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3D current source density imaging based on acoustoelectric effect: a simulation study using unipolar pulses

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Abstract

It is of importance to image electrical activity and properties of biological tissues. Recently hybrid imaging modality combining ultrasound scanning and source imaging through the acousto-electric (AE) effect has generated considerable interest. Such modality has the potential to provide high spatial resolution current density imaging by utilizing the pressure induced AE resistivity change confined at the ultrasound focus. In this study, we investigate a novel 3-dimensional (3D) ultrasound current source density imaging (UCSDI) approach using unipolar ultrasound pulses. Utilizing specially designed unipolar ultrasound pulses and by combining AE signals associated to the local resistivity changes at the focusing point, we are able to reconstruct the 3D current density distribution with the boundary voltage measurements obtained while performing a 3D ultrasound scan. We have shown in computer simulation that using the present method, it is feasible to image with high spatial resolution an arbitrary 3D current density distribution in an inhomogeneous conductive media.

Keywords

Current density imaging; acoustoelectric effect; tomography; ultrasound

1. Introduction

Hybrid imaging modality combining ultrasound scanning and electrical current source imaging through the acousto-electric (AE) effect may potentially provide solutions to imaging electrical sources and properties of biological tissues (Witte *et al* 2007, Olafsson *et al* 2009a). The acousto-electric modulation phenomenon has been found decades ago (Fox *et al* 1946) and has received considerable interest during recent years (Jossinet *et al* 1998, 1999, Lavandier *et al* 2000a, 2000b, Zhang and Wang 2004, Witte *et al* 2007, Olafsson *et al* 2008). This AE modulation describes the local electrical impedance changes in a conductive volume occupied by an ultrasonic packet. Physically, this type of impedance change is caused by the mechanical compression and rarefaction induced by the ultrasound packet. Applications of AE effect to medicine and biology have recently been proposed (Zhang and Wang 2004, Olafsson *et al* 2009a, Sumi 2009). For example, acousto-electric tomography has been suggested for high resolution electrical impedance imaging of breast tissue (Zhang and Wang 2004). The hybrid current density imaging technologies based on AE effect have also been pursued extensively (Witte *et al* 2007, Olafsson *et al* 2008). Initial experiments have been conducted to demonstrate the potential of AE imaging with sub-millimeter spatial

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resolution and good sensitivity to detect small current densities ($2 - 4 \text{ mA} / \text{cm}^2$) (Witte *et al* 2006, 2007, Olafsson *et al* 2006). This technology has also been used to detect or image current flow injected into tissues with physiologically realistic current densities (Witte *et al* 2006, 2007, Olafsson *et al* 2006). A general 3D AE equation and a 2-dimensional (2D) solution of ultrasound current source density imaging (UCSDI) were given by Olafsson *et al* (2008). Recently, cardiac activation mapping and the first experimental mapping of biological current in a live rabbit heart using this UCSDI technology were reported and a general 4D (3D+time) AE equation was reported as well (Olasson *et al* 2009a). Although it is much easier to obtain in real experiments, these 3D and 4D mappings of biological current only show the envelope of the collected AE signals instead of the reconstructed current density source itself. These mappings therefore are not direct representation of the current density source and the imaging results may need careful interpretations.

To our best knowledge, a 3D solution to the general AE equation (for general UCSDI) has not yet been developed in previous literatures. One of the challenges in deriving the 3D solution comes from the use of bipolar ultrasound pulses which makes the AE signal almost zero when the ultrasound wave packet travels in a slowly varying current field. As a consequence, the AE signal would emphasize the tissue boundaries if the dimension of the conductive tissue structure is larger than the ultrasound wavelength (Wang *et al* 2010a).

In the present study, we propose a 3D UCSDI solution to the general AE equation by using unipolar ultrasound pulse for reconstructing arbitrary current density distribution in a conductive volume. In the proposed method, specially designed unipolar ultrasound pulse (Holé and Lewiner 1996, 1998) is utilized to encode the current density distribution inside the ultrasound focal region (where the unipolar pulse maintains) in the collected voltage measurements. The 3D current density distribution can then be reconstructed with all the surface voltage measurements corresponding to each scanning point obtained while doing a 3D ultrasound scan. We have shown in computer simulations that using the proposed method, it is feasible to reconstruct an arbitrary 3D current density source distribution in an inhomogeneous conductive media. We have also investigated the impact on imaging performance by different parameters of the ultrasound beam and noise using computer simulation.

2. Methods

2.1. Theory

The diagram of the proposed 3D UCSDI approach based on AE effect is shown in figure 1. A focused ultrasound transducer at the bottom sends ultrasound pulses periodically to a conductive object. The acousto-electric (AE) signal caused by the local electrical impedance change with the ultrasound pulse packet traveling in the object is then measured at the boundary. A 3D ultrasound scan is performed in such a way that the ultrasound transducer's focusing point scans over the whole imaging object volume while the corresponding AE signal related to the periodic ultrasound pulse are collected at each scanning location. The 3D current density source distribution is then reconstructed with all the AE signal values collected at every scanning location when the ultrasound pulse packet passes its focusing point.

2.1.1. Acoustoelectric Effect—Acoustoelectric (AE) effect was first reported by Fox *et al* (1946) and initially used to characterize colloids in solution. AE effect describes the modulation of electric resistivity by acoustic pressure. A change in resistivity ρ due to a pressure wave with amplitude P can be expressed as

$$\frac{\Delta\rho}{\rho_0} = -KP \quad (1)$$

where $\Delta\rho$ is the resistivity change, ρ_0 the inherent resistivity, P the acoustic pressure and K is a constant of interaction which has order of 10^{-9} Pa^{-1} in a 0.9% NaCl solution (Jossinet *et al* 1998, 1999, Lavandier *et al* 2000a, 2000b). In addition, recent study has also shown that this interaction constant K has little variations in NaCl solutions with different concentrations and K in cardiac tissue is close to that measured in NaCl solutions (Li *et al* 2010). Furthermore, as shown in previous studies, the influence of this K variance on current density imaging is generally small (Olafsson *et al* 2009a and 2009b, Wang *et al* 2010a).

2.1.2. Ultrasound beam and ultrasound wave pattern—The ultrasound beam of a spherically focused transducer generally has a focal zone that can be calculated by

$F_z \approx NS_F^2 [2/(1+0.5S_F)]$, where N is the near field length and S_F is the normalized focal length. In the focal zone the ultrasound pressure wave pattern can be expressed as (Olafsson *et al* 2008)

$$P(x, y, z, t) = P_0 b(x, y, z) a(t) \quad (2)$$

where P_0 is the amplitude of ultrasound pressure wave, $b(x, y, z)$ is the beam pattern, and $a(t)$ describes the ultrasound waveform. For a focused spherical acoustic source, in a polar coordination system, the beam pattern can be calculated by (Lucas *et al* 1982, Chen *et al*

1993) $b(r, \theta) = ip_0(ka^2/r)\exp(-ikr) \int_{u=0}^1 \exp(-i\frac{ka^2}{2r}(1 - \frac{r}{R_0}\cos(\theta))u^2) u J_0(ka \sin(\theta)u) du$. In the focal zone the ultrasonic strength varies slowly along the beam axis. The -6dB beam diameter can be estimated as $B_D = 1.02 Fc/(fD)$, where F is the focal length, c is the sound velocity in the media, f represents the central frequency and D is the diameter of the transducer element. Generally, the higher the frequency or the larger the diameter, the smaller focus beam can be obtained.

In terms of ultrasound pulse waveform, for current density imaging based on AE effect, it is favorable to use unipolar ultrasonic pulse because bipolar or balanced oscillations tend to average any localized phenomena leading to a loss of AE signal. It is clear that unipolar ultrasonic pulse brings benefit when integrating over a special volume, which generates measurable signals in comparison to bipolar ultrasonic pulses. This advantage of unipolar ultrasonic pulse was previously introduced to quantitatively assess ultrasound induced resistance change in saline solution by Lavandier *et al* (2000a) and will be used in the present 3D UCSDI method. On the other hand, researches have shown that it is possible to generate unipolar pressure pulse using piezoelectric transducer in three different configurations (Brown and Weight 1974, Foster and Hunt 1978), i.e. a thick transducer electrically supplied by a uniform field (Eisenmenger and Haardt 1982, Maeda *et al* 1991), a thick transducer supplied by a divergent field (van der Pauw 1966), or a thin transducer with waveguide and backing medium coupled on both sides (Holé and Lewiner 1996, 1998). Holé and Lewiner (1996) presented a practical method to generate unipolar ultrasonic pulse using a single transducer with waveguide and backing medium coupled on both sides. Their experimental results were in good agreement with theoretical predictions. The transfer function of such transducer was expressed by (Holé and Lewiner 1996)

$$H(\omega) = \frac{i\omega Z_g e \epsilon^s \{Z[\cos(kl) - 1] + iZ_b \sin(kl)\}}{e^2 \{2Z[1 - \cos(kl) - i(Z_b + Z_g)\sin(kl)]\} + i\omega l \epsilon^s [Z(Z_b + Z_g)\cos(kl) + i(Z^2 - Z_b Z_g)\sin(kl)]} \quad (3)$$

where Z_g , Z , and Z_b are, respectively, the acoustical impedance of the waveguide, the transducer, and the medium. l is the thickness of the transducer, k is the wave number, e is the piezoelectric constant and ϵ^s is the dielectric constant (Holé and Lewiner 1996, 1998). The transducer acts as a derivator when Z_b is of the same order of magnitude as Z . Applying a step voltage to the transducer can generate a unipolar ultrasonic pulse at the transducer's focusing point (Holé and Lewiner 1996, Lavandier *et al* 2000a). The simulated unipolar waveform is shown in figure 2a. As reported in their experiments, unipolar ultrasonic pulse duration shorter than $0.09 \mu s$ with amplitude of 0.5 MPa or shorter than $0.14 \mu s$ with amplitude of 1.5 MPa could be obtained.

2.1.3. Boundary voltage measurement analysis—The measured voltage of lead i with lead field $\mathbf{J}_i^L = \mathbf{J}_i^L(x, y, z)$ due to a distributed current source distribution $\mathbf{J}^I(x, y, z)$ in the UCSDI system can be written as (Olafsson *et al* 2008, 2009a)

$$V^i = \iiint_V (\rho_0 - K\rho_0 P) \mathbf{J}_i^L \bullet \mathbf{J}^I dV = \iiint_V \rho_0 \mathbf{J}_i^L \bullet \mathbf{J}^I dV - \iiint_V K\rho_0 P \mathbf{J}_i^L \bullet \mathbf{J}^I dV. \quad (4)$$

As shown in (4), the voltage V^i consists of two terms. One is called the base current field

voltage $V_B = \iiint_V \rho_0 \mathbf{J}^L \bullet \mathbf{J}^I dV$ and the other term, generated due to the local resistivity change with focused ultrasound wave packet traveling in the media (AE effect), is called AE signal (Olafsson *et al* 2008, 2009a)

$$V_{AE} = \iiint_{V_p} K\rho_0 P \mathbf{J}^L \bullet \mathbf{J}^I dV. \quad (5)$$

In equation (5) we can restrain the integration within the ultrasound wave packet V_p , which is determined by the focused beam profile and pulse duration because the pressure level outside this volume is very small and can be ignored. When the ultrasound wave packet travels in the imaging object, both $\mathbf{J}^L$, $\mathbf{J}^I$ change, so V_{AE} is time and space dependent. Assuming the ultrasound pulse packet arrivals at position $z = 0$ when $t = 0$, so $z = ct$. When the focusing point of the transducer is moved to (x', y', z') the corresponding measured AE signal can be further expanded as

$$V_{AE}(x', y', z', t) = \int_{ct - T_d/2}^{ct + T_d/2} \iiint K\rho_0 P \mathbf{J}^L \bullet \mathbf{J}^I dV \quad (6)$$

Here T_d is the ultrasound pulse duration. Denoting V_f as the region around the transducer's focusing point, we can assume ρ_0 , $\mathbf{J}^L$, $\mathbf{J}^I$ to be constant in V_f as it is very small as compared to the imaging object volume. At time t' when the wave packet arrives at the transducer's focusing point we can simplify equation (6) as in equation (7)

$$V_{AE}(x', y', z', t') \approx K\rho_0 \mathbf{J}^L \cdot \mathbf{J}^I \iiint_{V_f} P dx dy dz. \quad (7)$$

In order to apply the unipolar pressure pulse to all the spatial locations inside the object, we can move the transducer so as to make the focusing point of the transducer scan over the object volume point by point and collect the AE signal V_{AE} generated around time t' that corresponds to the transducer's focusing point while the unipolar pressure pulse maintains. By combining equation (2) and (7), the voltage signal at the moment t' when the ultrasound pulse packet arrives at the focusing point z' can be expressed as

$$\begin{aligned} V_{AE}(x', y', z', t') &= V_{AE}(x', \\ y', z', \frac{z'}{c}) &\approx K\rho_0(x', \\ y', z') \mathbf{J}^L(x', \\ y', z') \cdot \mathbf{J}^I(x', y', z') &\iiint_{V_f} P_0 b(x \\ - x', y - y', z + F - z') a(t' \\ - \frac{z}{c}) dx dy dz \\ &= KA \varphi^L(x', \\ y', z') \cdot \mathbf{J}^I(x', y', z'), \end{aligned} \quad (8)$$

where $\varphi^L(x', y', z') = \rho_0(x', y', z') \mathbf{J}^L(x', y', z')$, $A = \iiint_{V_f} P_0 b(x - x', y - y', z + F - z') a(t' - \frac{z}{c}) dx dy dz$ and F denotes the focal length. Equation (8) gives the 3D mapping relationship between the 3D current source density distribution and the acquired AE signal with 3D scanning. From imaging perspective, this describes the forward problem of the 3D UCSDI by using unipolar ultrasonic pulse. In addition, we should note that if $a(t)$ is an oscillatory bipolar wave packet, the integration term A becomes very small. Furthermore, if the term $\rho_0 \mathbf{J}^L \cdot \mathbf{J}^I$ is constant in the volume of ultrasonic pulse wave packet, the acquired AE signal would then be close to zero. In order to achieve good SNR, it is therefore desired to use unipolar ultrasonic pulse for the ultrasound current source density imaging.

For 3D imaging, we can locate $V'_{AE}(x', y', z')$ according to the equation $t' = z' / c$ and the time reference of echo at the object boundary for each scanning point. So

$V'_{AE}(x', y', z') = V_{AE}(x', y', z', \frac{z'}{c}) = KA \varphi^L(x', y', z') \cdot \mathbf{J}^I(x', y', z')$ becomes a 3D point by point mapping of the 3D current density $\mathbf{J}^I(x, y, z)$ to the collected AE signal.

2.1.4. 3D current source density reconstruction—Here we rewrite the AE signal $V'_{AE}(x, y, z)$ corresponding to lead i and scanning point (x, y, z) as

$$V'_{AE}(x, y, z) = KA \varphi^L(x, y, z) \cdot \mathbf{J}^I(x, y, z) \quad (9)$$

Because of the vector nature of the current source density and its projection to the lead field, i.e. the dot product as in equation (9), in order to reconstruct the 3D current source density $\mathbf{J}^I$

(x, y, z), at least 3 independent measurements of boundary voltage $V_{AE}(x, y, z)$ are needed to obtain determined inverse equations. Of course more than 3 leads can be used to obtain a better inverse condition for 3D reconstruction in practice. If N leads are measured to obtain 3D reconstruction, these boundary measures can be written as a column vector

$$\mathbf{V}_{AE}(x, y, z) = [V_{AE}^1(x, y, z), V_{AE}^2(x, y, z), \dots, V_{AE}^N(x, y, z)]^T. \quad (10)$$

It is related to source current density $\mathbf{J}^I(x, y, z)$ by a $N \times 3$ transfer matrix

$$\boldsymbol{\psi}^L(x, y, z) = [\boldsymbol{\varphi}_1^L(x, y, z), \boldsymbol{\varphi}_2^L(x, y, z), \dots, \boldsymbol{\varphi}_N^L(x, y, z)]^T \quad (11)$$

by a determined matrix equation

$$\mathbf{V}_{AE}(x, y, z) = K A \boldsymbol{\psi}^L(x, y, z) \bullet \mathbf{J}^I(x, y, z). \quad (12)$$

Given $\boldsymbol{\psi}^L(x, y, z)$ the 3D current density $\mathbf{J}^I(x, y, z)$ can then be reconstructed from boundary measure $\mathbf{V}_{AE}(x, y, z)$ by minimum norm estimation. The reconstruction equation can be expressed as

$$\mathbf{J}^I(x, y, z) = \frac{1}{KA} [\boldsymbol{\psi}^L(x, y, z)^T \boldsymbol{\psi}^L(x, y, z)]^{-1} \boldsymbol{\psi}^L(x, y, z)^T \mathbf{V}_{AE}(x, y, z), \quad (13)$$

where $(\boldsymbol{\psi}^T \boldsymbol{\psi})^{-1} \boldsymbol{\psi}^T$ is a Moore–Penrose pseudoinverse (Moore 1920, Penrose 1955, Finke and Gulrajani 2003, He 2004, Gao and He 2008).

2.2. Computer Simulation

To evaluate the proposed imaging method, we have conducted a computer simulation study. The source current density distribution in a cubic conductive medium of $10 \times 10 \times 10 \text{ cm}^3$ was set to be the field generated by two or more needle-shaped electrodes. The ultrasound transducer emitted ultrasound pulse from the bottom of the cube and nonconductive deionized water was assumed to be the coupling media between the ultrasound probe and the imaged object. The AE signal was then collected at the object boundary. The origin of the coordinate system was set at the center of the imaged object. To get a 3D mapping, 3 pairs of electrodes were set on each two opposite surfaces in x , y and z directions, respectively. In total, there are 9 voltage measurements for each scanning point in the simulation. Centers of the nine pairs of needle-shaped electrodes of radii 0.04 cm and length 1.0 cm were positioned at $(-4, -3, 0; 4, -3, 0)$, $(-4, 0, 0; 4, 0, 0)$, $(-4, 3, 0; 4, 3, 0)$; $(-3, -4, 0; -3, 4, 0)$, $(0, -4, 0; 0, 4, 0)$, $(3, -4, 0; 3, 4, 0)$; $(-3, 0, -4; -3, 0, 4)$, $(0, 0, -4; 0, 0, 4)$, $(3, 0, -4; 3, 0, 4) \text{ cm}$. The shape and size of all electrodes are all the same in this study. All source and lead fields $\mathbf{J}^I$, $\mathbf{J}^L$ were computed by means of the finite element method (FEM) using COMSOL software (Comsol AB, Stockholm, Sweden) (Olafsson *et al* 2008). These fields were then used to calculate the transfer matrix $\boldsymbol{\psi}^L(x, y, z)$ in equation (11).

In the present study, we have conducted simulations using different target source distributions. First, imaging of a simple current density distribution generated by a pair of needle electrodes of radii 0.04 cm, length 1.0 cm in a homogeneous conductive medium of 0.9% NaCl solution ($0.568 \Omega \cdot m$) was simulated to illustrate the proposed imaging method. The centers of the pair of needle-shaped electrodes were positioned at $(-2, 0, 0)$ and $(2, 0, 0) \text{ cm}$ parallel to z axis. 200 Hz sinusoidal current of peak value 2.09 mA was injected through

the two needle electrodes. Current density imaging was then performed using the proposed 3D UCSDI method. Second, imaging of complex current density distribution generated from multiple current sources in an inhomogeneous conductive medium was tested to demonstrate the capability of this method to image arbitrary current source distribution. In addition, the comparison of imaging performance using unipolar ultrasonic pulse and bipolar ultrasonic pulse was conducted to demonstrate the benefit of the proposed 3D UCSDI method using unipolar ultrasound pressure pulse. Finally the influence of ultrasound parameters and noise level on the imaging performance was investigated. The correlation coefficient (CC) and relative error (RE), whose definitions are given by equation (14) and (15), respectively, were used to evaluate the imaging quality.

$$CC = \frac{\sum_n \sum_m (\hat{T}_{mn} - \bar{\hat{T}})(T_{mn}^0 - \bar{T}^0)}{\sqrt{[\sum_n \sum_m (\hat{T}_{mn} - \bar{\hat{T}})^2][\sum_n \sum_m (T_{mn}^0 - \bar{T}^0)^2]}} \quad (14)$$

$$RE = \frac{\|\hat{T} - T^0\|}{\|T^0\|} = \frac{\sqrt{\sum_n \sum_m (\hat{T}_{mn} - T_{mn}^0)^2}}{\sqrt{\sum_n \sum_m (T_{mn}^0)^2}} \quad (15)$$

Here T^0 , $\hat{T}$ are the original and reconstructed current density distribution, respectively. Method provided by Holé and Lewiner (1996, 1998) was adopted to simulate the unipolar ultrasonic pulse. A typical oscillatory bipolar ultrasonic pulse used in the simulation for comparison is shown in figure 2b as well (frequency 2 MHz, bandwidth 1.176 MHz, duration 0.85 μs) (Cobbold R S C 2006). The solution of acoustic beam pattern of a focusing source was reported and discussed by Lucas and Muir (1982), and Chen *et al* (1993). A numerically convergent solution with error smaller than 0.01 for the beam pattern of a focused spherical acoustic source was also provided by Chen *et al* (1993). We adopted Chen *et al*'s method to calculate the beam pattern in the simulation. The beam pattern calculated for a focused acoustic source with frequency of 2 MHz, diameter of the active element $D=3$ cm, the radius of curvature $R_0=15$ cm, beam diameter $0.4 B_D = cm$ was shown in figure 3. Equation (5) was then used to simulate the measured AE signal.

3. Results

3.1. Simulation results of a simple current density distribution

As shown in figure 4, the normalized AE signal collected by the lead placed at $(-4,3,0)$ and $(4,3,0)cm$ with the focal point of the unipolar ultrasonic pulse transducer scanning parallel to z axis from $(1, 1, -5)$ to $(1, 1, -5)cm$ is compared to the corresponding AE signal using bipolar ultrasound pulse. Here the beam diameter B_D is 0.2 cm. The pulse duration for the unipolar pulse is 0.125 μs . For the simulated bipolar pulse $T_d \approx 1/(bf)$ 0.85 μs where $b=0.588$ is the bandwidth factor. Identical peak pressure amplitude of 1 MPa was adopted for fair comparison between using the bipolar pulses and unipolar pulses in all simulations. From figure 4 we can see that AE signal generated by unipolar ultrasonic pulse is much stronger than that generated by bipolar ultrasonic pulse. Greater than 250 times of signal amplitude gain is obtained inside the imaged object. As shown in figure 4b, we can also see that when using bipolar ultrasonic pulse the AE signal is very weak in the internal region of the imaged object while strong AE signal can be seen at the object boundary where strong

current density gradient exists. This also agrees well with the reported experiment research results (Olafsson *et al* 2009a and 2009b, Wang *et al* 2010a and 2010b).

Using the proposed 3D UCSDI method, the reconstructed current density distribution with $B_D = 0.2\text{ cm}$, $T_d = 0.125\text{ us}$ under noise free condition is shown in the panels in the right column of figure 5 as compared to the target current density distribution shown in those left panels of figure 5. For comparison, the reconstructed current density distribution by using bipolar ultrasound pulse is shown in those middle panels of figure 5. From figure 5 we can see that the reconstructed current density distribution obtained by the proposed imaging method is consistent with the original distribution, and the reconstructed current density distribution by using bipolar pulses only gives a rough estimation of the original distribution. For a direct comparison, figure 6 shows the target current density, the reconstructed current density using bipolar ultrasonic pulses and the reconstructed current density using unipolar ultrasonic pulses along the line in the imaged object from $(-4, 1, -1)$ to $(4, 1, -1)\text{ cm}$. It is clear that the reconstruction using unipolar ultrasonic pulse gives better estimation of the target current source density distribution as compared to that using bipolar pulse. The quality of the image reconstruction is summarized in table 1. By using the proposed method, current density imaging with correlation coefficient between the target and reconstructed distributions greater than 0.9984, and relative errors smaller than 0.0497 can be achieved in this simple case. The reconstruction accuracy by using unipolar pulses is far better than that when using bipolar pulses.

3.2. Case of complex current density distribution in inhomogeneous conductive medium

We have conducted another simulation with two pairs of electrodes of radii 0.04 cm , length 1.0 cm positioned at $(-2, 0, 0; 2, 0, 0)$, $(0, 0, -2; 0, 0, 2)\text{ cm}$ in an inhomogeneous conductive media, which has resistivity distribution of $1/[1.76 + 200(x^2 + y^2 + z^2)^{0.5}]\text{ }\Omega\cdot\text{m}$. This simulation was studied to assess if the proposed imaging method can be applied to image complex current density distribution in an inhomogeneous conductive media. Here we assumed the AE interaction coefficient is uniform and $K = 1 \times 10^{-9}\text{ Pa}^{-1}$. The original current density distribution of this example is shown in the left panels of figure 7, while the reconstructed current density distribution obtained by the proposed imaging technique with $B_D = 0.2\text{ cm}$, $T_d = 0.125\text{ us}$ under noise free condition is shown in the right panels of figure 7. As shown in figures 7, the reconstructed current density distribution agrees well with the original distribution. For a direct comparison, figure 8 shows the target current density, the reconstructed current density using bipolar ultrasonic pulses and the reconstructed current density using unipolar ultrasonic pulses along the line in the imaged object from $(-4, 1, -1)$ to $(4, 1, -1)\text{ cm}$ as well. This result shows another example illustrating the better imaging performance of the proposed 3D UCSDI method using unipolar pressure pulse versus that using bipolar pressure pulse in an inhomogeneous media. As shown in table 2, current density reconstruction with correlation coefficients greater than 0.9987 and relative errors less than 0.05 can be obtained in this case by using the proposed method.

3.3. The influence of beam diameter, duration and noise

Finally the influence of ultrasound parameters and noise on the imaging performance was further investigated. We first tested the imaging performance with the ultrasound beam diameter changing from 0.06 to 0.6 cm , with fixed $T_d = 0.125\text{ us}$ under noise free condition. As shown in figures 9a and 9b, as the beam diameter increases, the correlation coefficient between the reconstructed current density distribution and the original distribution has a slight decrease and the relative error has a slight increase. However very good reconstruction with correlation coefficients greater than 0.993 and relative errors less than 0.12 can still be obtained even when the beam diameter B_D increases to 0.6 cm , as shown in figures 9c and 9d.

The ultrasound pulse duration T_d of the unipolar ultrasonic pulse used in this study is proportional to the time the elastic waves cross the transducer, so the duration T_d varies with the thickness of the transducer. We tested the imaging performance with T_d increases from 0.0068 to 2.703 μs , with fixed B_D 0.071 cm under noise free condition. From figures 10a and 10b, it can be seen that as pulse duration increases the correlation coefficient between imaged current density distribution and the original distribution decreases slowly and the relative error increases slowly. However good image reconstruction with correlation coefficient greater than 0.996 and relative error less than 0.10 can still be obtained when the pulse duration T_d increases to 2.7 μs , as shown in figures 10c and 10d.

The influence of noise to the proposed imaging technique was further investigated as shown in figure 11. Gaussian noise with zero mean was added to the simulated AE signal. The beam diameter and pulse duration were set as $B_D = 0.1\text{ cm}$, T_d 0.125 μs in this investigation. The ratio of the standard deviation of the added Gaussian noise over signal strength was denoted as noise index I_n . From figures 11a and 11b, we can see that the correlation coefficient between the reconstructed and target current source distribution decreases slowly and relative error increases gradually with the increase of noise index I_n . In addition, imaging results with corresponding correlation coefficients greater than 0.85 still can be obtained with the maximum noise index, as shown in figures 11c and 11d. However, relative error in the image reconstruction may reach 0.6 when the noise index increases to 0.2. As shown in this simulation study, unlike common ill-posed inverse problem, the proposed imaging technique is not vulnerable to noise.

4. Discussion

Imaging tissue electrical activities and properties with noninvasive measurements is an important research topic in the field of biomedical imaging. Ultrasound electrical property imaging, as a new imaging modality of imaging electrical properties, has been proposed to achieve noninvasive tissue electrical impedance and current density imaging with superior spatial resolution (Zhang and Wang 2004, Olafsson *et al* 2009). Previous studies have derived the 3D or 4D AE equations and a 2D inverse solution has also been developed (Olafsson *et al* 2008, 2009a). In this study, we propose a possible 3D solution to the general AE equation using unipolar ultrasound pulse. This kind of 3D inverse solution has been a big challenge when using general bipolar ultrasound pulses, as the AE signal tends to vanish if the ultrasound pulse packet is traveling in a slowly varying current density field. We have also conducted computer simulation to demonstrate that it is feasible to reconstruct an arbitrary 3D current source density distribution in an inhomogeneous media using the proposed method.

We should note that by using bipolar ultrasound pulse, 3D or 4D (3D+time) imaging of those strong gradient current field such as that generated by electrodes, lines or sheets of current on the order of an acoustic wavelength in tissue is still possible and confirmed by recent experiment studies (Olafsson *et al* 2009a and 2009b, Wang *et al* 2010b). These mapping studies however generally just use the envelope of the AE signal, thus the imaging result is an indirect representation of the current source density itself. In general, the AE signal generated using bipolar ultrasound pulse is hard to detect for an arbitrary current density distribution in the internal region of a volume conductive media especially when the current carrying media has larger dimension than the ultrasound wave length. Using the AE signal, the imaging result would thus emphasize the conductive object boundary (Wang *et al* 2010a). Therefore, using the proposed UCSDI method with unipolar pressure pulse gives a more general and accurate inverse solution and may give better imaging performance and better estimation of the real current source distribution. Of course, we should note that in conventional UCSDI method using bipolar pulse, only one pair of electrode measurement,

i.e. one lead field is required to obtain the AE signal envelope as a rough estimation of the current source density distribution. In comparison, the proposed new method needs at least 3 lead fields to fully estimate the 3D current source density vectors.

Of course, the unipolar pressure pulse is generally not easy to generate in practice using current ultrasound technology and it does not maintain through the whole wave propagation path. Therefore, in the proposed 3D UCSDI method, we need to do 3D ultrasound scan and real experiment system still need further careful development in the future.

In the proposed 3D UCSDI method, we assumed that the conductivity distribution inside the imaging volume is known. However, the exact in-vivo resistivity distribution of the human body is generally not known unless some other independent measurements are taken, for example using certain impedance imaging method such as EIT. However, literatures have also suggested the electrical resistivity ranges of different types of biological tissues. Therefore, most investigations on source localization in human body assume resistivity distributions based on some simplified model or based on tissue segmentation using CT/MRI images. The lead fields can be calculated by mean of FEM with known resistivity distributions and boundary conditions. In this proof of concept study, we assumed the resistivity distribution inside the imaging volume is known. Of course, in order to fully know how the uncertainties of resistivity distribution would affect the source localization performance using the acoustoelectric effect, further studies using more realistic conductive body model are needed, but it is beyond the scope of this study.

Seen from equation (6), it is notable that the spatial resolution of the imaging technique is mainly influenced by beam diameter and duration of the ultrasound pulse. So the lateral resolution can be improved by choosing transducer with tight focus. The beam axis resolution can be improved by using thin disc transducer because the pulse duration T_d of the unipolar ultrasonic pulse (Holé and Lewiner 1996, 1998) is proportional to the time the elastic waves to cross the transducer. In addition, in our simulation, we used 9 pairs of electrodes for measuring the AE signal. To obtain better imaging performance, the arrangement of electrodes needs further investigation. Generally, V_B is a low frequency signal, while V_{AE} is a high frequency signal and they can be easily separated using analog and digital filter. In addition, these two signals are generally recorded using two separate acquisition systems with a synchronization device (Witte *et al* 2007, Olafsson *et al* 2008, 2009a). In real experiment, the frequency of applied current should be different from AE signal V_{AE} for easy separation by using band pass filter.

In summary, we have proposed a 3D UCSDI solution to the general AE equation by using unipolar ultrasonic pulses. Utilizing specially designed unipolar ultrasound pulses and by confining the resistivity perturbation to the ultrasound focus, our simulation results indicate that we are able to reconstruct the 3D current density distribution with the boundary voltage measurements obtained while doing a 3D ultrasound scan. Our simulation results also show that the proposed method is feasible to image complex current density distribution in an inhomogeneous object. In addition, our simulation suggests that the proposed method is robust against noise and has stable imaging performance over a certain range of imaging parameter setups. Further experimental study will be needed in order to fully test the capability and performance of the proposed 3D current density imaging approach. The proposed method may give better estimation of the real current density source over the existing UCSDI mapping method for imaging functional electrical activities of biological tissues especially when the conductive tissue has larger dimension than the ultrasound wavelength.

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Figure 1. Schematic diagram of the 3D ultrasound current source density imaging based on acoustoelectric effect.

Figure 2.

(a) Simulated unipolar ultrasonic pulse. (b) Typical oscillatory bipolar ultrasonic pulse used in the simulation for comparison.

Figure 3.

The simulated beam pattern generated by a focused spherical transducer with central frequency of 2 MHz, diameter of the active element to be 3 cm and the radius of curvature of 15 cm.

Figure 4.

(a) Normalized AE signal (black solid line) collected by the lead placed at $(-4, 3, 0)$ and $(4, 3, 0)$ cm for current source density reconstruction with the focal point of the unipolar ultrasonic pulse transducer scanning parallel to z axis from $(1, 1, -5)$ to $(1, 1, -5)$ cm. The blue dashed line is the AE signal collected when using bipolar ultrasound pulse. (b) Enlarged AE signal collected when using bipolar ultrasound pulse.

Figure 5.

FEM based numerical calculation (the left column) of the current density distribution generated by a pair of needle electrodes in a cubic of $10 \times 10 \times 10$ cm³ homogeneous 0.9% NaCl solution, reconstructed current density distribution using bipolar ultrasound pulse (the middle column), and reconstructed current density distribution using the proposed method (the right column). (a), (b), (c), (d) are the x component, the y component, the z component and the norm of the current density in the x-y plane at $z = -1.0$ cm, respectively.

Figure 6. Line profiles of the target and reconstructed current density images as in Figure 5. (a), (b), (c), (d) are the x component, the y component, the z component and the norm of the current density along the line from $(-4, 1, -1)$ to $(4, 1, -1)$ cm (marked by the red dot line in figure 5), respectively.

Figure 7.

Simulated target current density distribution generated by two pairs of needle electrodes in a $10 \times 10 \times 10 \text{ cm}^3$ inhomogeneous conductivity medium (left) and reconstructed current density distribution using the proposed method (right). (a), (b), (c) and (d) are the x component, the y component, the z component and the norm of the current density in three x-y slices (i.e., $z = -3.5 \text{ cm}$, $z = -1.0 \text{ cm}$, $z = 3.5 \text{ cm}$), respectively.

Figure 8.

Line profiles of reconstructed current density using unipolar ultrasonic pulse as compared to target current density and reconstructed current density using bipolar ultrasonic pulse of the second example. (a), (b), (c), (d) are the x component, the y component, the z component and the norm of the current density along the line from $(-4, 1, -1)$ to $(4, 1, -1)$ cm, respectively.

Figure 9.

The correlation coefficient (a) and relative error (b) between the reconstructed and the target current density distribution versus ultrasound beam diameter. Curves marked by empty circles, solid circles, squares and stars indicate the values calculated using the x component, the y component, the z component and the norm of the reconstructed and target current density distributions in the x - y plane at $z = -1.0$ cm, respectively. (c) shows an example imaging result of the norm of the current density distribution when beam diameter reaches 0.6 cm, and (d) shows the comparison between the reconstructed and the target current density distribution along the line from $(-4, 1, -1)$ to $(4, 1, -1)$ cm (marked by the red dot line in (c)). The pulse duration is set to 0.125 μs .

Figure 10.

The correlation coefficient (a) and relative error (b) between the reconstructed current density distribution and the target distribution in the x-y plane at $z = -1.0$ cm versus ultrasound pulse duration. (c) shows an example imaging result of the norm of the current density distribution when ultrasound pulse duration reaches $2.703 \mu s$, and (d) shows the comparison between the reconstructed and the target current density distribution along the line from $(-4, 1, -1)$ to $(4, 1, -1)$ cm (marked by the red dot line in (c)). The ultrasound beam diameter is set to 0.071 cm.

Figure 11.

The correlation coefficient (a) and relative error (b) between the reconstructed current density distribution and the target distribution in the x-y plane at $z = -1.0$ cm versus noise index. (c) shows an example imaging result of the norm of the current density distribution when noise index reaches 0.2, and (d) shows the comparison between the reconstructed and the target current density distribution along the line from $(-4, 1, -1)$ to $(4, 1, -1)$ cm (marked by the red dot line in (c)). The ultrasound beam diameter is set to 0.1 cm and pulse duration is 0.125 μs .

Table 1

Evaluation of the ultrasound current source density reconstruction using the homogeneous conductive medium model

	x	y	z	norm
CC	0.9984	0.9987	0.9995	0.9994
RE	0.0497	0.0478	0.0277	0.0224
CC(Bipolar)	0.6425	0.8781	0.7872	0.8245
RE(Bipolar)	0.7832	0.5682	0.6601	0.5034

Table 2

Evaluation of the ultrasound current source density reconstruction by the proposed method using the inhomogeneous conductive medium model

	x	y	z	norm
CC	0.9994	0.9995	0.9987	0.9988
RE	0.029	0.0295	0.0446	0.0255