Intrinsic mode entropy based on multivariate empirical mode decomposition and its application to neural data analysis

Meng Hu · Hualou Liang

Received: 4 March 2011 / Revised: 9 June 2011 / Accepted: 9 June 2011 / Published online: 22 June 2011
© Springer Science+Business Media B.V. 2011

Abstract Entropy, a measure of the regularity of a time series, has long been used to quantify the complexity of brain dynamics. Given the multiple spatiotemporal scales inherent in the brain, traditional entropy analysis based on a single scale is not adequate to accurately describe the underlying nonlinear dynamics. Intrinsic mode entropy (IMEn) is a recent development with appealing properties to estimate entropy over multiple time scales. It is a multiscale entropy measure that computes sample entropy (SampEn) over different scales of intrinsic mode functions extracted by empirical mode decomposition (EMD) method. However, it suffers from both mode-misalignment and mode-mixing problems when applied to multivariate time series data. In this paper, we address these two problems by employing the recently introduced multivariate empirical mode decomposition (MEMD). First, we extend the MEMD to multi-channel multi-trial neural data to ensure the IMEn matched at different scales. Second, for the discriminant analysis of IMEn, we propose to improve the discriminative ability by including variance that has not been used before in entropy analysis. Finally, we apply the proposed approach to the multi-electrode local field potentials (LFPs) simultaneously collected from visual cortical areas of macaque monkeys while performing a generalized flash suppression task. The results have shown that the entropy of LFP is indeed scale-dependent and is closely related to the perceptual conditions. The discriminative results of the perceptual conditions, revealed by support vector machine, show that the accuracy based on

IMEn and variance reaches 83.05%, higher than that only by IMEn (76.27%). These results suggest that our approach is sensitive to capture the complex dynamics of neural data.

Keywords Entropy · Multivariate empirical mode decomposition · Neural signal analysis

Introduction

Neural signals are thought of containing the complex information related to the underlying dynamics in brain (Stam 2006). Entropy, as a commonly used information measure for quantifying the degree of complexity and irregularity of time series, has been widely used for extracting the information from the signal (Pincus 1991). Among the methods used for entropy estimate, sample entropy, SampEn, has attracted a great deal of attentions due to its insensitivity to the data length and immunity to the noise in the data (Richman and Moorman 2000; Lake et al. 2002). SampEn, however, is estimated only at a single scale, thus may fail to account for the multiple time scales underlying nonlinear dynamics. In contrast, multiscale entropy is an effective measure of complexity that takes into account the different scales of a time series, on which the SampEn is computed (Costa et al. 2002). This method, though powerful, is not well adapted for studying the nonlinear and non-stationary signals due to its linear extraction of scales (Peng et al. 2009).

Until recently, intrinsic mode entropy (IMEn) (Amoud et al. 2007) was developed to measure entropy over different scales corresponding to the different oscillatory levels of the given time series, as determined by empirical mode decomposition (EMD), a fully adaptive, data-driven time series decomposition method. Specifically, the EMD

M. Hu · H. Liang (✉)
School of Biomedical Engineering, Science and Health Systems,
Drexel University, 3141 Chestnut Street, Philadelphia, PA
19104, USA
e-mail: hualou.liang@drexel.edu

is first performed to decompose a time series into scale-dependent oscillations inherent to the data, referred to as intrinsic mode functions (IMFs) (Huang et al. 1998), and then the IMEn is obtained by computing SampEn of the accumulative sums of IMFs. This method is promising due to its robustness to changes in low-frequency components of a signal, and has been successfully used for sign language recognition (Kosmidou and Hadjileontiadis 2009).

As the IMEn relies on the EMD method, it inevitably suffers from both mode-misalignment and mode-mixing problems, particularly in the analysis of multivariate time series data. The mode-misalignment refers to a problem where the common frequency modes across a multivariate data appear in the different-index IMFs, thus resulting that the IMFs are not matched either in the number or in scale, whereas the mode-mixing is manifested by a single IMF containing multiple oscillatory modes and/or a single mode residing in multiple IMFs which may in some cases compromise the physical meaning of IMFs and practical applications (Wu and Huang 2009). Both problems can be simply illustrated by a toy example shown in Fig. 1. In this example, three synthetic tone signals (X, Y and Z) each with the superimposed white noises are individually subject to the EMD decomposition. As a result, three signals yield different numbers of IMFs that even with the same index exhibit the mode misalignment. In addition, the mode-mixing clearly occurs in the decomposition of signal Z.

In neuroscience, multivariate data such as multi-channel neural recordings over multiple trials becomes increasingly common. As such, the mode-misalignment and mode-mixing problems present a significant challenge for the

analysis of multi-channel multi-trial neural data. In the present work, we address these two problems simultaneously by employing the recently developed multivariate EMD (MEMD) (Rehman and Mandic 2010) for characterizing the IMEn of multivariate neural data. First, we extend the MEMD to multi-channel multi-trial data to yield the aligned IMFs not only in number, but also in frequency, and to relieve the mode-mixing problem. Second, we obtain the IMEn by computing the SampEn over the cumulative sums of the IMFs extracted by the MEMD. Finally, we apply our MEMD-based IMEn to the multi-electrode multi-trial local field potentials (LFPs) simultaneously collected from visual cortical areas of macaque monkeys while performing a generalized flash suppression task. We show that the entropy of LFPs is indeed scale-dependent and is closely related to different perceptual conditions (Visible vs. Invisible). To quantify the observed difference of two perceptual conditions, the discriminant analysis with support vector machine (SVM) is performed (Cortes and Vapnik 1995). Considering that the SampEn used in the calculation of IMEn is based on the standardized data where the variance is routinely discarded (Richman et al. 2004), which nonetheless may be important for the discriminant analysis, we thus propose to include the variance of the data in addition to the IMEn as features for the SVM classifier. Our results show that the accuracy based on the IMEn and variance reaches 83.05%, higher than that by IMEn (76.27%) alone, which confirms that the variance of the data indeed contains additional information for the discrimination. These results, taken together, suggest that our approach is sensitive to capture the complex dynamics of neural data.

Fig. 1 Decomposition of three synthetic signals X, Y and Z (top row) separately with EMD. The decomposed IMFs exhibit both the mode misalignment and the mode-mixing. For example, the 3rd IMFs of X and Y have different frequency scales, indicating the mode-misalignment problem. Most IMFs of Z show clear mixed frequencies, indicating the mode-mixing problem

Method

Background

Sample entropy

Entropy is an information measure for describing the complexity or irregularity of system. So far, many attempts have been made for estimation of entropy in the complex system, such as Kolmogorov entropy, Eckmann–Ruelle entropy and approximate entropy (Grassberger and Procaccia 1983; Eckmann and Ruelle 1985). Approximate entropy is an effective entropy measure for the short and noisy time series, but it introduces a bias as the approximate entropy counts self-match when it calculates the pairs of similar epochs (Richman and Moorman 2000). Sample entropy, SampEn, provides an improved version of approximate entropy to eliminate the bias. It is defined as the negative natural logarithm of the conditional probability that two sequences similar for m points remain similar at the next $m + 1$ point in the data set within a tolerance r , where self-matches are excluded in calculating the probability.

In order to compute SampEn, the time series $I = \{i(1), i(2), \dots, i(N)\}$ is embedded in a delayed m -dimensional space, where the m -dimensional vectors are constructed as $x_m(k) = (i(k), i(k+1), \dots, i(k+m-1))$, $k = 1 \sim N - m + 1$. The match of two vectors in the embedded space is defined as their distance lower than the tolerance r . The probability $B^m(r)$ that two sequences match is computed by counting the average number of vector pairs. Similarly, $A^m(r)$ is defined for an embedded dimension of $m + 1$ (Richman and Moorman 2000). The SampEn is then calculated as

$$\text{SampEn}(I, m, r) = -\ln\left(\frac{A^m(r)}{B^m(r)}\right).$$

Although the SampEn has been successfully applied to the analysis of physiologic signal (Lake et al. 2002), it is a single-scale measure which may fail to account for the multiple time scales inherent in such time series. In addition, it is severely affected by low-frequency components of the signal. As such, it is not well adapted for studying time series in different scales, particularly in the presence of local trends and low-frequency components (Amoud et al. 2007). The recently proposed intrinsic mode entropy (IMEn) based on the EMD is a promising method to estimate entropy over multiple scales.

EMD-based intrinsic mode entropy

Empirical mode decomposition (EMD) is a data-driven time–frequency technique which adaptively decomposes a signal, by means of the so-called the sifting algorithm, into

a finite set of amplitude- and/or frequency-modulated (AM/FM) modulated components, referred to as intrinsic mode functions (IMFs) (Huang et al. 1998). IMFs satisfy the requirements that the mean of the upper and lower envelopes is locally zero and the number of extrema and the number of zero-crossing differ by at most one, and they thus represent the intrinsic oscillation modes of data on the different frequency scales. The time series $x(t)$ can be decomposed by the EMD as $x(t) = \sum_{j=1}^k c_j(t) + r(t)$, where $c_j(t)$, $j = 1, \dots, k$ are the IMFs and $r(t)$ is the monotonic residue. The EMD algorithm is briefly described as follows.

- (1) Let $\tilde{x}(t) = x(t)$;
- (2) Find all local maxima and minima of $\tilde{x}(t)$;
- (3) Interpolate through all the minima (maxima) to obtain the lower (upper) signal envelop $e_{\min}(t)$ ($e_{\max}(t)$);
- (4) Compute the local mean $m(t) = \frac{e_{\min}(t) + e_{\max}(t)}{2}$;
- (5) Obtain the detail part of signal $c(t) = \tilde{x}(t) - m(t)$;
- (6) Let $\tilde{x}(t) = c(t)$ and repeat the process from step 2 until $c(t)$ becomes an IMF.
- (7) Compute the residue $r(t) = x(t) - c(t)$ and go back to step 2 with $\tilde{x}(t) = r(t)$, until the monotonic residue signal is left.

The IMEn is essentially the SampEn computed on the cumulative sums of the IMFs obtained by the EMD decomposition. In fact, the cumulative sums of the IMFs, starting from the finest scales and ending with the whole signal, yield a multiscale filtering of the original signal. The IMEn can be represented by

$$\text{IMEn}(k, m, r) = \text{SampEn}(S_{\text{IMF}}^k(t), m, r),$$

where m is the window length, r is the tolerance, and S_{IMF}^k corresponds to the cumulative IMF sum up to scale k , which is defined as $S_{\text{IMF}}^k(t) = \sum_{p=1}^k c_p(t)$.

The procedure for computing the IMEn is the same as for SampEn, but over different scales as specified by the EMD method. Computing the IMEn from the finest scales to the whole signal enables the analysis of the evolution of the signal regularity, as slower frequency components are gradually added until the SampEn of the whole signal is reached. Due to its reliance on the EMD method, the EMD-based IMEn suffers from both mode-misalignment and mode-mixing problems, which limit its application to multivariate time series data. In the following section, an improved IMEn based on the MEMD method is proposed.

MEMD-based IMEn for discriminant analysis

Multivariate EMD algorithm (MEMD) has been recently proposed to extract time–frequency information for more than two signals simultaneously (Rehman and Mandic

2010). The work proposed in this paper is based on the multivariate EMD that will be briefly reviewed in this section. The key for MEMD algorithm is the calculation of local mean. Unlike the EMD in which the local mean is computed by taking an average of upper and lower envelopes obtained by interpolating the local maxima and minima of the signal, MEMD directly works with multivariate data in which the local maxima and minima are not well defined. To deal with this problem, multiple n -dimensional envelopes are generated by taking signal projections along different directions in n -dimensional spaces. These envelopes are then averaged to obtain the local mean. The algorithm is briefly summarized as follows (Rehman and Mandic 2010):

- (1) Generate the point set based on the Hammersley sequence for sampling on an $(n - 1)$ -sphere
- (2) Calculate a projection $p^{\theta_k}(t) \}_{t=1}^T$ of the multivariate input data $\{v(t)\}_{t=1}^T$ along a direction vector x^{θ_k} for all k giving $p^{\theta_k}(t) \}_{k=1}^K$.
- (3) Locate the time points $t_i^{\theta_k}$ according to maxima of the set of projected signal $p^{\theta_k}(t) \}_{k=1}^K$.
- (4) Interpolate $\left[t_i^{\theta_k}, v(t_i^{\theta_k}) \right]$, for all values of k , to obtain multivariate envelope curves $e^{\theta_k}(t) \}_{k=1}^K$.
- (5) For a set of K direction vectors, calculate the mean $m(t)$ of the envelop curves

$$m(t) = (1/K) \sum_{k=1}^K e^{\theta_k}(t)$$

- (6) Extract the detail $c(t)$ using $c(t) = x(t) - m(t)$. If the detail satisfies the stoppage criterion for multivariate IMF, apply the above procedure to $x(t) - c(t)$, otherwise apply it to $c(t)$.

The stoppage criterion for multivariate IMF is similar to that for the univariate IMFs except that the equality constraint for number of extrema and zero crossings is not imposed, as extrema cannot be properly defined for the multivariate signal. By projection, MEMD directly processes multivariate signal to produce the aligned IMFs. As an example, the same synthetic signal used in Fig. 1 is subject to the MEMD. The result is shown in Fig. 2, where we can see that the IMFs of the same index from different channels show the aligned IMFs across all the channels both in number and in scale. This example demonstrates that the MEMD is an effective method to obtain the aligned IMFs for multivariate time series.

To characterize the complexity of multi-channel neural recordings collected over multiple trials, our proposed MEMD-based IMEn for discriminant analysis consists of the following seven steps:

- (1) Reshape the high-dimensional neural data into a multivariate form suitable for the MEMD analysis. For example, the neural data collected is typically represented as $Channels \times TimePoints \times Trials$. The data can be reshaped into such a two dimensional time series as $[Channels \times Trials] \times TimePoints$ before submitted for the MEMD analysis. It is a simple, yet important step to make sure that all the IMFs be aligned not only across channels, but also across trials.
- (2) Preprocess the multivariate data. Multi-channel neural recordings have a high degree of variability typically collected over many trials spanning from days to months, or even years (left column of Fig. 3). As the MEMD works via projections of the data in multidimensional spaces, such variability among individual time series has significant detrimental impact upon the final decomposition, and thus should be avoided. To alleviate the variability effect, we take such a critical preprocessing step as the normalization by temporal standard deviation of individual time series before MEMD (right column of Fig. 3), and then restore the standard deviations to the corresponding IMFs after MEMD.
- (3) Perform the MEMD on the multivariate data to obtain the aligned IMFs.
- (4) Compute the IMEn(k, m, r) based on the cumulative sums of the IMFs obtained by the MEMD decomposition. The parameters m and r in the calculation of SampEn are chosen as 2 and 0.15, respectively, to minimize the standard error of entropy estimation (Lake et al. 2002).
- (5) Determine the significant scales for IMEn between two different perceptual conditions, Visible vs. Invisible. A t -test of statistical significance is performed on IMEn at each scale for a given P value such as 0.01.
- (6) Selection the features for the discriminant analysis. We use the significant IMEn identified in Step (5) and the variance of the signal as the selected features. We note that in the calculation of SampEn the variance has been routinely discarded, which may nonetheless provide important information for the discriminant analysis. We therefore propose to combine the IMEn and the variance of the most discriminative scale, as the feature vector for the discrimination
- (7) Quantify the difference of two perceptual conditions using Support Vector Machine (SVM). We use decoding accuracy as a performance measure and calculate it via leave-one-out cross-validation. In particular, for a data set with N trials, we choose $N-1$ trials for feature extraction and training and use the remaining 1 trial for testing. This is repeated for N

Fig. 2 Decomposition of a synthetic trivariate signal [X Y Z] shown in the *top row* of Fig. 1 via MEMD. The decomposed IMFs exhibit the alignment of common modes within the input signal

Fig. 3 Effect of preprocessing on multivariate data. The *left column* of this figure shows an example of the high variability of neural recordings over multiple trials, where *Trial 1* has the standard deviation three times over that of *Trial 2*. To reduce the variability effect of neural recordings on the decomposition of MEMD, individual time series is normalized by its standard deviation before the MEMD is performed (*right column*), and the standard deviation is then restored to the corresponding IMFs after MEMD

times with each trial used for testing once. The decoding accuracy is obtained as the ratio of the number of correctly decoded trials over N .

Result

In this section, we use the local field potentials (LFPs) collected from visual cortex of a macaque monkey while performing a generalized flash suppression (GFS) task to

demonstrate the effectiveness of our proposed approach. The GFS task in which a salient visual stimulus can be rendered invisible despite continuous retinal input provides a rare opportunity to study neural mechanisms directly related to perception (Wilke et al. 2003). In the task, as soon as a monkey gained fixation for about 300 ms, the target stimulus indicated by a red disk was presented. At 1,400 ms after the target onset, small random-moving dots appeared as the surroundings. With the immediate presence of the surroundings, the red disk could be rendered

subjective invisible. If the target stimulus disappeared from perception, the monkey was trained to release a lever; otherwise, monkey was to hold the lever. Therefore, based on the responses of the animal, the trial was classified as either ‘Visible’ or ‘Invisible’. Note that the stimuli in these two conditions were physically identical. Multi-electrode LFP recordings were simultaneously collected from multiple cortical areas V1, V2, and V4 while monkeys performed the GFS task. The data were obtained by band-pass filtering the full bandwidth signal between 1 and 500 Hz, and then resampled at 1 kHz (Wilke et al. 2006). In this work, the LFP data from cortical area V4 is used to demonstrate our MEMD-based IMEn method in separating two visibility conditions.

The multi-channel multi-trial LFP data is first reshaped to multivariate form, and then MEMD is used for decomposing these signals. Figure 4 shows the typical example of decomposition by MEMD. In this figure, the first three columns show three LFP time series and their IMFs, and the last column displays the corresponding power spectrums of IMFs. We observe that the MEMD of three LFP time series produces the same number of IMFs and IMFs with the same index are confined in the aligned frequency scales. This result further confirms that the MEMD is able to decompose the multi-channel LFP data into the aligned IMFs.

The IMEn is then computed on the aligned IMFs. Figure 5 shows the IMEns as a function of scale for both ‘Visible’ and ‘Invisible’ conditions on two typical channels, shown in (a) and (b), respectively. From this figure, IMEns

on two channels show the similar pattern, in which two conditions both present the decreased trend as the slow oscillations are gradually added. IMEns of two conditions initially overlap at the two finest scales, then gradually diverge with maximum separation at the scale of 7 ($P < 0.01$), and finally reach plateaus starting at scale 9. We note that the last scale corresponds to the whole signal. The initial overlap at the fastest scales suggests that the high-frequency oscillations mainly contains noise, whereas the plateaus at the slowest scales indicates that the low-frequency oscillations (< 5 Hz) corresponding to the evoked potentials of stimuli that may have little contribution to the discrimination of two perceptual conditions. Since the stimuli are identical in GFS task, it is expected to have the same evoked potentials. As such, there is no significant change of the IMEns between two perceptual conditions as the low-frequency oscillations are gradually added.

The separation of two conditions at various intermediate scales underscores the multiscale dynamics underlying the complex neural processes, which is not clear at one single scale of the whole signal (the last scale). It is also evident that the ‘Invisible’ condition presented the significantly higher entropy than ‘Visible’ condition, indicating that perceptual suppression is likely to be related to the more complex neural processes than the normal visible condition.

With the most discriminative scale, we combine entropy and variance as the feature vector for the discrimination of visibility conditions. SVM with a linear kernel as a classifier is employed to decode two conditions for quantifying the discriminative ability of our approach. The result shows that

Fig. 4 An example of decomposition of a three-channel 1-s-long LFP by the MEMD with 0 indicating the surrounding onset (*Left*) and the corresponding power spectra of IMFs (*Right*). It is evident that the decomposed IMFs from multi-channel LFPs are aligned both in number and in scale; a result confirmed by the power spectra shown on the *right*

Fig. 5 Our proposed MEMD-based IMEn of the LFPs during GFS on two selected channels (a) and (b). The entropy estimates of each channel in two conditions show a decreased trend as the scale increases. It is clear that the entropy in the invisible condition is

higher than that in the visible condition, where the maximum separation is achieved at scale 7, as indicated by ‘*’ at $P < 0.01$. Note that the last scale is the raw data

Fig. 6 The comparison of the decoding accuracy obtained by SVM with different features. The *leftmost* bar shows the best decoding accuracy from the combination of the MEMD-based IMEn and the variance, the *middle* bar is from only the MEMD-based IMEn, and the *rightmost* bar gives the lowest decoding accuracy from the original IMEn based on the standard EMD method

our method provides the excellent performance for decoding the visibility conditions, whereby the decoding accuracy can reach 83.05% (Fig. 6). By contrast, we also compute the decoding accuracies based on the only MEMD-based IMEn as the feature and the original IMEn based on the standard EMD as the feature (Fig. 6). The former obtains the decoding accuracy of 76.27%, indicating that the variance indeed provides additional information for the discriminant analysis, whereas the latter gives the lowest decoding accuracy of 69.49%. In addition to the low decoding accuracy achieved by the original IMEn, the practical concern is that the comparison between the probably mismatched scales might lead to the misleading results.

Conclusion

In this paper, we propose an improved IMEn based on the MEMD to address both the mode-misalignment and the

mode-mixing problems in the original IMEn. Here, we extend the MEMD to multi-channel multi-trial neural data. With the aligned scales of data, we obtain the IMEn by computing the sample entropy over the cumulative sums of the IMFs extracted by the MEMD. For the discriminant analysis based on IMEn, we improve the discriminative ability of our approach by including the variance as a feature, which provides the additional information for discrimination. The application on the real cortical LFP data suggests that our approach is effective in capturing the complex dynamics of the nonlinear non-stationary neural signal.

Acknowledgments This work is partially supported by NIH. We thank Dr Melanie Wilke for proving the data, which were collected at the laboratory of Dr Nikos Logothetis at Max Planck Institute for Biological Cybernetics in Germany.

References

- Amoud H, Snoussi H, Hewson D et al (2007) Intrinsic mode entropy for nonlinear discriminant analysis. *IEEE Signal Proc Lett* 14(5):297–300
- Cortes C, Vapnik VN (1995) Support-vector networks. *Mach Learn* 20:273–297
- Costa M, Goldberger AL, Peng C (2002) Multiscale entropy analysis of complex physiologic time series. *Phys Rev Lett* 89(6):068102
- Eckmann JP, Ruelle D (1985) Ergodic theory of chaos and strange attractors. *Rev Mod Phys* 57:617–656
- Grassberger P, Procaccia I (1983) Estimation of the Kolmogorov entropy from a chaotic signal. *Phys Rev A* 28:2591–2593
- Huang NE, Shen Z, Long SR et al (1998) The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc R Soc Lond A* 454A:903–993
- Kosmidou VE, Hadjileontiadis LJ (2009) Sign language recognition using intrinsic-mod sample entropy on sEMG and accelerometer data. *IEEE Trans Biomed Eng* 56(12):2879–2890
- Lake DE, Richman JS, Griffin MP et al (2002) Sample entropy analysis of neonatal heart rate variability. *Am J Physiol* 283(3):R789–R797
- Peng CK, Costa M, Goldberger AL (2009) Adaptive data analysis of complex fluctuations in physiologic time series. *Adv Adapt Data Anal* 1(1):61–70

- Pincus SM (1991) Approximate entropy as a measure of system complexity. *Proc Natl Acad Sci USA* 88:2297–2301
- Rehman N, Mandic DP (2010) Multivariate Empirical Mode Decomposition. *Proc R Soc Lond* 466A:1291–1302
- Richman JS, Moorman JR (2000) Physiological time-series analysis using approximate entropy and sample entropy. *Am J Physiol Heart Circ Physiol* 278:2039–2049
- Richman JS, Lake DE, Moorman JR (2004) Sample entropy. *Method Enzymol* 384:172–184
- Stam CJ (2006) *Nonlinear brain dynamics*. Nova Science, New York
- Wilke M, Logothetis NK, Leopold DA (2003) Generalized flash suppression of salient visual target. *Neuron* 39(6):1043–1052
- Wilke M, Logothetis NK, Leopold DA (2006) Local field potential reflects perceptual suppression in monkey visual cortex. *Proc Natl Acad Sci USA* 103(46):17507–17512
- Wu Z, Huang NE (2009) Ensemble empirical mode decomposition: a noise-assisted data analysis method. *Adv Adapt Data Anal* 1(1):1–41