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Design Aspects of Focal Beams From High-Intensity Arrays

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Abstract

As the applications of ultrasonic thermal therapies expand, the design of the high-intensity array must address both the energy delivery of the main beam and the character and relevance of off-target beam energy. We simulate the acoustic field performance of a selected set of circular arrays organized by array format, including flat versus curved arrays, periodic versus random arrays, and center void diameter variations. Performance metrics are based on the -3 -dB focal main lobe (FML) positioning range, axial grating lobe (AGL) temperatures, and side lobe levels. Using finite-element analysis, we evaluate the relative heating of the FML and the AGLs. All arrays have a maximum diameter of 100λ , with element count ranging from 64 to 1024 and continuous wave frequency of 1.5 MHz.

First, we show that a 50% spherical annulus produces focus beam side lobes which decay as a function of lateral distance at nearly 87% of the exponential rate of a full aperture. Second, for the arrays studied, the efficiency of power delivery over the -3 -dB focus positioning range for spherical arrays is at least 2-fold greater than for flat arrays; the 256-element case shows a 5-fold advantage for the spherical array. Third, AGL heating can be significant as the focal target is moved to its distal half-intensity depth from the natural focus. Increasing the element count of a randomized array to 256 elements decreases the AGL-to-FML heating ratio to 0.12 at the distal half-intensity depth. Further increases in element count yield modest improvements. A 49% improvement in the AGL-to-peak heating ratio is predicted by using the Sumanaweera spiral element pattern with randomization.

I. Introduction

High-intensity array design has been pursued for many years. Early visionary use of therapeutic ultrasound appeared in 1942 [1] followed by landmark tissue absorption studies [2] in the 1950s. Currently, the field of therapeutic ultrasonic hyperthermia includes a very wide range of medical applications [3]. Strategies have been developed for the treatment of uterine fibroids [4], breast tumors [5], [6], prostatic hyperplasia [7], [8], thrombolysis [9], liver tumors [10], [11], and brain tumors [12]. Standard commercial imaging probes have been applied to guide ultrasound-based hyperthermia, and more recently, guidance has been built into the same probe [13], [14]. Magnetic resonance imaging (MRI) has also been applied to guide ultrasound hyperthermia because of the high quality of MRI 3-D imaging and temperature measurement [15].

Combining the physics of transducer design with thermal predictions of the bioheat equation has also been an important topic of interest. In 1981, Hynynen [16] described the relationships of focal intensity locations with frequency and tissue absorption effects with the use of the bioheat equation. Later in 1987, Lizzi *et al.* [17] described thermal model profiles for beams in brain and eye heating experiments. As arrays were developed in the 1980s, they also gained in popularity as thermal therapy devices. This advancement, which enabled electronic focusing of a beam, also produced the need for therapeutic beam design

and control. The main lobe and the inherent side and possible grating lobes all need attention in a good design, especially with target beam positioning away from the natural array focus.

Synthetic beam control has advanced to manipulate the center beam into multiple sub-beams to better distribute the heating [18] and to reduce the side lobes in a sparsely-sampled aperture [19]. A large general improvement was made as well with the advent of the randomized array, where Goss *et al.* [20] used a spherical array, and Hutchinson [21] used a 1-D array with randomized element widths. Gavrilov *et al.* described simulation results which clearly showed the superiority of element randomization [22] with synthetic matrices of foci. Filonenko *et al.* extended this work by showing the computed spatial temperature distributions produced from periodic and randomized arrays [23]. Kirkebo *et al.* demonstrated the benefit of curving a periodic array and the resulting suppression of lateral grating lobes (LGL) [24]. Ebbini and Cain recognized the seriousness of the axial grating lobes (AGLs) [25]; however, the attention given to these lobes has been limited in the literature.

The objective of this work is to reveal some of the critical characteristics in acoustic arrays designed for thermal therapy. Without loss of generality, we focus on the effects of array design on the acoustic field for typical mild-hyperthermia applications. Linear acoustic waves are assumed everywhere in the field. We study several array formats and characteristics as listed in Table I. The first part of this paper examines the expected target beam side lobe performance of an array with a central hole. Second, simulation performance comparisons are made for 1) the half peak intensity contour plots of focus positions in a 2-D axial symmetric plane; 2) the locations of the peak intensity focus; and 3) the peak intensities of common randomized element therapeutic array designs (e.g., highly spherical, mildly spherical, or flat). The final section of this paper uses finite-element modeling to study the thermal consequences of undesired AGLs which increase as the electronically controlled focus is positioned at increasing depths.

II. Methods

The majority of the work here is performed using numerical approximation because of the lack of simple analytical solutions; however, simulation comparisons to analytical solutions for circular apertures are also used for the analysis of the side lobe response of circular apertures (both flat and spherical) with a central void of varied size. Useful array design equations for the intensity gain of spherical arrays are summarized in the appendix; the intensity gain at the geometric focus is linearly related to the square of the active aperture area and the square of the frequency, and is inversely related to the square of the radius of curvature.

A. Framework for Modeling a Circular Array With Flexible Attributes

A modeling interface in MathCad (v14, Parametric Technology Corp., Needham, MA) was assembled to compute the pressure and intensity profiles. The geometries used in the simulations are illustrated in Fig. 1. The general construction methodology to arrange the elements on any circular surface is given in Fig. 2. The case shown is for a 256-element array.

A symmetrical and periodic array of element center locations is created first [Fig. 2(a)], followed by position randomization [Fig. 2(b)]. Because global optimization of element placement is mathematically challenging and beyond the scope of this work, a simple element arrangement by an arbitrary randomization routine has been applied to generate N radial shift-distances and directions to each of the element center locations in the periodic layout using spherical surface coordinates θ and ϕ .

Following randomization, a minimum gap requirement is enforced between elements, and between any element and the array boundaries. A gap of 0.2 mm is used, which is a typical spacing requirement in high-intensity arrays [Figs. 2(c) and 2(d)]. Using the new element positions, the array is populated with element sub-apertures with intra-element sampling always less than λ [Fig. 2(d)].

A 3-D array of the element positions on the array aperture was subsequently reassembled in Cartesian coordinates for the discrete implementation of the Rayleigh-Sommerfeld (RS) integral [26], which includes source phase focusing and tissue absorption. The complex field pressure $p(x, y, z)$ from an aperture of any shape is modeled by the summation of radiators from finely sampled elements as

$$p(x, y, z) = \frac{jk}{2\pi} \frac{f_a S_{ap}}{NM} p_{ap} \sum_{n=0}^{N-1} \exp(-j\varphi e_n) \sum_{q=0}^{M-1} \frac{\exp(jk_m R_{nq})}{R_{nq}}, \quad (1)$$

where pressure p_{ap} is the aperture surface pressure magnitude (173.25 kPa at 1 W/cm² intensity), and φe_n is the phase of the n th element, or kR_n (where wave number $k = \omega/c$), for electronic focusing from any n th element center to the desired focus. The vector magnitudes of R_n and R_{nq} are computed as functions of x, y, z . These two distances are the field point to the exact center of an element, and field point to each intra-element sample point respectively. Thus, each of N elements has M sample points which have the phase of the R_n distance but unique distances R_{nq} between a location on an aperture element and a point in the field. The wave number here is complex, where $k_m = \omega/c + j\alpha_m$ and factor α_m is the assumed absorption of the medium at the radian frequency ω and acoustic speed of c . The total element area is $f_a S_{ap}$, where surface fraction f_a is held constant at 40%.

Following the implementation of (1), the intensity was found for the XZ plane by

$$I(x, 0, z) = \frac{1}{2\rho c} |p(x, 0, z)|^2, \quad (2)$$

where ρ and c are the propagation medium density and speed, respectively.

B. Side Lobe Performance for Circular Arrays With Varied Annuli

For spherical therapeutic arrays with a center hole to accommodate a separate imaging array, the near-field beam side lobes produced by such aperture voids, and the reduction in the total available focusing power are important design considerations [10], [27], [28].

It is well known that the thin annulus side lobes show a $J_0(\kappa_1 x)/\kappa_1 x$ response [29], which exhibits a very high first lobe height (−8 dB, large gray square, Fig. 3) and a very slow side lobe decay (gray line, Fig. 3) of approximately $(\kappa_1 x)^n$, where the decay exponent n is −0.5. This is in contrast to the full aperture with $J_1(\kappa_1 x)/\kappa_1 x$ side lobes [30], which exhibits a −17.5 dB first side lobe and a lobe decay exponent n of −1.5. Based on the early work of O’Neil [31] and later contributions [32]–[35], the spherical aperture PSF side lobe response exhibits a very similar analytical form at its focus depth in comparison to the flat circular aperture. Thus, a difference function can be found with reasonable accuracy for the first few side lobes at the focus depth $z = d$ of a spherical aperture, which describes the normalized PSF of the annular spherical aperture as

$$\text{PSF}_{\text{norm}}(x, 0, z) = 20 \log |\pi a^2 J \text{inc}(\kappa_1 x) - \pi b^2 J \text{inc}(\kappa_2 x)|, \quad (3)$$

where the a and b are the radii of the large and small apertures, respectively, and κ_1 and κ_2 are ka/d and kb/d , respectively. Eq. (3) is applicable at the focus depth for either flat or spherical annuli with appropriate assignment of the κ values (i.e., flat: κ_1 and κ_2 are ka/z and kb/z ; spherical: κ_1 and κ_2 are ka/d and kb/d). Using either (3) or RS code computations, analytical metrics of first side lobe height and side lobe decay exponent as functions of annular radius fraction can be found.

C. Focal Main Lobe (FML) Characteristics for Flat and Curved Arrays

One of the most important performance criteria for a high-intensity array is its ability to both produce and focus ultrasound energy to useful target locations within a 3-D volume.

Because building and testing a set of significantly different arrays would be impractical, a general performance study of selected array types is very useful. A simulation study was performed with many common parameters to contrast and compare uniquely different high-intensity circular array designs: highly-curved ($\sim f/0.7$), mildly-curved ($\sim f/1.4$), and flat (f/∞) as a function of element number ranging from 64 to 1024 elements.

The simulation methodology described in Section II-A was used to estimate beams in the XZ-plane, from which several parameters related to the main lobe were quantified, including the location of the natural focal peak, the axial and lateral -3 -dB positioning range of the focus from the natural focal peak, the area of the -3 -dB positioning range around the natural focal peak, and the peak focal pressure and intensity. Validations of the simulations were performed (e.g., intensity at the natural foci and side lobe response) with methods described in Section II-B, and with (A2) in the appendix. The term natural focus peak is used because the true spatial peak of a spherical aperture appears at a depth closer to the aperture than the geometrical focus distance [31].

A generalized performance efficiency (GPE) metric, derived from the product of the -3 -dB positioning area in the XZ-plane and the peak focal intensity divided by the number of elements, was also quantified. This GPE, in units of watts per element, shows the efficiency of a particular array design to deliver acoustic power over a useful range of focal positions. The GPE metric uses both peak intensity and steering range as a product while normalizing for the number of elements (which is a cost of the array).

Although inaccurate for the higher pressures in the simulations, linear, non-scattering conditions were assumed for all focal intensities. An important parameter for this study is the tissue medium absorption characteristic (although signal scattering and tissue inhomogeneity effects are neglected), because without any tissue loss the true focal main lobe (FML) performance of the high-intensity array is overestimated. The tissue absorption is assumed to be 4 N p/m at 1.5 MHz throughout this work; a low value because nearly half of the transmission path in the convex cavity of a spherical array is assumed to be water. The use of different values of absorption is possible using analytical methods shown in the appendix and in [36].

D. Axial Grating Lobes (AGLs) and Relation to Temperature

For brevity, we focus on AGLs rather than LGLs because LGLs can be effectively suppressed with a 2-D randomized array using an effective element pitch (average distance between neighboring elements) of 1λ or less. This effective element pitch can be calculated by the well-known projection slice method [37]. Additionally, these LGLs can be further be reduced by the curvature in the spherical array, as described by [24]. Therefore, LGLs are of

somewhat diminished concern, except for designs with very low element counts [20], [38], [39]. Conversely, AGLs [22], [23], [25] (Fig. 4) have not been well addressed in the literature and can be significant because of the circular symmetry along the central axis of the array, even with the randomization of element positions.

The AGL and its production of undesired temperature are calculated based on the intensity from (2) translated into right plane thermal sources by

$$Q(x \geq 0, 0, z) = 2\alpha_m I(x \geq 0, 0, z). \quad (4)$$

Using only the (symmetric) right-hand plane $Q(x, 0, z)$, this heat function is applied as the driving function in an axisymmetric finite-element analysis (FEA) field implementation of the bioheat equation (Comsol Multiphysics v3.2, Comsol Inc., Burlington, MA). The FEA thermal boundary conditions are forced to be 37°C, which is reasonable if the spacing between boundaries and thermal sources in the tissue is more than 2 times the thermal diffusion length [40] for the entire insonation time. Bioheat FEA modeling was performed with the general parameters in Table II.

E. Proposed Method to Reduce AGLs

One method for grating lobe suppression was suggested by Sumanaweera [41] using a spiral element pattern in the design of sparse 2-D arrays. Establishing the inputs for the construction of a spiral, the result is shown in Fig. 5 and used in Section III-D. The constants used are $N = 256$, $r_{s\max} = 51.235$ mm, $r_{s\text{hole}} = 22.67$ mm, $C1 = 11.15$, $rs2 = 16.3$, $\delta = 0.015$, $rs1_n = (r_{s\text{hole}} + (r_{s\max} - r_{s\text{hole}})(n/N - 1))$ for $n = 0$ to $N - 1$, inserted into

$$\gamma_n = \frac{2\pi}{N} C1 \left(\frac{1 - (1 - rs1_n/rs2)\exp(-rs1_n/rs2)}{1 - (1 - rs_{\max}/rs2)\exp(-rs_{\max}/rs2)} \right)^{1-\delta} N^{1-\delta}, \quad (5)$$

which is then used to find the spherical coordinates of $x_n = r_n \cos(\gamma_n)$ and $y_n = r_n \sin(\gamma_n)$. The radian coordinates, θ_n and φ_n used in defining aperture position, are the x_n and y_n coordinates, respectively, divided by the radius of curvature (ROC) of the spherical aperture. The unmodified spiral (Fig. 5) design suppresses the AGL energy along the center axis. Randomization of this pattern further reduces AGL over a broader region around the axis.

III. Results

A. Side Lobe Performance for Spherical Arrays With Varied Annuli

The well-known side lobe height for three flat circular apertures with different annuli was shown in Fig. 3. Now, with the aid of the simulation model and analytical expression (3), we evaluate the side lobe height near the focus of spherical apertures as a function of annulus radius fraction. This annulus fraction (AF) is defined as $(a - b)/a$, where a is the total aperture radius, and b is the hole radius. The desired analytical metrics are the first side lobe height and side lobe decay exponent as functions of annular radius fraction. Three equal-diameter arrays with varied annuli are considered and the simulation results are shown in Fig. 6. All three apertures have a maximum Cartesian diameter of 100λ .

Based on both simulation results and comparison to (3), compared with its main lobe, the first side lobe peak of any circular aperture at its focus depth is approximately $SL_{\text{dB}}(\text{AF}) \approx$

$-17.5 + 9.5 (1 - AF^{2.2})$, and its side lobe decay exponent is approximately $n(AF) \approx -1.5 + \exp(-3.3AF)$; both are functions of AF only. The side lobes of a circular aperture of maximum radius a , flat or spherical, at a depth equal to its focus will decay as $(\kappa_1 x)^{n(AF)}$. From Fig. 6, a relatively thin annulus of only 30% offers more than double the exponential side lobe decay exponent compared with a thin annulus. Furthermore, although the first side lobe is relatively high, the side lobes of a spherical annulus of only 50% of its radius decay as a function of lateral distance with an exponent nearly 87% of the exponent of a 100% filled array.

B. Beam Focus Positioning of Randomized Element Arrays

Three circular array geometries were compared: highly-curved, mildly-curved, and flat arrays. By evaluating arrays ranging from 64 to 1024 elements, a total of 15 designs were compared; the peak intensity and focus positioning were calculated for each design. To obtain good comparison metrics between designs, the parameters in Table III were held constant.

The simulation code, using (1) and (2), was used to map the intensity in the XZ-plane. The peak focal intensities and focus positioning are presented in Table IV and Fig. 7. In addition to the -3 -dB contour plot for each case, reported values include the natural focal peak intensity on the center (z) axis, and also the near, far, and lateral half-intensity points relative to the natural focal peak on their respective axes. With 1024 elements, the highly spherical array produces more than 17 times the peak intensity compared with the flat array; however, the flat array produces a focus positioning range 6.3 times larger than the highly spherical array. For the designs studied, the GPE is at least 2 times greater for spherical than for flat arrays (Fig. 8). Because of the greater surface area of curved elements, the total transmitted powers are 38.8, 34.3, and 33.1 W for the highly spherical, mildly spherical, and the flat arrays, respectively. Correction for this power bias was not made in favor of a constant fill factor for all 15 designs. The peak positions for the curved arrays are included in Table IV.

C. AGLs and Their Relation to Temperature: Multiple Design Comparison Showing Grating to Focus Lobe Temperature Ratios

A modeling study was performed to examine the temperature ratio between the AGL lobe heating and the intended FML heating for designs with element count and element placement organization as variables. AGLs were examined because they have received little attention in the literature; they can create undesired heating just as LGLs, and due to their axial symmetry, are easily modeled for their temperature production with an FEA approach using (4). First, a general design comparison study is described using the ratio of undesired to desired lobe temperatures. To underscore the potential concern for AGLs, a detailed example with the 256-element highly spherical array with randomized elements is provided in the following section.

An axial grating lobe temperature ratio, defined as $(T_{AGL} - 37)/(T_{FML} - 37)$, can be used to compare the AGL temperatures to that of the FML temperature. Four pairs of spherical arrays (periodic and random; $N = 64, 128, 256, 512$) were examined for their undesired tendency to produce AGL heating. The results are plotted in Fig. 9 as a temperature ratio to normalize this undesired AGL temperature with the desired focal peak temperature. The eight array designs were simulated by taking beam field data for three depths of axial focus: one at the natural focus, one at the distal half-intensity depth on the z -axis beyond the natural focus, and a point between these two (see Fig. 9, inset). Note that for this part of the study, all three depths were within the half-intensity range of focus positioning. In these highly spherical array cases, the array parameters are the same as shown in Table IV, the total array

transmit power was constant at 38.8 W with total insonation time of 100 s, and the assumed tissue perfusion was 11 kg/m³/s.

The periodic 64-element array case produced a very high AGL temperature of approximately 1.4 times that of the FML when the focus was positioned to its half-intensity point distal to the natural focus of this array. With both 1) randomization of elements and 2) greater number of elements, the AGLs were diminished in their ability to produce these undesired temperatures.

D. AGLs and Their Relation to Temperature: Example of AGL Heating With Deep Focus Positioning

Three high-intensity spherical aperture designs, each with 256 randomized elements, are presented to illustrate potential undesired AGL heating when the desired focal position is far from the natural focus position. The three spherical arrays are shown in Fig. 10. As comparative design options, one array is designed without a central hole (Full), and one with a central hole (Annulus), and the last one with a central hole with element positions based on the Sumanaweera spiral pattern in Fig. 5 before the randomization and re-ordering processes were implemented (AnSpiral). The three designs have not been refined with respect to optimized element positions; however, the elementary cases here outline general trends to be explored and improved in future work. All arrays are designed for continuous wave 1.5 MHz frequency of operation with the parameters given in Table V. To maintain an equal total element area for all designs, the fill factor parameter is allowed to be (a high density) 64% for the designs with the central void, which permits a 39.5-mm Cartesian diameter hole for imaging probe guidance. As a consequence of constant element count and size, the cases with an annulus have a smaller average center to center spacing. In each of the three cases, the beam focus is shifted deeper into the tissue medium at about 1.59 times the differential distance Δz which is the natural focus depth to the distal half-intensity depth (see Fig. 9 inset).

The results simulate the effect of focus positioning far from the natural focus in depth by observing the temperature created by the resulting AGL. In these three cases, the focus is set to be at $z = 95$ mm on the center axis which was 10 mm deeper than the half-intensity limit at 85 mm [i.e., natural focus peak (68 mm), plus distance to half-intensity distal contour (17 mm), plus 10 mm] for the 4-mm element diameter case.

The beam field results are summarized in Table VI. To enable improved element spacing with the use of aperture annuli, these design examples use a slightly smaller element diameter (i.e., 4.0 mm) than used Table IV and in the study described in the previous section. Within each case shown in Fig. 10, the top middle panel is the axial pressure gain relative to array surface pressure; the bottom middle panel is the FEA temperature along the center line (z) axis after 100 s of insonation. The two right-side XZ plane plots (with depth 10 to 120 mm and width 30 mm) on the right side of the figure are the FEA temperature field with 38°C, 40°C, 42°C, 46°C, and 50°C contours, and the acoustic pressure field at 40 dB dynamic range in grayscale.

The Full, Annulus, and AnSpiral axial lobe temperature ratios are 0.43, 0.57, and 0.29, respectively. A significant improvement of 49% is seen with the use of the AnSpiral compared with the Annulus design.

IV. Discussion

A. Consequences of the Use of an Annulus Aperture

The side lobes produced by a central hole in the aperture do not create substantial problems in the high-intensity designs evaluated. In using an annulus, however, the total available power will vary with the active aperture area, and the grating lobe height with element placement in the aperture (Fig. 10). Although not described here for the sake of brevity, simulations have shown that the central hole does not negatively affect positioning of the FML and produces an equivalent intensity within a few percent in the same volumetric steering range as that of a complete aperture with the same element count and total element area.

B. General Array Design and Focus Positioning

Many array design realizations are possible; the plots in Fig. 7 are presented to reveal some of the critical performance aspects. The number of array elements, the size of elements, and frequency chosen are all within the range of typical values applied in high-intensity ultrasound.

As expected, the highly spherical array design provides the highest acoustic intensities, but with a reduced half-intensity boundary region. In all apertures, more elements allow better focus position range with improved focusing in the near field; this is due to improved individual element directivity because the elements are smaller. For a flat array with less than 1024 elements, deeper focusing produces signal attenuation and wider foci, which produces poor focal intensities, rendering these low-count arrays inadequate for high-intensity applications. Extrapolations from Fig. 8 suggest a GPE convergence of curved and flat arrays at array element counts in the range of 10 000, which indicates an element diameter of 4.385 times ($\sqrt{256/10,000}$) or 0.70 mm (0.68λ) while maintaining a constant total element area. This small (and admittedly impractical) element would produce a very wide directivity, which would essentially overcome effects of aperture shape.

In summary, the ability of an array to focus in a large volumetric region at a given frequency will depend greatly on the array element sizes as well as aperture shape and size. Further, both spherical array performances shown in Fig. 8 provide many times higher GPE than the flat array at the very reasonable array size of 256 elements. Although the spherical array GPE decreases with increasing element count, the GPE for the flat array increases with increasing element count.

C. Temperature Ratio Metric for AGLs

Undesired lobes produced by high-intensity arrays can be neglected if the undesired lobes produce a low enough intensity with respect to the FML. In the literature, acceptable side lobe levels have been variously given as 10% intensity (-10 dB) [21], 20% intensity (-7 dB) [42], or 10 to 16% intensity (-10 to -8 dB) [25] in homogeneous media. The temperature ratio in this work was shown to be 0.12 at the distal half-intensity axial depth for the 256-randomized-element case (Fig. 9); the corresponding intensity ratio for this case is 0.024 (-16 dB) which is well within an acceptable range according to past studies.

Fig. 9 indicates that randomization reduces the heating effect of AGLs; however, as demonstrated in Fig. 10, the effects of AGL contribution to unwanted heating can be an issue if the focal target is too far from the natural focus. The temperature ratios at the distal half-intensity depths for the randomized arrays (large gray circles) in Fig. 9 suggests that a randomized array with more than 256 elements is beyond the point of diminishing return for an array which will be used only within its half-intensity focus positioning limits.

Additionally, the negative element count trend for spherical arrays in Fig. 8 and the diminishing returns above 256 elements in Fig. 9 suggest a design of approximately 256 elements is optimal with regard to performance versus cost tradeoffs.

Table VI and Fig. 10 show the results of three similar spherical array designs with focal positioning pushed to a depth well beyond the -3 -dB focus intensity contour. At this depth, the AGL/FML intensity ratio for the Full aperture case is 66/268, or -6 dB; thus, this indicates a maximum permitted target depth for this design. The AnSpiral design improves the AGL -FML temperature ratio metric by 49%; however, optimized element positioning design may provide further improvement. One reason for the spiral pattern success is the asymmetry in this pattern. This reduction of symmetry relative to the center axis reduces AGL formation. Alternatively, steering beams (only) off the center axis reduces AGLs, but this method alone may not be a comprehensive and failsafe remedy.

AGL heating, and all spherical array grating lobes for that matter, are a particular concern because of the potential for a) a lack of AGL movement with steering or changes in the focal position, and b) the diffuse nature of random array lobes which may create higher sustained temperatures for a given peak intensity because of less-effective thermal diffusion of heat (i.e., less heat loss) from the grating lobe area. In general, the peak intensity of an undesired lobe should not be the only criterion for concern, but the acoustic intensity in the region around the peak is important to consider as well. Thorough simulation testing for GL behavior as a result of focus positioning within, and especially beyond, the FML half-intensity area is appropriate in a complete verification of any high-intensity array design.

D. Practical Issues and Study Limitations

There are many areas in this work where extensive results could not be presented. Specific areas in high-intensity array design which could not be covered, but are of continued interest are: 1) overall design optimization; 2) optimization in the use of the Sumanaweera spiral element pattern; 3) AGL formation as a result of the use of an annulus; and 4) LGLs and their behavior contrast between flat and spherical arrays.

The following discussion presents the rationales for some of the parameter selections shown in Table III. An array surface intensity of 1 W/cm^2 is generally accepted as an acoustic output level which requires no special cooling for a typical piezoceramic array; in contrast, a level of 4 W/cm^2 is an intensity which does require special cooling, especially for arrays based on piezoceramics. The 0.2 mm minimum element spacing was also applied to the exterior border of the array, and the interior border if a central hole is used.

The practical diameter for an array element is an important consideration. Because array inter-element decoupling is highly desired, the PZT-based designs have used 1–3 composites to diminish mechanical cross-coupling [43], [44]. As well, some concern for undesired intra-element transducer coupled modes can exist [45] for designs in which the diameter of the element is less than 4 times the thickness, which is addressed in part by using 1–3 piezoceramic composite. Element randomization is desirable; however, extreme randomness may create thermal hot spots in the array where elements (which typically exhibit some level of self-heating) are clustered together.

As a practical consideration, flat arrays do have an advantage over spherical arrays with regard to grating lobes. Flat array grating lobes move in accordance with focus positioning, which means the heat from these undesirable lobes can be spread over a wider region and reduce the peak temperature increase compared with the spherical grating lobes, which tend to collect near the geometric focus.

Finally, for reasons of brevity and for our intention to address non-ablative hyperthermia applications, the effects of non-linear acoustic behavior were omitted in this work. For the transmission intensities considered, the major non-linear region in the beams would occur at the focus and would (for most cases) not greatly affect the lobe structure. However, a thorough analysis should include significant heating effects at any high-intensity focus lobe [46], [47].

V. Conclusions

Simulations using a discrete implementation of the Rayleigh-Sommerfeld integral were performed to characterize and contrast the performance of a set of high-intensity ultrasound arrays.

For the three array shapes studied, the product of intensity and focus positioning area normalized by element count (i.e., the GPE metric) shows spherically shaped arrays produce a PDE at least twice that of flat arrays up to element counts of at least 1024. An assessment of the side lobe characteristics of spherical annuli reveals that their performance can be described with two simple relationships, which depend only on the annulus radius fraction. Good side lobe performance is possible with an aperture annulus of only 30% of the total radius, and side lobes for a 50% annulus decay similarly to a 100% filled array. Axial grating lobes (AGLs) can be a significant issue when an array combines low element count and focal lobe positioning at sites beyond the half-power steering range. AGLs can be diminished through the use of the Sumanaweera spiral. The ratio of the temperature rise of the AGLs and the FML can be a useful metric to establish grating lobe significance for a given array element count.

In summary, a high-intensity spherical array design with an element count of 256 may offer a good tradeoff between the benefit of its intensity-positioning metric against assembly cost from element count and the hazards of grating lobes, while limiting focal targets to the array's half-intensity focus positioning range. There are some reports of successful use with spherical arrays of this size [27], [28]. For applications for mild hyperthermia where lower intensities and larger regions of heat deposition are desired, it may be appropriate to select a less curved spherical array shape.

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Biographies

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Early in his career, he first developed electronics for the GE Medical Systems Division, then assisted two start-up companies in medical electronics, and in 1984 designed a motion control system for ultrasound scanning at SRI International. In 1985, he joined the founding technical group at EndoSonics Corporation as a senior electronic design engineer. As a key contributor at EndoSonics in solid state intravascular ultrasound (IVUS) technology he was responsible for all catheter electronics and analog signal processing. In 1990, Mr. Stephens led the technical effort in the creation of the world's first commercial 3.5-F solid-state ultrasound imaging catheter and was awarded the first EndoSonics Fellowship Award in that year. In 1994, he was a co-inventor for the means and method for IVUS color flow imaging, which allows a real-time intraluminal visualization of blood location and velocity. In 1995, he was promoted to vice president of strategic technology responsible for new designs of IVUS solid-state technology.

Mr. Stephens is currently in the department of Biomedical Engineering at the University of California, Davis. He is now working on methods of ultrasound-based targeted imaging and liposome-mediated drug delivery, and providing engineering design and management for a multisite research partnership developing novel intracardiac imaging catheters for use in electrophysiology procedures. His other research interests include piezoelectric transducer applications, novel methods for non-invasive applications for cancer drug therapies, and ASIC circuit designs for invasive medical imaging. He holds seven patents in the field of medical ultrasound.

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Katherine W. Ferrara (S'82–M'87–SM'99–F'10) received the B.S. and M.S. degrees in electrical engineering in 1982 and 1983, respectively, from the California State University, Sacramento, and the Ph.D. degree in electrical engineering and computer science in 1989 from the University of California, Davis. Prior to receiving her Ph.D., Dr. Ferrara was a project engineer for General Electric Medical Systems, involved in the development of early magnetic resonance imaging and ultrasound systems. Following an appointment as an Associate Professor in the Department of Biomedical Engineering at the University of Virginia, Charlottesville, Dr. Ferrara served as the founding chair of the Department of Biomedical Engineering at UC Davis, now a department of 24 faculty members. She is currently a Professor of Biomedical Engineering at UC Davis with research interests in imaging and drug delivery. She is a fellow of the IEEE, the American Association for the Advancement of Science, the Biomedical Engineering Society, the Acoustical Society of America, and the American Institute of Medical and Biological Engineering.

Appendix

Intensity Gain Relations for a Spherical Aperture

Using the theoretical development of O'Neil [31], [48] and the geometry in Fig. 1, we express several useful approximations for the ratio of geometric focus intensity to aperture

surface intensity gain, or G_I . Working in scalar magnitudes and with the assumptions of a lossless medium, linear acoustic propagation, and focal summation obeying superposition, the normalized intensity gain at the geometrical focus of a spherical array with a 100% active surface is approximated within a few percent error to be

$$G_I \cong \frac{S_{ap}}{S_{foc}} \cong \left(\frac{S_{ap}}{\lambda d} \right)^2 = (kh)^2, \quad (A1)$$

where the aperture surface area, effective geometrical focus area, and radius of curvature are S_{ap} , S_{foc} , and d , respectively. The aperture depth, h , is also $d(1 - \cos(\alpha))$, where α is the spherical aperture elevation angle (Fig. 1) which, to maintain the accuracy of these relationships in this appendix, should be limited to moderate angles, e.g., 0 to 45°. The transmitted signal is propagated through a medium with a speed of c and a wavelength of λ , and characterized by its wavenumber k of $2\pi/\lambda$.

Removing the continuous aperture and lossless medium requirements, the array intensity gain, G_{Ia} , at the geometric focus depth d of a reasonably uniform and well-populated spherical array of N elements, each element of circular diameter D , in a medium with a uniform absorption characteristic α_m in units of nepers per meter is estimated to be

$$G_{Ia} \cong \left(\frac{1}{\lambda d} \left(N\pi \frac{D^2}{4} \right) \right)^2 e^{-2\alpha_m d}, \quad (A2)$$

which may also be expressed as a function of kd with a frequency-normalized absorption coefficient, α_m' in units of nepers per meter-hertz, as

$$G_{Ia}(kd) \cong (kd)^2 (f_a(1 - \cos\alpha))^2 e^{-2\alpha_m' (c/2\pi)kd}, \quad (A3)$$

where f_a is the fraction of active aperture surface to total aperture area. The maximum intensity gain produced by (A3) at the geometric focus for any uniform spherical aperture with $h > 0$ can be found as a function of absorption, frequency, and geometric focus depth d . The condition for this maximum possible intensity at the geometric focus exists where the relation

$$kd = (\alpha_m' (c/2\pi))^{-1} \quad (A4)$$

is satisfied. Assuming 1) isotropic and homogeneous conditions; 2) a tissue medium acoustic speed of 1540 m/s; and 3) a frequency normalized absorption α_m' is 2.76E-6 (based on assumptions of 0.24 dB/cm/MHz, and with frequency exponent omitted for simplicity), the peak intensity gain at the geometric focus occurs where kd is equivalent to 1477. Thus, for 1.5 MHz operation, the maximum practical d (i.e., ROC) is estimated to be 241 mm. The ROC and frequency are linearly inverse factors in the determination of peak intensity performance at the geometric focus of a spherical aperture.

Fig. 1.

Graphical representations of (a) full and (b) spherical annulus apertures. The Cartesian total diameter is fixed at 100λ throughout this work; the characteristic fixed angle α subtends the z-axis and the ray from the radius of curvature (ROC) depth d to the edge of the aperture. The total aperture surface is S_{ap} , the spherical depth of the structure is h , and the points on the aperture surface are located by spherical coordinate angles θ and ϕ .

Fig. 2.

The stages used in the creation of a 256-random-element spherical array for beam focusing simulation. (a) An initial uniform and periodic assembly of element centers with nine element rings and their element count for each, (b) after initial randomization, (c) after application of element spacing rules, and (d) after population with individual elements.

Fig. 3.

Example comparisons of analytical (lines) and RS code (symbols) results of the PSF responses for three large-diameter (100λ is used here as the standard aperture) flat circular apertures in the far field ($z = 5 \text{ m} \sim 2a^2/\lambda$) with continuous wave transmission at 1.5 MHz. First side lobe height of the 100% (black circle), 63% (open circle), and 8% (open square) annular radius fraction apertures together with the side lobe decay (gray line) of the thin annulus.

Fig. 4.

A qualitative demonstration of both LGL and AGL distribution and relative intensities predicted for an $\alpha = 45^\circ$, 64-element periodic spherical array with a Cartesian diameter of 100λ and a frequency of 1.5 MHz with an FML location at 4.9 mm off the center axis at a depth of 70 mm (the FML position in this example is a location on the half-power contour for this design). The normalized display depth scale is limited to 12 dB.

Fig. 5. The spiral distribution for the array element centers in a 256-element aperture design. The pattern is produced directly from (5) with the parameters given in the text; the axes are shown in millimeters.

Fig. 6.

The RS model computation results (symbols) and empirical function estimates (lines) of the side lobe behavior for annulus apertures of various AF. The PSF first side lobe height (solid line) and side lobe decay exponent (dashed line) are shown as a function of a varied annulus size for a flat circular aperture focused in the far field at $z = 5$ m (boxes), a flat circular aperture focused in the near field at $z = 72.6$ mm (triangles), and a spherical aperture at focus distance $z = d = 72.6$ mm (circles). The left extreme of the plot shows the response for the thin annulus aperture, and the right extreme, the full aperture.

Fig. 7.

The estimated focal main lobe (FML) half-intensity XZ-plane contours for the highly spherical (top left), mildly spherical (top right), and flat (bottom) arrays using randomized element positions and the fixed design parameters in Table IV.

Fig. 8.

The generalized performance efficiency (GPE) metric defined as power distribution in watts per element. The GPE shown is the computed peak intensity times the half-intensity focus position area normalized by number of elements; it is plotted for three general high-intensity array shapes. The GPE metric is derived directly from the results in Table IV.

Fig. 9.

The summary of a study of the AGL/FML temperature ratios produced by lobes from both (P) periodic- and (R) random-element spherical arrays, with arrays in each group of 64, 128, 256, and 512 total elements. The maximum axial depth focus point shown in each array design is the distal half-intensity depth. Black and gray lines connect the axial lobe responses for each array as a function of focal depth. The inset example (256-element case) shows the three FML depths evaluated for each array geometry in the XZ-plane, all within the half-intensity focus area. The $1.59(\Delta z)$ distance is utilized for the axial lobe temperature results presented in Section III-D.

Fig. 10.

The simulation results for AGL heating (see dashed enclosed regions in both temperature plots within each panel) for a target focus of 1.59 times the distance from the natural focus to the distal half-intensity focus contour (see Fig. 9 inset) for three spherical aperture cases: (a) Full (without center hole), (b) Annulus (with hole and mild randomization), and (c) AnSpiral (with hole and spiral-seed randomization).

TABLE I

Summary of Array Formats and Parameters Studied.

Parameter studied	High-intensity array format		
	Flat vs. curved [†]	Periodic vs. random [#]	Center void vs. no void
Element count	•	•	NS*
Main lobe	•	•	•
First side-lobe	NS	NS	•
Axial grating lobes	NE	•	• [#]
Tissue heating	NE	•	• [@]

• studied

NS = not significant in this work.

NS* = element size is important, element count is not.

NE = not examined in this work.

[†] For random arrays only.

[#] For curved arrays only.

[@] Also studied the effect of a spiral pattern.

TABLE II

Common Bioheat Simulation Parameters for Axial Grating Lobe Heating.

Insonation time (s)	Tissue perfusion (kg/m ³ /s)	Thermal conductivity (W/m·K)	Tissue density (kg/m ³)	Tissue acoustic speed (m/s)	Tissue specific heat (J/kg·K)	Tissue baseline temperature (°C)
100	11	0.6	1060	1540	4100	37

TABLE III

Simulation Parameter Constants for the Beam Focus Positioning Study.

Frequency (MHz)	Cartesian diameter (mm)	Transmission intensity (W/cm ²)	Transmission pressure (kPa)	Element area fraction of total area (%)	Tissue medium absorption (Np/m)
1.5	102.7 (100 λ)	1.0	173.25	40	4

TABLE IV
Natural Peak Intensities and Focus Positioning Ranges for Convex and Flat Randomized Arrays With Varied Element Count.

α (°)	ROC (mm)	Element count	Element diameter (mm)	FML peak (mm)	-3-dB beam distance from FML peak (mm)			FML position area (cm ²)	FML peak intensity (W/cm ²)	FML peak pressure (MPa)
					(near)	(far)	(lat.)			
Case A: Highly spherical array										
45	72.597	64	8.72	71.3	7.3	8.7	5	1.20	1505	6.72
		128	6.20	69.7	9.4	12.4	6.9	2.08	1569	6.86
		256	4.385	67.4	11.5	16.8	9.2	3.70	1643	7.02
		512	3.099	64.5	13.5	20.8	11.5	6.35	1720	7.18
		1024	2.196	58.9	15.9	26.1	15.5	10.5	1911	7.57
Case B: Mildly spherical array										
21.5	140.1	64	8.199	126.1	21.3	32.1	8.6	7.9	212	2.52
		128	5.827	116.9	23.9	39.6	11.2	12.2	246	2.71
		256	4.124	105.3	25.6	44.7	14.1	16.7	296	2.98
		512	2.915	89.7	25.3	47.8	17.8	21.9	392	3.43
		1024	2.065	73.3	25.1	47	22.2	26.0	565	4.11
Case C: Flat array										
0	infinite	64	8.051	320	83	140	28.6	103	0.99	0.17
		128	5.724	244	66.8	121	28.5	93.9	3.98	0.34
		256	4.052	184	54.1	102	28.6	72.7	13.3	0.63
		512	2.863	132	41.3	84.7	30.0	62.3	42.9	1.13
		1024	2.028	94.5	39.5	73.5	34.2	65.9	108	1.80

$A = 102.7$ mm, 40% fill factor, $f = 1.5$ MHz, $I_{TX} = 1$ W/cm², $a_{TM} = 4$ Np/m.

Common Spherical Array Parameters for AGL Heating Example.

TABLE V

Cartesian diameter (mm)	$\alpha(^{\circ})$	Radius of curvature (mm)	Transmit intensity (W/cm ²)	Element count	Element diameter (mm)	Element locations random?	Minimum element spacing (mm)	Natural FML depth (mm)
102.7	45	72.597	1.0	256	4.0	yes	0.2	68

TABLE VI
Field Results for Axial Grating Lobe (AGL) Heating With Deep Axial Focus Positioning.

Array type aperture	Array fill factor (%)	At natural focus		At desired focus (FML)		At AGL	
		Focal intensity at 67 mm (W/cm ²)	Focal pressure at 67 mm (MPa)	Focal intensity at 95 mm (W/cm ²)	Focal pressure at 95 mm (MPa)	Lobe intensity at 65 mm (W/cm ²)	Temp. peak (°C)
Full	33.3	1141	5.85	268	2.84	66	43.5
Annulus	40	1135	5.84	254	2.76	113	44.8
AnSpiral	40	1146	5.86	275	2.87	25	42.4