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Fourier analysis in Magnetic Induction Tomography: Mapping of anisotropic, inhomogeneous resistivity

Steffan Puwal, PhD^{*} and Bradley J Roth, PhD

Abstract

Magnetic Induction Tomography is an electromagnetic-based technique for mapping the passive electromagnetic properties of conductors and has the potential for applications in biomedical imaging. In a previous analysis we approached the inverse problem of determining isotropic resistivity with a Fourier-based analysis. Here, we extend that analysis to anisotropic media. The proposed Fourier-based solution method, when properly filtered, robustly handles noise to accurately map the inhomogeneous terms of the resistivity tensor. We observe a random variation in the measure of accuracy (mean deviation) that is resolved with independent spatial frequencies in the x - and y -directions in the applied field. Further, the formation of improper images we noted in our previous analysis is addressed through this use of independent spatial frequencies and through the use of additional applied fields. We conclude with a discussion of computation time for the large system of linear equations this method requires and propose methods for limiting memory usage.

Introduction

Magnetic Induction Tomography (MIT) is an experimental technique used for mapping the passive electromagnetic (EM) properties of a medium, such as resistivity. The technique involves applying a known magnetic field to induce an eddy current, measuring the resulting magnetic field, and solving the inverse problem to determine the resistivity of the material [1,2,3,4]. A schematic diagram of an MIT experiment is shown in Fig. 1. MIT was developed recently and new image processing strategies are still forthcoming [1,2]. The central difficulty with image processing in EM-based tomographic techniques like MIT is the ill posed nature of the problem [2]. As such, proposed imaging strategies invoke regularization techniques or rely on *a priori* assumptions [2]. We propose a Fourier-based technique that involves series representations of the applied and measured fields and the resistivity. In a previous analysis [5] we considered an isotropic medium (with a resistance consisting of several Gaussian distributions [6]) and found that we were able to successfully reconstruct the resistivity. Under certain conditions we found that the resistivity was improperly constructed.

Here, we consider an anisotropic medium as a thin sheet in the xy -plane, the thickness of which is less than the skin depth of the material so that the induced eddy current density is independent of depth. We find, when properly filtered, the inverse solution accurately reproduces the components of the resistivity tensor, determines the direction of lowest resistivity, effectively recovers isotropic resistivity with the generalized equation. Others have recently proposed methods for determining anisotropic resistivity using a different method [7]. The main benefit of a Fourier-based approach is that the problem makes use of well developed and widely available digital signal processing methods that are

^{*}Contact: Department of Physics, Oakland University, Rochester, Michigan USA 48309, 1 (248) 370-3413, smpuwal2@oakland.edu.

computationally fast and are familiar to most scientists. In our analysis filtering plays a similar role as regularization does in other methods for solving the inverse problem.

One application of our method is for measuring the fiber geometry in nerve and muscle tissue, such as a sheet of cardiac tissue. The heart is known to have a complex fiber geometry, which impacts how the heart responds to an electric shock. As a test case, we use fiber geometry that has been used for simulations of cardiac electrophysiology [8]. Our method of mapping the resistivity tensor provides similar information as MRI diffusion tensor imaging (DTI) [9,10]. Just as in DTI, by determining all components of the resistivity tensor we can map fiber direction [11]. Although not addressed in this paper, the trace of the resistivity tensor also provides a measure of overall tissue resistivity that is independent of fiber direction.

Description

Consider a thin planar sheet at $z = 0$ of thickness δ with an anisotropic, inhomogeneous

resistivity distribution $\tilde{\rho} = \begin{pmatrix} \rho_{xx}(x, y) & \rho_{xy}(x, y) \\ \rho_{xy}(x, y) & \rho_{yy}(x, y) \end{pmatrix}$. Assuming the sheet is periodic with period L in both directions, the components of the resistivity tensor can be expanded as two-dimensional Fourier series [12]

$$\rho_{xx} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} p_{nm} e^{-i\frac{2\pi}{L}(nx+my)}, \quad (1a)$$

$$\rho_{xy} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} q_{nm} e^{-i\frac{2\pi}{L}(nx+my)}, \quad (1b)$$

$$\rho_{yy} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} r_{nm} e^{-i\frac{2\pi}{L}(nx+my)}. \quad (1c)$$

For our model of an applied field a set of field-producing coils are in a plane above the conductor at $z = z_{\text{applied}}$ [1,13]. As the plane of these applied field coils is parallel to the conducting plane and the normal (i.e. z -) component will induce the eddy current, we will concern ourselves only with the z -component of the applied magnetic field. The applied magnetic field $\mathbf{B}(\mathbf{x}, t)$ has an associated vector potential [14,15] $\mathbf{A}(\mathbf{x}, t)$ such that $\mathbf{B} = \nabla \times \mathbf{A}$. Because the applied coils are planar the z -component of $\mathbf{A}$ is zero everywhere. Below the sheet in the plane at $z = z_{\text{measured}}$ a set of measurement coils detect the resulting magnetic field [1,13]. Again, as the conducting plane is parallel to the plane of the measurement coils we shall concern ourselves only with the z -component of the measured magnetic field. (We use the term “measured field” to refer only to the magnetic field caused by the eddy current. In practice the field measured by the detection coil will be a combination of the applied field and the small eddy current field, but we assume that these two can be distinguished by examining the in-phase and out-of-phase components of the detected signal.) The measured field $\mathbf{b}(\mathbf{x}, t)$ (due only to the eddy current distribution) has an associated vector potential $\mathbf{a}(\mathbf{x}, t)$. Because the eddy current distribution is planar, the z -component of $\mathbf{a}$ is zero. For both vector potentials we shall use the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$ and $\nabla \cdot \mathbf{a} = 0$). With no currents in the region $|z| > 0$ the magnetic fields obey Laplace’s equation. Then we may write the z -components of the magnetic fields as Fourier series

$$b_z(x, y, z, t) = \sum_{N=-\infty}^{\infty} \sum_{M=-\infty}^{\infty} b_{NM} e^{\frac{2\pi}{L} \sqrt{N^2+M^2} z} e^{-i \frac{2\pi}{L} (Nx+My)}, \quad (2a)$$

$$B_z(x, y, z, t) = \sum_{N'=-\infty}^{\infty} \sum_{M'=-\infty}^{\infty} B_{N'M'} e^{\frac{2\pi}{L} \sqrt{N'^2+M'^2} z} e^{-i \frac{2\pi}{L} (N'x+M'y)}, \quad (2b)$$

where Eq. 2a is valid in the region between the conductor and the measurement coils and Eq. 2b is valid in the region between the conductor and the applied coils; the Fourier coefficients of the magnetic fields are the coefficients in the plane of the conductor. The eddy current distribution $\mathbf{j}(\mathbf{x}, t)$ induced in the sheet [14,15] (and obeying the continuity equation [14,15] $\nabla \cdot \mathbf{j} = 0$) can be represented with a stream function ξ such that

$\mathbf{j} = \frac{\partial \xi}{\partial y} \hat{\mathbf{i}} - \frac{\partial \xi}{\partial x} \hat{\mathbf{j}}$; ξ can be expanded as a Fourier series that relates the eddy current its magnetic field [16]

$$\xi(x, y, t) = \frac{-L}{\pi \mu_0 \delta} \sum_{n'=-\infty}^{\infty} \sum_{m'=-\infty}^{\infty} \frac{b_{n'm'}}{\sqrt{n'^2+m'^2}} e^{-i \frac{2\pi}{L} (n'x+m'y)}. \quad (3)$$

We consider the tissue to be Ohmic; the eddy current induced in the sheet may be related to the scalar electrical potential ϕ and magnetic vector potential $\mathbf{A}$ by $\rho \mathbf{j} = \mathbf{E} = -(\nabla \phi + \dot{\mathbf{A}})$, where $\dot{\mathbf{A}}$ is the time derivative of the vector potential [14,15]. If we take the curl of this equation, write $\mathbf{j}$ in terms of its stream function, and recognize $\dot{\mathbf{B}} = \nabla \times \dot{\mathbf{A}}$, then with ϕ eliminated [12] we obtain the relation [5]

$$\frac{\partial}{\partial x} \left(\rho_{xy} \frac{\partial \xi}{\partial y} - \rho_{yy} \frac{\partial \xi}{\partial x} \right) - \frac{\partial}{\partial y} \left(\rho_{xx} \frac{\partial \xi}{\partial y} - \rho_{xy} \frac{\partial \xi}{\partial x} \right) = -\dot{B}_z. \quad (4)$$

If we replace the terms of Eq. 4 by the Fourier series representations of Eqs. 1–3, integrate over x and y , and exploit orthogonality of the complex exponentials, then we obtain the linear system of equations 6

$$\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{b_{N-n, M-m} e^{-\frac{2\pi}{L} \sqrt{(N-n)^2+(M-m)^2} z_{meas}}}{\sqrt{(N-n)^2+(M-m)^2}} ((2NM-nM-Nm)q_{nm} - N(N-n)r_{nm} - M(M-m)p_{nm}) = \frac{\mu_0 L \delta}{4\pi} \dot{B}_{NM} e^{\frac{2\pi}{L} \sqrt{N^2+M^2} z_{appl}}, \quad (5)$$

which relates the Fourier coefficients of the components of the resistivity tensor to the Fourier coefficients of the z -components of the applied and eddy current magnetic fields. (This derivation is similar to our previous analysis of an isotropic, resistive medium. See [5] for a fuller derivation.) In practice, the relevant summations are finite results of a Discrete Fourier Transform (DFT) [12] and Eq. 5 is solved with least squares Gaussian elimination [17,18].

In MIT, noise may appear from the thermal motion of free electrons in the measuring apparatus [19] or in the conducting plane itself [16]. The measured field component due to the eddy currents is typically many orders of magnitude smaller than the applied field [5,13,20], so coping with even a small amount of noise can be a big problem. We model the noise as a Gaussian distributed random variable added to each spatial point of the measured

magnetic field. The random variable has a mean of zero and a standard deviation of $Q \times (\max(b_z) - \min(b_z))$, where $\max(b_z)$ and $\min(b_z)$ are the maximum and minimum values, respectively, of the measured magnetic field without noise and Q is the noise-to-signal ratio. Non-Gaussian noise, which can be observed in MIT systems [21], is not considered here.

Filtering of the signal is usually necessary to obtain a high fidelity image. With exponential growth (or decay) of the magnetic fields with increasing distance of the coils from the plane at $z = 0$, any high frequency noise will be amplified. We employ a Tukey filter to remove high frequency components of the noise (as well as high frequency components of the applied and measured magnetic fields) [22]

$$S' = S \times \begin{cases} \cos^2\left(\frac{L}{4F}k\right) & k < \frac{2\pi F}{L} \\ 0 & k > \frac{2\pi F}{L}, \end{cases} \quad (6)$$

where F is a filter threshold, k is the spatial frequency ($k = \Delta k \sqrt{p^2 + q^2}$ with p and q the relevant indices of summation), S is the signal in k -space before filtering, and S' is the signal in k -space after filtering. When filtering high frequency noise, high frequency information about the resistivity will also be lost. Consequently, filtering will produce best results with low spatial frequency magnetic fields and resistivities. In our numerical experiment we assume the applied field is completely known from the geometry of the applied coils. However, we still find it is necessary to filter both the applied and measured magnetic fields to get the most accurate solution for resistivity. This seems to be due, in a general sense, to a correlation between frequency components of the applied and measured magnetic fields. For example, if a low frequency magnetic field is applied to a medium with low frequency resistivity, one does not expect a high frequency magnetic field.

The experimentalist facing the problem of determining the resistivity by MIT would apply a time-varying magnetic field using a set of coils [1,2,13] (current loops). As a theoretical calculation, we are free to choose any applied field distribution. Note in Eq. 5 that there is at least one equation in the linear system with zero on the left side (where $N=M=0$). When formulated as a matrix-vector system for solution of the resistivity in frequency space, Eq. 5 results in a singular matrix [12]. Our approach is to repeat the experiment with multiple applied magnetic fields and use Gaussian elimination with least squares to solve the linear system [17,18]. The first derivative with respect to time of the applied magnetic fields we use are

$$\dot{B}_{z,u} = \dot{B}_0 \sin\left(v_u \pi \frac{x}{L} + \frac{\pi}{2} \phi_u\right) \sin\left(w_u \pi \frac{y}{L} + \frac{\pi}{2} \psi_u\right) \quad (7)$$

where $\dot{B}_{z,u}$ refers to the time derivative of the z -component of the u -th applied field ($u = \{1, 2, \dots, 10\}$), $\dot{B}_0 = 1 \text{ T}/\mu\text{s}$ and the fields are evaluated at $t=0$, v and w are the frequencies of the u -th applied field ($v = \{2, 4, 4, 2, 2, 6, 6, 2, 4, 6\}$ and $w = \{4, 2, 2, 4, 6, 2, 2, 6, 6, 4\}$), and ϕ and ψ express the phase shifts ($\phi = \{0, 0, 1, 1, 0, 0, 1, 1, 0, 0\}$ and $\psi = \{0, 1, 0, 1, 0, 1, 0, 1, 0, 1\}$) such that we have sines and cosines in Eq. 7. In our previous analysis of an isotropic medium [5] we used spatial frequencies having x - and y -components equal (i.e. $v_u = w_u$). This was predominantly responsible for the formation of the improper images. (We also observed that high value exponentials of Eqs. 2 evaluated outside of the range of floating-point numbers MATLAB was able to handle.) In the case of an anisotropic medium, instead of improper images this manifested itself as an instability in the solution (altering the values of the diagonal components of the resistivity tensor while leaving the trace unchanged) which, in turn,

resulted in a large variation in our measure of accuracy of the fiber angle. We avoid this problem here by requiring $v_u \neq w_u$.

Our experiments are repeated for different ratios of noise to signal Q , different levels of filtration F and different values of $z_{measured}$. In each case the ten fields given in Eq. 7 are applied. In every experiment, we use the same resistivity distribution shown in Fig. 2 and given by [8]

$$\tilde{\rho} = \frac{1}{g_L g_T} \begin{pmatrix} g_L \sin^2 \theta + g_T \cos^2 \theta & (g_T - g_L) \sin \theta \cos \theta \\ (g_T - g_L) \sin \theta \cos \theta & g_L \cos^2 \theta + g_T \sin^2 \theta \end{pmatrix} \quad (8a)$$

$$\theta = \frac{\pi}{2} \left(\cos^2 \left(\pi \frac{x}{L} \right) \cos^2 \left(\pi \frac{y}{L} \right) \right) \quad (8b)$$

where g_L and g_T are the longitudinal and transverse conductivities, respectively, and θ is the fiber angle. Our conducting plane is assumed to be periodic with $L=10$ cm, thickness $\delta=0.1$ cm, longitudinal conductivity $g_L = 1.0 \text{ k}\Omega^{-1} \text{ cm}^{-1}$, transverse conductivity $g_T = 0.25 \text{ k}\Omega^{-1} \text{ cm}^{-1}$, and the fiber angle map θ is shown in Fig. 2.

Our steps for simulating the MIT experiment and solving the inverse problem are as follows: given the applied magnetic field and resistivity distribution, we numerically solve the forward problem (Eq. 4) for the measured magnetic field using a relaxation method [14], repeating this step for each of the applied fields of Eq. 7; we add noise to the measured fields that result; given these applied magnetic fields and measured magnetic fields with noise, we solve the inverse problem (Eq. 5) to determine the components of the resistivity tensor; and we determine the principal axis (eigenvector) associated with the smallest eigenvalue of the resistivity tensor (the fiber axis) to determine the fiber angle. We characterize the fidelity of the calculated fiber angle to the true fiber angle using the Mean Deviation [23]

$$MD = \sqrt{\frac{\int \int (\theta_{true} - \theta_{calculated})^2 dx dy}{\int \int (\theta_{true})^2 dx dy}}. \quad (9)$$

Results

Using a set of applied magnetic fields given in Eq. 7 and shown in Fig. 3A we induce eddy currents in the simulated conducting plane, with resistivity given by Eq. 8A and shown in Fig. 2. As a result, we obtain a set of measured magnetic fields (with $\mathbf{b}_1$ shown in Fig. 3B). The Gaussian-distributed noise (with noise-to-signal ratio $Q=0.10$) that we add to $\mathbf{b}_1$ is shown in Fig. 3C; similar Gaussian distributed noise is also added to the other nine measured fields. Our applied fields are known in the conducting plane ($z_{applied} = 0$) and the measurement coils are quite close to the conducting plane (here $z_{measured} = -1$ cm). Fourier coefficients for the magnetic fields are obtained using the FFT routines provided by MATLAB and the inverse problem of Eq. 5 is solved with summations over -16 to $+15$. The Tukey filter applied here has a filtration level $F = 6$ (in Eq. 6). The components of the resistivity tensor obtained are shown in Figs. 3D, 3E and 3F and the fiber angle map obtained using this measured resistivity is shown in Fig. 3G. The fidelity of the measured fiber angle map to the true fiber angle seems quite reasonable for this simulated experiment;

repeated trials with random noise give MD values of 0.3079, 0.3578, and 0.2618 for this level of noise, filtration and measured coil distance.

In Table 1 we observe how MD varies with three parameters: the noise-to-signal level (Q), the filtration level (F), and the measurement coil distance ($z_{measured}$). With multiple applied fields of independent x and y spatial frequencies we obtain a relatively narrow range of MD values for the resistivity tensor components and for the fiber angle map for each (Q , F , $z_{measured}$). With a noise-to-signal ratio $Q=0.10$ we observe that we must filter the magnetic fields. When the measurement coils are close to the conducting plane we are able to obtain very good reproductions of the true resistivity and fiber angle, even for moderate levels of filtration (large F); when the measurement coils are far from the conducting plane a high valued exponential is obtained for high frequencies (in Eq. 2) that is outside of the range of floating-point numbers our implementation of MATLAB can handle, and very high filtration is necessary (small F). When there is no noise added to the system ($Q=0.0$) we obtain a stable solution (for the same (Q , F , $z_{measured}$) we obtain repeated MD values), though the MD values are non-zero. Information is lost through filtering and through the truncation of a finite series representation. We conclude, therefore, that we have a stable solution method that requires increasing levels of filtration for increasing levels of noise.

Discussion

Our analysis suggests that Fourier-based image processing in MIT is effective at reconstructing smoothly varying anisotropic resistivity. Abrupt changes in the angle of the principal axis (the fiber direction) contribute to high frequencies of the resistivity. With truncated series and the use of a low-pass filter on the magnetic fields, the reconstruction of the resistivity can be expected to be negatively affected; however, we see sufficiently accurate qualitative reconstruction of the resistivity (see Fig. 4). For anisotropic resistivity in general we note the following: The reconstruction is frustrated by increasing levels of noise added to the system, as one would expect. We, therefore, require increasing levels of filtration to cope with increasingly noisy systems, just as other methods require regularization to suppress noise. In Table 1 we observe for each (Q , F , $z_{measured}$) there appears to be a best level of filtration. We note, however, that this filtration level is a function of the resistivity we have chosen to model; experimentalists will need to consider the filtration level that is best for their system. With three resistivity tensor components and a singular contribution of each applied field ($N=M=0$) it is necessary to apply at least 3 or 4 fields to obtain data for the inverse solution. We have chosen to apply ten sinusoidal fields here, since applying more fields increases the accuracy of the reconstructed resistivity but more than ten applied fields increases the memory and computation time requirements when solving our inverse problem. The user is free to apply as many fields as their system may handle, keeping in mind aliasing issues present in Fourier systems. For us, the choice of ten applied fields was a satisfactory compromise between accuracy and computational limitations. We consider a two-dimensional medium. Future investigations will seek to determine if multiple axes of the applied and measured magnetic fields (x -, y - and z -axis and not just the z -axis as shown here) are capable of reconstructing a three-dimensional anisotropic resistivity. In such a case, with 6 components to the resistivity tensor, we will find it necessary to apply at least 6 or 7 applied fields. Our freedom of choice of the functional form of the applied fields allows us to apply fields of relatively simple form (Eq. 7). The experimentalist is, of course, limited by the geometry of the coils used to apply the magnetic field. Field coil distance and filtration seem to be the primary determinants of the fidelity of the reconstructed resistivity to the true resistivity (a result analogous to a similar Fourier method to obtain current images from measured magnetic fields [23]).

We note several limitations of this method of image processing. First, this system was assumed to be two-dimensional; a three-dimensional system may be more difficult to handle because of the non-uniqueness of the solution. Three-dimensional calculations may be further complicated by excessively high computational needs. Here, with two-dimensions, we solve the linear system in frequency space (with ten applied fields) using least squares. This method is computationally intensive (a calculation on the order of W^4 where W is the number of points along the x - or y -axis in the discrete mesh) and is a highly memory intensive algorithm. A three-dimensional system would necessitate a linear system on the order of W^6 . Further, we note the formation of improper images in the reconstruction of resistivity is due to high valued exponentials that exceed the range of floating-point integers for our implementation of MATLAB (We also note this explains the formation of improper images in our previous paper evaluating isotropic resistivity [5].); a larger bit representation of numbers should allow one to correctly evaluate the exponential and carry through the solution of the linear system of equations. However, this will further contribute to the already high memory usage of this algorithm. To properly consider a particularly large number of terms of the series expansions of the magnetic fields for a three-dimensional, anisotropic, resistive medium with field coils at a large distance, this algorithm may, therefore, require a petaflop system. Finally, while our method seems robust at handling noise with a noise-to-signal ratio of 0.10 or less, higher levels of noise are not well tolerated. A low noise-to-signal level is necessary. Measurement coils presented here are within a distance of 1 cm from the conducting plane (which is of length and width 10 cm) and the measured fields due to the eddy currents are extremely small when compared to the applied field. Discrimination of a measured field component this small is not unprecedented (see for example a discussion of SQUIDS for nondestructive evaluation [24]). We do not account for averaging of the measured field over the coil area (for each coil measures a region, on average, and not a single point). For some spatial frequencies this can result in the measured signal vanishing. This problem can be avoided by apodization [25]. When these limitations are considered, resistivity seems well reconstructed using Fourier methods in Magnetic Induction Tomography.

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Figure 1.

Diagram of the Magnetic Induction Tomography Experiment. In MIT a set of coils produces an applied magnetic field that varies with time; the coils, here, are located at a distance of $z_{applied}$ above the conducting plane. The conductor, of thickness δ , has an anisotropic resistivity distribution, with tensor components and principal fiber axis shown in Fig. 2. The applied field induces an eddy current in the plane, the distribution of which is a function of the resistivity of the material. At a distance below the conducting plane, here located at $z_{measured}$, a set of coils detects the measured field which includes the applied field and the field associated with the eddy currents. For the theoretical calculations considered here, we assume the applied field is known in the plane of the conductor and is a simple product of sines and cosines. In practice, the coil geometry is that of current loops.

Figure 2. Resistivity Distribution. The anisotropic resistivity distribution used in all of the experiments is given by Eq. 8. The units of resistivity are $k\Omega\text{ cm}$. In Panel A, B, and C we see maps of ρ_{xx} , ρ_{xy} and ρ_{yy} , respectively. To determine the path of least resistance—the cardiac fiber direction in heart tissue—one finds the eigenvector associated with the smallest eigenvalue. The map of the muscle fibers is shown in Panel D, where each line segment is at an angle θ with respect to the vertical x -axis (Eq. 8b).

Figure 3.

Sample Calculation of Image Processing of Resistivity. The z -component of the applied magnetic field (Panel A, units of $T/\mu s$) induces eddy currents which contribute to the z -component to the measured magnetic field (Panel B, with noise included, units of μT). We consider the measured magnetic field to be only due to the eddy current. We add Gaussian distributed random noise to the measured magnetic field with a noise to signal ratio $Q=0.10$ (Panel C, units of μT). In Panel D, E, and F we see maps of ρ_{xx} , ρ_{xy} and ρ_{yy} , respectively with units of $k\Omega\text{ cm}$, solved with a filter level ($F=6$). This allows us to map the angle of the principal axis associate with least resistance (Panel G) with an MD of 0.2618. Panel H is the fiber map reconstructed without filtration. The map found with filtration is clearly a better reconstruction of the true map (Fig. 2) than is found without filtration.

Figure 4.

Abrupt changes in the principal axis associated with fiber direction (here, from 0 to $\frac{\pi}{2}$) are qualitatively well reconstructed. With truncated series and the filtering of high frequencies out of the magnetic fields, the transition for fibers is not as abrupt in Panel B as it is in Panel A.

Table 1
MD variation with the noise-to-signal level (Q), the filtration level (F), and the measurement coil distance ($z_{measured}$)

With a somewhat high noise level ($Q=0.10$) we observe that we must filter the magnetic fields to obtain accurate reconstruction of the fiber axis. When there is no noise added to the system ($Q=0.0$) we obtain a stable solution (for the same $(Q, F, z_{measured})$ and conclude that we have a stable solution method that requires increasing levels of filtration for increasing levels of noise.

F	Q	MD, $z_{measured}=0.0$ cm					MD, $z_{measured}=-1.0$ cm				
		ρ_{xx}	ρ_{yy}	ρ_{xy}	θ	Improper Image	ρ_{xx}	ρ_{yy}	ρ_{xy}	θ	Improper Image
No Filtering	0	0.0012	0.0012	0.0036	0.0006	Improper Image	0.0012	0.0012	0.0036	0.0006	Improper Image
		0.0012	0.0012	0.0036	0.0006	Improper Image	0.0012	0.0012	0.0036	0.0006	Improper Image
		0.0012	0.0012	0.0036	0.0006	Improper Image	0.0012	0.0012	0.0036	0.0006	Improper Image
	0.1	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
12	0	0.0419	0.0228	0.0842	0.0327	Improper Image	0.0419	0.0228	0.0842	0.0327	Improper Image
		0.0419	0.0228	0.0842	0.0327	Improper Image	0.0419	0.0228	0.0842	0.0327	Improper Image
		0.0419	0.0228	0.0842	0.0327	Improper Image	0.0419	0.0228	0.0842	0.0327	Improper Image
	0.1	0.2682	0.2893	0.4782	0.6552	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
8	0	0.2717	0.2796	0.4320	0.6663	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
		0.2697	0.2530	0.4295	0.1947	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
		0.0891	0.0486	0.1758	0.0736	Improper Image	0.0891	0.0486	0.1758	0.0736	Improper Image
	0.1	0.0891	0.0486	0.1758	0.0736	Improper Image	0.0891	0.0486	0.1758	0.0736	Improper Image
4	0	0.1271	0.1492	0.3106	0.1258	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
		0.1548	0.1388	0.2853	0.1370	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
		0.1808	0.1263	0.3292	0.1307	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image	Improper Image
	0.1	0.2578	0.1481	0.4518	0.2385	Improper Image	0.2578	0.1481	0.4518	0.2385	Improper Image
	0	0.2578	0.1481	0.4518	0.2385	Improper Image	0.2578	0.1481	0.4518	0.2385	Improper Image
		0.2578	0.1481	0.4518	0.2385	Improper Image	0.2578	0.1481	0.4518	0.2385	Improper Image
		0.2578	0.1481	0.4518	0.2385	Improper Image	0.2578	0.1481	0.4518	0.2385	Improper Image
	0.1	0.2883	0.1593	0.4702	0.2570	Improper Image	0.2883	0.1593	0.4702	0.2570	Improper Image

F	Q	$MD, Z_{\text{measured}} = 0.0 \text{ cm}$						$MD, Z_{\text{measured}} = -1.0 \text{ cm}$					
		ρ_{xx}	ρ_{yy}	ρ_{xy}	θ	ρ_{xx}	ρ_{yy}	ρ_{xy}	θ	ρ_{xx}	ρ_{yy}	ρ_{xy}	θ
		0.2727	0.1774	0.4893	0.2458	0.2874	0.1726	0.5065	0.2605	0.2858	0.1684	0.4987	0.2554