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List-mode MLEM Image Reconstruction from 3D ML Position Estimates

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Abstract

Current thick detectors used in medical imaging allow recording many attributes, such as the 3D location of interaction within the scintillation crystal and the amount of energy deposited. An efficient way of dealing with these data is by storing them in list-mode (LM). To reconstruct the data, maximum-likelihood expectation-maximization (MLEM) is efficiently applied to the list-mode data, resulting in the list-mode maximum-likelihood expectation-maximization (LMMLEM) reconstruction algorithm.

In this work, we consider a PET system consisting of two thick detectors facing each other. PMT outputs are collected for each coincidence event and are used to perform 3D maximum-likelihood (ML) position estimation of location of interaction. The mathematical properties of the ML estimation allow accurate modeling of the detector blur and provide a theoretical framework for the subsequent estimation step, namely the LMMLEM reconstruction. Indeed, a rigorous statistical model for the detector output can be obtained from calibration data and used in the calculation of the conditional probability density functions for the interaction location estimates.

Our implementation of the 3D ML position estimation takes advantage of graphics processing unit (GPU) hardware and permits accurate real-time estimates of position of interaction. The LMMLEM algorithm is then applied to the list of position estimates, and the 3D radiotracer distribution is reconstructed on a voxel grid.

I. Introduction

Current state-of-the-art detectors in nuclear medicine are capable of collecting many attributes for each event. For example, the 3D position of interaction in a thick detector and the energy deposited can be estimated from the PMT outputs of a scintillation camera. There are at least two possible ways to organize the data for image-reconstruction purposes. First, the data can be binned into pixels/voxels and energy intervals to get a vector $\mathbf{g} = \{g_1, \dots, g_M\}$. In this approach, the number g_m counts events in the m^{th} bin. Alternatively, we can store the list of attributes and write a reconstruction algorithm that uses it directly, without binning. The advantage of this second approach is that the large data structures that would be necessary for binning are not needed. Also, in the second approach, the reconstruction

time grows with the number of events that are collected, and not with the number of attributes measured/estimated for each event and the bin size.

In this paper, we explore the use of list-mode (LM) reconstruction with a PET system. The object is placed between two thick detectors facing each other. For each coincidence event, the 3D locations of the interactions within each crystal are estimated and used for LM maximum-likelihood expectation-maximization (MLEM) reconstruction. We refer to the resulting algorithm as LMMLEM algorithm [1]–[4]. The 3D location of interaction estimates that the LMMLEM algorithm uses are calculated using maximum-likelihood (ML) [5]. So ML is used twice, once for position estimation and once for reconstruction. The statistical properties of ML estimation provide precise statistics for detector performance. For example, the asymptotic optimability of ML estimation allows us to get asymptotically optimal location of interaction estimates, which are the used in the LMMLEM algorithm. The LMMLEM algorithm takes advantage of the property of ML estimation, leading to reduced blur in the reconstructed data.

This paper is organized as follows. In Sec. II, we briefly describe the LMMLEM algorithm, and we compare its expression with the one for the traditional MLEM algorithm. In Sec. III we derive an expression for the probability density function needed for LMMLEM reconstruction and in Sec. IV we describe the ML estimation for position of interaction and we show that the model we used agrees with simulation. Some preliminary reconstructions from simulated data are reported in Sec. V and, finally, Sec. VI summarizes our work and presents possible future developments.

II. The LMMLEM Algorithm

Consider the PET setup shown in Fig. 1.

Assume that the field of view is subdivided into N voxels, centered at locations $\mathbf{r}_n$, for $n = 1, \dots, N$. The goal of any image reconstruction algorithm is to calculate an estimate $\hat{\mathbf{f}} = \{\hat{f}_1, \dots, \hat{f}_N\}$ for the discrete set of numbers $\mathbf{f} = \{f_1, \dots, f_N\}$. Here, $\mathbf{f}$ is a discrete approximation [5] to the unknown object $f(\mathbf{r})$. The object of interest has been injected with the radiotracer and pairs of photons are emitted from it. The two photons in each pair travel along the same straight line, but in opposite directions. We will refer to a coincidence event when both photons get collected by the corresponding detector, and we will assume that a mechanism for pairing photon collections is available. For example, this can be done by just recording the time of interaction and pairing events that occurred within a small time interval.

Each of the two detectors shown in Fig. 1 is made of a thick scintillation crystal with PMT optical readout. From the PMT outputs for the j^{th} coincidence event collected by each detector, the 3D location of interactions $\mathbf{R}_{1,j}$ and $\mathbf{R}_{2,j}$ within detectors D_1 and D_2 respectively are estimated. These two estimates of locations of interaction are paired to form the j^{th} attribute vector $A_j = (\mathbf{R}_{1,j}, \mathbf{R}_{2,j})$. During some acquisition time T , a total of J coincidence events are collected, and the attribute list $\mathbf{A} = \{A_1, \dots, A_J\}$ is formed. Given the list $\mathbf{A}$, the likelihood of $\mathbf{f} = \{f_1, \dots, f_N\}$ is defined as [4], [6]:

$$\mathcal{L}(\mathbf{f}; \mathbf{A}) = \prod_{j=1}^J \text{pr}(A_j | \mathbf{f}), \quad (1)$$

where we assume that A_j and $A_{j'}$ are statistically independent for all $j \neq j'$. The quantity $\text{pr}(A_j|\mathbf{f})$ is the probability density function for the attribute vector A_j , conditioned on $\mathbf{f}$. It is useful to write [4]:

$$\text{pr}(A_j|\mathbf{f}) = \sum_{n=1}^N \text{pr}(A_j|r_n) \text{Pr}(r_n|\mathbf{f}), \quad (2)$$

in which $\text{pr}(A_j|r_n)$ is the probability density function of estimating $A_j = (\hat{\mathbf{R}}_{1,j}, \hat{\mathbf{R}}_{2,j})$ when a pair of photons is emitted from the n^{th} voxel in the field of view and $\text{Pr}(r_n|\mathbf{f})$ is the probability that a pair of photons is emitted from voxel n when the discretized radiotracer distribution is $\mathbf{f}$. In particular, we can calculate this probability as [4]:

$$\text{Pr}(r_n|\mathbf{f}) = \frac{S_n f_n}{\sum_{n'=1}^N S_{n'} f_{n'}}, \quad (3)$$

in which the numbers S_n , with $n = 1, \dots, N$, are called sensitivities. Each S_n is the probability that the two photons emitted from voxel n are collected by the detectors. Notice that the quantities S_n can be measured or they can also be calculated from the system geometry.

The LMMLEM reconstruction algorithm takes the form [4], [7]:

$$\hat{\mathbf{f}}_n^{(k+1)} = \hat{\mathbf{f}}_n^{(k)} \left\{ \frac{1}{T} \sum_{j=1}^J \frac{\text{pr}(A_j|r_n)}{\sum_{n'=1}^N \text{pr}(A_j|r_{n'}) S_{n'} \hat{\mathbf{f}}_{n'}^{(k)}} \right\}, \quad (4)$$

where k is the iteration index. Relevant properties of LMMLEM reconstruction are presented in [4]. For example, under broad conditions, the estimate $\hat{\mathbf{f}}^{(k)}$ calculated according to (4) converges to the global maximum of the likelihood $\mathcal{L}(\mathbf{f}; \mathbf{A})$ in (1).

We can compare the expression in (4) with the binned form of the MLEM algorithm [8]–[11]:

$$\hat{\mathbf{f}}_n^{(k+1)} = \hat{\mathbf{f}}_n^{(k)} \left\{ \frac{1}{\sum_{m=1}^M H_{mn}} \sum_{m=1}^M \frac{g_m H_{mn}}{[H \hat{\mathbf{f}}^{(k)}]_m} \right\},$$

in which $\mathbf{g} = \{g_1, \dots, g_M\}$ is the data vector, g_m is the number of counts collected at the m^{th} detector voxel, and $H = (h_{m,n})$ is a $m \times n$ real matrix [5] used to characterize the imaging system (assumed here linear).

III. Derivation of $\text{pr}(A_j|r_n)$

From (4), we see that the evaluation of the LMMLEM iterative expression requires the calculation of $\text{pr}(A_j|r_n)$, which is the probability density function for the attribute vector A_j given that a photon pair is emitted from location r_n . By Bayes' rule, we can write:

$$\begin{aligned}\text{pr}(A_j|r_n) &= \text{pr}(\widehat{\mathbf{R}}_{1,j}, \widehat{\mathbf{R}}_{2,j}|r_n) \\ &= \int_{D_1} \int_{D_2} \text{pr}(\mathbf{R}_{1,j}, \mathbf{R}_{2,j}|r_n) \times \\ &\quad \times \text{pr}(\widehat{\mathbf{R}}_{1,j}, \widehat{\mathbf{R}}_{2,j}|\mathbf{R}_{1,j}, \mathbf{R}_{2,j}, r_n) d^3 \mathbf{R}_{2,j} d^3 \mathbf{R}_{1,j},\end{aligned}$$

where $\int_{D_1}$ and $\int_{D_2}$ denote integration over the 3D volume of detector 1 and detector 2, respectively. Notice that we have:

$$\begin{aligned}\text{pr}(\widehat{\mathbf{R}}_{1,j}, \widehat{\mathbf{R}}_{2,j}|\mathbf{R}_{1,j}, \mathbf{R}_{2,j}, r_n) &= \text{pr}(\widehat{\mathbf{R}}_{1,j}, \widehat{\mathbf{R}}_{2,j}|\mathbf{R}_{1,j}, \mathbf{R}_{2,j}) \\ &= \text{pr}(\widehat{\mathbf{R}}_{1,j}|\mathbf{R}_{1,j}) \text{pr}(\widehat{\mathbf{R}}_{2,j}|\mathbf{R}_{2,j}),\end{aligned}$$

because the estimation of $\widehat{\mathbf{R}}_{t,j}$, $t = 1, 2$ depends only on the true location of interaction $\mathbf{R}_{t,j}$ in detector D_t and estimation in one detector is independent on the estimation in the other detector. The expression for $\text{pr}(A_j|r_n)$ is now [12]:

$$\begin{aligned}\text{pr}(A_j|r_n) &= \int_{D_1} \int_{D_2} \text{pr}(\widehat{\mathbf{R}}_{1,j}|\mathbf{R}_{1,j}) \text{pr}(\widehat{\mathbf{R}}_{2,j}|\mathbf{R}_{2,j}) \times \\ &\quad \times \text{pr}(\mathbf{R}_{2,j}|\mathbf{R}_{1,j}, r_n) \text{pr}(\mathbf{R}_{1,j}|r_n) d^3 \mathbf{R}_{2,j} d^3 \mathbf{R}_{1,j}.\end{aligned}$$

The expression above contains probability density functions of the form $\text{pr}(\widehat{\mathbf{R}}|\mathbf{R})$. These densities are the densities for the location of interaction estimates $\widehat{\mathbf{R}}$, when the actual location of interaction is $\mathbf{R}$. In other words, $\text{pr}(\widehat{\mathbf{R}}|\mathbf{R})$ measures the detector performance. The concept of detector performance can be made more formal by introducing the Fisher information matrix $\mathbf{F}_R$ at location $\mathbf{R}$ [5], [13], [14]. More details concerning $\mathbf{F}_R$, including an approximated expression for $\text{pr}(\widehat{\mathbf{R}}|\mathbf{R})$ will be provided in the next section. For now, we will assume that an expression for $\text{pr}(\widehat{\mathbf{R}}|\mathbf{R})$ is available.

To derive an expression for $\text{pr}(\mathbf{R}_{1,j}|r_n)$, consider a small cube $V_{\mathbf{R}_{1,j}}$ of side ε centered at $\mathbf{R}_{1,j}$. We will first consider the probability that an interaction occurs within the cube $V_{\mathbf{R}_{1,j}}$, given that a photon is emitted at the point r_n in the field of view. We have [5]:

$$\text{Pr}\left(\text{interaction in } V_{\mathbf{R}_{1,j}}|r_n\right) = \varepsilon^3 \frac{\mu_{\text{pe}} e^{-\mu_{\text{tot}} L(\mathbf{R}_{1,j}; r_n)}}{4\pi |\mathbf{R}_{1,j} - r_n|^2},$$

in which μ_{pe} is the photoelectric attenuation coefficient for the detector crystal, μ_{tot} is its total attenuation coefficient, and the quantity $L(\mathbf{R}_{1,j}; r_n)$ measures how much the photon from r_n traveled within the detector crystal before reaching point $\mathbf{R}_{1,j}$. By simple probability properties:

$$\begin{aligned}\text{pr}(\mathbf{R}_{1,j}|r_n) &= \lim_{\varepsilon \rightarrow 0} \frac{\text{Pr}(\text{interaction in } V_{\mathbf{R}_{1,j}}|r_n)}{\varepsilon^3} \\ &= \frac{\mu_{\text{pe}} e^{-\mu_{\text{tot}} L(\mathbf{R}_{1,j}; r_n)}}{4\pi |\mathbf{R}_{1,j} - r_n|^2}.\end{aligned}\tag{5}$$

To calculate $\text{pr}(\mathbf{R}_{2,j}|\mathbf{R}_{1,j}, r_n)$, we start by recalling that the two photons that interacted at $\mathbf{R}_{1,j}$ and $\mathbf{R}_{2,j}$ travel in opposite directions along the same line, and this line must contain point r_n as well. Therefore, the point $\mathbf{R}_{2,j}$ must belong to the intersection between detector D_2 and the line

$$\mathbf{R} = \mathbf{R}_{1,j} + (\mathbf{r}_n - \mathbf{R}_{1,j})\ell, \quad -\infty < \ell < \infty \quad (6)$$

passing through $\mathbf{R}_{1,j}$ and $\mathbf{r}_n$. It is convenient to introduce the function:

$$\psi_{D_2}(\mathbf{R}) = \begin{cases} 1 & \text{if } \mathbf{R} \in D_2, \\ 0 & \text{otherwise,} \end{cases}$$

so that:

$$\begin{aligned} \text{pr}(\mathbf{R}_{2,j}|\mathbf{R}_{1,j}, \mathbf{r}_n) &= \mu_{\text{pe}} e^{-\mu_{\text{tot}} L(\mathbf{R}_{2,j}; \mathbf{r}_n)} \times \\ &\times \int_{-\infty}^{\infty} \psi_{D_2}(\mathbf{R}_{1,j} + (\mathbf{r}_n - \mathbf{R}_{1,j})\ell) \times \\ &\times \delta(\mathbf{R}_{2,j} - \mathbf{R}_{1,j} - (\mathbf{r}_n - \mathbf{R}_{1,j})\ell) \, d\ell, \end{aligned} \quad (7)$$

where $\delta(\mathbf{R})$ is the 3D delta function. Notice that the integrand in the expression above is nonzero only when the point $\mathbf{R}_{2,j}$ belongs to the intersection between the line defined in (6) with the detector D_2 . Along with the units of μ_{pe} , the 3D delta function integrated over a line makes the dimensions of $\text{pr}(\mathbf{R}_{2,j}|\mathbf{R}_{1,j}, \mathbf{r}_n)$ equal to inverse volume, as they should.

Putting the pieces together, we find:

$$\begin{aligned} \text{pr}(A_j|\mathbf{r}_n) &= \frac{\mu_{\text{pe}}^2}{4\pi} \int_{D_1} \text{pr}(\widehat{\mathbf{R}}_{1,j}|\mathbf{R}_{1,j}) \frac{e^{-\mu_{\text{tot}} L(\mathbf{R}_{1,j}; \mathbf{r}_n)}}{|\mathbf{R}_{1,j} - \mathbf{r}_n|^2} \times \\ &\times \int_{D_2} \text{pr}(\widehat{\mathbf{R}}_{2,j}|\mathbf{R}_{2,j}) e^{-\mu_{\text{tot}} L(\mathbf{R}_{2,j}; \mathbf{r}_n)} \times \\ &\times \int_{-\infty}^{\infty} \psi_{D_2}(\mathbf{R}_{1,j} + (\mathbf{r}_n - \mathbf{R}_{1,j})\ell) \times \\ &\times \delta(\mathbf{R}_{2,j} - \mathbf{R}_{1,j} - (\mathbf{r}_n - \mathbf{R}_{1,j})\ell) \, d\ell \, d^3 \mathbf{R}_{2,j} \, d^3 \mathbf{R}_{1,j}. \end{aligned} \quad (8)$$

Further manipulation of $\text{pr}(A_j|\mathbf{r}_n)$ is possible. For example, if the ML estimation of the position of interaction within the detector is accurate, we could replace some variables in the expression above with their estimates. These and further approximations (and their validity) are discussed in the next section.

IV. 3D Position Estimation

In this work, we use ML estimation [5] to estimate the location of interaction $\hat{\mathbf{R}}$ from PMT data. The ML estimate $\hat{\mathbf{R}}$ of $\mathbf{R}$ can be written in abstract form as [12]:

$$\hat{\mathbf{R}} = \underset{\mathbf{R}_0}{\text{argmax}} \text{pr}(\mathbf{G}(\mathbf{R})|\mathbf{R}_0),$$

in which $\text{pr}(\mathbf{G}(\mathbf{R})|\mathbf{R}_0)$ is the likelihood for measuring the PMT noisy outputs $\mathbf{G}(\mathbf{R}) = \{G_1(\mathbf{R}), \dots, G_K(\mathbf{R})\}$ given the true interaction location $\mathbf{R}$, where $\mathbf{R}$ varies over the whole detector. Further details and performance results concerning this estimation step can be found in [15].

It can be shown [16] that in the asymptotic limit of a large number of photoelectrons, the density $\text{pr}(\hat{\mathbf{R}}|\mathbf{R})$ of the estimate $\hat{\mathbf{R}}$ given the true interaction location $\mathbf{R}$ is of the form [12]:

$$\text{pr}(\widehat{\mathbf{R}}|\mathbf{R}) \approx \frac{\sqrt{\det(\mathbf{F}_{\mathbf{R}})}}{(2\pi)^{3/2}} \exp\left[-\frac{1}{2}(\widehat{\mathbf{R}} - \mathbf{R})^\top \mathbf{F}_{\mathbf{R}} (\widehat{\mathbf{R}} - \mathbf{R})\right]. \quad (9)$$

In other words, $\text{pr}(\widehat{\mathbf{R}}|\mathbf{R})$ is a Gaussian probability density function with mean $\mathbf{R}$ and variance given by the inverse of the Fisher information matrix $\mathbf{F}_{\mathbf{R}}$. The expression in (9) is derived in [16] by assuming Gaussian noise. However, even if in this work we assume that the noise in the PMT outputs follows a Poisson distribution, the approximation in (9) can still be used by dint of the central-limit theorem.

For Poisson noise in the PMT outputs, the Fisher information matrix can be calculated as [17]:

$$[\mathbf{F}_{\mathbf{R}}]_{mn} = \sum_{k=1}^K \frac{1}{\bar{G}_k(\mathbf{R})} \frac{\partial \bar{G}_k(\mathbf{R})}{\partial x_m} \frac{\partial \bar{G}_k(\mathbf{R})}{\partial x_n}, \quad (10)$$

in which x_m is the m^{th} component of $\mathbf{R} = (x_1, x_2, x_3)$ and the quantities $\bar{G}_k(\mathbf{R})$ denote the mean output for the k^{th} PMT when the interaction location is $\mathbf{R}$. We performed a simulation to support the claim in (9) and to verify the validity of the expression in (10). We used the same (discrete) calibration data of [18] and performed a 3D spline [19] approximation of the calibration data and obtained continuous functions $\bar{G}_k(\mathbf{R})$, for $k = 1, \dots, K$ for the mean detector response functions. The reason why we used spline approximation of the calibration data was twofold. First, it allowed us to calculate the partial derivatives needed in (10) without using finite differences. Second, we can obtain calibration data for any point within the detector, not just the points of the 3D grid the original calibration data was sampled on. This way, the algorithm from [20] can perform an arbitrary number of iterations and estimates $\widehat{\mathbf{R}}$ can be allowed to be on a grid as fine as desired.

For our simulation, we assumed a NaI(Tl) scintillation crystal of size 52.30 mm $\times$ 52.30 mm $\times$ 25 mm with $K = 64$ PMT outputs and we generated 10^7 noisy sets of PMT outputs for a known location of interaction $\mathbf{R} = (26.15 \text{ mm}, 26.15 \text{ mm}, 1.50 \text{ mm})$. In other words, $\mathbf{R}$ is at the center of the crystal's entrance face and 1.50 mm underneath it. Fig. 2 shows in blue a 1D section of the 3D probability density function in (9) and in red the probability density function we estimated from the simulated noisy sets of PMT outputs. The agreement between the two plots is striking, and it confirms that the noise model summarized by (9) and (10) is appropriate for our case.

The results of Fig. 2 allow us to perform some approximations so that an expression for $\text{pr}(A_j|\mathbf{r}_n)$ more viable to implementation than (8) can be obtained. For example:

$$\mathbf{F}_{\mathbf{R}} \approx \mathbf{F}_{\widehat{\mathbf{R}}}, \quad |\mathbf{R}_{1,j} - \mathbf{r}_n|^2 \approx |\widehat{\mathbf{R}}_{1,j} - \mathbf{r}_n|^2,$$

and:

$$L(\mathbf{R}_{t,j}; \mathbf{r}_n) \approx L(\widehat{\mathbf{R}}_{t,j}; \mathbf{r}_n), \quad t=1, 2.$$

With these approximations and performing the integral over $\mathbf{R}_{2,j}$ in (8), we get:

$$\begin{aligned} \text{pr}(A_j|r_n) &= \frac{\mu_{pe}^2}{4\pi} \frac{e^{-\mu_{tot}[L(\hat{R}_{1,j};r_n)+L(\hat{R}_{2,j};r_n)]}}{\|\hat{R}_{1,j}-r_n\|^2} \times \\ &\times \int_{D_1} \text{pr}(\hat{R}_{1,j}|R_{1,j}) \int_{-\infty}^{\infty} \psi_{D_2}(\hat{R}_{1,j} + (r_n - R_{1,j})\ell) \times \\ &\times \text{pr}(\hat{R}_{2,j}|R_{1,j} + (r_n - R_{1,j})\ell) d\ell d^3R_{1,j}, \end{aligned} \quad (11)$$

where now [cf. (9)]:

$$\text{pr}(\hat{R}|R) \approx \frac{\sqrt{\det(\mathbf{F}_{\hat{R}})}}{(2\pi)^{3/2}} \exp\left[-\frac{1}{2}(\hat{R} - R)^\top \mathbf{F}_{\hat{R}}(\hat{R} - R)\right]. \quad (12)$$

The expression in (11) is amenable to a fast implementation of the calculation of $\text{pr}(A_j|r_n)$. For example, because the density $\text{pr}(\hat{R}|R)$ is sharply peaked and is essentially zero a few millimeters away from the estimated value (see Fig. 2), we can replace the integration over D_1 with an integration over a small detector volume centered at $\hat{R}_{1,j}$. Also, the line integral that appears in (11) can be expressed in terms of $\text{erf}(u) = 2\pi^{-1/2} \int_0^u \exp(-v^2) dv$.

V. Simulation Results

We used simulated data to test the effectiveness of the reconstruction algorithm. We considered a discretized field of view of size $40 \times 40 \times 40$ voxels, each of them of size $0.50 \text{ mm} \times 0.50 \text{ mm} \times 0.50 \text{ mm}$. Sections of the real object are shown in top row of Fig. 3. Each high-activity block was of size $2 \times 2 \times 2$ voxels and the background activity was zero. Two detectors were assumed on opposite sides of the field of view and they were of size $52.30 \text{ mm} \times 52.30 \text{ mm} \times 25.00 \text{ mm}$ divided into $69 \times 69 \times 25$ voxels. We collected $J = 10^5$ events and we used them for reconstruction. The reconstructed object at iteration $k = 10$ is shown in the bottom row of Fig. 3. Some artifacts are present in the reconstructed data. This is due to the fact that we are considering a limited-angle system. Further tests are underway.

VI. Conclusions and Future Work

In this paper, we considered the problem of image reconstruction from list-mode data. Our approach consists of two separate steps: (1) estimation of position of interaction from PMT data, (2) reconstruction from position of interaction estimates. In both steps, we used maximum-likelihood approaches and we noticed that, thanks to the properties of maximum-likelihood estimation, an asymptotically optimal solution can be obtained from the input raw data.

As possible future studies, we want to mention the extension to other geometries (along with their optimization for a particular clinical task) and an objective study of system performance [5], [21].

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Fig. 1.
Setup for a PET system

Fig. 2. Comparison between a 1D section of the 3D probability density function $\text{pr}(\hat{\mathbf{R}}|\mathbf{R})$ in (9) with the probability density function we estimated from the simulated noisy sets of PMT outputs

Fig. 3.

Comparison between the original simulated object f (top row) and the reconstructed object $\hat{f}^{(k)}$ after $k = 10$ iterations of the LMMLEM algorithm. The total number of events collected was $J = 10^5$