Characterization of imaging performance in differential phase contrast CT compared with the conventional CT—Noise power spectrum NPS(k)

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Purpose: The differential phase contrast CT is emerging as a new technology to improve the contrast sensitivity of the conventional CT. Via system analysis, modeling, and computer simulation, the authors study the noise power spectrum (NPS)—an imaging performance indicator—of the differential phase contrast CT and compare it with that of the conventional CT.

Methods: The differential phase contrast CT is implemented with x-ray tube and gratings. The x-ray propagation and data acquisition are modeled and simulated with Fourier analysis and Fresnel analysis. To avoid any interference caused by scatter and beam hardening, a monochromatic x-ray source (30 keV) is assumed, which irradiates the object to be imaged by 360° so that no weighting scheme is needed. A 20-fold up-sampling is assumed to simulate x-ray beam's propagation through the gratings G_1 and G_2 with periods 8 and 4 μm , respectively, while the intergrating distance is 193.6 mm (1/16 of the Tabolt distance). The dimension of the detector cell for data acquisition ranges from 32×32 to $128 \times 128 \mu\text{m}^2$, while the field of view in data acquisition is $40.96 \times 40.96 \text{ mm}^2$. A uniform water phantom with a diameter 37.68 mm is employed to study the NPS, with its complex refraction coefficient $n = 1 - \delta + i\beta = 1 - 2.5604 \times 10^{-7} + i1.2353 \times 10^{-10}$. The x-ray flux ranges from 10^6 to $10^8 \text{ photon/cm}^2 \cdot \text{projection}$ and observes the Poisson distribution, which is consistent with that of micro-CT in preclinical applications. The image matrix of reconstructed water phantom is 1280×1280 , and a total of 180 regions at 128×128 matrix are used for NPS calculation via 2D Fourier Transform in which adequate zero padding is applied to avoid aliasing.

Results: The preliminary data show that the differential phase contrast CT manifests its NPS with a $1/|k|$ trait, while the distribution of the conventional CTs NPS observes $|k|$. This accounts for the significant difference in their noise granularity and the differential phase contrast CTs substantial advantage in noise over the conventional CT, particularly, in the situations where the detector cell size for data acquisition is smaller than 100 μm .

Conclusions: The differential phase contrast CT detects the projection of refractive coefficient's derivative and uses the Hilbert filter for image reconstruction, which leads to the radical difference in its NPS and the advantage in noise in comparison to that of the conventional CT. © 2011 American Association of Physicists in Medicine. [DOI: 10.1118/1.3602071]

Key words: CT, x-ray, x-ray phase CT, x-ray differential phase contrast CT, x-ray tube and grating-based phase CT, imaging performance, modulation transfer function, noise, noise power spectrum

I. INTRODUCTION

The conventional attenuation-based x-ray CT (namely conventional CT henceforth) has become one of the most popular modalities in the clinic for physicians to diagnose a wide spectrum of diseases. In comparison to other quantitative tomographic imaging modalities, the strength of the conventional CT lies in its balance over all the aspects of image quality,¹ while its contrast resolution or contrast sensitivity for detecting low contrast soft tissues is relatively low.² The major reasons underlying the conventional CTs weakness in contrast resolution are: (1) the subject contrast between the pathologic lesion and surrounding normal tissues generated by the energetic x-ray flux is relatively small and (2) the noise, especially the noise at high frequency, in the conventional CT images is boosted by the ramp filter required by

the filtered backprojection (FBP) reconstruction algorithm that has been preferably employed in diagnostic CT scanners offered by all the major CT vendors. Despite its wide usage in the clinic, the imaging performance, especially the contrast sensitivity, of the conventional CT still needs to be improved to meet the challenges imposed by the preclinical and clinical applications in which the differentiation of soft tissue is essential.² Meanwhile, the radiation dose associated with the conventional CT is inversely proportional to the fourth power of detector cell size, i.e., a ten times smaller detector cell may result in a 10^4 -fold increase in radiation dose.³ In the last two decades, the detector cell dimension of diagnostic CT has evolved from centimeter to submillimeter,¹ and the annual number of CT scans performed worldwide has continuously increased, which has drawn serious concern in the community about the radiation

risk rendered by CT scanners to patients and healthcare providers.⁴

Recently, the x-ray CT based on a new mechanism—refraction—has emerged as a new technology to substantially improve the conventional CTs capability of differentiating soft tissues.^{5,6} Unlike the conventional CT, which detects the amplitude variation (attenuation) of x-ray, the refraction-based CT detects its phase variation, in which the key is the retrieval of phase variation determined by the 3D refraction distribution of the object to be imaged. In physics, the refractive index of a material is complex and can be expressed as $n \equiv 1 - \delta + i\beta$.⁵⁻⁹ The amplitude of x-ray propagation is dependent on β , while the phase is determined by δ . In the conventional CT, one chiefly deals with β —the attenuation. In most biological tissues, β is approximately 10^{-10} and varies little, and this is the reason underlying the fact that the conventional CT is not good at imaging soft tissues. However, δ —the imaging parameter for x-ray phase CT—is 10^3 - to 10^4 -fold larger.⁵⁻⁹ It is hoped that a substantially larger parameter may provide a better chance to differentiate the pathologic lesion from surrounding biological tissues, and such optimism is being evaluated through extensive experimental effort,⁵⁻¹⁵ mainly because the contrast is actually dependent on the difference, rather than the magnitude, of the δ over biological tissues. Early attempts in x-ray phase-sensitive imaging were propagation-based,⁸ followed by the diffraction enhanced methods for better contrast property at increased x-ray energy.^{8,9} The latest progress in x-ray phase imaging is the advent of differential phase contrast CT implemented with x-ray tube and gratings in which the projection of the refractive coefficient's derivative is recorded in data acquisition.⁶

It has been a fundamental understanding that there exists an interpixel correlation in the noise of a conventional CT image though there is no intercell correlation in the noise of the x-ray detector for projection data acquisition. The noise correlation in the tomographic image is induced by the image reconstruction process, especially the reconstruction filter.¹⁶⁻²⁹ It has become a common practice that the noise behavior of a CT system can only be fully depicted by its noise power spectrum (or Wiener spectrum)—the distribution of noise power over spatial frequencies.¹⁶⁻²⁴ Such a practice has continued from the early CT scanners with a single row x-ray detector,¹⁶⁻²³ the state-of-the-art diagnostic CT scanners using one or two multirrow x-ray detectors,^{27,28} to the flat-panel imager-based cone beam CT.^{25,26,29} On the other hand, mainly due to its convenience and usefulness in many practical situations, the gross noise defined as the root-mean-square (RMS) or the standard deviation (SD) of image pixels within a region of interest in a CT image has been extensively employed to analyze the noise property of a CT system, especially in analyzing the noise variation as a function of radiation dose or detector cell size.^{3,30} Since the signal detected in the differential phase contrast CT implemented with x-ray source and gratings is the projection of the refraction's derivative,⁶ the ramp filter³⁰ that is notorious in its noise behavior can be replaced by the Hilbert filter^{31,32} that is much more tractable in noise. Consequently, the noise behavior, either the noise power spectrum or gross noise, or both, of the differential phase contrast CT

implemented with x-ray tube and gratings is anticipated to be substantially different from those of the conventional CT.

The investigation of the differential phase contrast CTs gross noise and its variation as a function of image's spatial resolution or detector cell size has been reported recently in the literature,¹¹⁻¹⁵ which has initiated continuous effort for the analysis of differential phase contrast CTs noise behavior. Meanwhile, we believe that it is imperative and more informative to fully describe the differential phase contrast CTs noise behavior through analyzing its noise power spectrum. In fact, under the ideal-observer framework,^{17,18,20,23} the figure of merit to assess the performance of an imaging system include the large area transfer function, spatial detail transfer function, noise power spectrum, and noise equivalent quanta. In this paper, we focus on the characterization of the noise power spectrum of the differential phase contrast CT implemented with x-ray tube and gratings. Through system analysis, modeling, and computer simulation, we conduct a quantitative investigation of its noise power spectrum and compare it with that of the conventional CT. We report the preliminary results obtained so far and hope they are informative and instructive to the readers who are interested in the design, evaluation, and optimization of differential phase contrast CTs imaging performance for preclinical and ultimately clinical applications.

II. MATERIALS AND METHODS

The goal of this work is to quantitatively characterize the potential noise power spectrum of the differential phase contrast CT implemented with x-ray tube and gratings and its comparison with that of the conventional CT. Hence, we constrain ourselves to conducting computer simulation study, so that the systematic and random errors that may exist in a physical differential phase contrast CT and compromise the accuracy and precision of noise power spectrum analysis can be excluded.

II.A. The differential phase contrast CT implemented with x-ray tube and gratings—Data acquisition and image generation

The architecture of a differential phase contrast CT implemented with a microfocus x-ray tube and gratings⁶ is shown in Fig. 1(a). To increase the x-ray source power for a shorter time to acquire the data, an x-ray tube with finite size can be utilized as shown in Fig. 1(b), in which a 1D grating G_0 is employed to meet the spatial coherence condition.⁶ G_1 is a phase grating and G_2 an absorption grating, which can be fabricated with photolithography, deep chemical etching and electroplating.³³ G_1 and G_2 work together as a shearing interferometer³⁴⁻³⁶ to detect the wavefront alteration caused by the object in the x-ray beam. A CCD x-ray detector is employed for data acquisition in which the x-ray tube irradiates the specimen that rotates along its own axis by a range satisfying the data sufficiency condition.

The key component of the imaging chain is grating G_1 , a diffraction interferometer based on the Talbot effect.³⁴⁻³⁶ Figure 2(a) shows how G_1 works by virtually decomposing it

FIG. 1. The schematic illustration of an x-ray grating-based differential phase contrast CT implemented with: (a) an x-ray tube of microfocus or (b) a commercial x-ray tube of finite size focal spot (see Ref. 6).

into absorption gratings A and B . The extra optical path of grating B relative to grating A is half wavelength, which is equivalent to a 180° or π phase shift. The beams corresponding to gratings A and B undergo different optical paths before they reach the gratings and recombine after they pass through. Interference fringe appears if the refraction is different between the paths. The Fresnel analysis^{34–37} unveils the mechanism behind Fig. 2(a) for phase retrieving

$$E_A(x, z) = \sum_{m=-\infty}^{\infty} c_m \exp\left(\frac{-i2\pi m^2 z}{z_T}\right) \times \exp\left(\frac{i2\pi m(x + \Delta x/2)}{g_1}\right), \quad (1)$$

$$E_B(x, z) = \sum_{m=-\infty}^{\infty} c_m \exp\left(\frac{-i2\pi m^2 z}{z_T}\right) \times \exp\left(\frac{i2\pi m(x - \Delta x/2)}{g_1}\right), \quad (2)$$

$$E_{A+B}(x, z) = E_A(x, z) + E_B(x, z), \quad (3)$$

$$I_{A+B}(x, z) \sim \phi(x + \Delta x/2, y) - \phi(x - \Delta x/2, y) \cong \frac{\partial \phi(x, y)}{\partial x} \Delta x, \quad (4)$$

where c_m is the Fourier series coefficients of the 1D grating A or B and Δx is their relative displacement. $\phi(x, y)$ is the phase, which is actually the projection of the refractive coefficient along the x-ray path Z , i.e.,

$$\phi(x, y) = \frac{2\pi}{\lambda} \int_Z \delta(x, y, z) dz. \quad (5)$$

$E(x, z)$ is the E -field of x-ray and $I(x, z)$ the irradiance. Equation (4) shows that the irradiance depends on the derivative of the phase variation along the x-axis. As shown in Fig. 2(a), the fringe period at location $(1/16)z_T$ is half that of G_1 , requiring that the period of G_2 be half that of G_1 .⁵ After the x-ray passes grating G_2 , the irradiance at detector D is¹⁰

FIG. 2. The diagram shows the schematic of virtual grating decomposition in which the upper and lower Talbot patterns associate with gratings A and B , respectively (a); 3D refraction distribution of a cylindrical object (b); the corresponding phase derivative retrieved at the detector behind G_2 (b').

$$I_{d_x, d_y}(x) = a_0(d_x, d_y) + \sum_{m=1}^{\infty} a_m(d_x, d_y) \times \cos\left(\frac{2\pi m x}{g_2} + \varphi_m(d_x, d_y)\right), \quad (6)$$

where (d_x, d_y) is the coordinate in the detector and g_2 the period of grating G_2 . If the object in the beam is not pure phase, i.e., it attenuates the x-ray beam, one needs to linearly shift G_2 along the x-axis. Via Fourier analysis, one can determine $a_0(d_x, d_y)$, $a_1(d_x, d_y)$, $\varphi_1(d_x, d_y)$ from Eq. (6). Then, one obtains

$$\frac{\partial \phi(x, y)}{\partial x} = \varphi_1(d_x, d_y) g_2 / \lambda z'_T, \quad (7)$$

where z'_T is a fraction of the Talbot distance z_T , and $a_0(d_x, d_y)$ and $\partial \phi(x, y) / \partial x$ are the foundation of x-ray attenuation and phase CT imaging, respectively. Shown in Fig. 2(b) is a cylindrical object with pure refraction, and that in Fig. 2(b') is an illustration of the detection of $\partial \phi(x, y) / \partial x$ at detector D shown in Fig. 1.

Note that, by substituting the $\phi(x, y)$ defined in Eq. (5) into Eq. (7), we further get

$$\begin{aligned} \varphi_1(d_x, d_y) &= \frac{\lambda z'_T}{g_2} \frac{\partial \phi(x, y)}{\partial x} = \frac{\lambda z'_T}{g_2} \frac{\partial}{\partial x} \left(\frac{2\pi}{\lambda} \int_Z \delta(x, y, z) dz \right) \\ &= \frac{2\pi z'_T}{g_2} \int_Z \frac{\partial}{\partial x} \delta(x, y, z) dz. \end{aligned} \quad (8)$$

This means that the phase retrieved through a Fourier analysis of Eq. (6) is the projection of the refractive coefficient's derivative, and this is the underlying reason that the phase CT implemented with x-ray tube and gratings is called the differential phase contrast CT. Once $\partial \phi(x, y) / \partial x$ and $a_0(d_x, d_y)$ are acquired, the corresponding tomographic images of refraction and attenuation can be, respectively, reconstructed using the filtered backprojection (FBP) algorithms that have been extensively employed in the conventional CT. Since the reconstruction of the refraction image can be carried out directly from the refraction derivative $\partial \delta(x, y, z) / \partial x$, the ramp kernel that is notorious in noise behavior can be replaced with the Hilbert filter (or actually the finite Hilbert filter)^{31,32} that is much more tractable in its noise behavior.

II.B. Characterization of noise behavior in differentiate phase contrast CT implemented with x-ray tube and gratings

In the early days of the conventional CT technology, the observation of the morphologic difference in the noise of CT images against the white noise suggests that the noise at one point in a CT image is correlated to that at another point in the same image.^{16–24} By analyzing the distribution of noise power over spatial frequencies, i.e., the noise power spectrum, it has been verified that there indeed exists an inter-pixel correlation in the conventional CT images, even though there is no intercell correlation in the noise of x-ray detector cells during projection data acquisition. Since then,

the groundwork of using the noise power spectrum to analyze the noise behavior of a CT system or CT imaging method has been laid out by the researchers in the field.^{16–24}

In this paper, we follow this convention to analyze the noise property of the differential phase contrast CT implemented with x-ray tube and gratings and compare it with that of the conventional CT. Prior to doing so, we concisely review the theoretical and practical aspects of the groundwork of noise power spectrum in the conventional CT imaging.

II.B.1. Noise power spectrum in conventional CT imaging

According to the Wiener–Khinchin Theorem, the noise power spectrum and autocorrelation function of a two-dimensional (2D) image is a Fourier transform pair.³⁸ Initially, the noise power spectrum of an image is defined as,¹⁸

$$\text{NPS}(k_x, k_y) = \frac{1}{A} \left\langle \left| \iint_A R(x, y) \exp(-2\pi i(xk_x + yk_y)) dx dy \right|^2 \right\rangle. \quad (9)$$

Here $R(x, y)$ is the spatial autocorrelation function of an image containing noise only and $\langle \rangle$ represents the ensemble average operation. In a way similar to the treatment of the central slice theorem,³⁰ Riederer *et al.* obtain the noise power spectrum of a CT image in the polar coordinate system by diagonally tiling the noise power spectrum of each projection along the spoke corresponding to its data acquisition angle with a weighting factor that is proportional to the spoke density¹⁶

$$\text{NPS}(k, \phi) = \frac{\pi}{m\bar{N}} \frac{|G(k)|^2}{k}, \quad (10)$$

where k represents the radial spatial frequency and is defined as

$$k = \sqrt{k_x^2 + k_y^2}. \quad (11)$$

Here m is the number of projections and $\bar{N}$ is the mean number of x-ray photons detected at a detector cell, which is assumed equal across all the detector cells. $G(k)$ is the filter associated with the algorithm to reconstruct the tomographic images.

In the FBP algorithm with the ramp filter utilized, Hanson gives a more specific expression¹⁸

$$G(k) = |k|H(k), \quad (12)$$

$$\text{NPS}(k) = \frac{\pi}{m\bar{N}} |k| |H(k)|^2, \quad (13)$$

where $|k|$ represents the well-known ramp filter. $H(k)$ is the overall modulation transfer function^{18,20} related to the numerous windowing and/or boosting techniques that have been adopted in CT for diagnostic imaging with adequate trade-off between noise and spatial resolution over clinical applications. Subsequently, a discrete form of Eq. (13) is given by Faulkner and Moores as²¹

$$\text{NPS}(n_k \Delta k) = \frac{\pi \Delta R |n_k \Delta k| |H(n_k \Delta k)|^2}{m \bar{N}}. \quad (14)$$

Later, the effect of spatial sampling on the noise power spectrum is pointed out and verified through simulation study by Kijewski and Judy in Ref. 22. If the ramp filter $|k|$ is replaced by the Hilbert filter as we are doing in the differential phase contrast CT implemented with x-ray tube and gratings, Eqs. (12)–(14) correspondingly become

$$G(k) = H(k), \quad (12')$$

$$\text{NPS}(k) = \frac{\pi}{m \bar{N}} \frac{|H(k)|^2}{k}, \quad (13')$$

$$\text{NPS}(n_k \Delta k) = \frac{\pi \Delta R}{m \bar{N}} \cdot \frac{|H(n_k \Delta k)|^2}{|n_k \Delta k|}. \quad (14')$$

II.B.2. Gross noise in conventional CT imaging

One can immediately get the gross noise in a CT image from the noise power spectrum by integrating $\text{NPS}(k)$ [or sum $\text{NPS}(n_k \Delta k)$] over all spatial frequency components in the Cartesian coordinate system

$$\sigma_r^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \text{NPS}(k_x, k_y) dk_x dk_y, \quad (15)$$

or equivalently in the polar coordinate system

$$\sigma_r^2 = \int_0^{2\pi} \int_{-\infty}^{\infty} \text{NPS}(k, \phi) k dk d\phi, \quad (16)$$

where the subscript r represents “reconstructed.”

Substituting Eq. (10) into Eq. (16), Riederer *et al.*¹⁶ give the following general expression of gross noise:

$$\begin{aligned} \sigma_r^2 &= \frac{\pi}{m \bar{N}} \int_0^{2\pi} \int_{-\infty}^{\infty} \frac{|G(k)|^2}{k} k dk d\phi \\ &= \frac{\pi}{m \bar{N}} \int_0^{2\pi} \int_{-\infty}^{\infty} |G(k)|^2 dk d\phi. \end{aligned} \quad (17)$$

Assuming the projection data be obtained from uncorrelated measurement by detector cells at equal space a , Hanson¹⁸ gives a more specific result by introducing the noise coefficient K_{noise}

$$\sigma_r^2 = \frac{\sigma_p^2}{ma^2} K_{\text{noise}}^2, \quad (18)$$

where σ_p^2 is the noise corresponding to each detector cell, a represents the aperture of a detector cell, and

$$K_{\text{noise}}^2 = \frac{\pi^2}{4k_N^3} \int_0^{k_N} |G(k)|^2 dk. \quad (19)$$

Note that in fact one has^{16–20}

$$\sigma_p^2 = 1/\bar{N}, \quad (20)$$

and the noise power spectrum of each projection measurement is assumed as band-limited in getting Eqs. (18) and (19), i.e.,¹⁸

$$\text{NPS}_p(k) = \begin{cases} \Delta \sigma_p^2 & |k| \leq k_N \\ 0 & |k| > k_N \end{cases}, \quad (21)$$

where k_N is the Nyquist frequency corresponding to the aperture of the detector cell and is in general equal to $(2a)^{-1}$.

With respect to the FBP reconstruction algorithm using the ramp filter, one has $G(k) = |k|H(k)$. Accordingly, if we assume $H(k) \cong 1$, the noise coefficient K_{noise}^2 approximately becomes a constant¹⁸

$$K_{n\text{-ramp}}^2 = \frac{\pi^2}{4k_N^3} \int_0^{k_N} |k|^2 |H(k)|^2 dk = \frac{\pi^2}{4k_N^3} \int_0^{k_N} |k|^2 dk = \frac{\pi}{12}. \quad (22)$$

As for the FBP reconstruction algorithm using the Hilbert filter, one has $G(k) = H(k)$. Again, if we assume $H(k) \cong 1$, the noise coefficient K_{noise}^2 approximately becomes

$$K_{n\text{-Hilbert}}^2 = \frac{\pi^2}{4k_N^3} \int_0^{k_N} |H(k)|^2 dk = \frac{\pi^2}{4k_N^3} \int_0^{k_N} dk = \frac{\pi}{4k_N^2}. \quad (23)$$

Substituting Eqs. (22) and (23) into Eq. (18), respectively, one gets

$$\sigma_{r\text{-ramp}}^2 = \frac{\pi}{12} \frac{\sigma_p^2}{ma^2}, \quad (24)$$

$$\sigma_{r\text{-Hilbert}}^2 = \frac{\sigma_p^2}{ma^2} \cdot \frac{\pi}{4k_N^2} = \frac{\pi}{4} \cdot \frac{\sigma_p^2}{m}. \quad (25)$$

Notice the absence of a^2 in the denominator of Eq. (25) in contrast to Eq. (24), which accounts for the substantial radiation dose advantages of the differential phase contrast CT over the conventional CT with increasingly finer detector cells as reported in the literature.^{11–15}

II.C. Measurement of noise power spectrum and gross noise in CT images

According to Eq. (9), the noise power spectrum can be calculated by taking the Fourier transform of the autocorrelation function that is obtained using a large number of noise images. An alternative approach that is more efficient in computation is to take the average of the squared Fourier transform of a large number of noise images or regions within the noise images. The obtained 2D noise power spectrum is in discrete form and circular symmetry about its origin, as predicted in Eqs. (14) and (14'). Once the noise power spectrum is obtained, the gross noise can be acquired in principle by integration according to Eq. (15) or Eqs. (16) and (17).

An approach that is much more frequently employed in practice is to calculate the gross noise of a CT image as the root-mean-square or the SD of the pixel's intensity in the image or a region within the image^{18,20}

$$\sigma_r = \sqrt{\sum_{l=1}^L (I_l - \bar{I})^2 / L - 1}, \quad (26)$$

where I_l is the value of a pixel in the image or a region of the image, L is the total number of pixels involved in the

calculation, and $\bar{I}$ is the mean of all the pixels' intensity defined as

$$\bar{I} = \sum_{l=1}^L I_l / L. \quad (27)$$

III. EVALUATION VIA SIMULATION STUDIES

Our effort in this work is dedicated to conducting the simulation study of the differential phase contrast CTs noise power spectrum in the parallel beam geometry, mainly because of the following reasons: (1) the x-ray beam in the differential phase contrast CT satisfies the paraxial condition^{34–37} and, thus, the beam is almost parallel, (2) the image reconstruction algorithms in the parallel beam geometry outperform those in the fan beam geometry from the perspective of noise uniformity significantly and, thus, almost all the clinical CT scanners based on the third generation geometry (fan beam or cone beam) adopt the parallel beam reconstruction algorithms via fan-to-parallel rebinning,³⁹ and (3) most of the simulation studies to investigate the noise power spectrum of CT imaging thus far have been carried out in the parallel beam geometry to exclude the influence of rebinning and weighting schemes. To avoid any interference caused by scatter and beam hardening, a monochromatic x-ray source (30 keV) is assumed, which irradiates an object by 360° at 1° steps so that no weighting effect can be induced to degrade the noise uniformity. At 30 keV, a 20-fold up-sampling is assumed to simulate the x-ray beam's propagation through the gratings G_1 and G_2 with periods 8 and 4 μm , respectively, while the distance between these two gratings is 193.6 mm, i.e., 1/16 of the Talbot distance. The size of each detector cell for data acquisition ranges from 32×32 to $128 \times 128 \mu\text{m}^2$, and 1280 detector cells constitute a detector array to make a $40.96 \times 40.96 \text{ mm}^2$ field of view for data acquisition. Grating G_1 shifts ten times at step 0.4 μm along the x-axis to retrieve the phase information corresponding to the refractive property of the object to be imaged.

The signals $\partial\phi(x, y)/\partial x$ specified in Eq. (7) and $a_0(x)$ in Eq. (6) are used to reconstruct the phase and attenuation images, respectively, in which the algorithm used in the image reconstruction is the classical FBP algorithm in the parallel geometry.³⁰ The finite Hilbert transform^{31,32} is used in the image reconstruction for differential phase contrast CT, while the classical ramp filter³⁰ is used for the conventional CT. Since the purpose of this work is to investigate the potential imaging performance of the differential phase contrast CT and compare it with that of the conventional CT, no windowing or boosting techniques^{27,28} are adopted in image reconstruction for both the differential phase contrast CT and the conventional CT.

Prior to analyzing the noise power spectrum, the well-known Shepp–Logan phantom³⁰ at outer dimension $37.68 \times 28.26 \text{ mm}^2$ is employed to evaluate and to verify the simulation accuracy of x-ray propagation, data acquisition and tomographic image generation of the differential phase contrast CT implemented with x-ray tube and gratings. Later, the

Shepp–Logan phantom is also employed to evaluate the contrast-to-noise ratio of the differential phase contrast CT and its comparison with that of the conventional CT. A uniform water phantom, with diameter 37.68 mm and the complex refraction coefficient of water $n = 1 - \delta + i\beta = 1 - 2.5604 \times 10^{-7} + i1.2353 \times 10^{-10}$, is employed to study the noise power spectrum. The x-ray flux observing the Poisson distribution is set at from 10^6 to $10^8 \text{ photon/cm}^2 \cdot \text{projection}$ in the simulation study, which is consistent with that of x-ray *micro*-CT in pre-clinical applications. The image matrix of reconstructed water phantom is 1280×1280 , and a total of 180 regions at 128×128 matrix dimension within the uniform water phantom are used for noise power spectrum analysis via 2D Fourier transform in which adequate zero padding to convert the data matrix from 128×128 to 256×256 is implemented to avoid aliasing effect. It should be indicated that two images of the same uniform water phantom reconstructed from the data acquired at identical x-ray techniques but sequentially (and thus with uncorrelated noise in data acquisition) are subtracted so that the DC component caused by the water in the noise power spectrum can be readily removed.

IV. RESULTS

IV.A. Data acquisition and image generation of differential phase contrast CT

The transaxial images of the Shepp–Logan phantom generated by the differential phase contrast CT implemented with x-ray tube and gratings and the conventional attenuation-based CT at the x-ray exposure $10^7 \text{ photon/cm}^2 \cdot \text{projection}$ are presented in Figs. 3(a) and 3(b), respectively. An inspection of Fig. 3 shows that, by using the Shepp–Logan phantom that is good at evaluating the overall performance of a tomographic imaging method, the correctness and accuracy in simulating the data acquisition and image reconstruction of both the differential phase contrast CT implemented with x-ray tube and gratings and the conventional CT are evaluated and verified appropriately. Moreover, it is observed that the contrast-to-noise ratio in the image generated by the differential phase contrast CT implemented with x-ray tube and gratings is substantially better than that in the image generated by the conventional CT, while their spatial resolution is quite comparable.

Fig. 3. Transaxial image of the Shepp–Logan phantom generated by: (a) the differential phase contrast CT and (b) the conventional CT (x-ray exposure: $10^7 \text{ photon/cm}^2 \cdot \text{projection}$, detector cell size: $32 \times 32 \mu\text{m}^2$).

FIG. 4. Transaxial image of the cylindrical water phantom generated by: (a) the differential phase contrast CT at x-ray exposure $5 \times 10^6 \text{ photon/cm}^2 \cdot \text{projection}$; (b) the conventional CT at x-ray exposure $10^8 \text{ photon/cm}^2 \cdot \text{projection}$ (detector cell size: $32 \times 32 \mu\text{m}^2$).

IV.B. Noise granularity/texture of differential phase contrast CT compared with the conventional CT

The transaxial images of the uniform water phantom generated by the differential phase contrast CT implemented with x-ray tube and gratings and the conventional CT at x-ray exposure $5 \times 10^6 \text{ photon/cm}^2 \cdot \text{projection}$ are presented in Figs. 4(a) and 4(b), respectively. It is obvious to observe again that the gross noise in the image generated by the conventional CT is significantly larger than that by the differential phase contrast CT. In addition, it should also be observed that there exist radical differences in the granularity/texture of noise in the images generated by the differential phase contrast CT and the conventional CT. The noise in the former looks clumpy, while that in the latter appears granular. There is no doubt that such an appearance difference in the noise morphology implies a dramatic difference in the frequency domain, i.e., in their noise power spectra.

IV.C. Noise power spectrum of differential phase contrast CT compared with the conventional CT

The noise power spectrum of the differential phase contrast CT implemented with x-ray tube and gratings and the conventional CT are displayed in Figs. 5(a) and 5(b), respectively, wherein the brightness is proportional to the magnitude of the noise power. Consistent with the anticipation from Eqs. (13) to (14), the noise power spectrum of the

FIG. 5. The noise power spectrum of: (a) the differential phase contrast CT and (b) the conventional CT (x-ray exposure: $5 \times 10^6 \text{ photon/cm}^2 \cdot \text{projection}$, detector cell size: $32 \times 32 \mu\text{m}^2$).

conventional CT appears as a torus, whereas that of the differential phase contrast CT looks like a fountain. This means that the noise in the differential phase contrast CT is abundant in low frequency components, while that in the conventional CT is present in the high frequency components. Such an observation in the spatial frequency domain is consistent with the noise morphology in the spatial domain as demonstrated in Fig. 4.

The profiles along the radial line at 0° of the noise power spectra in Figs. 5(a) and 5(b) are plotted in Figs. 6(a) and 6(b), respectively. Note that the fluctuation in the profiles of the conventional CTs noise power spectrum is quite severe, because only 180 regions at matrix dimension 128×128 in the uniform water phantom are engaged in the noise power spectrum calculation. With an increasing number of ensemble samples (images or regions within the images of uniform phantom), smoother profiles corresponding to the noise power spectrum can be obtained. Actually, as demonstrated in Fig. 6(b), we carry out a fifth-order polynomial fitting of the noise power spectrum profile corresponding to the conventional CT, which asymptotically predicts the trend of noise power spectrum in the conventional CT if a larger number of ensemble samples, e.g., more than 1000, are available for the noise power spectrum analysis.¹⁸

IV.D. The gross noise of differential phase contrast CT compared with the conventional CT

The variation of contrast-to-noise ratio in the differential phase contrast CT as a function over the x-ray exposure and detector cell size is plotted in Fig. 7(a), and that of the conventional CT is in Fig. 7(b). Since the contrast does not vary over detector cell size, the variations in the contrast-to-noise ratios illustrated in Figs. 7(a) and 7(b) are exclusively caused by the variation in the gross noise. According to Eq. (25), the gross noise in the differential phase contrast CT $\sigma_{\text{phase-CT}}$ varies with σ_p , where σ_p is the noise existing in each detector cell and is inversely proportional to the aperture a of each detector cell, i.e.,

$$\sigma_{\text{phase-CT}} \propto \sigma_p \propto 1/a, \quad (28)$$

which is exactly what we observe in Fig. 7(a). On the other hand, according to Eq. (24), the gross noise in the conventional CT $\sigma_{\text{conventional-CT}}$ varies with σ_p/a . Hence, one has

$$\sigma_{\text{conventional-CT}} \propto \sigma_p/a \propto 1/a^2, \quad (29)$$

which is what we observe in Fig. 7(b).

V. DISCUSSION

The differential phase contrast CT implemented with x-ray tube and gratings is emerging as a new CT technology to improve the contrast sensitivity of the conventional CT imaging substantially, which is definitely of significance for preclinical and clinical applications, wherein the differentiation of low contrast structure is essential. The potential performance of a CT imaging system is jointly determined by its signal and noise properties. Recognizing the fact that the noise power spectrum is a more complete and objective metric than the

FIG. 6. The profiles along a radial line at 0° cross the noise power spectrum of (a) the differential phase contrast CT [shown in Fig. 5(a)] and (b) the conventional CT [shown in Fig. 5(b)].

gross noise to assess the noise property of a CT system or imaging method, we characterize the noise behavior of the differential phase contrast CT via its noise power spectrum and compare it with that of the conventional CT. The preliminary results obtained so far show that the noise power spectrum of the differential phase contrast CT is drastically different from that of the conventional CT. Below, we give a concise summary on what we have observed thus far and discuss the implication behind the observation.

The most distinctive difference in the noise power spectrum between the differential phase contrast CT implemented with x-ray tube and gratings and the conventional CT is that the former manifests itself with the $1/|k|$ trait, whereas the latter exhibits the $|k|$ trait. These two traits are predicted in Eqs. (13)–(14) and (13')–(14') and then verified in Figs. 5(a) and 5(b) and Figs. 6(a) and 6(b), respectively, which leads to the radical difference in the noise morphology and noise power spectrum between the differential phase contrast CT and the conventional CT. The root cause for these radical differences in the noise power spectrum and noise granularity/texture is that the differential phase contrast CT detects the projection of refraction's derivative in data acquisition, which results in the replacement of the ramp filter that is notorious in noise behavior by the Hilbert transform that is much more tractable in noise for image reconstruction.

The gross noise of an image σ_r is the square root of the integration of its noise power spectrum over all frequency components. Note that the projection noise σ_p corresponding to each detector cell is in general proportional to the inverse of the aperture of an x-ray detector cell. Hence, as predicted in Eqs. (24) and (25), the noise σ_r in the reconstructed image of the differential phase contrast CT and conventional CT is proportional to $1/a$ and $1/a^2$, respectively. Or, equivalently, the x-ray doses corresponding to the differential phase contrast CT and conventional CT are proportional to $1/a^2$ and $1/a^4$, respectively. These predictions are verified by the results presented in Figs. 7(a) and 7(b) and consistent with what has been published in the literature.^{11–15} Overall, the variation of the gross noise in the differential phase contrast CT varies over detector cell size in a relatively tame manner in contrast to the conventional CTs variation. In addition, as demonstrated in Fig. 7(c), the contrast-to-noise ratio of the differential phase contrast CT is approximately 30- and 150-fold larger than that of the conventional CT at detector cell size 128 and 32 μm , respectively, which may substantially

benefit the management of x-ray radiation dose in practical situations.

It should be pointed out that the noise power spectrum indeed provides a much wider perspective for us to assess the noise behavior of the differential phase contrast CT system. For example, two images with the identical gross noise may appear substantially different in noise granularity and, thus, in their noise power spectrum as we experienced in the conventional CT imaging. Moreover, the noise power spectrum provides valuable instructive information to guide the suppression of noise and maintenance of detected signal in an imaging system, whereas the gross noise itself provides no such information at all. Based on the noise power spectrum of the differential phase contrast CT shown in Figs. 5(a) and 6(a) and their comparison with those of the conventional CT presented in Figs. 5(b) and 6(b), we believe that the windowing and boosting techniques that are frequently utilized in the conventional CT to optimize the trade-off between noise and spatial resolution by tweaking the reconstruction filter kernel can also be adopted in the differential contrast phase CT. More work along this technical path is under way and the results will be published promptly in our future publication.

Before ending this section, we would like to make two important points:

- (1) The noise behavior possessed by the differential phase contrast CT implemented with x-ray tube and gratings, including the noise power spectrum and gross noise, is a common feature of all the tomographic imaging systems or methods as long as the detected signal is the projection of the derivative of the physical parameters that generate contrast in the tomographic images. In other words, if any other imaging system or method can detect the projection of the differential “X” contrast, where X can be attenuation, refraction, or any physical, chemical, or other biological parameters that are of relevance for imaging, its noise behavior would be very similar to that of the differential phase contrast CT implemented with the x-ray tube and gratings unveiled in this manuscript.
- (2) The noise behavior of the differential phase contrast CT and its comparison with that of the conventional CT is characterized via computer simulation in which the imaging chains of both the differential phase contrast CT and

FIG. 7. The variation of contrast-to-noise ratio as a function over detector cell size corresponding to: (a) the differential phase contrast CT; (b) the conventional CT (x-ray exposures at 10^6 , $10^{6.5}$, 10^7 , $10^{7.5}$, 10^8 photon/cm²·projection), and (c) the gain in contrast-to-noise ratio by the differential phase contrast CT over the conventional CT (x-ray exposures at $10^{7.5}$, 10^8 photon/cm²·projection).

the conventional CT are assumed ideal. This means that the noise power spectrum of the differential phase contrast CT presented in this paper is the best possible performance under the ideal-observer model.^{17,18,23} In practice, it is a common understanding that the imaging performance, including the noise behavior, of a physical CT system may be degraded by the imperfection in its imaging chain in practical situations.

VI. CONCLUSIONS

The differential phase contrast CT implemented with x-ray tube and gratings detects the projection of refractive coefficient's derivative and uses the Hilbert filter for image reconstruction, which leads to the radical difference in its noise power spectrum and the substantial advantage in its gross noise over the conventional CT. The characterization of the differential phase contrast CTs noise power spectrum unveiled in this study is instructive to guide the design, evaluation, and optimization of differential phase contrast CTs imaging performance for preclinical and ultimately clinical applications. More importantly, the noise behavior of the differential phase contrast CT unveiled in this work is an exemplification of the noise behaviors of the tomographic imaging systems or methods wherein the signal detected is the projection of the derivative of the physical parameters that generate the contrast in the tomographic images.

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