Hyperbolic-cosine waveguide tapers and oversize rectangular waveguide for reduced broadband insertion loss in W-band electron paramagnetic resonance spectroscopy

R. R. Mett,^{1,2} J. W. Sidabras,¹ J. R. Anderson,¹ and J. S. Hyde^{1,a)}

¹*Department of Biophysics, Medical College of Wisconsin, Milwaukee, Wisconsin 53226-0509, USA*

²*Milwaukee School of Engineering, Milwaukee, Wisconsin 53202-3109, USA*

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The two-way insertion loss of a 1 m length of waveguide was reduced by nearly 5 dB over a 4% bandwidth at W-band (94 GHz) for an electron paramagnetic resonance (EPR) spectrometer relative to WR10 waveguide. The waveguide has an oversize section of commercially available rectangular WR28 and a novel pair of tapers that vary in cross section with axial position according to a hyperbolic-cosine (HC) function. The tapers connect conventional rectangular WR10 waveguide to the WR28. For minimum loss, the main mode electric field is parallel to the long side of the WR28. Using mode coupling theory, the position of maximum flare (inflection point) in the taper was optimized with respect to the coupling to higher order modes and the reflection of the main mode. The optimum inflection point position is about one-tenth of the taper length from the small end of the taper. Reflection and coupling were reduced by about 20 dB relative to a pyramidal (linear) taper of the same length. Comb-like dips in the transmission coefficient produced by resonances of the higher order modes in the oversize section were about 0.03 dB. Specially designed high-precision, adjustable WR28 flanges with alignment to about 5 μm were required to keep higher order mode amplitudes arising from the flanges comparable to those from the HC tapers. Minimum return loss was about 30 dB. This paper provides a foundation for further optimization, if needed. Methods are not specific to EPR or the microwave frequency band. © 2011 American Institute of Physics. [doi:10.1063/1.3607432]

I. INTRODUCTION

Electron paramagnetic resonance (EPR) spectroscopy can be carried out at an arbitrary radio or microwave frequency in accordance with the equation for resonance of the free electron: $2\pi f = \gamma B$, where the frequency is given by f , the magnetic field by B , and γ is the gyromagnetic ratio. In the range of frequencies from about 10 to 150 GHz, microwave rectangular waveguide is customarily used to construct the required microwave bridge that is used for detection of EPR signals. At lower frequencies, coaxial transmission-line technology is used; at higher frequencies, quasi-optical technology is used. At the upper end of the microwave range, insertion loss becomes troublesome. For example, it is 2.60 dB/m in WR10 (W-band—~94 GHz) at room temperature.

The sample to be investigated by high-frequency EPR spectroscopy is generally placed in a resonator that, in turn, is supported inside a cryogenic magnet. A typical distance between the resonator and the microwave bridge is 1 m. Thus, at 94 GHz, nearly half of the power incident on the transmission line is lost. Moreover, since the microwave bridge is sensitive to voltage, the signal-to-noise ratio, as determined from the microwave power reflected from the resonator when

magnetic resonance occurs, is reduced by about $2^{1/2}$ because of insertion loss. This paper is concerned with development of technology to minimize insertion loss in the context of high-frequency EPR spectroscopy. The methods are general and may find uses in other applications of high-frequency microwave devices.

Known methods for reduction of insertion loss in high-frequency EPR include use of corrugated cylindrical waveguide,¹ cylindrical waveguide propagating in the TE₀₁ mode,² and so-called “tall waveguide.” All of these waveguide types can be called oversize² since they can support higher order propagating modes in addition to the main mode. We have concluded that whatever the method used to reduce insertion loss relative to standard rectangular waveguide, it is necessary to employ a mode-conversion device at both ends of the low-insertion-loss waveguide segment—one to couple to the bridge and one to couple to the resonator. A problem in the use of oversize waveguide is that any mode converter or any mechanical imperfection between mode converters will excite higher order modes that can propagate in the oversize waveguide. Since these higher order modes reflect from the place they are cut off² (in the mode converters), the modes will resonate at particular frequencies determined by the length of the oversize waveguide. The longer the oversize section, the closer in frequency the resonances occur. Because the modes are coupled to the main mode, these resonances can be seen in the transmission and reflection coefficients of the main mode through an oversize waveguide section. The higher the amplitude of the higher order mode, the more strongly the

^{a)}Author to whom correspondence should be addressed. Electronic mail: jshyde@mcw.edu. Present address: Ph.D. Department of Biophysics Medical College of Wisconsin, 8701 Watertown Plank Road, Milwaukee, Wisconsin 53226-0509, USA. Telephone: (414) 456-4005. Fax: (414) 456-6512.

coefficients of the main mode are perturbed at resonance. Such narrow-band perturbations in the signal cannot be tolerated in many types of EPR experiments.

Several versions of the W-band microwave bridge used by us have been described in the literature.^{3–7} In short, we use multiple arms to enable several microwave frequencies to be incident on the sample. Moreover, the W-band loop-gap resonator⁸ (LGR) that is used in the experiments that have been described has a Q -value of about 100, corresponding to a separation between 3 dB points of about 1 GHz at 94 GHz. This high bandwidth has enabled us to explore the use of microwave frequency modulation and also microwave frequency sweeps as alternatives to magnetic field modulation and magnetic field sweep. Thus, because of multiple incident frequencies, frequency modulation, and frequency sweep, we require that the low-loss transmission-line section, which consists of two mode-converter devices and the oversize waveguide section, be broadband.

There are no broadband rectangular-to-cylindrical mode-conversion devices known to us. Rectangular-to-tall waveguide-transition design seems to be proprietary. Our general idea was to use simple tapers between WR10 and WR28, noting that the latter has an insertion loss of 0.5 dB/m. Our final solution to the problem was to use tapers that vary in cross section with axial position according to a hyperbolic-cosine (HC) function, with the roles of broad and narrow faces of WR10 and WR28 waveguide interchanged. This was found to reduce coupling to spurious modes by about 20 dB relative to pyramidal (linear) tapers. Our final solution also included the use of specially designed high-precision waveguide flanges between the tapers and oversize waveguide section. Residual misalignment in these flanges is about 5 μm . With the HC tapers and the high-precision flanges, the resonance-produced comb-like dips in the transmission coefficient of the main mode were reduced to about 0.03 dB over a bandwidth of 4 GHz in a 132 mm oversize waveguide section. This compares to dips of about 1 dB for a linear taper. In a 1 m section with HC tapers, the dips produced by the tapers are smaller. Broadband performance of the low-loss transmission-line section is enhanced accordingly.

The paper organization follows our taper design and development strategy. In Sec. II, the theory of Sporleder and Unger⁹ is used to predict transmission coefficients of higher order modes and the reflection coefficient of the main mode for different taper shapes. A review of the literature of electromagnetic waves in tapers up to 1979 is given in the book. The theory reduces Maxwell's equations to a coupled system of first order differential equations for the mode amplitudes, one equation for each waveguide mode. The method of solution is single-pass, which means that only two modes are considered at a time, the main mode with an amplitude of 1 and a coupled mode with a smaller amplitude. The coupled mode amplitude is calculated at the exit of the taper assuming zero amplitude at the entry. Taper shape influences the mode amplitude because the coupling coefficient is proportional to the rate of taper flare and the mode undergoes spatial interference as it propagates through the taper. Although apparently new, the HC function seemed a natural choice for the taper shape for two rea-

sons. First, since the coupling coefficient is proportional to the logarithmic derivative of the taper dimension with respect to axial position, the exponential functional dependence tends to minimize the overall magnitude of the coupling coefficient for a given taper length. Second, the HC function as defined gives zero flare at each end of the taper. This causes the broadband spatial interference that minimizes the coupled mode amplitude on exit. Although approximate, the single pass solution method permits broadband optimization of the taper shape because minimizing single pass excitation tends to minimize multiple pass excitation. The point of maximum flare in the taper was varied and the coupled mode amplitude observed as a function of frequency. Optimum performance was obtained with the point of maximum flare near the small end of the taper (12% of the total taper length) because this maximizes space for higher order mode interference on the large end and gives sufficient space for reflected mode interference at the small end. For the chosen shape, results of the single pass theory were compared to those of a finite element computer program, which solved Maxwell's equations for a single taper.

Then in Sec. III, the performance of two identical tapers with an oversize rectangular waveguide in between is discussed. Following a summary of the attenuation of the various modes, the existence and characterization of resonances of higher order modes between the tapers is discussed and the size of the resonance-produced dips in the transmission coefficient of the main mode is related to the coupled mode amplitude. Finite element simulations of the performance of this system with HC and linear tapers are shown and compared with the results of Sec. II.

In Sec. IV, the fabrication of components and their experimental characterization is presented. The fabrication of the tapers is discussed along with the fabrication of specially designed high-precision adjustable flanges, which were found to be necessary. Network analyzer measurements of the performance of two tapers with an oversize rectangular waveguide between are shown. Performance of the HC tapers is compared to the linear tapers and to the finite-element simulations of Sec. III. Finally, the performance of a 1 m oversize section with HC tapers is shown and compared to the equivalent length of WR10. We make concluding remarks in Sec. V.

II. REFLECTION AND MODE COUPLING IN A RECTANGULAR TAPER

A. Theory

As shown by Sporleder and Unger,⁹ coupling between modes can occur in a waveguide where the waveguide size varies with position along the waveguide axis. The modes can be propagating or evanescent. Reflection of a mode is equivalent to coupling to a mode with the same index propagating in the opposite direction. When the waveguide size varies continuously, the mode coupling is described by a system of coupled first order differential equations, one equation for each mode, with coefficients that are functions of the waveguide axial position z . When the amplitude of one mode is significantly larger than the others, the system

reduces to two coupled equations,

$$\frac{dA_1^p}{dz} = (-\gamma_1^p + \kappa_{11}^{pp}) A_1^p + \kappa_{12}^{pq} A_2^q \quad (1)$$

and

$$\frac{dA_2^q}{dz} = \kappa_{21}^{qp} A_1^p + (-\gamma_2^q + \kappa_{22}^{qq}) A_2^q. \quad (2)$$

In these equations, A represents the amplitude of the mode with the subscripts one corresponding to the main mode and two corresponding to a coupled mode. A subscript designates a mode and index (e.g., TE_{10}). A superscript corresponds to the direction of propagation: p and q take only values of ± 1 for propagation toward positive and negative z , respectively. In addition, γ_i^p represents the propagation constant $p\gamma_i$ with

$$\gamma_i = \sqrt{k_{ci}^2 - k^2}, \quad (3)$$

where the free space wavenumber is

$$k = \frac{\omega}{c}, \quad (4)$$

and k_{ci} is the cutoff wavenumber of mode i . For a rectangular waveguide of dimensions $2a$ (using the notation of Ref. 9; see Fig. 1) and b ,

$$k_{ci} = \sqrt{\left(\frac{m\pi}{2a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}, \quad (5)$$

where (m, n) represent the mode indices (number of half wavelength variations of the mode fields in x and y , respectively; Fig. 1). Equation (5) is valid for the TE and TM modes. The propagation constant is imaginary for modes above cutoff, $k^2 > k_c^2$ (propagating) and real for modes below cutoff (evanescent). Time harmonic fields of the form $e^{j\omega t}$ are assumed. In a uniform waveguide, the coupling coefficients, κ , are zero; the differential equations decouple; and the solution is $A \sim e^{-p\gamma z}$. For a nonuniform waveguide, Eq. (1) is valid for a wide range of waveguide types including curved and those filled with nonuniform inhomogeneous anisotropic material.⁹ For a rectangular waveguide with dimensions a and b that vary with z (see Fig. 1), the coupling coefficients are proportional to the logarithmic derivatives of a and b . Expressions for the coupling coefficients as functions of a and b are shown in Appendix A.

FIG. 1. Waveguide taper cross section and coordinate system definition. The waveguide dimensions a and b vary in z according to Eqs. (9) and (10) for the HC taper or Eqs. (B1) and (B2) for the linear taper.

B. Properties of coupled wave equations

The number of propagating modes is determined by the size of the oversize waveguide, Sec. III A. Because the coupling coefficient is an integral of the product of the main mode and the coupled mode over the waveguide perimeter (Eq. (A3)), many modes have a zero coupling coefficient. It can be shown that the coupling coefficient is zero for modes with any index that differs by an odd integer from the corresponding index in the main TE_{10} mode. These modes have spatial interference nulls in the waveguide cross section. Only second order spatial harmonics of the main mode are produced.

Influence of taper shape on the coupled wave amplitude can be understood from properties of the coefficients and solutions of Eqs. (1) and (2). In the rectangular taper,

$$\kappa_{11}^{pp} = \kappa_{22}^{qq} = 0 \quad (6)$$

and

$$\kappa_{12}^{pq} = -\kappa_{21}^{qp}, \quad (7)$$

which make the coupled system of equations anti-symmetric. Further, when the coupling to other modes is not strong, the main mode, A_1^p (TE_{10}), has a nearly constant amplitude through the taper, $A_1^p \cong e^{-\gamma_1 z}$. Under these conditions, Eqs. (1) and (2) reduce to the single differential equation for the coupled mode,

$$\frac{dA_2^q}{dz} = \kappa_{21}^{qp} e^{-\gamma_1 z} - \gamma_2^q A_2^q. \quad (8)$$

The first term on the right-hand side of Eq. (8) is a forcing term due to the main mode and causes the coupled mode to grow or decay.

Although the solution of Eq. (8) can be expressed analytically, the main characteristics of the solution are shown in Fig. 2, which contains plots of the solutions of Eq. (8) for three simple forms of κ and for constant $\gamma_1 = j1.679 \text{ mm}^{-1}$ and $\gamma_2 = j0.756 \text{ mm}^{-1}$ (representative values for the middle of the taper). The boundary condition is taken as $|A_2^q| = 0$ at $z = 0$ and propagation is toward positive z , $p = 1$. When κ is constant (Fig. 2(a)), the coupled wave $|A_2^q|$ oscillates with z and maintains a constant average value. The oscillation is a spatial beating between the main mode and the coupled mode. The beat wavenumber seen in Fig. 2(a) is the difference in wavenumbers, producing the relatively long beat wavelength of about 7 mm in the taper. This result is independent of the sign or phase of κ . At some frequencies, one half beat wavelength of coupled mode growth will exit the taper, in this case an amplitude of 0.22, which represents non-optimal taper performance.

When $|\kappa|$ monotonically increases (Fig. 2(b)), $|A_2^q|$ also monotonically increases with z . The wave growth is proportional to $|\kappa|$. When $|\kappa|$ decreases (Fig. 2(b)) after wave growth, the wave decreases in amplitude as $|\kappa|$ decreases. This is the optimum type of spatial interference since the mode amplitude exiting the waveguide is small, independent of frequency, and, therefore, broadband. Notice that the mode amplitude is 0.023, nearly 20 dB smaller than Fig. 2(a). The coupling coefficients for each part of Fig. 2 correspond to the same waveguide cross section ratio from one side of the

FIG. 2. Plots of three basic forms of coupling coefficient κ and the corresponding approximate wave solutions according to Eq. (8). Constant values of $\gamma_1 = j1.679 \text{ mm}^{-1}$ and $\gamma_2 = j0.756 \text{ mm}^{-1}$ were assumed (see discussion below Eq. (8)).

taper to the other since equal coefficient areas produce equal $b(L)/b(0)$. Finally, if $|\kappa|$ is abruptly high but monotonically decreases through the taper (Fig. 2(c)), the initial step in $|\kappa|$ creates an oscillation that decays with the decreasing $|\kappa|$, but a significant residual wave amplitude remains at the end of the taper. The amplitude is the same as 0.22 of Fig. 2(a) and is also the same for a monotonically increasing taper of the same length.

In a real taper, γ_1 and γ_2 vary through the taper and change from real when the mode is cut off to imaginary when propagating. Near the large end of the taper, the wavelengths of the main and higher order modes become close, implying that the beat wavelength becomes long. The results are more complicated, but the qualitative behavior described above persists. Based on the solution properties, the main feature of an optimum taper is a $|\kappa|$ that gradually decreases to zero on both ends in order to produce broadband spatial interference of the coupled modes along the taper.

C. Hyperbolic-cosine taper

For the HC taper, we wanted to satisfy three criteria: (1) to have both a and b vary exponentially with z ; (2) to have zero flaring (da/dz or db/dz) on each end of the taper, $z = 0$ and $z = L$; and (3) to be able to choose the axial position of maximum flare, which is also the inflection point. A suitable functional dependence is (see Fig. 1)

$$a = \begin{cases} b_W \cosh(\alpha_1 z), & 0 < z < d \\ \frac{b_Q}{2} \{2 - \cosh[\alpha_2 (z - L)]\}, & d < z < L \end{cases} \quad (9)$$

FIG. 3. (Color) Magnitude of the coupling coefficient from Eqs. (A1)–(A9) for the largest coupled and reflected modes (TE_{12} , TE_{10} , and TM_{12}) through the HC taper. Plots are shown for values of the inflection point d in Eqs. (9) and (10) of 2 mm (red), 5 mm (orange), 8 mm (green), 11 mm (blue), and 14 mm (purple) and at a frequency of 94 GHz.

and

$$b = \begin{cases} b_W \cosh(\beta_1 z), & 0 < z < d \\ 2b_Q \{2 - \cosh[\beta_2 (z - L)]\}, & d < z < L \end{cases}, \quad (10)$$

where b_W represents the short waveguide dimension of WR10, 1.27 mm; b_Q represents the short waveguide dimension of WR28, 3.556 mm; L is the taper length, which was chosen to be 76.2 mm; and d represents the position of maximum flare in the taper. The constants α_1 , α_2 , β_1 , and β_2 are functions of b_W , b_Q , L , and d and were determined by making a , b , $\frac{da}{dz}$, and $\frac{db}{dz}$ continuous at d . In Eqs. (9) and (10), the short dimension of WR10 goes into the long dimension of WR28. This orientation minimizes Ohmic losses in the over-size waveguide (Sec. III).

When Eqs. (9) and (10) are substituted into Eqs. (A2)–(A8) and Eq. (5), γ and κ become explicit functions of z . Shown in Fig. 3 are plots of the magnitude of the coupling coefficients (Eq. (A1)) for modes that have the highest excitation from the TE_{10} main mode, the TE_{10} reflection, the TE_{12} , and the TM_{12} . The coupling coefficients are zero at each end of the taper (because the flaring is zero) and are peaked at the point of inflection. In addition, the coupling coefficient for the TE_{12} has a dip at cutoff and the TM_{12} an infinity. The coefficients are shown for values of the point of inflection varying between 2 and 14 mm and at a frequency of 94 GHz.

FIG. 4. (Color) Magnitude of the coupled and reflected modes predicted by Eqs. (1) and (2) corresponding to the coupling coefficients of Fig. 3 through the HC taper.

Shown in Fig. 4 are the wave solutions corresponding to the coefficients of Fig. 3. The coupled equations (1) and (2) were solved numerically using Mathematica (version 8, Wolfram, Champaign, IL). All modes exhibit a peak excitation near the inflection point and decay toward the ends of the taper. The decay is caused by spatial interference between the coupled mode and the main mode and the decay of the coupling coefficients at the taper ends, Eq. (8) discussion.

D. Optimization of inflection point

Reflection and transmission coefficients vary with frequency and also with the position of the inflection point d . The strategy of the placement of the inflection point can be summarized as follows. Placing d near the small end of the taper ($z = 0$) minimizes the coupling coefficients for the higher order modes where they propagate on the opposite side. This is because the derivatives of a and b are minimized there. However, this will also increase the reflection coefficient for the main mode as it enters from the small end because of the increase of derivatives of a and b . The optimal value of d is a compromise between these competing effects. The inflection point tends to be optimum near the small end of the taper because only a few wavelengths are required to null the reflection coefficient.

This behavior can be seen in the reflection and transmission coefficients shown in Table I, which are obtained from

TABLE I. Taper single-pass reflection and transmission coefficients.

Type	Coupled mode	d (mm)	R or T (dB)		
			92 GHz	94 GHz	96 GHz
Linear	TE ₁₀ (R)	...	−33.9	−33.1	−34.0
	TE ₁₂ (T)	...	−27.9	−27.6	−27.3
	TM ₁₂ (T)	...	−34.8	−35.1	−35.3
HC	TE ₁₀ (R)	2	−49.7	−56.0	−75.2
		5	−46.1	−45.9	−46.4
		8	−53.9	−62.8	−65.2
		9.72	−52.1	−56.0	−67.5
		11	−56.1	−53.5	−54.3
		14	−56.8	−68.2	−62.6
	TE ₁₂ (T)	2	−47.8	−47.2	−46.6
		5	−47.4	−46.8	−46.2
		8	−47.0	−46.4	−45.8
		9.72	−46.7	−46.1	−45.5
		11	−46.5	−45.9	−45.3
		14	−46.0	−45.4	−44.8
	TM ₁₂ (T)	2	−54.7	−54.7	−54.6
		5	−54.4	−54.3	−54.2
		8	−54.0	−53.9	−53.9
		9.72	−53.7	−53.7	−53.6
		11	−53.5	−53.5	−53.4
		14	−53.1	−53.0	−53.0

numerical solutions of Eqs. (1) and (2). An optimum value for d can be chosen for a band of frequencies from this data. It can be seen that the transmission coefficient of the TM₁₂ mode is about 7 dB below the TE₁₂ over the entire range of d values and frequencies. Also, the transmission coefficients decrease slowly and monotonically with decreasing d . They are also slightly higher at the high end of the frequency band. On the other hand, the reflection coefficient of the TE₁₀ mode shows oscillations with d and frequency. If one wants the reflection coefficient and the transmission coefficients to be of similar magnitude and also minimized, one would choose $d = 2$ mm. However, at this value, the reflection coefficient varies by 25 dB over the frequency band. This is because there are very few wavelengths between the point of maximum reflection, $z = 2$ mm, and the short end of the waveguide, $z = 0$. This gives little space for interference. By choosing $d = 8$ mm, a decrease of 4 dB is obtained in the reflection coefficient over the frequency band at an expense of less than 1 dB in the TE₁₂ transmission coefficient. The values are −54 dB reflection and −46 dB transmission. Also shown in Table I are results for $d = 9.72$ mm, the dimension picked for fabrication. This value is obtained by scaling the waveguide cross sections on each side of the taper, $d = Lb_w^2/b_Q^2$.

Coupled mode amplitudes as a function of frequency were also calculated for a linear taper as modeled in Appendix B. Results can be compared to the HC taper in Table I.

E. Comparison of theoretical results to predictions of Ansoft high frequency structure simulator

The HC taper with $d = 9.72$ mm and the linear taper were modeled by the finite-element computer program Ansoft

high frequency structure simulator (HFSS) (version 13, Pittsburgh, PA) running on a Dell Precision 690 workstation with 64-bit Intel Xeon dual dual-core processors running at 3 GHz with 6 MB of L3 cache per chip and 16 GB of random access memory (RAM). The HC taper was created in Ansoft HFSS by starting with Eqs. (9) and (10) to define the edges of the taper as lines. The lines were modeled in Mathematica as a three-dimensional list of position points in space. This edges list was exported as a data file and then imported into AutoDesk Inventor 2010, where a built-in feature called “loft” was used to create a smooth body. Using the edges as guides, one end of the taper was lofted to the other to produce a smooth mandrel volume. This mandrel was then exported as a “step” file.

After the mandrel was imported into Ansoft, it was found that the default meshing scheme did not adequately fill the volume because of the smooth boundaries. To solve this problem, the mandrel was split in Ansoft into 20 subsections, 10 sections on each side of the point of inflection d . The meshing scheme effectively treated these split sections as 20 linear tapers butted end-to-end.

A 10 mm length of waveguide was added to each end of each taper in order to minimize evanescent modes at the port excitation.¹⁰ The conducting boundaries were chosen to be the perfect electric conductor in order to compare with the theory, which assumes infinite conductivity. To reduce computation time, the structure was split and a symmetry electric field boundary (tangential electric field zero) was placed at $y = b/2$ (Fig. 1). This reduced the meshing volume by a factor of two. Thirteen passes were performed on the HC taper and ten on the linear. The HC taper solution had an error in the S parameter magnitude of 0.004 and the linear of 0.0008.

Because the split WR28 waveguide was square, it was found that the TE_{02} mode numerically coupled to the main TE_{10} mode in Ansoft HFSS. According to the theory, these modes do not couple because $\kappa = 0$. By adding $1\mu\text{m}$ to b in the Ansoft simulations of both tapers, the coupling to this mode was reduced to below -70 dB in the HC and -80 dB in the linear taper. This small dimensional change does not influence the reported results significantly. Other modes with theoretically zero coupling, such as the TE_{04} , were also observed to have very low transmission coefficients in Ansoft HFSS and therefore are not reported.

The transmission and reflection coefficients for the modes with the highest coupling are shown in Table II. Also shown for comparison are the corresponding theoretical single-pass values from Table I. It can be seen that the overall behavior of the HC and linear tapers is consistent between the two simulations. However, the coefficients are different by 0–11 dB for the HC taper and 5–6 dB for the linear. A reason for the differences is that the Ansoft HFSS simulations have coupling between all modes simultaneously and also permit propagation in both directions in the taper and waveguide (positive and negative z). The single-pass values only account for coupling between the main mode and one coupled mode propagating in one direction. From Table II, it can be seen that the transmission coefficient for the TM_{12} mode predicted by Ansoft HFSS is 8 dB higher than the TE_{12} mode for the HC taper and 3 dB higher for the linear taper, unlike the

TABLE II. Taper reflection and transmission coefficients predicted by Ansoft HFSS at 94 GHz.

Type	Coupled mode	$d(\text{mm})$	R or $T(\text{dB})$	
			Ansoft	Theory ^a
Linear	TE_{10} (T)	...	−0.007	...
	TE_{10} (R)		−39	−33
	TE_{12} (T)		−33	−28
	TM_{12} (T)		−30	−35
HC	TE_{10} (T)	9.72	−0.0001	...
	TE_{10} (R)		−56	−56
	TE_{12} (T)		−57	−46
	TM_{12} (T)		−49	−54

^aFrom Table I.

single-pass theory. Backward coupling of the TE_{12} and TM_{12} modes to the main mode, reflection from cutoff, and subsequent forward coupling can account for the difference. Although the coupling coefficients between the TE_{12} and TM_{12} modes are of comparable magnitude to those shown in Figs. 3 and 9, the influence of this coupling is weak compared to the main mode because of the weak TE_{12} and TM_{12} mode amplitudes.

Also from Table II, the reflection coefficient of the main mode predicted by Ansoft HFSS for the linear taper is 6 dB lower than the single-pass theory, whereas they are the same for the HC taper. Because the single-pass theory accurately models the reflected mode coupling, we expect the reflection coefficients of the single-pass theory to match the Ansoft values. This is true for the HC taper but not for the linear. Investigation reveals that this difference is due to the fact that the single-pass coupled wave equations are only accurate to first order changes in the waveguide cross section with respect to the axial position dz .⁹ Second order terms are small throughout the HC taper because of the smooth, continuous transition in cross section from the WR10 to WR28. However, for the linear taper, the second derivative of the cross section is singular at each end because of the abrupt change in slope. It was verified that by smoothing the edge at each end of the taper using a tangent arc (fillet), the reflection coefficient for the linear taper was reduced below the value reported in Table II. This was done by appropriately modifying Eqs. (B1) and (B2), recalculating the coupling coefficients and solving the modified system of equations. Due to dimensional differences, the axial size of the fillet on the b dimension is larger than for a (see Fig. 1 and Eqs. (B1) and (B2)) for a given radius. When the axial fillet size on b was increased to ~ 1 mm (with a corresponding radius of 50 mm), the single-pass reflection coefficients for the linear taper matched the Ansoft predictions. This fillet size satisfies

$$\Delta z \frac{d^2 b}{dz^2} \lesssim \frac{db}{dz}, \quad (11)$$

which is the condition that must be satisfied to neglect second order terms in dz .

According to Table II, the overall performance of both tapers according to Ansoft HFSS is 3 dB better than predicted by the single-pass theory.

III. THEORETICAL BEHAVIOR OF OVERSIZE WAVEGUIDE BETWEEN TAPERS

A. Attenuation and higher order modes

Ohmic losses are reduced in oversize waveguide because the surface currents are reduced at the walls. As shown in Table III, theoretical Ohmic copper waveguide loss is 2.60 dB/m for TE₁₀ in WR10 and 0.52 dB/m for TE₁₀ (see Table III footnote) in WR28. Calculations were done using Eqs. (9)–(11) of Sec. 8.02 of Ref. 2. (Equation (10) of Ref. 1 cannot be used, if any index is zero.) Consequently, for a 1-m run, there is a 4.2 dB theoretical maximum decrease in insertion loss, there and back, in the substitution of WR10 with WR28. The TE₁₀ is the lowest order mode in WR10, but is *not* the lowest order mode in WR28 when the long side of WR28 is oriented parallel to the short side of WR10. However, as shown in Table III, this mode does have the lowest attenuation of any mode in WR28. Higher order propagating modes can exist in WR28 at W-band. These include the following TE and TM modes: 01, 02, 03, 04, 10, 11, 12, 13, 14, 20, 21, and 22, of which only the 12, 14, and TE₁₀ are nonorthogonal in the cross section (Sec. II B). Attenuation constants for higher order modes with the highest excitation levels are shown in Table III.

B. Resonances

Because only the TE₁₀ mode propagates in WR10 and because this mode continues as the main mode in the WR28, other modes excited by the tapers are cut off in the WR10 and therefore reflect back and forth between the two tapers. The region between the tapers can act like a large high- Q cavity for these modes. If the reflection of the higher order mode has a different phase than the coupled higher order mode, destructive interference of the mode occurs. This is analogous to driving a cavity off-resonance and is true at most frequencies. However, at certain frequencies, the phases match and the reflected mode adds constructively to itself each time it reflects from cutoff in a taper. The coupled wave amplitude in the oversize waveguide is larger at this frequency and can be expressed by

$$A(\text{dB}) \cong 20 \log_{10} [10^{T/20} (1 + 10^{-\alpha Z/20} + 10^{-2\alpha Z/20} + \dots)] \\ = 20 \log_{10} \left(\frac{10^{T/20}}{1 - 10^{-\alpha Z/20}} \right), \quad (12)$$

where T is the single-pass transmission coefficient of the coupled mode in the taper (Table I), α is the attenuation per

TABLE III. Wavelengths and attenuation lengths for Cu waveguide at 94 GHz.

Waveguide	Mode (Ref. 17)	Wavelength (mm)	Attenuation (dB/m)
WR10	TE ₁₀	4.10	2.60
WR28	TE ₁₀	3.57	0.523
	TE ₀₁	3.27	0.559
	TE ₁₂	4.12	1.41
	TM ₁₂	4.12	1.01

unit length of the coupled mode in the oversize waveguide (Table III), and Z is the distance between tapers. For example, for the linear taper, the amplitude of the TE₁₂ mode is

$$A(\text{dB}) \cong 20 \log_{10} \left(\frac{10^{-27.3/20}}{1 - 10^{-1.41/20}} \right) = -10.8 \text{ dB}, \quad (13)$$

and for the HC taper,

$$A(\text{dB}) \cong 20 \log_{10} \left(\frac{10^{-45.5/20}}{1 - 10^{-1.41/20}} \right) = -29.0 \text{ dB}. \quad (14)$$

This resonance effect produces comb-like dips in the transmission coefficient of the main mode as a function of frequency (see also Sec. III C below). As the length of WR28 increases, the more closely spaced in frequency these resonances become. The size of the fluctuations in the transmission coefficient can be estimated from the mode amplitude of Eq. (12) by conservation of power,

$$\Delta T_{\text{main}}(\text{dB}) \cong 20 \log_{10} \sqrt{1 - (10^{A/20})^2} \\ = 10 \log_{10} (1 - 10^{A/10}). \quad (15)$$

For the linear taper, we obtain $\Delta T_{\text{main}} \cong -0.38 \text{ dB}$, and for the HC taper, $\Delta T_{\text{main}} \cong -0.0055 \text{ dB}$. These correspond to S parameters, which are amplitude ratios. This model suggests that a higher order mode grows incrementally, taking power from the main mode each time it passes through a taper. However, destructive interference prevents the mode amplitude from growing unless there is constructive interference between tapers. Then the mode amplitude is limited by the Ohmic loss between the tapers. The more passes it makes, the more power is removed from the main mode, making the dips in T deeper. This is confirmed by Ansoft simulations and measurements (see also Sec. IV A). We find that this calculation tends to overestimate the size of the transmission coefficient dips because the higher order modes are more strongly attenuated near cutoff. This higher attenuation is not modeled in the calculation.

The reflection of the main mode from the tapers does not influence the transmission coefficient of the main mode in a way similar to that described above because the reflected mode does not go through cutoff and makes only a single pass through the oversize section.

C. Ansoft HFSS simulations

In the oversize section, a higher order mode is observed only during one of the resonances described in Sec. III B. Such behavior is shown in the electric field magnitude plots for a taper pair with 132 mm of WR28 waveguide between, see Fig. 5. Figure 5(a) is for a pair of linear tapers and a frequency where there is no resonance of a higher order mode. Here, the mode has the same form in WR28 as in WR10 with reduced field strength due to the larger size. Due to the large number of wavelengths in the structure, the simulations require a large amount of RAM. The 16 GB of available computer memory constrained the simulations to be done for the relatively short 132 mm section of WR28. Figure 5(b) shows the same simulation at a different frequency, where there is a higher order mode resonance. Between the tapers, the electric

FIG. 5. (Color) Ansoft HFSS simulation of the electric field magnitude in a pair of tapers with a 132 mm length of WR28 waveguide between. Only the left side is shown in order to view the field profile more clearly. (a) Brass linear tapers off resonance, 94 GHz. (b) Brass linear tapers on resonance, 95.048 GHz, transmission coefficient 0.7 dB below non-resonance. (c) Cu HC tapers off resonance, 94.1 GHz. (d) Cu HC tapers on resonance, 95.694 GHz transmission coefficient 0.015 dB below non-resonance. See also Secs. III B, III C, and IV C.

field magnitude is a mixture of the propagating TE_{10} and the standing higher order modes. The amplitude of the higher order mode is larger than the main mode in the oversize waveguide. Dimensions and materials chosen for the simulation match those discussed in Sec. IV A. No offsets or imperfections of the WR28 waveguide at the flanges were included in the simulation. The solution method was driven mode, with only the fundamental mode, the TE_{10} , on each end of the structure. A discrete frequency sweep was done over the frequency range 92–96 GHz with 1 MHz steps. The simulation took about 36 h. The transmission coefficient corresponding to this simulation is shown in Fig. 6(a). For non-resonance, the transmission coefficient is flat at about -0.39 dB, and where there is a resonance, there is a dip. The dips vary in size from about 0.2–0.7 dB.

Figures 5(c) and 5(d) show the same simulation as Figs. 5(a) and 5(b), except with HC tapers. Dimensions and materials used in the simulation match those discussed in Sec. IV A. The simulation included a $5\text{ }\mu\text{m}$ symmetrical step offset at each of the WR28 waveguide flanges. The taper was $5\text{ }\mu\text{m}$ larger than the WR28 waveguide section. The setup was similar as for the linear tapers except that in order for the simulation to detect the small step we configured the initial mesh to permit 180° vertices in the tetrahedral faces. The amplitude of the higher order mode resonance of Fig. 5(d) is considerably smaller than for the linear taper. Scaling of the electric field is the same for all plots shown in Fig. 5. The largest resonance in the frequency range 92–96 GHz was chosen for Figs. 5(b) and 5(d). The corresponding transmission and reflection coefficients are shown in Fig. 6. Notice that the average level of attenuation is smaller, -0.28 dB, and the dips are considerably smaller, about 0.01 dB compared to the linear taper simulations. The reflection coefficients vary strongly with resonances of higher order modes. Each resonance produces a peak and dip in R . The peaks in R determine the overall quality of the reflection coefficient. According to Fig. 6(c), the peak reflection coefficient of the HC tapers is -38.7 dB, which is considerably lower than for the linear tapers, -22.6 dB, Fig. 6(b). Also shown in Fig. 6 are the measured T and R , which are further discussed in Sec. IV C.

FIG. 6. (Color) (a) Transmission and (b) and (c) reflection coefficients of a pair of tapers with a 132 mm length of WR28 waveguide between. The colored lines show network analyzer measurements (Sec. IV C), and the black lines indicate Ansoft HFSS simulations (Sec. III C). Blue indicates HC tapers and red linear tapers. The measurement averages and standard deviations are (a) -0.105 ± 0.015 dB for HC and -0.68 ± 0.16 dB for linear, and (b, c) -37.1 ± 5.6 dB for HC and -32.1 ± 6.7 dB for linear. The comb-like dips in T due to resonance of higher order modes between the tapers are less than -0.03 dB for HC compared to -0.8 dB for linear tapers.

IV. FABRICATION AND CHARACTERIZATION

A. Taper fabrication

Two pair of HC tapers were electroformed by A. J. Tuck Company, Brookfield, CT. First, we used Autodesk Inventor to produce initial graphics exchange specification (IGES) files of the HC taper (see step file description in Sec. II E), which were sent to Tuck's engineering staff. The objects were converted to G-code for computer-controlled fabrication of the mandrel. The taper consists of an interior curved part 76.2 mm long and 10 mm straight rectangular ends. An aluminum mandrel was precision machined from the IGES files. Copper was electroplated onto the mandrel to create the taper walls. Finally, aluminum was chemically dissolved, leaving only copper. End sections were attached and the product was shipped back to our laboratory. The ends of the tapers were subsequently machined flat using electric

FIG. 7. (Color) Photographs of the HC taper pair. (a) End detail showing, from left to right, UG-599 WR28 flange (Ref. 12), custom high-precision WR28 flange (Sec. IV B), ALMA WR10 flat flange (Ref. 11). (b) Assembled taper pair with 132 mm WR28 oversized waveguide section.

discharge machining (EDM). The interior of each taper was cleaned and deburred. Waveguide flanges were made according to the ALMA WR10 flat flange¹¹ specification or the standard UG-599 WR28 flange¹² specification. Alignment of the WR28 flanges was accomplished by specially designed high-precision flanges on the waveguide section as discussed in Sec. IV B. They are shown in Fig. 7.

A pair of linear tapers was fabricated by first grinding a pyramidal electrode out of a tungsten copper alloy. On each end of the electrode, the appropriate size interior waveguide cross section was made. A round hole was bored into a solid 3/4 in square bar from each end. The purpose of the hole was to permit flushing of material during subsequent machining. Then the electrode was plunged into the bar, and material removed by EDM. The ends of the bar were cut flat by EDM and shaped according to the appropriate flange specification.^{11,12} The interior of the taper was cleaned with TarnexTM. The finished tapers were a total of 72.4 mm in length, which included 2.0 mm of straight WR10 on one end and 1.8 mm of straight WR28 on the other. Brass 360 was used to fabricate the linear tapers because the material is easy to machine by EDM. The brass tapers were used for comparison to the HC tapers only and not for EPR due to the higher loss. Ansoft HFSS simulations of copper linear tapers with 132 mm WR28 between, Fig. 6(a), show a reduced insertion loss of 0.157 dB compared to brass. At resonance, the sizes of the dips in the transmission coefficient are 0.348 dB *larger* because the reduced loss produces a larger amplitude of the higher order modes, consistent with the resonance theory of Sec. III B.

B. High-precision flange fabrication

It was found that the UG-599 WR28 flanges¹² were not aligned well enough to characterize the HC taper perfor-

mance. Positional tolerance for the alignment pins of these flanges is $\pm 50 \mu\text{m}$. This is usually not a problem when the waveguide supports only the fundamental propagating mode. However, when the waveguide is overmoded, the mismatch produces propagating higher order modes that can strongly influence the transmission and reflection coefficients. Measurements and simulations indicate that mismatch between WR28 flanges should be less than $5 \mu\text{m}$ to produce higher order mode amplitudes comparable to the HC taper itself. To accomplish this, a special high-precision flange was designed and fabricated by J.R.A. as shown in Fig. 7. The flanges were made in brass 360. Since the brass is not in the presence of the fields, the additional loss is not a performance factor. The flange surrounds the UG-599 flange (with no pins) on five sides. Twelve silver-tipped set screws, three on each flange edge, permit alignment adjustment in two dimensions. Once alignment is made, four other screws butt the flanges together and form a permanent fixture once tightened. Although there are 12 set screws, alignment is straightforward. Alignment of the interior WR28 waveguide faces was first done optically, then further optimized on the bench by minimizing the resonance-produced dips in the transmission coefficient seen on the network analyzer. Adjustments were repeatable. For the HC taper with the 132 mm WR28 straight section, the alignment between interior waveguide faces was estimated to be about $5 \mu\text{m}$. This cannot be decreased further because the difference in corresponding waveguide dimension between mating parts is $10 \mu\text{m}$ in both the short and long dimensions. This is smaller than the standard WR28 waveguide interior dimensional tolerance of $\pm 38 \mu\text{m}$.

For the linear tapers with 132 mm WR28, adjustment of the flanges after optical alignment made the dips in the transmission coefficient larger. This confirms that resonances are produced by the tapers themselves, and these resonances are larger than those produced by misalignment of the waveguide faces at the waveguide flanges.

C. Measurements

Measurements of the transmission and reflection coefficients of tapers with 132 mm WR28 between are shown in Fig. 6, with comparison to the corresponding Ansoft HFSS simulations (Sec. III C). Measurements were done using an Agilent Technologies N5242A PNA-X network analyzer with corresponding millimeter wave controller and two V10VNA2 T/R 75–110 GHz millimeter wave VNA extenders. Measurements compare favorably with the simulations, but with significant differences. First, the overall insertion loss of the HC tapers is 0.2 dB less than the simulation, a result that was repeated after network analyzer recalibrations. The ± 0.04 dB sinusoidal pattern in the measured transmission coefficient is likely due to mismatch in the WR10 flanges since the pattern is seen in measurements of sections of WR10. The small $\cong 0.03$ dB dips in the transmission coefficient are caused by higher order mode resonances and are seen in both the simulation and the measurements. The half-width of these resonances is about 10 MHz and independent of oversized waveguide length. The measured transmission coefficient for

FIG. 8. (Color) Transmission and reflection coefficient comparison between 96 mm HC tapers with 711 mm section of WR28 (blue) and 902 mm section of WR10 (red). (a) T . (b) R . (c) R with slightly detuned LGR (Ref. 8) with 38 mm section of WR10. The measurement averages and standard deviations are (a) -0.44 ± 0.02 dB for WR28 and -2.80 ± 0.03 dB for WR10, and (b) -38.5 ± 5.1 dB for WR28 and -40.4 ± 4.8 dB for WR10.

the linear tapers show an overall 0.2 dB larger insertion loss compared to the simulation. The sizes of the larger dips in transmission coefficient are of similar magnitude and similar spacing in frequency as the simulation. However, there is an overall sinusoidal pattern of ± 0.1 dB due to the WR10 flanges and not simulated.

The overall measured reflection coefficients compare favorably to simulations. The linear tapers show stronger peaks and dips than the HC. The maximum measured reflection is -20.5 dB for the linear tapers and -29.6 dB for the HC. A reflection coefficient below -30 dB meets the requirements of our spectrometer.

Finally, measurements for the 1 m section of WR28 are shown in Fig. 8 and compared to a similar length of WR10. The overall insertion loss of the oversize section is about 0.5 dB, nearly six times smaller than the WR10. Sinusoidal fluctuations in T caused by misalignment of the WR10 flanges is apparent in the WR10 curve. Similar fluctuations

occur in the oversize section, but there are also sharp dips caused by higher order mode resonances, which do not occur in the WR10. Fluctuations in the oversize section caused by flange misalignment and higher order mode resonances are no worse than for the straight section of WR10. The overall insertion loss of each section compares favorably to the attenuation numbers shown in Table III. In Fig. 8(b), the reflection coefficients follow similar behavior. The WR10 is slightly better, but overall performance is less than -30 dB. Averages and standard deviations of the measured transmission and reflection coefficients shown in Figs. 6 and 8 are given in the captions. In Fig. 8(c), reflection coefficients with a slightly detuned LGR (Ref. 8) on the end of a 1 m section are shown. Performance of the oversize section is significantly better than the WR10. Two-way insertion loss is nearly 5 dB less and the overall fluctuation level is smaller, achieving the theoretical goal of 4.2 dB (Sec. III A) based on the attenuation lengths shown in Table II. The resonance-produced dips are attenuated as the frequency goes through resonance. The LGR resonance dip is deeper.

V. CONCLUSIONS

In this paper, we have shown that return loss for a 1 m length of waveguide can be reduced by nearly 5 dB and the reflection coefficient kept below -30 dB over a 4% bandwidth by using a pair of novel HC tapers and commercially available WR28 oversize waveguide at W-band. For minimum loss, the main mode electric field must run parallel to the long side of the WR28. The HC shape permitted the optimization of the position of maximum flare (inflection point) in the taper with respect to the coupling to higher order modes and the reflection of the main mode. The optimum inflection point position is about 12% of the taper length from the small end of the taper. Reflection and coupling were reduced by about 20 dB over a linear taper of the same length. Comb-like dips in the transmission coefficient produced by resonances of the higher order modes were about 0.03 dB. Specially designed high-precision adjustable WR28 flanges with alignment to about $5 \mu\text{m}$ were required to keep higher order mode amplitudes from the flanges comparable to those from the HC tapers. Alignment precision is nearly ten times smaller than the interior dimensional tolerance of standard WR28 waveguide. EDM has sufficient accuracy to make the taper mandrels, but not standard machining methods.

Further optimization of the taper might be possible by an exhaustive exploration of parameter space. However, increasing the taper length would also increase insertion loss. In the present design, we chose to make the inflection point positions of each pair of sides of the taper the same. This is because the dominant contribution to the coupling coefficients comes from the change in cross section from only one pair, the WR10 short side to the WR28 long side. Therefore, little optimization can be done by separating the inflection points of the two pairs of sides. The paper provides a foundation for further optimization, if needed. Methods are not specific to W-band and can be applied to other frequency ranges.

We also have experience with the TE₀₁ mode in circular oversize waveguide at W-band.¹³ A mode converter was designed using Ansoft HFSS and found to be relatively narrow band compared to the present design. The transmission coefficient was not flat over any range of frequencies. Spurious mode generation was high compared to the present design. The mode converter was complicated, and fabrication was attempted. The response was found to be too sensitive to machining tolerances to be practical.

The broadband spurious mode generation in the present design is much lower than when corrugated waveguide is used for the oversize section. Resonance-produced dips in the transmission coefficient as a function of frequency are typically about 2 dB for a straight section of corrugated guide with two mode converters on each end; this compares to 0.03 dB for the present design. Most experiments using corrugated waveguide are done at high power and are not sensitive to small variations in reflected power.¹

There are a number of ways in which a more level response in reflected voltage as a function of microwave frequency is of benefit in W-band CW EPR spectroscopy. An immediate and practical benefit is improved performance of the automatic frequency control (AFC) system, particularly when using LGRs, which have a relatively low Q -value. It is important that the reactivity of the resonator dominates other sources of reactivity in the microwave bridge if the AFC system is expected to lock the microwave frequency to the resonant frequency of the LGR.

Two types of single-arm EPR spectroscopy have been investigated by us: replacing the normal sinusoidal magnetic field modulation by the equivalent microwave frequency modulation and replacing the normal sweep of the magnetic field across the EPR spectrum by sweep of the equivalent microwave frequency across the spectrum with the magnetic field held constant. In both applications, one must consider the real and imaginary parts of the reflection coefficient of the LGR, which necessarily vary slowly across the W-band spectrum. It is expected that these classes of experiments will benefit from the methods of this paper.

Pulse saturation recovery experiments at W-band have been reported from the author's laboratory. It is customary in these experiments to collect data both at EPR resonance and off resonance. Although we have not yet compared the alternatives of stepping off resonance by frequency or by field jumps in order to suppress spurious transient response, the methods of this paper will be useful in the frequency-jump alternative. The higher power at the

sample that is available using the methods of this paper is also advantageous.

There are several multiarm EPR experiments where manipulation of the incident microwave frequencies is advantageous compared with magnetic field sweeps, modulations, or jumps. A principal benefit of microwave frequency manipulation in these double-resonance experiments compared with magnetic field manipulation is that microwave frequencies and levels in each arm can be independently varied. With magnetic field manipulation, all resonance conditions are affected. These experiments include frequency-swept electron-electron double resonance (ELDOR),^{14,15} multi-quantum ELDOR,¹⁶ and pulse ELDOR. Each of these methods benefits from decrease of loss in the incident microwave fields as well the reflected signal, and each is further benefited by a more level frequency response curve.

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APPENDIX A: COUPLING COEFFICIENTS

For a rectangular waveguide filled with vacuum (air) with the dimensions a and b that vary with z , see Fig. 1, the coupling coefficients in Eqs. (1) and (2) are given by⁹

$$\kappa_{ij}^{pq} = \begin{cases} \frac{1}{2} \left(C_{ij} \sqrt{\frac{Z_j}{Z_i}} - pq C_{ji} \sqrt{\frac{Z_i}{Z_j}} \right), & i \neq j \\ C_{ii} - \frac{1}{\sqrt{Z_i}} \frac{d\sqrt{Z_i}}{dz}, & i = j, p \neq q \\ 0, & i = j, p = q \end{cases} \quad (\text{A1})$$

where the wave impedance $Z_i = (j\omega\mu_0/\gamma_i)$ for TE modes and $Z_i = (\gamma_i/j\omega\epsilon_0)$ for TM modes. As indicated in Sec. II A, the subscript i or j refers to the mode type (TE or TM) and mode index. Therefore, for example, the top equation in Eq. (A1) holds for TE₁₀-TM₁₀ coupling. In these expressions, μ_0 represents the magnetic permeability of free space, ϵ_0 is the electric permittivity of free space, and γ_i is the propagation constant given by Eq. (3). The C values in Eq. (A1) depend on whether the modes are TE or TM. In Eqs. (A2) and (A3), the top equation is true for unequal mode indices and the bottom equation holds for equal mode indices. They are given by, for TE-TE coupling,

$$C_{ij} = \begin{cases} \frac{k_{cj}^2}{k_{ci}^2 - k_{cj}^2} \left[\int_0^b dy \frac{da}{dz} \left(T_j \frac{d^2 T_i}{dx^2} \right)_{x \rightarrow a} - \int_b^0 dy \frac{da}{dz} \left(T_j \frac{d^2 T_i}{dx^2} \right)_{x \rightarrow -a} + \int_{-a}^a dx \frac{db}{dz} \left(T_j \frac{d^2 T_i}{dy^2} \right)_{y \rightarrow 0} - \int_a^{-a} dx \frac{db}{dz} \left(T_j \frac{d^2 T_i}{dy^2} \right)_{y \rightarrow b} \right] \\ - \frac{1}{2} \left[\int_0^b dy \frac{da}{dz} \left(\frac{dT_i}{dy} \right)_{x \rightarrow a}^2 - \int_b^0 dy \frac{da}{dz} \left(\frac{dT_i}{dy} \right)_{x \rightarrow -a}^2 + \int_{-a}^a dx \frac{db}{dz} \left(\frac{dT_i}{dx} \right)_{y \rightarrow 0}^2 - \int_a^{-a} dx \frac{db}{dz} \left(\frac{dT_i}{dx} \right)_{y \rightarrow b}^2 \right] \end{cases}; \quad (\text{A2})$$

for TM-TM coupling,

$$C_{ij} = \begin{cases} \frac{k_{ci}^2}{k_{cj}^2 - k_{ci}^2} \left[\int_0^b dy \frac{da}{dz} \left(\frac{dT_i}{dx} \frac{dT_j}{dx} \right)_{x \rightarrow a} - \int_b^0 dy \frac{da}{dz} \left(\frac{dT_i}{dx} \frac{dT_j}{dx} \right)_{x \rightarrow -a} + \int_{-a}^a dx \frac{db}{dz} \left(\frac{dT_i}{dy} \frac{dT_j}{dy} \right)_{y \rightarrow 0} - \int_a^{-a} dx \frac{db}{dz} \left(\frac{dT_i}{dy} \frac{dT_j}{dy} \right)_{y \rightarrow b} \right] \\ - \frac{1}{2} \left[\int_0^b dy \frac{da}{dz} \left(\frac{dT_i}{dx} \right)^2_{x \rightarrow a} - \int_b^0 dy \frac{da}{dz} \left(\frac{dT_i}{dx} \right)^2_{x \rightarrow -a} + \int_{-a}^a dx \frac{db}{dz} \left(\frac{dT_i}{dy} \right)^2_{y \rightarrow 0} - \int_a^{-a} dx \frac{db}{dz} \left(\frac{dT_i}{dy} \right)^2_{y \rightarrow b} \right] \end{cases}; \quad (\text{A3})$$

for TM-TE coupling,

$$C_{ij} = - \int_0^b dy \frac{da}{dz} \left(\frac{dT_i}{dx} \frac{dT_j}{dy} \right)_{x \rightarrow a} + \int_b^0 dy \frac{da}{dz} \left(\frac{dT_i}{dx} \frac{dT_j}{dy} \right)_{x \rightarrow -a} + \int_{-a}^a dx \frac{db}{dz} \left(\frac{dT_i}{dy} \frac{dT_j}{dx} \right)_{y \rightarrow 0} - \int_a^{-a} dx \frac{db}{dz} \left(\frac{dT_i}{dy} \frac{dT_j}{dx} \right)_{y \rightarrow b}; \quad (\text{A4})$$

and for TE-TM coupling,

$$C_{ij} = C_{ji} \quad \text{of Eq. (A4)}. \quad (\text{A5})$$

In Eq. (A4), the index i corresponds to TM and j to TE. In Eqs. (A2)–(A5), the normalized scalar potential T satisfies the transverse Helmholtz wave equation

$$\nabla_{\perp}^2 T_i + k_{ci}^2 = 0, \quad (\text{A6})$$

where k_{ci} are given by Eq. (5) and the transverse Laplacian $\nabla_{\perp}^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$. For the rectangular waveguide, they are given by, for TE_{mn} modes,

$$T_i = \frac{1}{\pi} \left[\frac{2ab\epsilon_m\epsilon_n}{(mb)^2 + (2na)^2} \right]^{1/2} \cos \frac{n\pi}{b} y \times \begin{cases} (-1)^{\frac{m-1}{2}} \sin \frac{m\pi}{2a} x & m \text{ odd}, \\ (-1)^{\frac{m}{2}} \cos \frac{m\pi}{2a} x & m \text{ even} \end{cases} \quad (\text{A7})$$

and for TM_{mn} modes,

$$T_i = \frac{2}{\pi} \left[\frac{2ab}{(mb)^2 + (2na)^2} \right]^{1/2} \sin \frac{n\pi}{b} y \times \begin{cases} \cos \frac{m\pi}{2a} x & m \text{ odd} \\ \sin \frac{m\pi}{2a} x & m \text{ even} \end{cases}. \quad (\text{A8})$$

In Eq. (A7),

$$\epsilon_m = \begin{cases} 0 & m = 0 \\ 2 & m \neq 0 \end{cases}. \quad (\text{A9})$$

APPENDIX B: LINEAR TAPER

For the linear taper, the waveguide dimensions vary linearly with axial position according to

$$a = b_W + (b_Q/2 - b_W)z/L \quad (\text{B1})$$

and

$$b = b_W + (2b_Q - b_W)z/L, \quad (\text{B2})$$

where b_W represents the short waveguide dimension of WR10, 1.27 mm; b_Q represents the short waveguide dimension of WR28, 3.556 mm; and L is the taper length, 76.2 mm. In Eqs. (B1) and (B2), the short dimension of WR10 goes into the long dimension of WR28, which minimizes Ohmic

FIG. 9. (Color) Coupling coefficients and corresponding wave amplitudes in linear taper. (a) Magnitude of the coupling coefficient from Eqs. (A1)–(A9) for the largest coupled and reflected modes (TE_{12} , TE_{10} , and TM_{12}) at frequencies of 92 GHz (red), 94 GHz (green), and 96 GHz (blue). (b) Magnitude of the coupled and reflected modes predicted by Eqs. (1) and (2) corresponding to the coupling coefficients of (a).

losses in the oversize waveguide (Sec. III). When Eqs. (B1) and (B2) are substituted into Eqs. (A2)–(A8) and Eq. (5), γ and κ become explicit functions of z .

Shown in Fig. 9(a) are plots of the magnitude of the coupling coefficients (Eq. (A1)) for modes that have the highest excitation from the TE₁₀ main mode. The coefficients are shown for the three frequencies 92, 94, and 96 GHz. The coupling coefficient for main mode reflection monotonically decreases through the taper and is nearly independent of frequency. The coefficients for the TE₁₂ and TM₁₂ also monotonically decrease except in the vicinity of cutoff. The TE mode coefficient has a zero at cutoff, and the TM mode, an infinity. A few higher order modes also couple to the main mode in the linear taper, but are excited at smaller levels because they are cut off through most of the taper.

Wave solutions for the modes corresponding to the coefficients of Fig. 9(a) are shown in Fig. 9(b). Propagation of the coupled mode is toward $z > 0$, $p = 1$, and the boundary condition is taken as $A_2^q = 0$ at $z = 0$. The mode is cut off for $z \lesssim 33$ mm and propagates for $z \gtrsim 33$ mm. A dip is seen in the mode amplitude at cutoff, but once the wave propagates, it grows monotonically. For the reflected TE₁₀ mode, propagation is through the entire taper and toward negative z , $p = -1$. The boundary condition is taken as $A_2^q = 0$ at $z = L$ (76.2 mm). Overall behavior is similar to the coupled TE₁₂ mode except the wave grows toward negative z . There is also an oscillation. The coupled TM₁₂ mode has different behavior because $|\kappa|$ is not monotonic in z . Most of the mode growth occurs due to an increasing $|\kappa|$, while the mode is evanescent. After cutoff, the TM₁₂ mode amplitude decreases with decreasing $|\kappa|$. However, a substantial wave amplitude exits the taper. From Fig. 9(b), it can be seen that the reflection coefficient is about 2% (−33 dB) and the highest mode coupling is for the TE₁₂ mode, which has a transmission coefficient of about 4% (−27 dB). More exact values as a function of frequency are shown in Table I. Overall performance of the linear taper is about an order of magnitude (20 dB) worse than the HC taper.

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