t</i>	<i>B</i>	<i>SE</i>	<i>Beta</i>	<i>t</i>
Constant	7.669	2.65		2.894	2.253	1.299		-1.734
Incoming calculation skill	.189	.019	.372	9.939 ^f	.063	.009	.256	6.775 ^f
Language	.003	.050	.002	.055	.079	.024	.134	3.234 ^f
Nonverbal reasoning	.055	.018	.113	3.125 ^e	.026	.009	.109	2.994 ^e
Concept formation	.052	.017	.122	3.018 ^e	.025	.009	.119	2.907 ^e
Processing speed	.005	.006	.032	.881	.004	.003	.049	1.316
Working memory-sentences	.069	.037	.072	1.882	.012	.018	.026	.661
Working memory-numerical	.090	.040	.080	2.256 ^d	.066	.020	.120	3.362 ^f

^aWhole-number Skill Outcome: $R^2 = .312$, $F(7, 678) = 43.89$, $p < .001$.

^bRational-number Skill Outcome: $R^2 = .296$, $F(7, 678) = 40.75$, $p < .001$.

^c R^2 change after all other variables have been entered.

^d $p < .05$

^e $p < .01$

^f $p < .001$.