Published in final edited form as:

IEEE Trans Ultrason Ferroelectr Freq Control. 2009 September ; 56(9): 1888–1902. doi:10.1109/TUFFC.2009.1265

A Post-Beamforming 2-D Pseudoinverse Filter for Coarsely Sampled Ultrasound Arrays

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Abstract

Beamforming artifacts due to coarse discretization of imaging apertures represent a significant barrier against the use of array probes in high-frequency applications. Nyquist sampling of array apertures dictates center-to-center spacing of $\lambda/2$ for elimination of grating lobes in the array pattern. However, this requirement is hard to achieve using current transducer technologies, even at the lower end of high-frequency ultrasonic imaging (in the range 25–35 MHz). In this paper, we present a new design approach for 2-D regularized pseudoinverse (PIO) filters suitable for restoring imaging contrast in systems employing coarsely sampled arrays. The approach is based on a discretized 2-D imaging model for linear arrays assuming scattering from a Cartesian grid in the imaging field of view (FOV). We show that the discretized imaging operator can be represented with a block Toeplitz matrix with the blocks themselves being Toeplitz. With sufficiently large grid size in the axial and lateral directions, it is possible to replace this Toeplitz-block block Toeplitz (TBBT) operator with its circulant-block block circulant (CBBC) equivalent. This leads to a computationally efficient implementation of the regularized pseudoinverse filtering approach using the 2-D fast Fourier transform (FFT). The derivation of the filtering equation is shown in detail and the regularization procedure is fully described. Using FIELD, we present simulation data to show the 2-D point-spread functions (PSFs) for imaging systems employing linear arrays with fine and coarse sampling of the imaging aperture. PSFs are also computed for a coarsely sampled array with different levels of regularization to demonstrate the tradeoff between contrast and spatial resolution. These results demonstrate the well-behaved nature of the PSF with the variation in a single regularization parameter. Specifically, the 6 dB axial and lateral dimensions of the PSF increase gradually with increasing value of the regularization parameter. On the other hand, the peak grating lobe level decreases gradually with increasing value of the regularization parameter. The results are supported by image reconstructions from a simulated cyst phantom obtained using finely and coarsely sampled apertures with and without the application of the regularized 2-D PIO.

I. Introduction

High-frequency ultrasound (HFUS) has been used for noninvasive visualization of living tissues at or near microscopic levels in many clinical or biological applications, such as dermatology [1], [2], ophthalmology [3]–[5], intravascular intracardiac imaging [6]–[10], nonvascular endoluminal imaging [11], cartilage imaging [12], [13], and small-animal studies [14], [15]. Researchers have recently achieved some success in extending the resolution to microscopic levels [3], [16], [17] using high-frequency systems with single-element transducers. At least one high-frequency ultrasound imaging system of this kind has been commercialized for small-animal research applications.

Various array transducers for HFUS imaging applications have been proposed recently [16]–[23]. These efforts are motivated by the success of array transducers in clinical applications

in the frequency range of 2 to 16 MHz. The benefits of arrays in ultrasonic imaging are well understood and amply demonstrated by their wide use in clinical scanners. Linear probes with center frequencies up to 15 MHz are available on some commercial scanners. A typical linear array is implemented with element-to-element spacing in the 1 to 1.2λ range with minimal grating lobe artifacts. Several transducer materials and fabrication methodologies have been developed in recent years [17], [21], [23], [24]. The benefits of these advances in materials and fabrication techniques have not been fully realized for high-frequency arrays above 20 MHz. One of the primary challenges for current transducer technologies is the element size requirement in this frequency range. Smaller size elements in the 30 to 60 μm range may be difficult to realize using existing techniques. Even if the fabrication problems are solved, additional problems due to increased cross coupling, increased element impedance, and increased variability in element sensitivity may arise. Therefore, based on the existing knowledge of transducer materials and fabrication methodologies, it is expected that high-frequency arrays in the range of 25 to 35 MHz will be realized with 1.5 to 3λ pitch (rather than the desirable range of 1 to 1.2λ for linear array imaging). Conventional beamforming with these coarsely sampled arrays results in increased grating lobe levels leading to reduced dynamic range and loss of contrast.

Pseudoinverse and matched filtering has been successfully used in ultrasonic imaging to restore axial resolution, primarily in conjunction with coded excitation [25]–[29]. The algorithm in [25], [30] employs a filter bank for parallel processing of echo data from multiple directions from a single beamforming operation. The coefficients of the filters were computed based on a regularized inverse of a discretized 2-D (axial-lateral) pulse-echo propagation operator from the array to the region of interest (ROI). For a single A-line, the sampled beamformer output is related to the scatter distribution within the ROI by a discretized propagation operator (matrix) as described in [25]. The propagation operator was represented as a block row matrix with each block representing the array response in a given lateral direction. Using the range-shift invariance assumption [25], the matrix elements of the propagation operator were represented as Toeplitz. This allowed for the use of the computationally efficient discrete Fourier transform (DFT) in finding a pseudoinverse operator resulting in the filter bank implementation for the parallel imaging in multiple directions from a single beamforming operation. In principle, the filter bank could be designed to remove some of the beamforming artifacts, but this is limited to the removal of uncorrelated grating-lobe components from the desired beam direction(s). Therefore, the PIO is more effective in decoupling echoes from different directions in conjunction with multi-modal coded excitation, i.e., several distinct codes are transmitted simultaneously with single receive beamforming of echoes from the ROI.

In this paper, we propose a post-beamforming filtering algorithm derived from a multiple-scan linear-array imaging system model. The single-line model is extended to include data from multiple A-lines (up to a frame) by axial and lateral shifting of the 2-D impulse response of the single-line beamforming operation. This discretized propagation model results in a system matrix with attractive properties for computationally efficient inversion. Specifically, the matrix is block Toeplitz with the matrix elements themselves being Toeplitz due to the range-shift invariance assumption [25]. A direct inversion of the Toeplitz-block block Toeplitz (TBBT) matrix is possible with Akaike's algorithms [31]. For a matrix with $M \times M$ blocks each with $N \times N$ elements, the Akaike algorithm requires $O(N^2M)$ memory storage and $O(N^3M^2)$ operations. The values of N and M are determined by the extent of the ROI, the sampling frequency, and the line density of the imaging system. These result in excessive memory requirement (typically above 100 GB) and number of operations for inversion. Our approach takes advantage of the fact that a banded Toeplitz matrix is asymptotically equivalent to its associated circulant matrix given that both matrices are bounded in the strong norm [32]. Similarly, the Toeplitz-block block-banded Toeplitz

matrices can be approximated by their associated circulant-block block-circulant (CBBC) matrices [33], [34]. Not only does this approach improve the computational efficiency and reduce the storage cost of the implementation, but it also transforms a usually ill-posed inverse problem to a well-posed FFT problem. Therefore, we are able to use the CBBC matrix to approximate the original propagation matrix; in addition, we extend an efficient inversion algorithm for block circulant matrices [35] specifically for elements being circulant blocks. Consequently, the original inversion operation in spatial-temporal domain is transformed into multiplication operation in 2-D frequency domain, preceded and followed by FFT-order DFT operations. Furthermore, regularization is used to avoid the noise amplification due to the inversion of small eigenvalues of the operator (or Fourier coefficients in the frequency domain). As demonstrated by the results given below, our regularization approach leads to well-behaved PSFs with properties that change gradually with increasing values of the regularization parameter, β . In particular, the axial and lateral dimensions of the PSF increase approximately as a sigmoid function with respect to $\log\beta$. Similarly, the mainlobe-to-gratinglobe (energy) ratio (MGR) increases approximately as a sigmoid function with respect to $\log\beta$.

The remainder of this paper is organized as follows. The proposed 2-D pseudoinverse filtering algorithm is derived in Section II. In Section III, we present simulation results of imaging cyst phantoms using 2 linear arrays operating at 25 and 35 MHz with different aperture-sampling geometries and bandwidth characteristics. Quantitative measurements of the spatial resolutions and contrast ratios are also given in Section III. Finally, discussions and conclusions are given in Section IV and Section V, respectively.

II. 2-D Pseudoinverse Filter Design

A. System Model

The system model for a single A-line acquisition using a 1-D linear array is similar to the model described in [25], but defined on a Cartesian grid. We begin by defining a uniform grid in the imaging field as shown in Fig. 1 and assuming scatterers with random amplitude distribution at the grid points. The number of grid points within a range interval $[z_{\min}, z_{\max}]$ in the axial direction, N , determines the axial sampling rate. Similarly, the number of grid points in the lateral direction (or the number of scan lines) within $[x_{\min}, x_{\max}]$, M , determines the lateral sampling rate. We assume the grid spacing is small with respect to the correlation cell size of the imaging system in both directions. Note that the grid shown in Fig. 1 represents a subregion of the imaging volume for purposes of reconstructions. Scattering from outside this region must be accounted for by the simulation model for the received echo data.

Define $s_{m,q}$ as the scatterer strength at grid location (m,q) ($m \in [1,M]$, $q \in [1,N]$), $\mathbf{s}_m$ as a vector of size $N_i \times 1$ containing amplitudes of scatterers along scan line m ($N_i \gg N$). Let $\mathbf{S}$ be a vector containing all lines of scatterers in the imaging field as

$$\mathbf{S} = \begin{bmatrix} \cdots & \mathbf{s}_1^T & \mathbf{s}_2^T & \cdots & \mathbf{s}_M^T & \cdots \end{bmatrix}^T, \quad (1)$$

where T denotes matrix/vector transpose and the number of lines is defined as M_i ($M_i \gg M$). Then the discretized version of the received echo for scan line m in the range interval $[z_{\min}, z_{\max}]$ can be represented in vector form as $\mathbf{f}_m$ of size $N \times 1$ and all the M scan lines can be grouped in $\mathbf{F}$ as follows:

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_1^T & \mathbf{f}_2^T & \cdots & \mathbf{f}_M^T \end{bmatrix}^T. \quad (2)$$

The impulse response of the system at a grid point is the echo from a single unit-strength scatterer positioned at that point. For illustration purposes, we assume the point scatterer is at the center of the specified grid. Then the generated pulse-echo impulse response at grid point q along line i is denoted as vector $\mathbf{g}_{i,q}$ of size $N \times 1$, which contains the discretized samples in time-domain. The pulse-echo impulse responses at the grid points along line i can be grouped as

$$\mathbf{G}_i = \begin{bmatrix} \cdots & \mathbf{g}_{i,1} & \mathbf{g}_{i,2} & \cdots & \mathbf{g}_{i,N} & \cdots \end{bmatrix}. \quad (3)$$

Assume $\mathbf{G}_0$ is the matrix containing the pulse-echo impulse responses from grid points along the line through the focus and $\{\mathbf{G}_{\pm i}\}_{i=1}^{\infty}$ are the matrices containing pulse-echo impulse responses from the grid points along the lines on right/left sides of focus. Typically, $\mathbf{G}_i = \mathbf{G}_{-i}$ due to the symmetry of the linear array aperture or apodization. Based on the above notations and definitions, the received echo signal from one scan line is obtained by superposition, which can be represented in matrix form:

$$\mathbf{f}_m = \underbrace{\begin{bmatrix} \cdots & \mathbf{G}_{-i} & \cdots & \mathbf{G}_{-1} & \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_i & \cdots \end{bmatrix}}_{\Psi} \mathbf{S}, \quad (4)$$

where Ψ is the spatial-temporal impulse response of the system. We can group the received signals from M scan lines as follows

$$\mathbf{F} = \underbrace{\begin{pmatrix} \ddots & & & \\ & \Psi & & \\ & & \Psi & \\ & & & \Psi \\ & & & & \ddots \end{pmatrix}}_{\bar{\mathbf{G}}_T} \mathbf{S}. \quad (5)$$

Eq. (5) provides a basis for an inverse solution to determine the scatterer strength $\mathbf{S}$ from the measurement $\mathbf{F}$. However, this is not computationally feasible due to the extremely large sizes of the (matrix) operator involved. Furthermore, due to the finite aperture and finite bandwidth of the system, it will not be possible to recover the scatter strength exactly. Therefore, we would like to develop an algorithm for solving (5) using a *regularized inverse* obtained through computationally efficient operations. The following properties of the operator $\bar{\mathbf{G}}_T$ provide a basis for such an algorithm:

1. The matrix $\bar{\mathbf{G}}_T$ is block Toeplitz.
2. Strictly speaking, the energy and the shape of the impulse responses in the matrices $\mathbf{G}_i$ are gradually changing along the axial direction. However, this change is moderate in the vicinity of the focus where we can establish computationally that the impulse responses can be approximated by the time-shifted versions of each

other [25], [30]. This range shift-invariance (RSI) approximation implies that the matrices $\mathbf{G}_i$ themselves being Toeplitz.

3. The array beam pattern in the axial and lateral directions justifies truncation of the impulse responses from grid points far away from the focus. This implies $\bar{\mathbf{G}}_T: \mathbf{G}_i \rightarrow 0, |i| > M_f; \mathbf{g}_{i,q} \rightarrow 0, q > N_f$ where M_f and N_f are some finite values.

Using the shifted version of $\mathbf{g}_{i,c}$, the pulse-echo impulse response at the focus, the RSI approximation version of $\mathbf{G}_i$ is a banded Toeplitz matrix of size $N \times N$:

$$\mathbf{G}_i = \mathcal{T}(0, \dots, g_{i,c}(0), g_{i,c}(1), \dots, g_{i,c}(N_f), \dots, 0), \quad (6)$$

where N_f is the number of nonzero samples of the impulse response along the i th direction and $\mathcal{T}$ stands for the operator of forming a Toeplitz matrix using a specified sequence. Therefore, $\bar{\mathbf{G}}_T$ is a block banded Toeplitz matrix with banded blocks, i.e., $\bar{\mathbf{G}}_T$ can be described as a Toeplitz-block block Toeplitz matrix. With the knowledge of the spatial-temporal impulse response Ψ and the output $\mathbf{F}$, retrieving the input $\mathbf{S}$ is an inverse problem. In principle, this solution can be obtained using matrix inversion assuming M_f and N_f are sufficiently large to minimize the truncation errors. However, inverting the matrix $\bar{\mathbf{G}}_T$ is still a challenging computational problem due to its large dimensions ($MN \times MN$). While some savings in the storage requirements and number of operations can be realized by exploiting the Toeplitz structure of the matrix, the requirements remain prohibitively large, even on powerful computers used in modern scanners. To illustrate this point, we describe the storage and computational requirements for a well-known inversion algorithm. Specifically, Akaike [31] derived an efficient inversion algorithm for general block Toeplitz matrices. His algorithm is a generalization of the Toeplitz matrix inversion algorithm developed by Trench [36] and refined by Zohar [37]. The order of magnitude of the number of operations is $\mathcal{O}(N^3M^2)$ for Akaike's algorithm. The storage requirement for the block Toeplitz matrix $\bar{\mathbf{G}}_T$ is reduced to $\mathcal{O}(N^2M)$ compared with the original general matrix. However, the memory needed in this case exceeds 100 GB for typical values of M and N in high-frequency ultrasound. Therefore, direct inversion of block Toeplitz matrices is not feasible for the ultrasonic imaging problems of interest.

B. Circulant-Block Block Circulant Matrix Approximation

1) Asymptotically Equivalent Circulant Matrices—The computational challenges of inverting the matrix $\bar{\mathbf{G}}_T$ described above can be circumvented by using the fact that a banded Toeplitz matrix is asymptotically equivalent to its associated circulant matrix given that both matrices are bounded in the strong norm [32]. Asymptotic equivalence of the 2 matrices means they approximate each other as the matrix dimension becomes large. For a banded Toeplitz matrix, its associated circulant matrix can be simply formed by filling in the upper right and lower left corners with the appropriate entries, e.g., for $\mathbf{G}_i$, the asymptotically equivalent circulant matrix is:

$$\mathbf{C}_i = \mathcal{C}(0, \dots, g_{i,c}(0), g_{i,c}(1), \dots, g_{i,c}(N_f), \dots, 0), \quad (7)$$

where $\mathcal{C}$ stands for the operator of forming a circulant matrix using a specified sequence. With $\mathbf{C}_i$ as the associated circulant matrix for $\mathbf{G}_i$, the eigenvalues of $\mathbf{G}_i$ and the eigenvalues of $\mathbf{C}_i$ are asymptotically equally distributed [32]. In addition, as long as the strong norm of the inverse of $\mathbf{G}_i$ and the strong norm of the inverse of $\mathbf{C}_i$ are bounded, the inverse of $\mathbf{C}_i$ is asymptotically equivalent to the inverse of $\mathbf{G}_i$ [32].

Note that an $N \times N$ circulant matrix can be diagonalized using a size- N DFT matrix and the diagonal values are the DFT coefficients, which are samples of the discrete-time Fourier transform (DTFT) of the underlying sequence. Therefore, the asymptotic equivalence between Toeplitz matrices and circulant matrices can be understood in terms of the equivalence between linear and circular convolutions with zero-padding applied appropriately to avoid time-domain aliasing.

2) Asymptotically Equivalent CBBC Matrices—Similar to the Toeplitz matrices, the TBBT matrices can be approximated by their associated CBBC matrices [33], [34]. This approximation has the obvious advantages of improving the computational efficiency and reducing the storage cost of inverting the TBBT matrix. In addition, it transforms a usually ill-posed inverse problem to a well-posed FFT problem. Bose [33] extended the Szegő theorem for the TBBT matrices. Specifically, the sequence of eigenvalues of a Hermitian block Toeplitz with Toeplitz-block matrix is asymptotically equally distributed as the sequence of eigenvalues of its associated CBBC matrix. Using similar arguments to those given in Section II-B-1, the asymptotic equivalence between TBBT matrices and CBBC matrices can be understood in light of the relationship between the 2-dimensional circular convolution and the 2-D linear convolution.

The 2-D convolution TBBT matrix, $\bar{\mathbf{G}}_T$, can be replaced by its associated CBBC matrix with $M \times M$ blocks and the truncated version of the original convolution equation. Eq. (5) can be expressed as:

$$\mathbf{F} = \underbrace{C(0, \dots, C_{-M_f}, \dots, C_{-1}, C_0, C_1, \dots, C_{M_f}, \dots, 0)}_{\bar{\mathbf{G}}_C} \mathbf{S}_M, \quad (8)$$

where each element in $\bar{\mathbf{G}}_C$ is a circulant matrix, and $\mathbf{S}_M$ of size $M \times 1$ is defined as the truncated version of $\mathbf{S}$:

$$\mathbf{S} = \begin{bmatrix} s_1^T & s_2^T & \dots & s_M^T \end{bmatrix}^T. \quad (9)$$

C. Inversion of the CBBC Matrix

Eq. (8) is the CBBC matrix equivalent of the convolutional system model to be inverted. It provides the basis for the regularized inversion algorithm described by the following steps:

1) Step I. Diagonalize the Circulant Blocks—The circulant blocks C_i s in $\bar{\mathbf{G}}_C$ can be diagonalized using the direct sum of DFT matrices. A DFT matrix of size N is defined as

$$\mathbf{F}_N = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & W_N^{1 \cdot 1} & \dots & W_N^{1 \cdot (N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{(N-1) \cdot 1} & \dots & W_N^{(N-1) \cdot (N-1)} \end{pmatrix}, \quad (10)$$

where $W_N = e^{-j(2\pi/N)}$. The direct sum of the DFT matrices is

$$\mathbf{Q} = \oplus_{m=1}^M \mathbf{F}_N = \begin{pmatrix} \mathbf{F}_N & & & \\ & \mathbf{F}_N & & \\ & & \ddots & \\ & & & \mathbf{F}_N \end{pmatrix}. \quad (11)$$

The circulant blocks $\mathbf{C}_i$ s can be diagonalized by multiplying $\mathbf{Q}$ and $\mathbf{Q}^{-1}$ from left and right sides of $\bar{\mathbf{G}}_C$:

$$\bar{\mathbf{H}} = \mathbf{Q} \bar{\mathbf{G}}_C \mathbf{Q}^{-1}, \quad (12)$$

where $\mathbf{H}$ retains the block circulant structure as $\bar{\mathbf{G}}_C$, but every block $\mathbf{H}_i$ is a diagonal matrix obtained by

$$\mathbf{H}_i = \mathbf{F}_N \mathbf{C}_i \mathbf{F}_N^{-1}. \quad (13)$$

That is,

$$\bar{\mathbf{H}} = \mathbf{C}(\mathbf{0}, \dots, \mathbf{H}_{-M_f}, \dots, \mathbf{H}_{-1}, \mathbf{H}_0, \mathbf{H}_1, \dots, \mathbf{H}_{M_f}, \dots, \mathbf{0}), \quad (14)$$

and

$$\begin{aligned} \mathbf{H}_i &= \begin{pmatrix} h_{i,0} & & & \\ & h_{i,1} & & \\ & & \ddots & \\ & & & h_{i,N-1} \end{pmatrix} \\ &= \text{diag}(h_{i,0}, h_{i,1}, \dots, h_{i,N-1}), \end{aligned} \quad (15)$$

where $\text{diag}(\cdot)$ is defined as the operator of forming a diagonal matrix using the specified input series and $\{h_{i,n}\}_{n=0}^{N-1}$ is the DFT coefficients from the Fourier transform of $\{g_{i,c}(l)\}_{l=0}^{N-1}$ forming the circulant matrix $\mathbf{C}_i$:

$$h_{i,n} = \sum_{l=0}^{N-1} g_{i,c}(l) W_N^{nl}. \quad (16)$$

2) Step II. Block-Diagonalize the Block Circulant Matrix—The circulant block matrix $\bar{\mathbf{H}}$ can be diagonalized using the direct product of a DFT matrix and an identity matrix:

$$\begin{aligned} \mathbf{P} &= \mathbf{F}_M \otimes \mathbf{I}_N \\ &= \begin{pmatrix} \mathbf{I}_N & \mathbf{I}_N & \dots & \mathbf{I}_N \\ \mathbf{I}_N & W_M^{1 \cdot 1} \mathbf{I}_N & \dots & W_M^{1 \cdot (M-1)} \mathbf{I}_N \\ \vdots & & \ddots & \vdots \\ \mathbf{I}_N & W_M^{(M-1) \cdot 1} \mathbf{I}_N & \dots & W_M^{(M-1) \cdot (M-1)} \mathbf{I}_N \end{pmatrix}, \end{aligned} \quad (17)$$

where $\mathbf{F}_M$ is a DFT matrix of size M and $\mathbf{I}_N$ is an identity matrix of size N . The block diagonal matrix $\bar{\mathbf{D}}$ can be obtained by

$$\bar{\mathbf{D}} = \mathbf{P} \bar{\mathbf{H}} \mathbf{P}^{-1}, \quad (18)$$

which is simply a diagonal matrix:

$$\bar{\mathbf{D}} = \text{diag}(\mathbf{D}_0, \mathbf{D}_1, \dots, \mathbf{D}_{M-1}), \quad (19)$$

and

$$\mathbf{D}_k = \sum_{i=0}^{M-1} W_M^{ik} \mathbf{H}_i. \quad (20)$$

Note that $\mathbf{H}_i$ is already a diagonal matrix, so $\mathbf{D}_k$ is a diagonal matrix too,

$$\mathbf{D}_k = \text{diag}(d_{k,0}, d_{k,1}, \dots, d_{k,N-1}), \quad (21)$$

where

$$d_{k,n} = \sum_{i=0}^{M-1} h_{i,n} W_M^{ik}. \quad (22)$$

Substituting (16) into (22), we have

$$d_{k,n} = \sum_{i=0}^{M-1} \sum_{l=0}^{N-1} g_{i,c}(l) W_N^{nl} W_M^{ik}. \quad (23)$$

Eq. (23) shows that the diagonal terms are the 2-D DFT of the sequence $g_{i,c}(l)$ or the k -space representation of our system model. In this formulation, diagonalizing the block matrices $\mathbf{C}_i$ is a Fourier transform in the axial direction (time) and block-diagonalizing $\bar{\mathbf{G}}_C$ is a Fourier transform in lateral direction. Eq. (23) can be written in matrix form,

$$\bar{\mathbf{D}} = \mathbf{P} \mathbf{Q} \bar{\mathbf{G}}_C \mathbf{Q}^{-1} \mathbf{P}^{-1}. \quad (24)$$

Alternatively, the matrix $\bar{\mathbf{G}}_C$ can be written in terms of the diagonal matrix $\bar{\mathbf{D}}$ and DFT (and inverse DFT) operations

$$\bar{\mathbf{G}}_C = \mathbf{Q}^{-1} \mathbf{P}^{-1} \bar{\mathbf{D}} \mathbf{P} \mathbf{Q}. \quad (25)$$

3) Step III. k-Space Inverse Filtering—Substituting (25) in (8), we obtain

$$\mathbf{F} = \mathbf{Q}^{-1} \mathbf{P}^{-1} \bar{\mathbf{D}} \mathbf{P} \mathbf{Q} \mathbf{S}_M, \quad (26)$$

which, upon multiplying by $\mathbf{PQ}$ from the left, yields the k -space representation of (8),

$$\underbrace{\mathbf{PQF}}_{\mathbf{F}_K} = \underbrace{\bar{\mathbf{D}} \mathbf{P} \mathbf{Q} \mathbf{S}_M}_{\mathbf{S}_K}. \quad (27)$$

The vectors $\mathbf{F}_K$ and $\mathbf{S}_K$ are simply the 2-D DFTs of $\mathbf{F}$ and $\mathbf{S}_M$, respectively. From (27), the inversion of (8) in k -space is simplified as the inversion of the diagonal matrix $\bar{\mathbf{D}}$:

$$\widehat{\mathbf{S}}_K = \bar{\mathbf{D}}^\dagger \mathbf{F}_K, \quad (28)$$

where $\widehat{\mathbf{S}}_K$ is the estimate of $\mathbf{S}_K$ and the $(\cdot)^\dagger$ indicates the *regularized inverse* or *pseudoinverse*. The design of this 2-D pseudoinverse operator (2-D PIO) is discussed in Section II-D below.

4) Step IV. Inversion—Finally, we obtain $\widehat{\mathbf{S}}_M$ using an inverse 2-D DFT:

$$\widehat{\mathbf{S}}_M = \mathbf{P}^{-1} \mathbf{Q}^{-1} \widehat{\mathbf{S}}_K. \quad (29)$$

D. 2-D PIO Design

In principle, the inverse filtering problem can be achieved by inverting the diagonal matrix $\bar{\mathbf{D}}$ in (27). However, this will result in very high gains at frequencies where the magnitude response of the system is very low. In practice, the inverse filter amplifies noise components rendering the restoration problem useless. Therefore, a regularized inverse solution is usually sought; one that attempts to incorporate the signal-to-noise (SNR) in the inversion problem. A simple solution is given by

$$d_{k,n}^\dagger = \frac{d_{k,n}^*}{|d_{k,n}|^2 + \beta}, \quad (30)$$

where β is a regularization parameter and $(\cdot)^*$ denotes complex conjugation. One can see that, for $\beta \ll |d_{k,n}|^2$, $d_{k,n}^\dagger \approx d_{k,n}^{-1}$; inverse filtering. On the other extreme, when $\beta \gg |d_{k,n}|^2$, $d_{k,n}^\dagger \propto d_{k,n}^*$; matched filtering. The value of β could be determined based on SNR considerations. Other considerations include controlling the behavior of the resulting filter in the space domain, e.g., to minimize ringing. One can also consider frequency-dependent regularization (i.e., $\beta = \beta_{k,n}$), but this is usually coupled with specific knowledge of frequency dependence of the SNR.

In this paper, the simple regularization scheme suggested by (30) was found to be adequate for our purposes. This is justified based on the observation of the degradation pattern in k -space for different levels of spatial sampling as can be seen in Fig. 2. The figure shows the

2-D frequency response of a generic focused linear array pattern (no steering) using the same aperture and temporal bandwidth, but with element-to-element spacing, d_e , between 0.5λ and 4λ as shown. One can see that the bandwidth in the axial (temporal) direction is largely unaffected by the lateral sampling up to $d_e = 3\lambda$. For this range of element-to-element spacing, one can also see that the effect of increasing d_e introduces aliased lateral frequency components. Larger values of d_e produce significant ripple in the lateral frequency response in the pass-band of the system, which reduces both the lateral and the axial bandwidth of the imaging system. Our interest, however, is in the restoration of the imaging contrast for imaging systems employing arrays with d_e in the range of 1 to 3λ . In this range, the coarser sampling effects are most pronounced in the lateral frequency direction. One would expect the single-parameter regularization approach described by (30) to affect the array response mainly in the lateral frequency direction. This can be seen in Fig. 3, which shows the frequency response of the regularized system for $d_e = 2\lambda$ and $\beta = 0.01, 0.3$, and 10. One can see that the smallest value of β produces relatively high gains at frequencies outside the main passband of the system. This is likely to amplify noise components in the received signal, which can result in poor performance. One can also see that the highest value of β produces a 2-D PIO that behaves like a matched filter. The matched filter is known to maximize the SNR (resulting in improved contrast), but possibly at the expense of reduced spatial resolution. The intermediate values of β are more likely to achieve a more appropriate tradeoff between spatial and contrast resolutions. These observations are quantified further using simulated images from speckle-generating cyst phantoms given below.

III. Simulation Results

In this section, we present simulation results of 2 HFUS apertures of interest:

- A 25 MHz HFUS linear-array prototype for imaging tissue-engineered heart-valve equivalent (HVE) [10] *in vitro*. This prototype is intended to replace a 25 MHz, mechanically scanned, single-element imaging system used for imaging the HVE [38].
- A 35 MHz HFUS linear-array prototype that has been fabricated by the NIH Ultrasonic Transducer Resource Center at the University of Southern California [39]. This prototype is intended to represent the state-of-the-art in HFUS linear-array imaging.

Both prototypes were simulated in Matlab (MathWorks, Natick, MA) using Field II [40]. The speed of sound was set to 1480 m/s. A Gaussian impulse response model was assumed for the array elements with impulselike excitation.

A. 25 MHz HFUS Linear-Array Prototype

The objective is to image the HVE *in vitro* and ultimately *in vivo* using HFUS. The array prototype described here serves as a reference for future prototypes using coded-aperture and/or coded excitation. For the purposes of this paper, we have simulated a linear array prototype with the following parameters:

- Center frequency: $f_0 = 25$ MHz
- Fractional bandwidth: FBW = 60%
- Active aperture size: 6 mm
- Focal distance: 12 mm
- Element height: 2.5 mm

- Mechanical focal distance: 15 mm
- Element-to-element spacing: 0.5λ ($\approx 30 \mu\text{m}$) and 2λ ($\approx 118 \mu\text{m}$). For each array design, the element width was approximately equal to the element-to-element spacing (minus $10 \mu\text{m}$ kerf).

The operating frequency was determined based on single-element imaging results of HVE samples [38]. The remaining parameters of the prototype were chosen based on recent publications on the design and manufacture of HFUS arrays [16]–[23] and potential HFUS imaging applications [3], such as intravascular and intracardiac imaging [6]–[10]; skin imaging [1], [2]; ocular imaging [4], [5]; nonvascular endoluminal imaging [11]; cartilage imaging [12], [13]; and small-animal study: developmental biology [14] and tumor biology [15]. The wavelength is $59.2 \mu\text{m}$ given the specified speed of sound. The finely sampled array is assumed to have $d_e = 0.5\lambda$, which uses elements with width of $\approx 30 \mu\text{m}$. This is currently very challenging and expensive even for the transducer manufacture technology in the research stage. The coarsely sampled array is assumed to have $d_e = 2\lambda$, which uses elements with width of $\approx 120 \mu\text{m}$. This can be easily achieved using modern transducer technology.

1) Point Spread Functions—Fig. 4 shows 70 dB gray-scale images of the PSFs for the imaging systems using the 6 mm aperture with (a) 0.5λ and (b) 2λ sampling, respectively. The images show the PSFs in a $2.8 \times 19 \text{ mm}^2$ region with the unit scatterer positioned at the focus $[0 \ 0 \ 12] \text{ mm}$ (f -number = 2). One can see the increased level of the grating lobes due to the coarser sampling case, shown in Fig. 4(b), compared with Fig. 4(a), where the sidelobes are well below the -70 dB level. The line graphs in Fig. 5(b) give a direct comparison between the 2 PSFs in the lateral direction. The grating lobes are shown to be nearly 54 dB below the main beam and more than 40 dB above the sidelobes of the PSF resulting from the Nyquist-sampled aperture.

To illustrate the effect of the regularized solution described in Section II-D, we calculated the lateral and axial PSF plots for $\beta = 0.001, 0.1$, and 1 . These results are shown in Fig. 6 and Fig. 7, which demonstrate the nature of the regularized inverse solution. In particular, one can see that higher values of β result in wider mainlobe width of the PSF in both axial and lateral directions, i.e., worse spatial resolution. On the other hand, the axial and lateral sidelobe levels, including grating lobes due to coarse aperture sampling, are reduced by increasing the value of β . In Fig. 8, the grayscale image of the PSF filtered by 2-D PIO filter with $\beta = 0.1$ gives one example of the regularization process.

Fig. 9 illustrates the dependence of the resolution and contrast parameters on the regularization parameter. In particular, Fig. 8(a) shows the 6 dB axial and lateral resolutions as functions of $\log_{10}\beta$ (approximately sigmoid). We have also calculated the MGR as a function of $\log_{10}\beta$ as a predictor of contrast ratios achievable with various levels of regularization. Fig. 8(b) shows a similar behavior of the MGR as a function of $\log_{10}\beta$. Specifically, the MGR increases with $\log\beta$ with approximate sigmoid function. The results of Fig. 8(a) are quite significant in that the range of achievable spatial resolutions using the regularized inverse filtering approach can be predicted as a function of $\log_{10}\beta$. The significance of the MGR dependence on $\log_{10}\beta$ is in the gradual nature of the change, as with the resolution. Furthermore, as we show below, the MGR is a useful predictor of the contrast resolution (and dynamic range) of the imaging system.

2) Imaging Cyst Phantoms—The PSFs shown in the previous section provide useful information on the expected performance of the imaging system. Specifically, the 6 dB widths of the PSF in the axial and lateral direction provide a direct measure of spatial resolution in the respective directions. In addition, the MGR provides an indirect measure of

contrast resolution and dynamic range (for high SNR scenarios) of the imaging system. These issues are best addressed when imaging contrast targets within speckle-generating targets. We have used Field II for simulating imaging experiments of cyst phantoms using linear arrays with different aperture sampling with and without regularized inverse filtering. The details of the phantom are as follows:

We used a 2-D uniform phantom (average 25 scatterers per resolution cell) with a 1 mm radius spherical cyst at the center of a region with axial and lateral extent of 4 and 20 mm, respectively. The lateral extent of the phantom was chosen sufficiently large to include first-order grating lobes of the imaging beams used to acquire the pulse-echo data from the region of interest (ROI). With reference to Fig. 1, $x_{\min} = -10$ mm, $x_{\max} = 10$ mm, $z_{\min} = 10$ mm, $z_{\max} = 14$ mm, with the cyst centered at $(x_c, y_c, z) = (0, 0, 12)$ mm. The ROI is defined by $-4.2 \leq x \leq 4.2$ mm and $10.1 \leq z \leq 13.9$ mm.

Fig. 10 shows 50 dB grayscale images of the 3.8×8.4 mm² ROI obtained using the (a) Nyquist-sampled aperture and (b) coarsely sampled aperture. To provide a quantitative measure of the imaging performance for the different arrays, we computed the contrast ratio (CR) for 2 circular regions of radius 0.9 mm: one centered at the center of the cyst, (0,0,12), and one outside the cyst at the same axial distance from the imaging aperture, (3.4,0,12). The CR was computed as follows:

$$CR = 10 \log_{10} \left[\frac{\bar{I}_2}{\bar{I}_1} \right]. \quad (31)$$

The contrast ratios obtained from the images shown in Fig. 10(a) and (b) are 28.8 dB and 20.4 dB, respectively. This result demonstrates that the increase in grating lobe level due to coarse sampling of the aperture results in reduction of the CR by 8.4 dB for this imaging target. This provides a quantitative measure for the assessment of the performance of our regularized postbeamforming inverse filtering approach for contrast restoration.

Fig. 11 shows image reconstructions (50 dB dynamic range) obtained by applying the 2-D PIO approach described in Section II-D with $\beta = 0.01, 0.1, 0.3, 1$, and 10. The corresponding CR values computed from these images are 22.7 dB, 26.9 dB, 28.8 dB, 30 dB, and 29.2 dB, respectively. Table I summarizes the CR values obtained using the 25 MHz aperture with Nyquist and coarse sampling (without and with the application of the regularized 2-D PIO). By comparing the images in Fig. 11, one can see the improvement in contrast with increased β values as shown from Figs. 11(a)–(d). The small reduction in CR for the $\beta = 10$ case, shown in Fig. 11(e), is due to the noticeable loss in lateral resolution that results in the visible blurring of the cyst boundary in this case. This is best illustrated by the line plots shown in Fig. 12, which shows the image intensity along a lateral line through the center of the cyst for the coarsely sampled array with conventional beamforming and using the 2-D PIO with β values of 0.01, 0.3, and 10. Fig. 13 shows the image intensity along an axial line through the center of the cyst (similar to Fig. 12). The results are consistent with the behavior of the range mainlobe width and sidelobe levels with various values of β as can be seen in Fig. 6. In particular, the range sidelobes exhibit significant dependence on β while the mainlobe width changes only slightly with the regularization parameter.

The cyst phantom imaging results suggest that the regularization parameter β should be chosen to provide a tradeoff between the contrast and the spatial resolution. They also suggest that the single parameter regularization primarily affects the lateral resolution (lateral mainlobe width) with minimal effect on the axial resolution (range mainlobe width). At the same time, the regularization has a significant effect on the contrast resolution (both

lateral and range sidelobes of the PSF depend strongly on β). For example, a good compromise is achieved by a β value of 0.3, shown in Fig. 11(c), which recovers all the 8.4 dB loss in the contrast due to the coarse sampling artifact as in Figs. 10(a) and (b). At the same time, the spatial resolutions are kept about the same as the original system as plotted in Fig. 8(a). Table II summarizes the spatial resolution values achieved using conventional beamforming with the Nyquist-sampled and coarsely sampled ($d = 2\lambda$) arrays. It also shows the values achieved after the application of the 2-D PIO filter ($\beta = 0.3$) to the beamformed data from the coarsely sampled array.

Fig. 8(b) sheds some light on the relationship between the MGR and CR as a function of the regularization parameter. For the cyst phantom images obtained using the coarsely sampled aperture and formed using the 2-D PIO with different values of β , one can see that the MGR serves as a predictor of the CR performance. It is also significant to note the smooth monotone increase in the CR as a function of $\log_{10}\beta \in [-8, 0]$. This suggests that the user will be able to determine directly the level of regularization necessary to achieve the desired levels of spatial and contrast resolutions. This can be implemented in much the same way as the TGC is implemented on current scanners.

B. Simulation of a 35 MHz Linear Array Prototype

To demonstrate the applicability of the regularized 2-D PIO approach to imaging arrays of practical interest, we present simulation results of an HF linear array that has been recently introduced [39]. Researchers at the NIH Transducer Research Center at the University of Southern California (USC) have designed and fabricated a 35 MHz linear array using 2–2 composite for medical imaging applications. We present simulation results from a prototype array with the same aperture sampling in the lateral direction operating at the same center frequency with similar fractional bandwidth. We have modified the element dimensions in the elevation direction to maintain an appropriate f -number in this direction. Images of a cyst phantom similar to the one described in Section III-A-2 were simulated using the same aperture with element spacing of $\lambda/2$ (for reference), 1.2λ (corresponding to the USC design), and 2λ and an f -number of 2 at the center of the cyst, (0,0,6.4) mm. For each array design, the element width was approximately equal to the element-to-element spacing (we allowed for a 10 μm kerf between elements). The larger element size may preserve the transducer bandwidth and minimize the cross coupling between the array elements. Of course, these benefits may be realized only if the image artifacts due to coarse aperture sampling could be removed or significantly reduced by appropriate signal processing of the beamformed data. This is illustrated by the simulation results given below for a transducer aperture with the following parameters:

- Center frequency: $f_0 = 35$ MHz
- Fractional bandwidth: FB = 55%
- Active aperture size: 3.2 mm
- Focal distance: 6.4 mm
- Element height: 1.5 mm
- Mechanical focus: 7.5 mm
- Element-to-element spacing: 0.5λ (≈ 21 μm), 1.2λ (≈ 51 μm), 2λ (≈ 85 μm).

The characteristics of the PSF for the 35 MHz array prototype are similar to those of the 25 MHz array described above (with appropriate frequency scaling) and will not be repeated here. A summary of the spatial resolution results can be seen in Fig. 14(a), which shows the axial and lateral resolution values as a function of $\log\beta$. The result follows the same trend

seen in Fig. 8(a) for the 25 MHz array with 2λ element-to-element spacing. The MGR dependence on $\log\beta$ also follows the same trend seen for the 25 MHz array as shown below.

To evaluate the array performance in imaging contrast targets, we simulated a $4 \times 12 \text{ mm}^2$ speckle-generating cyst phantom (2 mm diameter cyst at the center [0 0 6.4] mm). The lateral extension of the cyst phantom is chosen to include the 1st-order grating lobe due to coarse sampling. Fig. 15 shows the 50 dB grayscale images of the $3.8 \times 12 \text{ mm}^2$ region in the cyst phantom obtained using 35 MHz aperture with element spacing: (a) 0.5λ , (b) 1.2λ , and (c) 2λ . In addition, Fig. 15(d) shows the grayscale image obtained after the application of the 2-D PIO with $\beta = 0.3$ to the beamformed data collected with the coarsely sampled array (2λ element spacing). The results show that conventional beamforming with the Nyquist-sampled aperture produces an image with a CR of 30.4 dB. Conventional beamforming with the 1.2λ and 2λ sampled apertures produce images with CR values of 25.8 dB and 17.1 dB, respectively. The loss in image quality is quite significant in the case of the coarsely sampled aperture. The loss in contrast was completely recovered by applying the 2-D PIO to the beamformed RF data from this array as can be seen in Fig. 15(d) with minimum loss in spatial resolution. Table III summarizes the spatial resolution values achieved using conventional beamforming with the Nyquist-sampled and coarsely-sampled ($d = 2\lambda$) arrays. It also shows the values achieved after the application of the 2-D PIO filter ($\beta = 0.3$) to the beamformed data from the coarsely sampled array as listed in Table IV.

To further illustrate the well-behaved nature of the CR as a function of the regularization parameter, Fig. 14(b) shows the MGR and CR as a function of $\log\beta$ for $\beta = 0.001 \dots 10$. Both MGR and CR increase with $\log\beta$ gradually in a manner that suggests easy implementation on modern scanners. Although the CR is not directly proportional to the MGR as a function of the regularization parameter, the results suggest that the MGR is a good predictor of the performance of the imaging system in terms of contrast.

In Section II, the proposed approach is derived as a regularized pseudoinverse filtering operation in k -space preceded and followed by the 2-D FFT operations. The 2-D regularized imaging operator was applied to the full frame data in k -space using computationally efficient 2-D FFT. Furthermore, the FFT size is at least the size of the full frame data plus the size of the system response to avoid aliasing. The computational efficiency of the 2-D filter can be further improved by implementing the filter in the spatial domain provided the filter has finite region of support (ROS). In Fig. 16, an example of the 2-D kernel with size of $266.4 \times 651.2 \text{ }\mu\text{m}$ is obtained from the ROS of the original filter with $\beta = 0.3$.

The algorithm was verified in imaging the same speckle-generating cyst phantom using the same 25 MHz linear array with 2λ element spacing. The image of the cyst phantom is shown in Fig. 17(b) by applying the 2-D kernel of the post-beamforming 2-D pseudoinverse filter in spatiotemporal domain to the RF data obtained using the coarsely sampled array. The contrast ratio was measured as 28.2 dB, 2% less than the results filtered in k -space. In addition, the spatial resolutions were $71 \text{ }\mu\text{m}$ in the axial direction and $131 \text{ }\mu\text{m}$ in the lateral direction, respectively. The results verify that the 2-D spatial filter with finite ROS can achieve the same level of restoration in CR while maintaining the same level of spatial resolutions achieved by the full k -space filtering approach. The size of the 2-D kernel is only 0.03% of the size of original filter, which significantly reduces the computational load, allowing for real-time implementation especially with modern hardware platforms optimized for digital filtering.

IV. Discussion

The simulation results shown in this paper demonstrate that 2-D postbeamforming filtering can be effectively used in recovering the loss in contrast due to beamforming artifacts with minimum loss in spatial resolution. In particular, grating-lobe artifacts can be removed by applying a properly designed 2-D PIO to the beamformed data acquired using arrays with coarse sampling (element spacing in the range of 1.2 to 2.5λ). In principle, loss in contrast due to even coarser sampling can be restored, but the spatial frequency loss due to increased element directivity becomes excessive for element spacing $>3\lambda$. The regularization approach described in this paper may not be sufficient for recovering some or all of the nulls that appear in the passband defined by the aperture (see Fig. 2). It is important to point out, however, the significance of the contrast recovery achieved for element spacing in the range of 2λ demonstrated in this paper. As suggested by the 35 MHz prototype example, the same aperture could be sampled at 2λ ($\approx 85\text{ }\mu\text{m}$) and still achieve the same CR levels as the Nyquist sampling case ($\approx 1\text{ }\mu\text{m}$) and exceed the performance of conventional beamforming with the existing prototype sampled at 1.2λ ($\approx 51\text{ }\mu\text{m}$). Given the current state of transducer technology, easing the requirements on array sampling may allow higher fidelity imaging by reducing element cross coupling and/or preserving transducer bandwidth.

The imaging system model shown in this paper applies to linear-array imaging, which is useful given the wide use of this imaging mode in ultrasound. However, extending this approach to phased-array imaging may be of significant value in high-frequency ultrasound, e.g., forward-looking catheter arrays in intravascular applications. In addition to the obvious formulation of the problem in the $r \sin\theta$ coordinate system, one has to account for the spatially varying nature of the array PSF with the steering angle. This approach is currently being investigated.

We have shown that, although k -space provides an ideal domain for deriving the 2-D PIO, the filter can be implemented in either k -space or directly in the spatial domain. We have also shown that the filter kernel for spatial implementation has a finite ROS, which offers the promise of real-time implementation. An added advantage for the implementation in the spatial domain is the reduced sensitivity to motion by operating the filter on a limited number of A-lines at a time.

The formulation of the 2-D PIO described in this paper applies to conventional pulse excitation. For imaging systems employing coded excitation, our formulation applies to the beamformer output before pulse compression. In practice, however, it will be possible to design one postbeamforming filter for contrast enhancement and pulse compression. A complete treatment of this topic is beyond the scope of this paper due to the plethora of possible codes and coding schemes.

In this paper, we have presented the 2-D PIO as a tool for restoring contrast without a significant loss in spatial resolution. The results show that such a restoration, based on single-parameter regularization of the imaging operator, can be achieved by simple controls on modern imaging scanners in much the same way TGC is implemented. Future work will address the regularization problem in more detail to investigate the feasibility of simultaneous optimization of the contrast and spatial resolutions in ultrasound imaging systems.

V. Conclusion

We presented a 2-D postbeamforming filtering algorithm for restoration of contrast resolution in pulse-echo ultrasonic imaging systems employing coarsely sampled apertures. The algorithm is based on a discretized receive signal model of the imaging FOV on a

Cartesian grid in linear-array imaging. The formulation of the operator was shown to allow a computationally efficient implementation of the filter in k -space using 2-D fast Fourier transform or in the spatial domain using a 2-D impulse response with small ROS. In addition, a robust implementation of the PIO restoration filter was obtained through a single-parameter regularization in k -space. Measures of spatial and contrast resolution were shown to vary smoothly (approximately as a sigmoid) with the logarithm of the regularization parameter. This behavior was demonstrated for 2 high-frequency ultrasound arrays operating at 25 and 35 MHz with fractional bandwidths of 60% and 55%, respectively. The latter array has been described in the recent literature [39] and represents an example of the latest technology in HFUS. The simulation results clearly show that, for a given array sampling geometry, one can design a 2-D PIO with a specific regularization parameter value to restore the contrast ratio to levels achieved by Nyquist-sampled arrays without sacrificing spatial resolution. Therefore, the 2-D postbeamforming PIO approach described in this paper offers the promise of realizing the benefits of high-resolution ultrasonic imaging using coarsely sampled apertures.

Acknowledgments

Funded by Grants HL 71538 and EB 6893 from the National Institutes of Health.

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Fig. 1. A 1-D linear array system model where the scattering is assumed to result from scatterers with random amplitudes on the Cartesian grid.

Fig. 2.

Frequency responses in k -space for generic apertures with different values of d_e : 0.5λ to 4λ , f -number = 2, and fractional bandwidth of 60%. Patterns are displayed using a 50 dB dynamic range with lateral direction along the horizontal axis and axial direction along the vertical axis.

Fig. 3. The frequency response of a 2-D PIO filter in k -space for a generic linear array with $d_e = 2\lambda$ using $\beta = 0.01, 0.3$, and 10 .

Fig. 4. Grayscale representations (70 dB) of the PSF. The vertical coordinate is the axial direction, the horizontal coordinate is the lateral direction, and both are in millimeters. Lateral pitch: (a) 0.5λ and (b) 2λ .

Fig. 5. Comparison of (a) axial and (b) lateral profiles of the PSF for 0.5λ pitch and 2λ pitch.

Fig. 6. Axial profiles of the PSF, filtered by 2-D PIO filter with regularization parameter $\beta = 0.001$, 0.1, and 1.

Fig. 7. Lateral profiles of the PSF, filtered by 2-D PIO filter with regularization parameter $\beta = 0.001, 0.1, \text{ and } 1$.

Fig. 8.
70 dB gray scale representation of the PSF filtered by 2-D PIO filter with $\beta = 0.1$.

Fig. 9.

(a) Spatial resolutions and (b) MGR and CR resulting from the PSFs produced by 2-D PIO filter as a function of $\log\beta$.

Fig. 10.

Grayscale images (50 dB) obtained from the cyst phantom. The vertical coordinate is the axial direction, the horizontal coordinate is the lateral direction, and both are in millimeters; (a) conventional beamforming using a Nyquist sampled array; (b) conventional beamforming using a coarsely sampled array (2λ).

Fig. 11.

Grayscale images (50 dB) obtained from the cyst phantom, filtered by the regularized 2-D PIO filter. The vertical coordinate is the axial direction, the horizontal coordinate is the lateral direction, and both are in millimeters. (a) $\beta = 0.01$, (b) $\beta = 0.1$, (c) $\beta = 0.3$, (d) $\beta = 1$, and (e) $\beta = 10$.

Fig. 12.

Lateral line plots through cyst center at $z = 12$ mm for different values of β . The loss of lateral resolution can be clearly seen for $\beta = 10$ while the loss of contrast is clear for $\beta = 0.01$.

Fig. 13.

Axial line plots through cyst center at $x = 0$ mm for different values of β . One can observe a minimal loss in resolution for $\beta = 10$ in the axial direction while the loss of contrast is clear for $\beta = 0.01$.

Fig. 14.

(a) Spatial resolutions and (b) MGR and CR of results filtered by 2-D PIO filter with sweeping regularization parameter β .

Fig. 15.

Grayscale images (50 dB) obtained from the cyst phantom. The vertical coordinate is the axial direction, the horizontal coordinate is the lateral direction, and both are in millimeters. Lateral pitch: (a) 0.5λ , (b) 1.2λ , (c) 2λ , and (d) 2λ , $\beta = 0.3$.

Fig. 16.

The kernel of the 2-D postbeamforming pseudoinverse filter. The vertical coordinate is the axial direction, the horizontal coordinate is the lateral direction, and both are in millimeters.

Fig. 17.

Grayscale image (50 dB) obtained from the cyst phantom. The vertical coordinate is the axial direction, the horizontal coordinate is the lateral direction, and both are in millimeters. (a) Full k -space with $\beta = 0.3$ and (b) 2-D kernel with $\beta = 0.3$.

TABLE I

Contrast Ratio of the Images from the Cyst Phantom.

		2-D PIO filter, β					
0.52	22	0.001	0.01	0.1	0.3	1	10
28.8	20.4	18.1	22.7	26.9	28.8	30	29.2

TABLE II

Resolution Measurements from the PSF.

	0.5λ	2λ	$\beta = 0.3$
Axial resolution (μm)	61.2	62.2	71
Lateral resolution (μm)	111.8	120	128

TABLE III

Resolution Measurements from the PSF.

	0.5λ	2λ	$\beta = 0.3$
Axial resolution (μm)	47.7	47.7	55.7
Lateral resolution (μm)	80.4	84.7	88.7

TABLE IV

Contrast Ratio of the Images from the Cyst Phantom.

2-D PIO filter, β									
0.52	1.22	22	0.001	0.01	0.1	0.3	1	10	
30.4	25.8	17.1	17.2	22	28.3	30.4	32	32.6	