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A framework for generalized reference image reconstruction methods including HYPR-LR, PR-FOCUSS, and k-t FOCUSS

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Abstract

Purpose—To investigate the relationships among HYPR-LR, PR-FOCUSS, and k-t FOCUSS by showing how each method relates to a generalized reference image reconstruction method. That is, the generalized series model employs a fixed reference image and multiplicative corrections - that model is extended here to consider reference images more broadly, both in image space and in transform spaces (x-t and x-f spaces), and that can evolve with iteration.

Materials and Methods—Theoretical relationships between the methods are derived. Computer simulations are done to compare HYPR-LR to one iteration of PR-FOCUSS. The generalized reference approaches applied in the x-t or x-f domain are compared using computer simulation, five cardiac cine imaging datasets and six myocardial perfusion datasets.

Results—PR-FOCUSS and HYPR-LR gave comparable errors, with PR-FOCUSS slightly outperforming HYPR-LR. The baseline image is important to the performance of k-t FOCUSS and x-t FOCUSS as demonstrated by results from cardiac cine imaging. For cardiac perfusion reconstructions with the use of a temporal average image as the baseline image, k-t FOCUSS gave lower errors than x-t FOCUSS.

Conclusion—HYPR-LR and PR-FOCUSS are closely related: both work for radial sampling and use reference images in the x-t domain; HYPR-LR is an approximate implementation of the generalized reference framework, while PR-FOCUSS is a conjugate gradient implementation of the generalized reference framework. The superiority of generalized reference approaches applied in the x-t or x-f domain was sensitive to the characteristics of the acquired data and to the baseline image used.

Keywords

generalized series model; compressed sensing; HYPR; PR-FOCUSS; k-t FOCUSS

Introduction

A Generalized Series model (GS) was proposed in order to improve image quality by including prior or reference images for dynamic MRI applications (1). Various related methods have been proposed since then, and it has been shown that many of them, such as the keyhole method (2) and the Reduced-encoding Imaging by Generalized-series

Reconstruction (RIGR) (3), can be considered as special cases of a formulation similar to the GS method (4).

Recently, Highly Constrained Back Projection (HYPR) and related methods such as HYPR-LR and I-HYPR were proposed for dynamic reconstruction of undersampled radial k-space magnetic resonance imaging (5–8). Another type of approach for radial k-space reconstruction is the Projection Reconstruction FOCal Underdetermined System Solver (PR-FOCUSS) (9), which was designed to minimize the L1 image norm to constrain the image sparsity. This is a compressed sensing type of method.

HYPR-LR and PR-FOCUSS deal with each time frame separately (here denoted as “x-t methods”), and use a composite image (or images) reconstructed from multiple time frames as a reference image. Methods such as k-t BLAST/k-t SENSE (10) include spatiotemporal correlations in a different way, by working in the x-f domain (denoted as “x-f method”). k-t FOCUSS (11–13), which is the reweighted L2 norm implementation of compressed sensing, has been shown to be an extension of k-t BLAST/k-t SENSE for general sampling patterns. k-t FOCUSS constrains the image sparsity in the x-f domain.

In this work, it is shown that HYPR-LR, PR-FOCUSS, k-t BLAST/k-t SENSE and k-t FOCUSS can be included in a unified generalized reference framework that employs a reference image and multiplicative corrections. Previously, k-t SENSE and k-t FOCUSS have been presented in a unified framework (11), but HYPR-LR and PR-FOCUSS have not previously been shown to have specific relationships to each other. The theory is presented first. Demonstrations of the differences of these methods are then presented as well as similarities. Real data examples are also used to show the tradeoffs of different methods, in particular, if the reconstruction operates in the x-t or x-f domain.

Materials and Methods

Theory

Review of Generalized Series Model—The MRI signal equation relates acquired k -space samples $d(\vec{k})$ to the image domain object $\rho(\vec{x})$ as follows:

$$d(\vec{k}) = \int \rho(\vec{x}) e^{-j2\pi \vec{k} \cdot \vec{x}} d\vec{x} \quad [1]$$

In matrix form, this can be written as

$$\mathbf{d} = \mathbf{E} \rho \quad [2]$$

where $\mathbf{E}$ is the encoding matrix, and $\mathbf{d}$, ρ are column vectors of k -space data and image pixel values, respectively. The elements of the encoding matrix are given by:

$$[\mathbf{E}]_{k_i, x} = e^{-j2\pi \vec{k}_i \cdot \vec{x}}$$

The generalized series (GS) approach suggests that the image may be obtained by multiplicative correction of the reference or composite image $\mathbf{r}(\vec{x})$. The following model for image reconstruction was proposed (1,3,4,14):

$$\rho(\vec{x}) = \mathbf{r}(\vec{x}) \times \mathbf{m}(\vec{x}) = \mathbf{r}(\vec{x}) \times \sum_{i=1}^{N_{terms}} c_i e^{j2\pi \vec{k}_i \vec{x}} \quad [3]$$

where the reference image $\mathbf{r}(\vec{x})$ is typically a magnitude-only image (3), multiplication “ $\times$ ” is a pixel-by-pixel operation here, and c_i is the i th GS basis coefficient. The basis functions can be general (1), but a Fourier basis is typically used (3). N_{terms} is the number of basis coefficients and typically matches the number of k-space samples (14). Note that this formulation (with an additional additive reference image term or “baseline” image) can represent at least 14 different reconstruction methods such as feature-recognizing MRI (15) and RIGR, as shown in (14).

Written in matrix form, equation [3] becomes:

$$\rho = \mathbf{R}\mathbf{m} = \mathbf{R}\mathbf{E}^H \mathbf{c} \quad [4]$$

where $\mathbf{R}$ is a diagonal matrix with $\mathbf{r}$ on the main diagonal; and superscript H is the Hermitian transpose operator.

Generalized Reference Image Framework—In the original GS method and other methods listed in (14), $\mathbf{r}$ is limited to the image domain (x-t space). Here $\mathbf{r}$ is generalized to other domains, such as the x-f domain, and is permitted to be updated at each iteration. Correspondingly, the encoding matrix $\mathbf{E}$ is generalized to encode in the appropriate domain. This new more general framework is referred to here as “generalized reference framework”.

For Cartesian acquisitions undersampled in the phase encode direction, $\mathbf{E} = \mathbf{W}\mathbf{F}$, where $\mathbf{W}$ is a binary diagonal matrix choosing which k-space points that are sampled, and $\mathbf{F}$ is the forward Fourier transformation matrix. To adapt the formulation for non-Cartesian sampling, interpolation should be included, $\mathbf{E} = \mathbf{T}\mathbf{F}$, $\mathbf{E}^H = \mathbf{F}^H \mathbf{T}^H$, where $\mathbf{T}$ is the interpolation and resampling matrix. To include coil sensitivity profiles, the encoding matrix

$$\mathbf{E} = \begin{bmatrix} \mathbf{TFS}_1 \\ \vdots \\ \mathbf{TFS}_{N_c} \end{bmatrix}$$

may be extended to include coil sensitivity profiles as follows: $\mathbf{E} = \begin{bmatrix} \mathbf{TFS}_1 \\ \vdots \\ \mathbf{TFS}_{N_c} \end{bmatrix}$, where $\mathbf{S}_i$ are diagonal matrices with values of the i^{th} coil sensitivity on the main diagonal ($i = 1, \dots, N_c$). For x-f domain methods, the encoding matrix is $\mathbf{E} = \mathbf{E}_k \mathbf{F}_t$ where $\mathbf{F}_t$ is used to Fourier transform images from the x-f domain to the x-t domain, and $\mathbf{E}_k$ is used to transform images from the x-t domain to the k-t domain.

Substituting equation [4] into [2], we have $\mathbf{E}\mathbf{R}\mathbf{E}^H \mathbf{c} = \mathbf{d}$. If a regularization term is added to provide more resistance to noise (16), $(\mathbf{E}\mathbf{R}\mathbf{E}^H + \lambda \mathbf{I})\mathbf{c} = \mathbf{d}$, or equivalently

$$\mathbf{c} = (\mathbf{E}\mathbf{R}\mathbf{E}^H + \lambda \mathbf{I})^{-1} \mathbf{d} \quad [5]$$

In the original GS paper, no regularization term was added, and $\mathbf{c}$ was found using a Toeplitz solver (1). Here we include a regularization term and solve for $\mathbf{c}$ using conjugate-gradient methods. Note that the regularization term is not necessarily a diagonal matrix; it is diagonal only when the noise and signal covariance matrices are chosen to be diagonal matrices by fixing the transform domain.

Substituting back into equation [4] gives

$$\tilde{\rho} = \mathbf{R}\mathbf{E}^H(\mathbf{E}\mathbf{R}\mathbf{E}^H + \lambda\mathbf{I})^{-1}\mathbf{d} \quad [6]$$

This has a related mathematical form as noted in (17)

$$\tilde{\rho} = (\mathbf{E}^H\mathbf{E} + \lambda\mathbf{R}^{-1})^{-1}\mathbf{E}^H\mathbf{d} \quad [7]$$

The next sections will show how these equations [5–7] relate to more recent reconstruction techniques.

Relation to PR-FOCUSS, k-t FOCUSS, and k-t BLAST/k-t SENSE—PR-FOCUSS is a specific case of equation [6] when the reference image $\mathbf{r}$ is in x-t space and is updated at each iteration. PR-FOCUSS uses radial sampling and the inverse Fourier transform (IFT) and projection operations for encoding.

Also very similar to equation [6] is k-t FOCUSS, although for this reconstruction method, $\mathbf{r}$ is in the x-f domain and is updated at each iteration. k-t FOCUSS has been shown to generalize k-t BLAST/k-t SENSE (11), and thus only k-t FOCUSS is referred to in this section.

From equation [6], when a baseline term is added, we get

$$\tilde{\rho} = \rho_0 + \mathbf{R}\mathbf{E}^H(\mathbf{E}\mathbf{R}\mathbf{E}^H + \lambda\mathbf{I})^{-1}(\mathbf{d} - \mathbf{E}\rho_0) \quad [8]$$

Equation [8] is the same as the k-t FOCUSS equation [17] of (11) when p in (11), which is the coefficient adjusting the weighting matrix, is set to be $\frac{1}{2}$ to provide minimization of the L1 norm of x-f space and the pseudoinverse is solved using the conjugate gradient method. ρ_0 is the baseline term that can be a temporal average image (11) or images that are reconstructed using more sophisticated techniques such as RIGR, or a motion estimation/compensation scheme (12,13).

Derivation of HYPR-LR from the generalized reference framework—Two types of HYPR have been proposed. The first type of HYPR is similar to the first iteration of the maximum likelihood method (8) and weights the composite image by the backprojection of the ratio of the 1D inverse Fourier transform of each ray of k-space data and the projections of the reference image:

$$\tilde{\rho} = \frac{1}{N_p} \mathbf{r}(\vec{\mathbf{x}}) \times \mathbf{B} \left(\frac{\mathbf{F}_{1d}^{-1}(\mathbf{d})}{\mathbf{P}(\mathbf{r}(\vec{\mathbf{x}}))} \right)$$

where N_p is the number of projections, $\mathbf{r}(\vec{\mathbf{x}})$ is the reference (or composite) image, $\mathbf{x}$ denotes a pixel-wise multiplication, $\mathbf{P}$ and $\mathbf{B}$ are projection and backprojection operators, $\mathbf{F}_{1d}^{-1}$ is the 1D inverse Fourier transform along the radial direction that transforms the k-space measurements $\mathbf{d}$ into a sinogram. This mixed matrix/vector and functional notation is used

in an attempt to make the equations relatively intuitive and match some of the notation used in other papers.

The second type of HYPR weights the composite image by the ratio of the backprojection of the 1D inverse Fourier transform of each ray of k-space data and the backprojection of the composite or reference image's projection:

$$\tilde{\rho} = \mathbf{r}(\vec{\mathbf{x}}) \times \frac{\mathbf{B}(\mathbf{F}_{1d}^{-1}(\mathbf{d}))}{\mathbf{B}(\mathbf{P}(\mathbf{r}(\vec{\mathbf{x}})))}$$

In this paper, only the second type HYPR is investigated and this type of HYPR is a specific case of HYPR-LR (7) (refer to below).

In order to express HYPR-LR in the generalized reference framework, we multiply both sides of equation [2] by $\mathbf{E}^H$ with a density compensation function $\mathbf{D}$ and substitute equation [3], which gives

$$\mathbf{E}^H \mathbf{D} \mathbf{d} = \mathbf{E}^H \mathbf{D} \mathbf{E} \rho = \mathbf{E}^H \mathbf{D} \mathbf{E} (\mathbf{r}(\vec{\mathbf{x}}) \times \mathbf{m}(\vec{\mathbf{x}})),$$

here the multiplication operator is a pixel-by-pixel operation. The matrix operation $\mathbf{E}^H \mathbf{D} \mathbf{E}$ is equivalent to a convolution operation with point spread function $\mathbf{h}$ and using notation with the point spread function $\mathbf{h}$ instead of the matrix $\mathbf{E}^H \mathbf{D} \mathbf{E}$, along with using functional forms of $\mathbf{r}$ and $\mathbf{m}$, makes the derivations below more clear.

$$\mathbf{E}^H \mathbf{D} \mathbf{d} = \mathbf{h} \otimes (\mathbf{r}(\vec{\mathbf{x}}) \times \mathbf{m}(\vec{\mathbf{x}})) \quad [9]$$

where $\mathbf{r}$ and $\mathbf{m}$ in are both vectors, and the multiplication is pixel-by-pixel operation.

When $\mathbf{h}$ is a Dirac delta function or $\mathbf{m}$ is a constant:

$$\mathbf{h} \otimes (\mathbf{r} \times \mathbf{m}) = (\mathbf{h} \otimes \mathbf{r}) \times \mathbf{m}$$

If $\mathbf{m}$ does not change significantly over the effective spatial support of $\mathbf{h}$, this relationship forms a good approximation, as has been exploited for homodyne detection (18) and fast RIGR(19). That is,

$$\mathbf{h} \otimes (\mathbf{r} \times \mathbf{m}) \approx (\mathbf{h} \otimes \mathbf{r}) \times \mathbf{m} \quad [10]$$

From this approximation and equation [7],

$$\mathbf{E}^H \mathbf{D} \mathbf{d} \approx (\mathbf{h} \otimes \mathbf{r}(\vec{\mathbf{x}})) \times \mathbf{m}(\vec{\mathbf{x}}) = (\mathbf{E}^H \mathbf{D} \mathbf{E} \mathbf{r}(\vec{\mathbf{x}})) \times \mathbf{m}(\vec{\mathbf{x}}).$$

Substituting into equation [3],

$$\tilde{\rho} = \mathbf{r}(\vec{\mathbf{x}}) \times \mathbf{m}(\vec{\mathbf{x}}) = \mathbf{r}(\vec{\mathbf{x}}) \times \frac{\mathbf{E}^H \mathbf{D} \mathbf{d}}{\mathbf{E}^H \mathbf{D} \mathbf{E} \mathbf{r}} \quad [11]$$

Here the multiplication and division operators are pixel-by-pixel operations.

From equation [11], when $\mathbf{r}$ is set to be the composite image(s) which has high resolution, and $\mathbf{d}$ is the undersampled radial data, the equation is similar to the HYPR-LR method. The only difference is that equation [11] is written with gridding or inverse gridding with a density compensation function, rather than the notation in HYPR-LR where a projection or filtered backprojection image convolved with a low-pass filter (7) is used. Compared to the fast RIGR method (19), $\mathbf{h}$ is slightly different: HYPR-LR uses the filtered backprojection of projection convolved with a low-pass filter; while fast RIGR uses the inverse Fourier transform of the low resolution (truncated in Cartesian k-space) data. These are essentially the same, but fast RIGR was originally created for keyhole-type undersamplings, while HYPR-LR was designed for an undersampling pattern spread throughout k-space.

In order for results from the multiplicative correction method of equation [6] and HYPR-LR to be most similar, equation [10] should be a good approximation. In order to obtain a locally smooth $\mathbf{m}$ over the effective spatial support of $\mathbf{h}$, the terms $\mathbf{E}^H \mathbf{D} \mathbf{d}$ and $\mathbf{E}^H \mathbf{D} \mathbf{E} \mathbf{r}$ should be locally smooth. This is consistent with the fact that HYPR-LR uses a filtered backprojection with a low frequency filter; in order to reduce the effective spatial support of $\mathbf{h}$, a Gaussian filter was applied in (7). Note that the phase of $\mathbf{m}$ is usually locally smooth and can be included in $\mathbf{m}$, and $\mathbf{r}$ is typically set to be the magnitude image without a phase

term for HYPR-LR. When $\mathbf{h} = \frac{1}{|\mathbf{r}|}$, HYPR-LR is equivalent to the second type of HYPR

$$\tilde{\rho} = \mathbf{r}(\vec{\mathbf{x}}) \times \frac{\mathbf{B}(\mathbf{F}_{1d}^{-1}(\mathbf{d}))}{\mathbf{B}(\mathbf{P}(\mathbf{r}(\vec{\mathbf{x}})))}.$$

Sparsity in x-f and x-t domains—From compressed sensing theory, the sparsity of an image or the sparsity of its transformation is essential to recover the image from undersampled k-space data. It was reported that a sparse image is necessary for HYPR-LR and k-t FOCUSS to be effective (11,20). Strategies can be applied to make the images sparser. Current literature has applied a DC baseline image to enhance the sparsity, such as in (10,11). Other methods have also been proposed to obtain a baseline image, such as using RIGR (21), or employing a motion estimation/compensation scheme (12,13).

k-t FOCUSS essentially minimizes the L1 norm of the image in the x-f domain by minimizing the reweighted L2 norm iteratively, which assumes image sparsity in the x-f domain. The question arises as to how well this works when the image in the x-t domain is constrained instead. Here we compare the x-f method and the x-t method (the x-f method is denoted as “k-t FOCUSS”, and the x-t method is denoted as “x-t FOCUSS”, which is the same as PR-FOCUSS when the sampling pattern is radial) from the generalized reference framework perspective.

For k-t FOCUSS, equation [3] can be written as

$$\rho_{x-f} = \Gamma_{x-f} \times (\mathbf{F}_t^H \mathbf{E}_k^H \mathbf{c}_{k-t}) \quad [12]$$

where $\mathbf{F}_t^H$ is the Fourier transform in time dimension, which will change x-t domain to x-f domain, and $\mathbf{E}_k$ is the transformation operator that changes the x-t domain to the k-t domain. This equation sets up the minimization problem for the first iteration of k-t FOCUSS.

Taking the inverse Fourier transform in the time dimension, we get an alternate expression for k-t FOCUSS:

$$\rho_{x-t} = \mathbf{r}_{x-t} \otimes (\mathbf{E}_k^H \mathbf{c}_{k-t}) \quad [13]$$

where the convolution is along the time dimension for each image pixel.

On the other hand, for x-t FOCUSS,

$$\rho_{x-t} = \mathbf{r}_{x-t} \times (\mathbf{E}_k^H \mathbf{c}_{k-t}) \quad [14]$$

where the multiplication is an element-by-element operation.

The difference between equation [13] and equation [14] is that equation [13] is a convolution operation, while equation [14] is a multiplication operation. For equation [13], $\mathbf{E}_k^H \mathbf{c}_{k-t}$ acts as the weighting term of $\mathbf{r}_{x-t}$ in the time dimension and sums up contributions from different time frames to get ρ_{x-t} . Equation [14] deals with each time frame separately, and $\mathbf{r}_{x-t}$ can be used to effectively bring temporal correlations into the reconstruction.

Intuitively, when there are more local changes in the images over many time frames, then equations [12] and [13] might not provide optimal weighting since the term $\mathbf{E}_k^H \mathbf{c}_{k-t}$ is not tailored for each time frame. In such cases, using x-t FOCUSS with $\mathbf{r}_{x-t}$ reconstructed using sliding windows (SW) or similar methods may be a better strategy. The trade-offs are difficult to gauge, however, and are further complicated by the updates of $\mathbf{r}_{x-t}$ and $\mathbf{r}_{x-f}$ at each iteration. Comparisons of the two reconstruction strategies are described in the Methods.

Inclusion of Coil Sensitivity profiles—For parallel imaging, the locality of receive coil sensitivity profiles can make the images of each coil have a small finite support even though the original image has large finite support. The encoding matrices $\mathbf{E}$ can include sensitivity profiles in the reconstruction in all of the previous derivations. Recently, an adaptive regularized iterative SENSE (AR-SENSE) method was applied in MR angiography (22). Essentially, AR-SENSE is the same as equation [7] in this paper if image pre-whitening was applied to remove noise correlations between the coils (23).

The different methods that can be fit in the generalized reference framework are compared in table 1 in terms of their sampling pattern and reconstruction details.

Methods

HYPR-LR Simulation—PR-FOCUSS was shown to be a specific case of the multiplicative correction formulation (equation [6]), which also covers GS and HYPR-LR. PR-FOCUSS was compared to HYPR-LR using a dynamic computer simulation with motion and contrast changes over 40 time frames, with a matrix size of 256×256. The simulated object was composed of several elliptical or circular structures that imitate the heart or vessel motion and contrast enhancement. Twenty-four rays k-space data were simulated for each time frame in an interleaved fashion with four adjacent time frames composing an evenly distributed 96 rays. The reference images were reconstructed using

SW methods that combined four adjacent time frames. HYPR-LR was implemented with complex filtered backprojection with a Ram-Lak filter and a Gaussian kernel with $\sigma=3$ (kernel size =19). PR-FOCUSS used the same SW images as the reference images, and the conjugate-gradient (CG) method was implemented to solve for the generalized coefficients, using 20 CG iterations. For comparison, one iteration without updating the reference image was done for PR-FOCUSS (note: for PR-FOCUSS, when the reference image was initialized with SW images, iterations past the first one resulted in small changes for this dataset), and projection and backprojection were used in the implementation.

k-t FOCUSS and x-t FOCUSS Comparison—To compare k-t FOCUSS and x-t FOCUSS, simulation data and fully-acquired in-vivo k-space data were used. Both data types were 2D dynamic Cartesian acquisitions and an undersampling of $R=4$ in the phase encode direction was simulated. The undersampling was random with a Gaussian distribution that had more frequent sampling at low frequency regions (11). The reconstruction was performed by k-t FOCUSS and x-t FOCUSS and both used the temporal average image as the baseline image. For k-t FOCUSS, the reference image was initialized to a constant value of one as in (11–13). For x-t FOCUSS, the reference image was initialized to be the SW image. For both methods, the reference image was updated five times by the previous results. At each iteration, the conjugate-gradient (CG) method was implemented to solve equation [8], and the CG iteration number was set to be 20.

The simulation used a disc without motion but with contrast changes in the disc over 40 time frames, with matrix size of 256×256 . Different temporal contrast patterns (as shown by figure 2) were used to investigate the characteristics of the k-t and x-t FOCUSS reconstruction methods.

For the in-vivo studies, cine imaging and myocardial perfusion imaging datasets were acquired under an IRB-approved protocol on a 3T Siemens Trio. For cine imaging, a SSFP sequence with $TR=42.45\text{--}45.92$ ms, $TE=1.22\text{--}1.64$ ms, flip angle= $40\text{--}44^\circ$, and slice thickness= $6\text{--}8$ mm was used, and five datasets were acquired with fields of view between $256\text{--}385 \times 165\text{--}256$. 14–26 cardiac phases were acquired from three volunteers (2 male and one female; age 55, 81 and 59 years; weight 66, 102 and 68 kg). To study the effect of different baseline images (p_0 in equation [8]) on the k-t and x-t FOCUSS methods, the sliding window image of the diastolic period was used as the baseline image, and compared to the reconstruction using the temporal average image as the baseline image.

For myocardial perfusion imaging, gadolinium doses ranged from 0.015 to 0.04 mmol/kg, and a saturation recovery turboFLASH sequence with $TR \approx 2.2$ ms, $TE=0.98\text{--}1.36$ ms, and slice thickness= $7\text{--}8$ mm was used. Six datasets with a Cartesian acquisition matrix varied from $(192\text{--}256) \times (90\text{--}168)$ with 70–80 time frames were acquired from six subjects (three male and three female, 56 ± 10 years; weight 83 ± 25 kg).

Results

HYPR-LR and PR-FOCUSS results for the dynamic simulation are compared in figure 1. One iteration of PR-FOCUSS and the non-iterative HYPR-LR are comparable in terms of normalized RMSE (nRMSE), and the differences in the nRMSE between PR-FOCUSS and HYPR-LR are on the order of 0.4%. Both methods have more error near the edges of objects.

Figure 2 shows the k-t FOCUSS and x-t FOCUSS simulation results. Different contrast change patterns show the different performance of the two reconstruction methods. For the linear curve (curve 2), x-t FOCUSS has lower MSE than k-t FOCUSS; for the periodic

curve with a period of 2, k-t FOCUSS has slightly lower MSE than x-t FOCUSS. This shows that the reconstruction performance of k-t FOCUSS and x-t FOCUSS is content-dependent.

Five cine imaging datasets were used for comparison of k-t FOCUSS and x-t FOCUSS, and the performance of the two methods differed depending on the dataset and depending on the baseline images used. Typical results are shown in figures 3–5. Figure 3 shows that x-t FOCUSS had lower RMS error compared to k-t FOCUSS when using the sliding window image of the diastolic period as the baseline image. Figure 4 shows the results when using the same dataset as figure 3 and setting the baseline image to be the temporal average image. The two methods show comparable results in this case, and both methods show aliasing artifacts. Figure 5 shows in a different dataset that k-t FOCUSS has less error than x-t FOCUSS when using the temporal average image as the baseline image.

Six myocardial perfusion imaging datasets were used for comparison of k-t FOCUSS and x-t FOCUSS. k-t FOCUSS had lower RMSE in all cases, and the RMSE values are listed in table 2. One result is shown in figure 6.

Discussion

The generalized reference image framework can include HYPR-LR, PR-FOCUSS, k-t BLAST/SENSE, and k-t FOCUSS. Among these reconstruction methods, PR-FOCUSS and k-t FOCUSS are based on compressed sensing methods that minimize an L1 norm of the image through a re-weighted L2 norm minimization method. This implies that the generalized reference framework implicitly encourages image sparsity. This is consistent with the fact that HYPR-LR requires images to be sparse. In addition, all of these methods are prone to signal zeroing problems that stems from using multiplicative corrections. In practice, it is typically not difficult to ensure the reference images do not have zeros in the region of support, which will avoid this problem.

Both HYPR-LR and PR-FOCUSS work with radial sampling and in the x-t domain, and both equation [7] and equation [11] can be derived from generalized reference framework, demonstrating HYPR-LR is closely related to the first iteration of PR-FOCUSS, which is based on compressed sensing. PR-FOCUSS and HYPR-LR are comparable in terms of nRMSE (figure 1(bottom)). Both methods have some errors near edges. PR-FOCUSS includes a matrix inversion operation which was solved by a conjugate gradient method, while HYPR-LR only included element-by-element division and multiplication operations which were more computationally efficient.

From figure 2, k-t FOCUSS and x-t FOCUSS show different performance for different datasets, and it demonstrates that x-t FOCUSS works better than k-t FOCUSS in some cases. Intuitively, a smooth signal works well for both x-t FOCUSS and k-t FOCUSS. However, the linearly changing curve 2 in figure 2(b), although “smooth”, seemingly has enough temporal frequency components that x-t FOCUSS gives lower error. On the other hand, curve 1 is periodic which is relatively sparse in k-space. From compressed sensing theory, sparsity in the x-f domain image is beneficial to k-t FOCUSS, so k-t FOCUSS gives lower error for curve 1 in figure 2(b).

While the performance of the two methods may be determined by the features of the dataset as demonstrated in figure 2, the baseline image also plays an important role. In figure 4, a temporal average image with some temporal blurring was used as the baseline image; in this case, k-t FOCUSS and x-t FOCUSS were comparable. When using a sliding window image of the diastolic period as the baseline image, x-t FOCUSS had less error than k-t FOCUSS. The baseline image of a sliding window image of the diastolic period helps to make the x-t

image sparser, and the SW reference image helps to bring in the correlations of the adjacent time frames. This test shows the importance of the baseline image.

In contrast, we found that the initial reference image estimate used for k-t FOCUSS had less effect on the reconstruction result, while the x-t FOCUSS method needed a good initial estimate such as the SW image for the reference images.

For myocardial perfusion imaging, the contrast in the heart changes nonlinearly and there is often respiration motion present. In this case, neither method was expected to work well. The use of a temporal average image as the baseline image likely made the x-f image sparse, which could explain the result that k-t FOCUSS has lower error than x-t FOCUSS (figure 6). A limitation of k-t FOCUSS is that it computes all the image series together, which demands relatively large memory, especially when there are a large number of image time frames.

Parallel MRI makes the images of each coil have fewer locations with appreciable signal which benefits the generalized reference methods. The inclusion of sensitivity encoding makes it possible to select only one regularization parameter λ in equation [6]. Here, the coils were reconstructed separately and then combined with the square root of the sum of the squares procedure. A single small lambda was used for all of the datasets and all of the coils. The results changed little with small changes in lambda. In theory, however, if each coil is reconstructed separately, then the parameter lambda should be selected differently according to coil characteristics.

In conclusion, the generalized reference framework encompasses a rich variety of reconstruction methods. Derivations were given to include k-t BLAST/SENSE, HYPR-LR, PR-FOCUSS and k-t FOCUSS. These relationships are useful to better understand differences between the methods — for example, this work clarifies that HYPR-LR is related to PR-FOCUSS. The methods unified by the generalized reference framework can be divided into x-t or x-f domain methods. The performance of these methods was sensitive to the characteristics of the acquired data, and the choice of the baseline image was important as demonstrated by cine imaging results. The perfusion reconstructions used a temporal average image as the baseline image, and had a lower RMSE compared to x-t FOCUSS in the six perfusion datasets studied.

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Figure 1. Comparison of images reconstructed from different methods. (top) upper row from left to right, Filtered Backprojection, HYPR-LR ($\sigma=3$), PR-FOCUSS; bottom row are the difference images from the true image. (bottom) normalized RMSE plot for the different methods. The circle in the top right image indicates the region of interest for the normalized RMSE calculation.

Figure 2.

Comparison of images reconstructed from simulated dataset using k-t FOCUSS and x-t FOCUSS. (a) Image of one of the 40 time frames. (b) Contrast change curves for two different cases: red is a periodic curve and blue is a linear curve with a slight slope. (c) RMSE plot of the images of the red periodic contrast change curve reconstructed from k-t FOCUSS and x-t FOCUSS. (d) RMSE plot of the reconstruction of the blue linear curve. This shows that the reconstruction performance of k-t FOCUSS and x-t FOCUSS is content-dependent.

Figure 3.

Comparison of images reconstructed from one cine imaging dataset of multiple coils using k-t FOCUSS and x-t FOCUSS with the sliding window image at the diastolic period as the baseline image. (a) The 20th image reconstructed from fully sampled data of all coils. (b,c) Images reconstructed using k-t FOCUSS and x-t FOCUSS with $R=4$, respectively. (d-e) Difference images of (b,c) from the fully-sampled image in (a). (f) RMSE plot of the images reconstructed from k-t FOCUSS and x-t FOCUSS. (d-e) are brightened 2 times for visibility.

Figure 4.

Comparison of images reconstructed from the same cine imaging dataset of multiple coils as figure 3 using k-t FOCUSS and x-t FOCUSS but now with the temporal average image as the baseline image. (a) The 20th image reconstructed from fully sampled data of all coils. (b,c) Images reconstructed using k-t FOCUSS and x-t FOCUSS with R=4, respectively. (d–e) Difference images of (b,c) from the fully-sampled image in (a). (f) RMSE plot of the images reconstructed from k-t FOCUSS and x-t FOCUSS. (d–e) are brightened 2 times for visibility.

Figure 5.

Comparison of images reconstructed from another cine imaging dataset of all coils using k-t FOCUSS and x-t FOCUSS with the temporal average image as the baseline image. (a) The 10th image reconstructed from full sampled data of all coils. (b,c) Images reconstructed using k-t FOCUSS and x-t FOCUSS with R=4, respectively. (d-e) Difference images of (b,c) from the fully-sampled image in (a). (f) RMSE plot of the images reconstructed from k-t FOCUSS and x-t FOCUSS. (d-e) are brightened 4 times for visibility.

Figure 6.

Comparison of images reconstructed from a myocardial perfusion imaging dataset using k-t FOCUSS and x-t FOCUSS using 5 coils with the temporal average image as reference image. (a) The 25th image reconstructed from fully sampled data of all coils. (b,c) Images reconstructed using k-t FOCUSS and x-t FOCUSS with R=4, respectively. (d-e) Difference images of (b,c) from the fully-sampled image in (a). (f) RMSE plot of the images reconstructed from k-t FOCUSS and x-t FOCUSS. (d-e) are brightened 4 times for visibility.

Table 1

Comparison of different GS model-derived methods

	HYPR-LR	PR-FOCUSS	k-t BLAST/SENSE	k-t FOCUSS
Sampling Pattern	Radial	Radial	Arbitrary	Arbitrary
Iterative Updating reference data	No	Yes	No	Yes
x-f or x-t domain	x-t	x-t	x-f	x-f

Table 2

RMSE of six myocardial perfusion imaging dataset using k-t FOCUSS and x-t FOCUSS with the temporal average image as baseline image

	k-t FOCUSS	x-t FOCUSS
Dataset 1	13.05	16.59
Dataset 2	12.93	14.34
Dataset 3	13.11	14.78
Dataset 4	15.67	17.45
Dataset 5	15.74	19.01
Dataset 6	12.35	14.92