Effect of detector lag on CT noise power spectra

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Purpose: The authors examined the effect of detector lag on the noise power spectrum (NPS) of CT images reconstructed with filtered backprojection (FBP).

Methods: The authors derived an analytical expression of the NPS with detector lag, and then verified it using computer simulations with parallel beam and fan beam geometries. The dependence of the NPS on the amount of lag, location within the scanned field of view (FOV), and the number of views used in the reconstruction (samples per rotation) was investigated using constant and view dependent noise in the raw data.

Results: Detector lag introduces noise correlation in the azimuthal direction. The effect on the NPS is a frequency dependent reduction in amplitude. In small regions of the image, the effect is primarily in the frequencies corresponding to the azimuthal direction. The noise blurring and NPS filtering increases with increasing radial distance, and therefore regions at larger radial distances have lower noise power. With the same detector lag response function, the amount of noise correlation and NPS filtering decreases with increasing number of views.

Conclusions: The shape of the NPS depends on the detector lag coefficients, location of the region, and the number of views used in the reconstruction. In general, the noise correlation caused by detector lag decreased the amplitude of the NPS. © 2011 American Association of Physicists in Medicine. [DOI: 10.1118/1.3589135]

Key words: detector lag, NPS, noise correlation, CT

I. INTRODUCTION

Detector lag is defined as signal present in detector samples subsequent to the one in which it was generated¹ and is present in many imaging applications [e.g., radiographic modalities, fluoroscopy, image guided radiotherapy, and computed tomography (CT) systems].^{2–6} The main cause of detector lag is believed to be charge trapping and release in the x-ray detector. In this paper, we study the effect of lag on the noise power spectrum (NPS) of CT images. We deal only with CT systems in which the source and detector rotate together around the object and in which a single detector element samples the same radial distance. In those systems, lag results in correlation between projections. After reconstruction, the signal correlation between adjacent projections can cause image artifacts (e.g., “comet” and “streak” artifacts),¹ and the noise in different projections becomes correlated which changes noise structure in the reconstructed image. Several correction algorithms (e.g., hardware based^{7,8} and software based^{9–12}) were developed and the effects of detector lag on image signal were investigated.

The amount of lag depends, of course, on the type of detector. Multidetector CT (MDCT) systems using high performance scintillator and photodiodes have very low lag. Large area detector based on active matrix, flat panel imagers, and amorphous silicon (a-Si:H) thin-film transistors have significantly higher lag, much of it due to the a-Si. Indirect conversion panels were reported to have lag as high as

10% in the first frame¹ though more recent models have residual signal of 2%–3% for the first frame.^{12,13} These detectors have relatively low signal level per interacting x-ray, however, which is a limitation in cone beam CT.^{14–16} Direct conversion detectors based on photoconductive materials such as HgI₂, PbO, and PbI₂ can achieve higher DQE and conversion efficiency which is desirable for CT, but they have higher lag ranging from 5 to 75% in the first frame.^{17–19} Thus, while some current systems may have low lag detectors, others do not, and understanding the effect of lag may be necessary when future detectors are considered for CT applications.⁴

The effect of detector lag on fluoroscopic systems was investigated using the NPS.^{20–22} In CT systems, the effect of detector lag on contrast-to-noise ratio has been reported,¹² but this metric does not fully describe the noise properties (e.g., noise correlation) in the reconstructed CT image. The NPS can be a better metric since it provides noise information over all frequencies. It was conjectured that the low frequency NPS might increase as the lag increases,²³ but our simulation study did not show this effect. Rather, the noise power at high frequency decreases as the lag increases. Here, we examine the effect of lag on the noise in CT images reconstructed with FBP using parallel beam and fan beam geometries and quantify it with the NPS. We derive an analytical expression of the NPS with detector lag and then verify it using computer simulations. The dependence

of NPS on lag coefficients, location in the field of view (FOV), and the number of views used in the reconstruction is investigated.

II. METHOD

II.A. Theoretical analysis

We define a “lag-free” signal and a detector lag response as follows:

$$\begin{aligned}\vec{p}_{1:n} &= [p_1, \dots, p_n] \\ \vec{h}_{0:k} &= [1, L_1, \dots, L_k] / \left(1 + \sum_{j=1}^k L_j\right)\end{aligned}\quad (1)$$

where p_i is the lag-free i th view projection composed of noisy line integrals, n is the number of views used in the reconstruction, $\vec{h}_{0:k}$ is the detector response, and L_j is the j th lag coefficient.

From linear system theory, the measured x-ray intensity for a single detector channel is the convolution of the lag-free intensity with the detector response.^{10,24} Although $\vec{p}_{1:n}$ is related to the logarithm of the measured signal, we assume we are in the high signal-to-noise ratio regime, and the signal variation between adjacent views is small. Then, to a first order approximation, the effect of lag can be expressed as the convolution of the lag-free projections $\vec{p}_{1:n}$ with the detector response $\vec{h}_{0:k}$ (see Appendix).

We first analyze the simpler case with a single lag coefficient and then generalize it. For $k = 1$, the (approximate) projections with lag are

$$\begin{aligned}\vec{y} &= \vec{p}_{1:n} \otimes \vec{h}_{0:1} = [p_1 + L_1 \times p_0, p_2 + L_1 \times p_1, \dots, p_n \\ &\quad + L_1 \times p_{n-1}] / (1 + L_1) \\ &= \vec{p}_{1:n} / (1 + L_1) + L_1 \times \vec{p}_{0:n-1} / (1 + L_1)\end{aligned}\quad (2)$$

Even though views 1 through n will be used in the reconstruction, we allow for a zeroth view, p_0 , which may be present as the x-ray generator stabilizes and which contributes to the lag in view 1.

In Eq. (2), the second term represents the contribution of detector lag to the output signal. Here, we have assumed that the detector response (i.e., lag coefficients) is uniform for all channels. This is a good approximation. For example, the spatial variation of detector lag is less than 1% in a flat panel detector.² Since the reconstruction process using FBP is linear, the reconstructed image is the weighted summation of two images: an original image reconstructed with $\vec{p}_{1:n}$ and a rotated image reconstructed with $\vec{p}_{0:n-1}$. By defining $B(p_i)_\psi$ as an FBP operator for the i th view projection at view angle ψ , the lag-free image, $i_0(\vec{r})$, reconstructed with $\vec{p}_{1:n}$ is expressed as

$$i_0(\vec{r}) = B(\vec{p}_{1:n})_{0;\theta:(n-1)\theta}, \quad (3)$$

where θ is the angle between views (i.e., $2\pi/n$) and $\vec{r}$ is a two dimensional vector in image space defined by $\vec{r} = \vec{r}_x + \vec{r}_y$ [shown in Fig. 1(a)]. Then, the image reconstructed with data containing a single lag term, $i(\vec{r})$, is

FIG. 1. Original and rotated images for (a) a single lag coefficient and (b) k lag coefficients.

$$\begin{aligned}i(\vec{r}) &= \frac{B(\vec{p}_{1:n})_{0;\theta:(n-1)\theta} + L_1 B(\vec{p}_{0:n-1})_{0;\theta:(n-1)\theta}}{1 + L_1} \\ &= \frac{B(\vec{p}_{1:n})_{0;\theta:(n-1)\theta} + L_1 [B(\vec{p}_{1:n})_{0;\theta:n\theta} + B(p_0)_0 - B(p_n)_{n\theta}]}{1 + L_1} \\ &= \frac{i_0(\vec{r}) + L_1 R(i_0(\vec{r}), \theta) + L_1 B(p_0 - p_n)_0}{1 + L_1},\end{aligned}\quad (4)$$

where $R(g, k)$ is the rotation of 2-dimensional image g by k in the clockwise direction, and $B(\vec{p}_{1:n})_{0;\theta:n\theta}$ is replaced by the rotation of $i_0(\vec{r})$ by an angle θ since the first view angle of the backprojection is θ . We expect $p_0 - p_n$ to be small, and further, the effect of the last term will be diluted by the large total number of views. As a result, we will ignore the startup effects contained in the last term in the numerator to approximate the image with lag.

For a small off-centered region of the image at location $\vec{r}$ from the isocenter (shown in Fig. 1), the rotation in Eq. (4) can be approximately expressed as a translation in the direction orthogonal to the direction of $\vec{r}$

$$i(\vec{r}) = \frac{i_0(\vec{r}) + L_1 i_0(\vec{r} + \vec{r}_1)}{1 + L_1}, \quad (5)$$

where $\vec{r}_1$ represents the translation direction and distance between original and rotated images.

Note that the noise correlation in the reconstructed image, $i(\vec{r})$, is mainly caused by the rotation angle and the amplitude of the lag coefficient. Then, by assuming that the noise image in a small region is stationary and ergodic, the NPS in this small region is expressed as

$$\begin{aligned}
S_1(\vec{f}) &= I(\vec{f})I(\vec{f})^* \\
&= I_0(\vec{f})[1 + L_1 \exp(j2\pi\vec{f} \cdot \vec{r}_1)]I_0(\vec{f})^* \\
&\quad \times [1 + L_1 \exp(-j2\pi\vec{f} \cdot \vec{r}_1)]/(1 + L_1)^2 \\
&= S_{i_0}(\vec{f})(1 + L_1^2 + 2L_1 \cos(2\pi\vec{f} \cdot \vec{r}_1))/(1 + L_1)^2, \quad (6)
\end{aligned}$$

$$\begin{aligned}
S_I(\vec{f}) &= S_{i_0}(\vec{f})W_{(L_1:L_k, \vec{r}_1:\vec{r}_k)}(\vec{f}) \\
W_{(L_1:L_k, \vec{r}_1:\vec{r}_k)}(\vec{f}) &= \frac{\left(1 + \sum_{j=1}^k L_j^2 + 2 \sum_{j=1}^k L_j \cos(2\pi\vec{f} \cdot \vec{r}_j) + 2 \sum_{j=1}^{k-1} \sum_{l=j+1}^k L_j L_l \cos(2\pi\vec{f} \cdot (\vec{r}_j - \vec{r}_l))\right)}{\left(1 + \sum_{j=1}^k L_j\right)^2}, \quad (7)
\end{aligned}$$

where we assumed that the k projections prior to the first one used in the reconstruction, $\vec{p}_{-(k-1):0}$, contribute to the lag in $\vec{p}_{1:k}$, and we are again ignoring the start-up effects.

The NPS with lag is equal to the lag-free NPS multiplied by a lag dependent function. Equations (6) and (7) show that the lag dependent function is mainly determined by the amount of detector lag and translation direction and distance (i.e., $\vec{r}_k$) between original and rotated images.

We now consider the NPS that would be calculated from a larger finite region of interest. When the NPS is estimated by the Fourier transform of the image of this region, an assumption of stationarity and ergodicity is implicit. We can extend our analytical derivation to a finite

where $S_{i_0}(\vec{f})$ is the NPS of the lag-free image of the small region, $\vec{f}$ is a two dimensional vector in frequency space defined by $\vec{f} = \vec{f}_x + \vec{f}_y$, and $I(\vec{f})$ and $I_0(\vec{f})$ are Fourier transform of the small region of the images with and without lag, respectively.

By generalizing the derivation from one detector lag coefficient to any number of coefficients k , the NPS in a small region is expressed as

region. Assuming that noise correlation among small regions is negligible, the NPS of a finite region can be calculated by averaging the NPS of small regions.²⁵ Note that if the region of interest is large enough, the amount and direction of noise blurring will vary among the small regions within it. For the comparisons described below, the region of interest was divided into 400 (i.e., 20×20) small subregions. The 2D NPS within each subregion was the expected NPS for parallel beam or fan beam reconstruction (see below) multiplied by the effect of lag [Eq. (7)] in the azimuthal direction, where the term “azimuthal” refers to the polar angle coordinate. The assumed parallel beam and fan beam NPS were^{25,26}

$$S_{i_0}(\vec{f}) = \begin{cases} \left(\frac{\pi}{n}\right)^2 \sum_{m=1}^n R \left[S_{fp,m}(f) \delta(f_y), \frac{2\pi}{n} m \right] & \text{for parallel beam} \\ \left(\frac{\pi}{n}\right)^2 \sum_{m=1}^n R \left[\frac{S_{fp,m}(f/a_{i_0,m})}{a_{i_0,m}^2 u_{i_0,m}^4} \delta(f_y), \alpha_{i_0,m} - \beta_{i_0,m} \right] & \text{for fan beam} \end{cases}, \quad (8)$$

where $S_{fp,m}(f)$ is the NPS for the filtered projection in the m th view, $\alpha_{i_0,m}$ and $\beta_{i_0,m}$ are a view angle and the angle of rays passing through the center of a small region in the m th view, $a_{i_0,m}$ and $u_{i_0,m}$ are the scaling factor and the backprojection weighting in the m th view, and R is the same rotation operator as in Eq. (4). The 2D NPS in the subregions, modified for the local lag effect, were averaged to produce the expected 2D NPS.

II.B. Computer simulations

To examine the effect of detector lag on CT image noise, we used a parallel beam and a fan beam geometry. The parallel beam geometry produces stationary image noise when the measurement noise is stationary over all views, the effect of the square of the ramp filter in the spatial domain is approximated by a delta function, and

the effect of interpolation in the backprojection step is neglected.²⁷ With the same conditions, the fan beam geometry produces locally stationary image noise.²⁵ Table I summarizes geometry parameters for both systems. Since the noise structure of the reconstructed images is mainly determined by the first few lag coefficients,¹⁰ we used 10

TABLE I. System geometry parameters.

	Parallel beam geometry	Fan beam geometry
Source-to-isocenter distance	N/A	100 cm
Source-to-center detector distance	N/A	150 cm
Detector array element spacing	1 mm	
Number of detector cells	512	1024

Fig. 2. Simulated view dependent noise statistics. Views at 0 and π radians are in the horizontal direction.

lag coefficients with two settings “Nominal” and “High” lag. For the “Nominal” setting, $L_{j(1:10)} = [0.1 \ 0.033 \ 0.023 \ 0.017 \ 0.01 \ 0.007 \ 0.005 \ 0.003 \ 0.002 \ 0.001]$ was used.¹ For the “High” lag setting the values were four times higher. While our derivation above made approximations, such as applying the lag coefficients to the projection data and ignoring start-up effects, the simulations properly modeled these factors. We assumed that noise was due only to quantum statistics. Energy weighting in the detector was not included in the simulation. Doing so would primarily increase the overall noise level but not change the lag effects. Thus, x-ray intensity data generated from Poisson statistics were convolved with the detector lag coefficients and then normalized. To avoid apodization or aliasing caused by the interpolation in the backprojection step, the noise-only projections were filtered (with an unweighted ramp filter) with 20-fold Fourier interpolation and then pixel-driven backprojection with linear interpolation was performed. For both systems, local, noise-only images were reconstructed onto a 128×128 array ($64 \times 64 \text{ mm}^2$ for the parallel beam geometry and $43 \times 43 \text{ mm}^2$ for the fan beam geometry), and a small pixel spacing (0.5 mm for the parallel beam geometry and 0.33 mm for the fan beam geometry) was used to avoid image spatial aliasing. 1000 different noise realizations were conducted and the 2D NPS were estimated from the Fourier transform (squared) of noise-only images²⁸

$$S(\vec{f}) = \frac{b_x b_y}{N_x N_y} \left\langle |DFT\{i(\vec{r})\}|^2 \right\rangle, \quad (9)$$

where N_x and N_y are the number of elements in two dimensions, and b_x and b_y are the corresponding pixel size in each direction. The symbol $\langle \rangle$ indicates the ensemble average of realizations ($m = 1000$). Full field of view 1024×1024 (i.e., $512 \times 512 \text{ mm}^2$ for the parallel beam geometry and $683 \times 683 \text{ mm}^2$ for the fan beam geometry) reconstructions were also performed.

The expected standard deviation in the NPS estimate is²⁹

$$\sigma_{\text{NPS}} = \frac{\text{NPS}}{\sqrt{m}}. \quad (10)$$

Experimentally, we further reduced the standard deviation by calculating the average of the NPS of seven adjacent lines in frequency space. The standard deviation was calculated across the 1000 independent realizations from the frequency space average of each realization using Eq. (10).

The dependence of the NPS on location within the scanned FOV, the amount of lag, and the number of views was investigated using constant and view dependent noise statistics. We used 10 000 photons per detector cell for constant noise statistics. For the view dependent noise statistics, the intensity variation depicted in Fig. 2 was used, where the noise level was view dependent but uniform across each single projection (i.e., stationary within each view). Other simulation parameters are summarized in Table II. We assumed the amount of lag per sample was independent of the number of views, so the same lag coefficients were used for the 500 and 1000 view simulations.

III. RESULTS

Figure 3 shows full FOV noise images of the parallel beam geometry, and estimated and analytic NPS of the parallel beam and the fan beam geometry for the constant noise statistics. Figures 3(c)–3(f) show that, compared to the NPS with “Nominal” lag, the NPS with “High” lag shows lower amplitude in the f_y direction due to the increased noise correlation in the azimuthal direction [visible in Fig. 3(a)]. The dependence on location, amount of lag, and number of views is consistent with the expected increase in noise correlation and NPS reduction with increasing radial distance and angular increment between views [c.f. Eq. (6)]. For example, as the radial distance increases or the number of view decreases, the translation distance [e.g., $\vec{r}_1$ in Eq. (6)] increases and therefore the NPS at larger radial distance shows a more oscillatory

TABLE II. Simulation parameters.

Noise statistics	Number of views over 360°	Location of the ROI	Lag setting
Constant	500 and 1000	Centered at -10 cm and -20 cm in x	High and nominal
View dependent	500	Centered at -10 cm in x and 10 cm in y	High

FIG. 3. Noise images and estimated and analytic NPS with constant noise statistics. Noise images with (a) 500 views and (b) 1000 views for the parallel beam geometry. Estimated and analytic NPS with (c) 500 views and (d) 1000 views for the parallel beam geometry. Estimated and analytic NPS with (e) 500 views and (f) 1000 views for the fan beam geometry. (g) Central horizontal and vertical profile of the estimated and analytic NPS at -20 cm with 500 views and high lag for the parallel beam geometry. The size of the noise images are: 512×512 mm² for full FOV, 64×64 mm² for local FOV parallel beam geometry, and 43×43 mm² for local FOV fan beam geometry. The NPS of the parallel beam geometry were estimated from local regions indicated by white boxes in Figs. 3(a) and 3(b) and are displayed with scale of $[0 \text{ to } 2.6 \times 10^{-7}]$ for 500 views and $[0 \text{ to } 1.4 \times 10^{-7}]$ for 1000 views. The NPS of the fan beam geometry are displayed with scale of $[0 \text{ to } 7 \times 10^{-8}]$ for 500 views and $[0 \text{ to } 3.5 \times 10^{-8}]$ for 1000 views.

pattern [see the NPS at -10 and -20 cm in Figs. 3(c) and 3(e)]. Because of this effect, the NPS at -10 cm with 500 views is similar to that at -20 cm with 1000 views. In addition, the amount of lag also changes the overall amplitude of the noise power. Note the complex dependence of the NPS at -20 cm with 500 views [e.g., Fig. 3(c)], resulting from the side lobes of the lag filter (which arise from the discrete angles used in backprojection) falling within

the bandwidth of the image. Figures 3(e) and 3(f) show the fan beam results for 500 and 1000 views, respectively, for the no lag, high lag, and nominal lag cases. By comparing these to the parallel ray results, we see that the multiplicative effect of lag is the same. The fan beam NPS appear to have more structure in the f_y direction, but this is only because the higher noise bandwidth of the underlying lag-free fan beam NPS allows the higher side lobes to

FIG. 3. (Continued).

be observed. Figure 3(g) compares the central vertical and horizontal profile of the estimated and analytic NPS at -20 cm with 500 views and high lag for the parallel beam geometry, and shows excellent agreement. The central vertical (horizontal) profile was calculated by averaging central seven columns (rows) of the NPS image to reduce the statistical error in the estimation. We show quantitative comparison only at -20 cm with 500 views and high lag for the parallel beam geometry, but the accuracy of the analytical NPS expression was verified for all other cases.

Figure 4(a) shows the reconstructed noise image from the parallel beam geometry without and with “High” lag for the view dependent noise statistics. Since the noise level is high in the horizontal view directions (i.e., 0 and π radians), the noise image without lag shows horizontally structured noise, which results in high noise power in the f_y direction [shown in Fig. 4(b)]. Although not shown, a region offset in the y direction would have identical NPS in this case without lag due to the approximate stationarity of parallel beam recon-

struction with noise that is constant within projections. Lag introduces noise structure that is different in different local regions. For the region centered at $(-10, 0)$, the azimuthal blurring generated between original and rotated images increases noise correlation in the vertical direction, which decreases noise power at high f_y frequencies. By contrast, for the region centered at $(0, 10)$ cm, azimuthal blurring accentuates the effect of view dependent noise by further decreasing noise power in the f_x direction, making the high relative noise power in the f_y direction more prominent. Figure 4(c) shows the estimated and analytic NPS at the same local regions for the fan beam geometry. While the NPS showed a locally varying shape due to the nonstationarity of fan beam CT, the effect of the detector lag on the NPS was the same as that of the parallel beam geometry at the same location. Figure 4(d) compares the central vertical and horizontal profile of the estimated and analytic NPS centered at $(0, 10)$ cm with high lag for the fan beam geometry and shows the excellent agreement.

FIG. 3. (Continued).

IV. DISCUSSION AND CONCLUSIONS

To investigate the effect of detector lag on the NPS of CT images reconstructed with FBP, we derived an analytical expression and tested it with computer simulations using parallel beam and fan beam geometries. The impact of lag on the NPS is determined by a local weighting function that depends on lag coefficients, the location within the scanned FOV, and the number of views used in the reconstruction. The lag effect on the NPS was also investigated with constant and view dependent noise statistics. Contrary to the speculation in Ref. 23, in all cases, the effect of uncorrected lag was a decrease in the amplitude of the NPS.

Lag introduces noise correlation in the azimuthal direction that increases with stronger lag and with increased radial distance. The angular extent of the correlation depends on the angular rotation during the evolution of the lag which in our case was controlled by the number of lag coefficients

and the number of views per rotation. In small local regions, the lag causes an attenuation in noise power at frequencies corresponding to the azimuthal direction. For a given lag performance (in units of percent signal retained per sample) the effect also increases with the angular increment between views (the inverse of the number of views in 2π). While it might be expected that the effect of a smoothly varying function like signal lag would be slowly varying and monotonic in frequency space, we found that the effect can be quite complex when the number of views is small (e.g., even with 500 views in our case, Fig. 3). This is particularly relevant in C-arm CT systems which tend to have relatively small number of views compared to their resolution and FOV. While our simulation study assumed an equal-time per projection view scheme, the constant total acquisition time scheme can also be considered. Since the detector lag would increase as the scanning speed increases,¹³ the detector lag effect with the latter case may be analogous to the comparison of NPS

FIG. 4. Noise images and estimated and analytic NPS with view dependent noise statistics. (a) Noise images without and with “High” lag for the parallel beam geometry. NPS without lag and with “High” lag centered at $(-10\text{ cm}, 0)$ and $(0, 10\text{ cm})$ for (b) the parallel beam geometry and (c) the fan beam geometry. (d) Central vertical and horizontal profile of the estimated and analytical NPS at $(0, 10\text{ cm})$ with high lag for the fan beam geometry. The sizes of the noise images are: $512 \times 512\text{ mm}^2$ for full FOV, $64 \times 64\text{ mm}^2$ for local FOV parallel beam geometry, and $43 \times 43\text{ mm}^2$ for local FOV fan beam geometry. The local region NPS were estimated from regions indicated by white boxes in Fig. 4(a) and are displayed with scales of $[0\text{ to }8.3 \times 10^{-7}]$ and $[0\text{ to }2 \times 10^{-7}]$ for the parallel and fan beam cases, respectively.

images between Nominal lag setting with 500 views in Fig. 3(c) and High lag setting with 1000 views in Fig. 3(d).

In our study, we depicted the NPS in small local regions. Since the NPS in any size of region can be approximated by averaging the NPS of small regions within it, the shape of the NPS in larger or smaller regions can be easily predicted. An effect that is highly directional in the NPS of a small region is likely to become blurred as the region size increases due to more directional variations of noise structure. In the limit of a region that encompasses all of the FOV, the effect of lag would be a reduction of noise power at high frequencies in all directions. If not analyzed separately in different regions, this could be confused with the effect of stationary blurring from the reconstruction algorithm (e.g., backprojection interpolation).

Many of the lag effects were demonstrated in this paper using a parallel beam geometry since this system is expected to produce approximately stationary noise in a local small region when noise is uniform within projections, allowing the effect of lag to be visualized most effectively. However,

as long as the reconstruction process is linear, the analytical expression [Eq. (7)] can be generalized to other system geometries. This was demonstrated here for fan beam geometry. The work can be applied to cone beam by using an appropriate initial NPS. For example, Fig. 5 shows the estimated and analytic NPS of a small local volume ($3.3 \times 3.3 \times 3.3\text{ cm}$) centered at $(-11\text{ cm}, 0, 0)$ with the simulation parameters in Ref. 30 and “High” lag for a cone beam CT system, where the FDK algorithm was used for reconstruction. Since lag only affects the azimuthal direction, the weighting function calculated for the central plane was used for all f_z slices of the NPS. Application to helical scanning is somewhat more complicated. In a helical cone beam CT system, lag affects both azimuthal and longitudinal directions, and therefore the weighting function calculated for the cone beam CT system will be reoriented in the translation direction between original and rotated images.

In our study, detector lag coefficients were selected to accentuate the effect on the NPS and make it easily observed. While current MDCT and flat panel cone beam

FIG. 4. (Continued).

FIG. 5. Estimated and analytic NPS at a local volume without and with "High" lag for a cone beam CT system.

CT systems have relatively low detector lag, lag effects could be more severe in applications using a direct conversion detector, or those requiring higher scan speed and/or very different exposure levels. In addition, future candidate detector technologies may present lag challenges. The framework presented here allows potential effects to be readily examined.

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APPENDIX

Let N_{in} be the incident intensity (assumed to be constant) and N_j the detected intensity for a single detector channel in the j th view. Then, the lag-free projection data, p_j , can be expressed as

$$p_j \log \left(\frac{N_{in}}{N_j} \right). \quad (A1)$$

With k detector lag coefficients, the measured intensity is expressed as the convolution of the detected intensity (i.e., N_j) with detector lag coefficients, and therefore the projection data with lag is

$$\begin{aligned} p_{j,lag} &= \log \left(\frac{(1 + \sum_{i=1}^k L_i) N_{in}}{N_j + L_1 N_{j-1} + \dots + L_k N_{j-k}} \right) \\ &= \log \left(\frac{(1 + \sum_{i=1}^k L_i) N_{in}}{N_j (1 + L_1 N_{j-1}/N_j + \dots + L_k N_{j-k}/N_j)} \right) \\ &= \log \left(\frac{N_{in}}{N_j} \right) + \log \left(\frac{(1 + \sum_{i=1}^k L_i)}{1 + L_1 N_{j-1}/N_j + \dots + L_k N_{j-k}/N_j} \right). \end{aligned} \quad (A2)$$

In Eq. (2), using convolution of lag-free projections, the approximated projection data with lag is

$$\begin{aligned} p'_{j,lag} &= \left(\log \left(\frac{N_{in}}{N_j} \right) + L_1 \log \left(\frac{N_{in}}{N_{j-1}} \right) + \dots + L_k \log \left(\frac{N_{in}}{N_{j-k}} \right) \right) / \\ &\quad \times \left(1 + \sum_{i=1}^k L_i \right) \\ &= \log \left(\frac{N_{in}}{N_j} \right) + \left(\sum_{i=1}^k L_i \log \left(\frac{N_j}{N_{j-i}} \right) \right) / \left(1 + \sum_{i=1}^k L_i \right) \end{aligned} \quad (A3)$$

The difference between Eqs. (A2) and (A3) is from the second terms. Starting with the second term in Eq. (A2), adding and subtracting the same term in the numerator, using the

approximation that $\log(1+x) \approx x - x^2/2$, defining $\Delta N_j = N_j - N_{j-i}$ and assuming signal variation is small ($N_j \approx N_{j-i} \approx N$), the second term in Eq. (A2) can be expressed

$$\begin{aligned} &\log \left(\frac{(1 + \sum_{i=1}^k L_i)}{1 + L_1 N_{j-1}/N_j + \dots + L_k N_{j-k}/N_j} \right) \\ &= \log \left(\frac{1 + \sum_{i=1}^k L_i N_{j-i}/N_j - \sum_{i=1}^k L_i N_{j-i}/N_j + \sum_{i=1}^k L_i}{1 + \sum_{i=1}^k L_i N_{j-i}/N_j} \right) \\ &= \log \left(1 + \frac{\sum_{i=1}^k L_i - \sum_{i=1}^k L_i N_{j-i}/N_j}{1 + \sum_{i=1}^k L_i N_{j-i}/N_j} \right) \\ &= \log \left(1 + \frac{\sum_{i=1}^k L_i (N_j - N_{j-i})}{N_j + \sum_{i=1}^k L_i N_{j-i}} \right) \\ &\approx \frac{\sum_{i=1}^k L_i (N_j - N_{j-i})}{N_j + \sum_{i=1}^k L_i N_{j-i}} - \frac{1}{2} \left(\frac{\sum_{i=1}^k L_i (N_j - N_{j-i})}{N_j + \sum_{i=1}^k L_i N_{j-i}} \right)^2 \\ &\approx \frac{\sum_{i=1}^k L_i \Delta N_{ji}/N}{1 + \sum_{i=1}^k L_i} - \frac{1}{2} \left(\frac{\sum_{i=1}^k L_i \Delta N_{ji}/N}{1 + \sum_{i=1}^k L_i} \right)^2. \end{aligned} \quad (A4)$$

Similarly, the second term in Eq. (A3) leads to

$$\begin{aligned} &\frac{L_1 \log \left(\frac{N_j}{N_{j-1}} \right) + \dots + L_k \log \left(\frac{N_j}{N_{j-k}} \right)}{1 + \sum_{i=1}^k L_i} \\ &= \frac{L_1 \log \left(\frac{N_j - N_{j-1} + N_j}{N_{j-1}} \right) + \dots + L_k \log \left(\frac{N_j - N_{j-k} + N_j}{N_{j-k}} \right)}{1 + \sum_{i=1}^k L_i} \\ &= \frac{L_1 \log \left(1 + \frac{N_j - N_{j-1}}{N_{j-1}} \right) + \dots + L_k \log \left(1 + \frac{N_j - N_{j-k}}{N_{j-k}} \right)}{1 + \sum_{i=1}^k L_i} \\ &\approx \frac{L_1 \log(1 + \Delta N_{j1}/N) + \dots + L_k \log(1 + \Delta N_{jk}/N)}{1 + \sum_{i=1}^k L_i} \\ &\approx \frac{\sum_{i=1}^k L_i \Delta N_{ji}/N}{1 + \sum_{i=1}^k L_i} - \frac{1}{2} \frac{\sum_{i=1}^k L_i (\Delta N_{ji}/N)^2}{1 + \sum_{i=1}^k L_i}. \end{aligned} \quad (A5)$$

Equations (A4) and (A5) are the same to first order, showing that the lag effect can be approximated by a convolution of the log signals when the variation in adjacent readings of the same detector channel is small.

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