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Magnetoacoustic Tomography with Magnetic Induction (MAT-MI) for Breast Tumor Imaging: Numerical Modeling and Simulation

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Abstract

Magnetoacoustic tomography with magnetic induction (MAT-MI) was recently introduced as a noninvasive electrical conductivity imaging approach with high spatial resolution close to ultrasound imaging. In the present study, we test the feasibility of the MAT-MI method for breast tumor imaging using numerical modeling and computer simulation. Using the finite element method, we have built three dimensional numerical breast models with varieties of embedded tumors for this simulation study. In order to obtain an accurate and stable forward solution that does not have numerical errors caused by singular MAT-MI acoustic sources at conductivity boundaries, we first derive an integral forward method for calculating MAT-MI acoustic sources over the entire imaging volume. An inverse algorithm for reconstructing the MAT-MI acoustic source is also derived with spherical measurement aperture, which simulates a practical setup for breast imaging. With the numerical breast models, we have conducted computer simulations under different imaging parameter setups and all the results suggest that breast tumors that have large conductivity contrast to its surrounding tissues as reported in literature may be readily detected in the reconstructed MAT-MI images. In addition, our simulations also suggest that the sensitivity of imaging breast tumors using the presented MAT-MI setup depends more on the tumor location and the conductivity contrast between the tumor and its surrounding tissues than on the tumor size.

Index Terms

Magnetoacoustic tomography; MAT-MI; acoustic source; reconstruction; breast model; conductivity

I. INTRODUCTION

Noninvasive electrical impedance tomography of biological tissues has gained considerable research interests in biomedical imaging area during the last decades. Imaging modalities for impedance tomography that have been developed or under active investigation include electrical impedance tomography (EIT) (Metheral *et al* 1996, Mueller *et al* 1999, Cheney *et al* 1999), magnetic resonance electrical impedance tomography (MREIT) (Kwon *et al* 2002, Lee *et al* 2006, Woo and Seo 2008) and magnetic induction tomography (MIT) (Griffiths, 2001). In EIT, measurements of surface voltages due to injected current are used to reconstruct the impedance distribution of biological tissues. Compared to other tomography methods, EIT has advantages of low cost, safety and high speed. However, EIT has its limitations of low spatial resolution and the “shielding effect” (Wen 2000) caused by low

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conductive tissue layers surrounding the imaging object. To solve the problem of “shielding effect” and contact resistance between skin and surface electrodes, magnetic induction tomography (MIT) was developed. Using dynamic magnetic field sent through coils, eddy current is induced in the object in MIT. Secondary magnetic field is then detected by coils located around the object for impedance image reconstruction. Imaging resolution of the MIT technique, however, is also quite limited due to the employed boundary magnetic measurements and the need to solve an ill-posed inverse problem. To achieve high resolution impedance imaging, magnetic resonance electrical impedance tomography (MREIT) was pursued based on the magnetic resonance current density imaging (MRCDI) (Joy *et al* 1989) technique. In MREIT, the distribution of electrical conductivity is reconstructed using the measurements of magnetic flux density induced by injected current. This approach does not need to solve an ill-posed inverse problem and high spatial resolution electrical conductivity imaging has been achieved in *in vivo* experiments (Woo and Seo 2008). However, currently the need of high amplitude current injection still limits the clinical applications of MREIT.

Besides those electromagnetic imaging methods, magnetoacoustic tomography (MAT) (Towe and Islam 1988, Roth *et al* 1994) and Hall effect imaging (Wen *et al* 1998, Wen 2000) have been introduced as alternative approaches for noninvasive impedance imaging with high spatial resolution. Utilizing the coupling between electromagnetic field and acoustic field through the Lorentz force, it is possible to obtain the electrical conductivity information of the imaging sample with acoustic measurements. However, these two methods still suffer from the shielding effect due to the use of surface electrodes for current injection or voltage measurements.

In order to avoid the shielding effect, magnetoacoustic tomography with magnetic induction (MAT-MI) (Xu and He 2005, Li *et al* 2007, Xia *et al* 2007, Ma and He 2008, Xia *et al* 2009, Li *et al* 2009, Li and He 2010) was proposed for electrical impedance imaging of biological tissues with high spatial resolution. In MAT-MI, a time-varying and a static magnetic field are setup around a tissue sample and electrical conductivity distribution of the imaging sample is reconstructed with acoustic signals measured using ultrasound transducers. The time-varying magnetic field induces eddy current in the conductive sample and the Lorentz force acting on the moving charges is induced due to the existing static magnetic field. The Lorentz force caused motion will then lead to acoustic wave emitted from the sample. Using acoustic pressure signals collected around the tissue sample, its electrical impedance distribution can be reconstructed (Cassereau and Fink 1992, Fink 1992, Wu *et al* 1992, Xu and Wang 2004, Xu and He 2005). In comparison with other impedance imaging approaches, MAT-MI eliminates the shielding effect and is able to provide images containing electrical impedance related contrast with resolution close to ultrasound imaging.

In earlier works, Xu and He solved both the forward problem and the inverse problem of MAT-MI for tissues with homogeneous conductivity (Xu and He 2005), and Li *et al.* investigated a three-dimensional MAT-MI forward problem using the finite element method (Li *et al* 2009). However, the calculation of MAT-MI acoustic source at conductivity boundaries in these works are either ignored or replaced by a linear approximation. In the present study, we first derived an integral method for calculating MAT-MI acoustic sources over the whole imaging volume in order to avoid the errors caused by singular MAT-MI acoustic sources at conductivity boundaries. An inverse algorithm for reconstructing the MAT-MI acoustic source was also derived using Green's function technique. This inverse solution is derived with spherical measurement aperture, which simulates a practical setup for breast imaging. Using three dimensional (3D) numerical breast models with a variety of embedded tumors, we tested the feasibility of the MAT-MI method for breast tumor imaging.

II. THEORY

A. Forward problem

In solving the forward problem of MA-MI, with a known volume conductor model and other imaging parameters, we calculate the acoustic pressure signal over space and time, especially at locations on certain detection surface. Usually, the forward problem of MAT-MI involves two processes: generation of the Lorentz force induced acoustic sources and acoustic wave propagation.

In the first process, an excitation coil gives a time-varying magnetic field with flux density $\vec{B}_1(\vec{r}, t)$ in a static magnetic field with flux density $\vec{B}_0(\vec{r})$ (the arrow over a variable indicates a vector). The induced eddy current in the sample is denoted as $\vec{J}(\vec{r}, t)$. In MAT-MI, with around or under MHz frequency, the displacement current in a conductive tissue sample is much smaller than the conductive current and is usually ignored (Li *et al* 2009). Using the notation of magnetic vector potential $\vec{A}(\vec{r}, t)$, where $\vec{B}_1(\vec{r}, t) = \nabla \times \vec{A}(\vec{r}, t)$, we got Eq. (1) (Li and He 2010):

$$\nabla \cdot \vec{J}(\vec{r}, t) = \nabla \cdot (-\sigma(\vec{r}) \nabla \phi(\vec{r}) - \sigma \frac{\partial \vec{A}(\vec{r}, t)}{\partial t}) = 0 \quad (1)$$

Here σ is the electrical conductivity of the sample and ϕ is the electrical potential. Considering the Neumann boundary condition $\vec{J}(\vec{r}, t) \cdot \vec{n} = 0$ ($\vec{n}$ is the norm of outer boundary surface), unique solution of $\vec{J}(\vec{r}, t)$ can be calculated with an arbitrarily chosen reference position with zero potential (Wang and Eisenberg 1994).

With the eddy current density $\vec{J}(\vec{r}, t)$ and the total magnetic field $\vec{B}(\vec{r}, t)$, the Lorentz force acting on the eddy current can be defined as $\vec{J} \times \vec{B}$. $\vec{B}(\vec{r}, t)$ equals $\vec{B}_0(\vec{r})$ because the field strength of $\vec{B}_1(\vec{r}, t)$ is much smaller than that of $\vec{B}_0(\vec{r})$ (Roth *et al* 1994). The MAT-MI acoustic source over unit volume can be further expanded as in Eq. (2)

$$\nabla \cdot (\vec{J} \times \vec{B}_0) = \sigma \left(-\frac{\partial \vec{B}_1}{\partial t} \right) \cdot \vec{B}_0 + (\nabla \sigma \times \vec{E}) \cdot \vec{B}_0 \quad (2)$$

As we can see from Eq. (2) that the MAT-MI acoustic source over unit volume is related to both the conductivity distribution and its spatial gradient. For numerical calculation of the MAT-MI acoustic source in a piecewise homogeneous conductivity distribution, the conductivity gradient term in Eq. (2) is singular at boundaries between pieces of different conductivity values. Because of this, the gradient term was usually ignored or replaced by a linear approximation (Li *et al* 2009, Li and He 2010). However, as we will see in the following, using an integral method in a finite volume, we can avoid such kind of singularity in the forward calculation of the MAT-MI acoustic sources.

As shown in figure 1(a), we consider a very small and thin volume V laid at the conductivity boundary between two pieces with conductivity values of σ_1 and σ_2 . The two surfaces of volume V parallel to the conductivity boundary surface are S_1 and S_2 . The thickness of the volume is l and $l \ll \sqrt{S_1}$. As the slab volume is very small, we can consider the MAT-MI acoustic sources in this volume to be homogenous. $\vec{n}$ is a normal vector pointing to the σ_1 area. $\vec{t}$ is the tangential vector of boundary current $\vec{J}$, and $\vec{t}'$ is perpendicular to both $\vec{t}$ and $\vec{n}$. Integral of the Lorentz force over the bounding surface of V can be written as in Eq. (3):

$$\oint_s \vec{J} \times \vec{B} dS = J_{1t} B_{t'} \vec{n} \cdot \vec{S}_1 + J_{2t} B_{t'} \vec{n} \cdot \vec{S}_2 = (\sigma_1 - \sigma_2) E_{1t} B_{t'} S_1 \quad (3)$$

where J_{1t} and J_{2t} are the tangential components of the boundary current on surface S_1 and S_2 , respectively. $B_{t'}$ is the magnetic field component along the t' direction. Note here the integral over the other four surfaces of volume V can be ignored because the surface sizes are much smaller than S_1 . Using the divergence theorem and due to the homogenous distribution of acoustic sources in volume V , the left side of Eq. (3) can be written as $\nabla \cdot (\vec{J} \times \vec{B})V$. Therefore, Eq. (3) can be rewritten as:

$$\nabla \cdot (\vec{J} \times \vec{B})V = (\sigma_1 - \sigma_2) E_{1t} B_{t'} S_1 \quad (4)$$

In the following, we describe the numerical method for calculating the MAT-MI acoustic sources without singularity. We consider an arbitrarily small rectangular volume element dV in the conductive object as the shadowed volume element shown in figure 1(b). This element has conductivity of σ . Its neighboring elements in the x and y directions have conductivity of σ_x and σ_y , respectively. We denote the MAT-MI acoustic source in this volume element as in Eq. (5):

$$AS = \nabla \cdot (\vec{J} \times \vec{B})dV = \left[\sigma \left(-\frac{\partial \vec{B}_1}{\partial t} \right) \cdot \vec{B}_0 + (\nabla \sigma \times \vec{E}) \cdot \vec{B}_0 \right] dV \quad (5)$$

For $\vec{B}_0$ pointing in the Z direction, and using numerical difference, we can get Eq. (6):

$$AS = \sigma \left(-\frac{\partial \vec{B}_1}{\partial t} \right) \cdot \vec{B}_0 dV + (\sigma_x - \sigma) E_y B_{0z} dS_x - (\sigma_y - \sigma) E_x B_{0z} dS_y \quad (6)$$

where E_x and E_y are the x and y components of the induced electrical field, respectively. With Eq. (6) we are then able to calculate the MAT-MI acoustic source in every volume element without singularity problem.

In the second process of wave propagation, the generated MAT-MI acoustic pressure p obeys the following wave equation as in Eq. (7) (Fink 1992)

$$\nabla^2 p(\vec{r}, t) - \frac{1}{c_s^2} \frac{\partial^2 p(\vec{r}, t)}{\partial t^2} = -Q(\vec{r}, t) \quad (7)$$

where $-Q$ is the acoustic source function with respect to $\vec{r}$ and t , and $Q = -\nabla \cdot (\vec{J} \times \vec{B}_0)$ (Xu and He 2005). c_s is the velocity of wave propagation in the tissue. Here we assume the tissue is acoustically homogenous and c_s is a constant and is set to be $1500 \text{ m} \cdot \text{s}^{-1}$ (Xu and He 2005).

Using Green's function, we can derive the discrete formula for calculating the pressure signal as in Eq. (8)

$$p_0(\vec{r}, t) = - \sum_V (\sigma(-\frac{\partial \vec{B}_1}{\partial t}) \cdot \vec{B}_0 dV + (\sigma_x - \sigma) E_y B_0 dS_x - (\sigma_y - \sigma) E_x B_0 dS_y) G_+(\vec{r}, t | \vec{r}_0, t_0) \quad (8)$$

Where $G_+(\vec{r}, t | \vec{r}_0, t_0)$ is the corresponding the Green's function which equals

$$\frac{\delta(t - t_0 - |\vec{r} - \vec{r}_0|/c_s)}{4\pi |\vec{r} - \vec{r}_0|} \text{ and usually } t_0 = 0. p_0(\vec{r}, t) \text{ is defined as the acoustic pressure in the generated acoustic field.}$$

B. Inverse Problem

As shown in previous simulation and experiment studies, using MAT-MI method with single type of magnetic excitation we can only reconstruct images representing conductivity boundaries (Xia *et al* 2007, Li *et al* 2007). Generally, this kind of boundary images can also be obtained by reconstructing the MAT-MI acoustic source using backprojection algorithm (Xu and He 2005). In the present study, using the Green's function technique (Morse and Ingard 1968, Norton and Linzer 1981) we derive the exact formula for reconstructing the MAT-MI acoustic source map from pressure measurements collected on a spherical surface. The corresponding algorithm in time domain is also proposed. As illustrated in our computer simulation model, the half spherical detection surface is a setup especially suitable for breast imaging in practice and therefore we only consider this type of signal collection scheme in this study. Reconstruction algorithms with other types of signal collection scheme can also be derived using similar techniques (Norton and Linzer 1981).

We first define in spherical coordinate the source and field position vectors $\vec{r}_0(\theta_0, \varphi_0, r_0)$ and $\vec{r}(\theta, \varphi, r)$, respectively. We use $\vec{n}_0$ and $\vec{n}$ to denote the unit vectors pointing in the directions (θ_0, φ_0) and (θ, φ) , respectively.

Taking the Fourier transform on both sides of (8) and $\tilde{p}_0$ can be expanded as a sum of spherical harmonic basis functions as in Eq. (9) in spherical coordinate (the tilde over a variable indicates a function after the Fourier transform).

$$\tilde{p}_0(\vec{r}, k) = \frac{ik}{4\pi} \iiint_V d\vec{r}_0 \tilde{Q}(\vec{r}_0, k) \cdot \sum_{l=0}^{\infty} (2l+1) j_l(kr_0) h_l^{(1)}(kr) P_l(\vec{n}_0 \cdot \vec{n}) \quad (9)$$

where $k = \frac{\omega}{c_s}$, ω is the angular frequency, $j_l(kr_0)$, $h_l^{(1)}(kr)$ and $P_l(\vec{n}_0 \cdot \vec{n})$ are spherical Bessel, Hankel functions and Legendre polynomial of order l , respectively (Norton and Linzer 1981).

Here we give three identities which will be used during the flowing derivation (Xu and Wang 2002, Xu *et al* 2003):

$$\iint_{\Omega} d\Omega P_l(\vec{n}_0 \cdot \vec{n}) P_n(\vec{n}' \cdot \vec{n}) = \frac{4\pi}{2n+1} \delta_{ln} P_n(\vec{n}_0 \cdot \vec{n}') \quad (10)$$

$$\int_{-\infty}^{+\infty} dk(k^2) j_l(kr_0) j_l(kr') = \frac{\pi}{r_0^2} \delta(\vec{r}_0 - \vec{r}') \quad (11)$$

$$\sum_{n=0}^{\infty} (2n+1) P_n(\vec{n}_0 \cdot \vec{n}') = \frac{4\pi \delta(\varphi_0 - \varphi') \delta(\theta_0 - \theta')}{\sin \theta_0} \quad (12)$$

Here $\vec{n}_0$ and $\vec{n}'$ are unit vectors pointing in the directions of (θ_0, φ_0) and (θ', φ') , respectively. δ_{ln} equals to one when l equals n , otherwise it is zero. $d\Omega = \sin \theta d\theta d\phi$ is an element of solid angle on the normalized sphere Ω .

Multiplying both sides of (9) by $\frac{(2n+1)}{h_n^{(1)}(kr)} P_n(\vec{n}' \cdot \vec{n})$ and integrating with respect to $\vec{n}$ over the normalized spherical detection surface, we can derive Eq. (13):

$$\iint_{\Omega} d\Omega \tilde{p}_0(\vec{n}, k) \frac{(2n+1)}{h_n^{(1)}(kr)} P_n(\vec{n}' \cdot \vec{n}) = ik \iiint_V d\vec{r}_0 \tilde{Q}(\vec{r}_0, k) j_n(kr_0) P_n(\vec{n}_0 \cdot \vec{n}') (2n+1) \quad (13)$$

Multiplying both side of Eq. (13) with $k \cdot j_n(kr')$ and integrating with respect to k from minus infinite to positive infinite, and then sum n from 0 to positive infinite, we can have

$$\begin{aligned} \iint_{\Omega} d\Omega \int_{-\infty}^{+\infty} dk(k) \tilde{p}_0(\vec{r}, k) \cdot \sum_{n=0}^{\infty} \frac{(2n+1)}{h_n^{(1)}(kr)} P_n(\vec{n}' \cdot \vec{n}) j_n(kr') &= i \iiint_V d\vec{r}_0 \tilde{Q}(\vec{r}_0, k) \frac{4\pi^2 \delta(\varphi_0 - \varphi') \delta(\theta_0 - \theta') \delta(\vec{r}_0 - \vec{r}')}{\sin \theta_0 r_0^2} \\ &= 4\pi^2 i \tilde{Q}(\vec{r}', k) \end{aligned} \quad (14)$$

The exact formula for reconstructing the MAT-MI acoustic source with spherical measurement scheme can then be written as in Eq. (15):

$$\tilde{Q}(\vec{r}', k) = \frac{1}{4\pi^2 i} \iint_{\Omega} d\Omega \int_{-\infty}^{+\infty} dk k \tilde{p}_0(\vec{r}, k) \cdot \sum_{n=0}^{\infty} \frac{(2n+1)}{h_n^{(1)}(kr)} P_n(\vec{n}' \cdot \vec{n}) j_n(kr') \quad (15)$$

However, the exact algorithm is computationally very time-consuming. If the radius of detection surface is much larger than the corresponding acoustic wavelength, i.e. $|k| \cdot r \gg 1$, we can simplify Eq. (15) to a form of modified backprojection (See Appendix).

$$\nabla \cdot (\vec{J}(\vec{r}') \times \vec{B}(\vec{r}')) = \frac{r^2}{2\pi c_s^3} \iint_{\Omega} d\Omega \frac{\partial^2 p_0(\vec{r}, t)}{\partial t^2} \frac{\delta(t - \frac{|\vec{r}' - \vec{r}|}{c_s})}{|\vec{r}' - \vec{r}|} \quad (16)$$

Eq. (16) can be calculated in time domain by means of back projection (Porter 1969, Fink 1992) and integral over the spherical detection surface. As compared to Eq. (15), this simplified algorithm is more computationally efficient.

III. SIMULATION

A. Simulation Model

The breast model used in the present computer simulation study is shown in figure 2. This model contains two major layers simulating the mammary tissue and fat tissue of the breast, respectively. It is well known that the conductivity of biological tissue depends on the frequency of electrical field, injected current, temperature, and mainly varies with the frequency of the applied electrical field (Zou and Guo, 2003). When the frequency is $20\text{kHz} - 100\text{MHz}$, the conductivity is $0.2 - 0.8\text{S/m}$ for malignant tissues and $0.03 - 0.04\text{S/m}$ for normal tissues (Surowiec *et al* 1988). When the frequency is between 3MHz and 3GHz , the conductivity is $0.75 - 1.2\text{S/m}$ for malignant tissues and $0.15 - 0.3\text{S/m}$ for normal tissues (Chaudhary *et al* 1984). In the present simulation study, the conductivity value of mammary tissue is set to be 0.2 S/m and the conductivity of fat tissue is set to be 0.04 S/m . In addition, five spheres with different radii and higher conductivity values ($0.5 - 0.9\text{ S/m}$) are embedded in the mammary layer of the breast model to simulate different tumor tissues. Furthermore, in order to see how the MAT-MI acoustic source and pressure signals are affected by those tumor parameters such as the conductivity value, radius and position relative to the coil center, we have conducted simulations with various tumor parameters. The ranges of different parameters explored in these simulations are summarized in Table I. In figure 1, we also illustrate the half spherical surface enclosing the breast where MAT-MI acoustic signals are collected for image reconstruction. This half spherical surface provides a 2π view angle for image reconstruction.

The forward simulation was conducted using finite element method with a full model containing an excitation coil, the aforementioned breast model and the surrounding water and air media. The solid models are built and meshed with tetrahedrons, pentahedrons and hexahedrons (Zhang *et al* 2010). Important objects which need high accuracy including the coil and breast model are meshed with hexahedrons. Other regions are combined with tetrahedrons and pentahedrons with much bigger size. The numbers of elements and nodes of different pieces of the breast model are summarized in Table II. Figure 2 further illustrates the finite element meshes of the breast model and the excitation coil. The excitation coil which generates the dynamic magnetic field $\vec{B}_1$ is laid 60mm below the breast model. The height of the coil is 10mm and its inner and outer radii are 85mm and 100mm , respectively. A current pulse with $5\mu\text{s}$ pulse width is injected into the excitation coil. The current density in the coil is $3 \times 10^6\text{ A/m}^2$. A uniform static magnetic field $\vec{B}_0$ is also set to be in the z direction in the entire space and the flux density of B_0 is set to 1 Tesla .

In the simulation of MAT-MI pressure signals, each finite element is considered as a point source and the center position of the element is considered as where the point source is located. The collected pressure signal at each sensing location is simulated to have a sampling frequency of 5MHz and 7.5 milliseconds long.

B. Simulation Procedure

The following procedures are followed in the simulation of both the MAT-MI forward and inverse problem for breast imaging in the present study.

1. Calculate the magnetically induced electrical field $\vec{E}$ in the whole conductive volume using electromagnetic finite element analysis
2. Calculate the MAT-MI acoustic source in each finite element using Eq. (6).
3. Calculate the acoustic pressure signals p_0 at uniformly distributed locations on the detection surface using Eq. (8).

4. With the simulated pressure data, reconstruct the MAT-MI acoustic source map using Eq. (16).

C. Results

With the introduced computer simulation model and taking the simulation procedure, we have tested the feasibility of applying the MAT-MI method for breast tumor imaging. Figure 4(a) shows a typical forward simulation of the magnetically induced eddy current in the breast model. The corresponding simulated MAT-MI acoustic source distribution is shown in figure 4(b). As shown in figure 4, larger amplitude of eddy current can be seen flowing in those high conductive tumor regions as compared to the surrounding mammary and fat tissues. In addition, as expected, the generated MAT-MI acoustic sources mainly focus on the conductivity boundary between the mammary tissue and fat tissue. Large MAT-MI acoustic sources are also observed at those boundaries between the high conductive tumor tissues and their surrounding tissues. Therefore, an accurate reconstruction of such kind of MAT-MI acoustic source distribution might provide useful diagnostic information for breast tumor detection.

With the simulated ultrasound signal data at 819 different sensing locations, we have reconstructed the MAT-MI acoustic source distribution under noise free condition. The image reconstruction result is shown in figure 5(a). The reconstructed image has voxel size of $2 \times 2 \times 2 \text{ mm}^3$. As compared to figure 4(b), the reconstructed acoustic source map agrees well with the target acoustic source distribution and the conductivity boundaries can be easily identified. This is also the case when the radius of the tumor is as small as 2.5 mm (tumor 1). Figure 5(b) shows the reconstructed acoustic source images using noisy pressure data with $\text{SNR} = 3.5$. Here the SNR is defined as $\text{SNR} = A/s_n$, where A is the average amplitude of the noise free signal and s_n is the standard deviation of the added white noise (Zhang *et al* 2010). As shown in this figure, under $\text{SNR}=3.5$, the MAT-MI acoustic source image can still be very well reconstructed and the conductivity boundaries around those tumor tissues can be clearly seen. In order to better illustrate the simulated noisy acoustic pressure data with $\text{SNR}=3.5$, three typical pressure data sets are shown in figure 6. In this figure, blue lines represent the noise-free data and red lines are the signal with added white noise.

We have also conducted similar computer simulations under different SNR levels to test the noise tolerance of breast tumor imaging using the MAT-MI method. Figure 7 shows reconstructed MAT-MI acoustic source image slices at $Z=50 \text{ mm}$ under SNR levels of infinite, 8.7, 3.5 and 1.7. As shown in this figure, although the image quality and contrast become worse as the SNR level decreases, it is possible to identify the high conductive tumor regions even at relatively low SNR levels of 3.5 and 1.7.

In order to better understand how the data acquisition parameters such as the number of sensing locations (transducer numbers) and the radius of the detection surface influence the MAT-MI imaging of breast tumors, we have also compared the imaging results using different settings for these two parameters. The comparison results are illustrated in figure 8. Here we use an average reconstruction error ARE to quantitatively evaluate the imaging

performance. The average reconstruction error is defined as $ARE = \left| 1 - \frac{1}{M} \sum_M \frac{AS_r}{AS_0} \right|$, where AS_r is the reconstructed acoustic source in each element; AS_0 is the target acoustic source and M is the total number of elements in the region of interest. As shown in figure 8(a), the average reconstruction error goes up to 30% as we use about 300 transducers or sensing locations. This level of reconstruction errors, however, does not significantly deteriorate the image contrast for identifying those high conductive tumor regions, as illustrated in figure 9,

in which we present the reconstructed acoustic source images at $Z = 50\text{mm}$ with different number of transducers. In addition, the average reconstruction errors as a function of the radius of the detection surface is also shown in figure 8(b), which indicate a better imaging performance can be obtained when the ultrasound signal is collected at a closer distance to the breast surface.

Finally, we evaluate the sensitivity of imaging different types of breast tumors by comparing the generated MAT-MI acoustic sources around these breast tumors and the results are shown in figure 10. Figure 10(a) shows that if tumor 1 appears at a larger distance to the z axis in the range of 0.018m to 0.054m , there would be larger MAT-MI acoustic sources. This is mainly because a larger excitation magnetic field can be obtained there. In other words, we would have a better imaging sensitivity for tumors appearing at those regions with relatively large stimulating magnetic field. In addition, as shown in figure 10(b), the conductivity contrast between the tumor tissue and the surrounding mammary tissue have big influence on the generated MAT-MI acoustic sources and it is expected that tumor tissues with higher conductivity contrast to its surroundings are more easily detected by the MAT-MI method. Furthermore, figure 10(c) shows that increasing the radius of tumor 3 does not significantly change the generated MAT-MI acoustic source around it.

IV. DISCUSSION

In this study, we have developed a new numerical algorithm for accurate forward modeling of MAT-MI problem, and evaluated the feasibility of the MAT-MI method for breast tumor imaging using computer simulations. Using the finite element method, we have evaluated the MAT-MI imaging method with a more realistic numerical breast model than those models used in previous MAT-MI computer simulation studies. The present promising simulation results suggest that the MAT-MI method has the potential to become an important approach for breast tumor imaging.

MAT-MI was originally proposed to reconstruct the conductivity contrast of biological tissues with spatial resolution close to ultrasound image. In this computer simulation study, however, we use the MAT-MI acoustic source images as resultant images for possible diagnostic interpretation. As shown in our previous simulation and experiment studies (Xia *et al* 2007, Li *et al* 2007), this kind of acoustic source images obtained by using MAT-MI method with single type of uniform magnetic excitation generally allow us to identify conductivity boundaries. Therefore, for imaging well defined breast tumors with significantly higher conductivity than the surrounding tissues as reported in literatures, the MAT-MI acoustic source images may provide useful information to help identify suspicious tumor regions.

The numerical integral method developed in this study solved the singularity problem in the forward calculation of MAT-MI acoustic sources. This singularity makes the simulated MAT-MI acoustic sources at conductivity boundaries not convergent when finer spatial grid or finite element meshes are utilized. With the developed integral method, a generalized convergent solution can then be obtained at any locations in the entire imaging volume.

In the present computer simulation, we assume the ultrasound signal is collected by an ideal point detector at more than 300 sensing locations (transducer numbers) for image reconstruction. In practice, over hundred of sensing locations can be easily obtained by mechanically scanning an ultrasound transducer or by using a sensor array. However, ultrasound transducers with acceptable sensitivity generally have finite aperture size which may introduce certain blurring in the reconstructed MAT-MI acoustic source images.

About the MAT-MI imaging sensitivity, generally, it is believed to be proportional to both the static and dynamic magnetic field strength and the ultrasound receiving sensitivity. With certain MAT-MI system for breast imaging, however, the sensitivity of imaging different types of tumors may be affected by the tumor geometry including shape and size, its location and its conductivity contrast to its surrounding tissues. This kind of sensitivity may also depend on the coil geometry and its relative position to the breast.

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Appendix. Simplify equation (15) to a form of modified backprojection

Take the approximation as in Eq. (A.1) (Norton and Linzer 1981)

$$h_m^{(1)}(kr) \approx \frac{1}{h_m^{(2)}(kr) \cdot (kr)^2} \quad \text{if } |k| \cdot r \gg 1 \quad (\text{A.1})$$

We can rewrite Eq. (15) as Eq. (A.2):

$$\tilde{Q}(\vec{r}', k) = -\frac{1}{\pi} \int_{\Omega} d\Omega \int_{-\infty}^{+\infty} dk \tilde{p}_0(\vec{r}, k) \cdot \frac{ik}{4\pi} \sum_{n=0}^{\infty} P_n(\vec{n}' \cdot \vec{n}) j_n(kr') (2n+1) h_n^{(2)}(kr) \cdot (kr)^2 \quad (\text{A.2})$$

In addition, because

$$\tilde{G}_k(\vec{r}', \vec{r}) = \frac{e^{-ik|\vec{r}' - \vec{r}|}}{4\pi|\vec{r}' - \vec{r}|} = \frac{-ik}{4\pi} \sum_{n=0}^{\infty} (2n+1) j_n(kr') h_n^{(2)}(kr) P_n(\vec{n}' \cdot \vec{n}) \quad (\text{A.3})$$

we can get Eq. (A.4)

$$\tilde{Q}(\vec{r}', k) = \frac{r^2}{4\pi^2} \iint_{\Omega} d\Omega \int_{-\infty}^{+\infty} dk k^2 \tilde{p}_0(\vec{r}, k) \frac{e^{-ik|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|} \quad (\text{A.4})$$

Taking the inverse Fourier transform, we can then derive the modified backprojection algorithm in time domain as in Eq. (16)

FIG. 1.

(a) shows a thin slab volume V with a small height of l at a conductivity boundary. S_1 and S_2 are the top and bottom surface of the volume V , respectively. The shadowed plane is the boundary surface which separate two types of tissue having conductivity values of σ_1 and σ_2 . (b) shows three arbitrarily small neighboring conductivity elements sitting in the same Z plane. Each element is regarded as homogeneous in its conductivity distribution.

FIG. 2.

(a) shows the top view of a typical breast model. The outermost circle represents the half spherical detection surface enclosing the breast. (b) shows the lateral view of the breast model and the detection surface. The impedance values of each piece are shown on the right of this figure.

FIG. 3.
3D view of the finite element mesh of the developed breast model and excitation coil model.

FIG. 4. (a) shows the amplitude of the simulated eddy current in the breast model in the unit of A/m². Slice images at different Z planes between 0mm and 90mm are shown. (b) shows the corresponding simulated MAT-MI acoustic source distribution on similar image slices.

FIG. 5.

(a) shows the reconstructed MAT-MI acoustic source images with simulated pressure signal and under noise free condition. (b) shows the corresponding image reconstruction using noisy data with SNR=3.5.

FIG. 6.

The simulated acoustic pressure data at three selected locations on the detection surface at (a) ($X=105\text{mm}$, $Y=0\text{mm}$, $Z=0\text{mm}$) (b) ($X=0\text{mm}$, $Y=105\text{mm}$, $Z=0\text{mm}$) (c) ($X=0\text{mm}$, $Y=0\text{mm}$, $Z=105\text{mm}$). Blue line represents the noise free signal and red one is the signal with added white noise which corresponds to SNR of 3.5.

FIG. 7.

Comparison of the reconstructed acoustic source images at $Z=50\text{mm}$ under different noise levels with SNR to be infinite, 8.7, 3.5 and 1.7, respectively.

FIG. 8.

(a) shows the average reconstruction error as a function of the number of transducers (sensing locations) where acoustic pressure signals are collected for image reconstruction. (b) shows the average reconstruction error as a function of the radius of the half spherical detection surface.

FIG. 9.

Reconstructed acoustic source images at $Z=50\text{mm}$, with noise free pressure data collected by different numbers of transducers.

FIG. 10.

(a), (b) and (c) are the average intensity of the generated MAT-MI acoustic sources around tumor 1, tumor 2 and tumor 3 as a function of its distance to z-axis, the tumor conductivity value and the tumor radius, respectively.

Table I

Ranges of different parameters used in the simulation study

	Distance to z axis (Tumor 1)	Electrical conductivity (Tumor 2)	Radius (Tumor 3)
Range	0.018m~0.054m	0.5S/m~0.9 S/m	0.006m~0.014m

Table II

Numbers of elements and nodes in each piece of the breast model

	Fatty	Mammary	Tumor 3	Tumor 2	Tumor 1	Tumor 4	Tumor 5
Element	46769	124823~126198	112~1487	36	4	8	56
Node	63234	134867~135886	214~2004	84	18	27	118