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Noise Modeling From Conductive Shields Using Kirchhoff Equations

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Abstract

Progress in the development of high-sensitivity magnetic-field measurements has stimulated interest in understanding the magnetic noise of conductive materials, especially of magnetic shields based on high-permeability materials and/or high-conductivity materials. For example, SQUIDs and atomic magnetometers have been used in many experiments with mu-metal shields, and additionally SQUID systems frequently have radio frequency shielding based on thin conductive materials. Typical existing approaches to modeling noise only work with simple shield and sensor geometries while common experimental setups today consist of multiple sensor systems with complex shield geometries. With complex sensor arrays used in, for example, MEG and Ultra Low Field MRI studies, knowledge of the noise correlation between sensors is as important as knowledge of the noise itself. This is crucial for incorporating efficient noise cancelation schemes for the system. We developed an approach that allows us to calculate the Johnson noise for arbitrary shaped shields and multiple sensor systems. The approach is efficient enough to be able to run on a single PC system and return results on a minute scale. With a multiple sensor system our approach calculates not only the noise for each sensor but also the noise correlation matrix between sensors. Here we will show how the algorithm can be implemented.

Index Terms

Kirchhoff equations; noise modeling; rf shields

I. Introduction

Progress in the development of high-sensitivity magnetic-field measurements has stimulated interest in understanding the magnetic noise of conductive materials, especially of magnetic shields based on high-permeability materials and/or high-conductivity materials. For example, mu-metal shields are routinely used in experiments using SQUIDs and atomic magnetometers. Additionally, SQUID systems frequently have radio frequency shielding based on thin conductive materials. Apart from shielding, conductive materials can be present in experiments due to a variety of reasons, and it may be necessary to evaluate the magnetic noise from these materials. For example, the cell of an atomic magnetometer can contain some amount of alkali-metal forming a thin film on the cell wall. Thermal currents in this film can introduce significant noise in magnetic measurements due to proximity to the active measurement volume. In ultra-low field (ULF) MRI experiments copper coils are located in the vicinity of the SQUIDs, and thus it is important to understand the possible effect of these coils on the noise in the system. Dewars for low-transition temperature (low- T_c) SQUIDs contain some conductive materials for reflecting thermal radiation to reduce the boiling off rate of helium. There are many other situations where the understanding of noise

from conductive surfaces can be important: microchips, atomic magnetometers with electric heating, electronic circuits, shielded cables, etc.

In non-magnetic conductive materials, the main source of noise is due to the thermal fluctuations of carriers. This noise is similar to that of resistors, called either Johnson (owing to its thermodynamic explanation [1]) or Nyquist (owing to its experimental investigation [2]) noise. While it is a well-known fact that the power spectral density of the noise voltage v_n^2 across the resistor R given by

$$v_n^2 = 4kTR \quad (1)$$

where k is the Boltzmann constant and T is the resistor temperature (see for example [2]), calculation of the noise currents in volume conductor is complicated, especially for an arbitrary shaped conductor.

Previously, work has been done on calculations of magnetic noise from conductive materials [3]–[7]. In some simple geometries analytical expressions were obtained allowing simple estimates of magnetic field noise and its dependence on the size and thickness of walls, but for a general shape of the conductive object, the solution can be found only numerically.

The problem is further complicated in that for many applications it is not enough to know statistical properties of magnetic field noise at a point, but it is necessary to estimate statistical properties of the noise induced in the system of magnetic field sensors.

In this work we develop numerical algorithms for calculations of the statistical properties of magnetic noise induced in arbitrary shaped magnetic field sensors by thermal currents in arbitrary shaped non-magnetic conductive materials at low frequency. We have verified our approach by comparing our results with those previously obtained with other methods for conductors with simple geometry. The method presented here has been used to evaluate the sensitivity and optimize the design of a multi-channel SQUID-based MRI scanner for ultra-low field (ULF) MRI applications, used, for example, for detection of liquid explosives [8]. Lastly we note that our method is computationally efficient compared to numeric solution of the Maxwell equations.

II. Model Principles

To calculate magnetic noise from a conducting surface we represent the surface as a network of discrete resistors and voltage noise sources corresponding to the thermal noise of the resistors, see Fig. 1. The resistance of each leg of the network is assigned so that the resistance of the equivalent square mesh is equal to that of the surface. Then we calculate statistical properties of the leg currents associated with those noise sources using Kirchhoff equations. Lastly we calculate statistical properties of the magnetic field generated by those leg currents using the Biot-Savart law.

We assume that the shields are thin enough to be approximated as a surface system instead of a volume system. This is, however, not a limitation of the code, but only a sign of what we use the algorithm for. Our systems have an 80 nm thick gold Mylar shield. The shield surface is on the order of 1 m^2 so the area approximation is valid. The changes in the model for handling volume systems is minor and only means that more memory and processing power are needed. The shield mesh is shown in Fig. 1. As shown, each leg on each triangle can be seen as a part of a network of discrete electric components. Each leg includes a

voltage source (ε) and a resistive load (R). By ignoring the inductive load of the impedance we are limiting our model to low frequencies, see Section III.

With the complex systems that are used today in for example ULF-MRI and MEG where interest lies in the noise properties of magnetic field flux through the pickup loop of multiple sensors there are advantages to go from looking at edge currents in the legs to treating them as loop currents of triangles. For this we use the relationship between the two, through a transfer matrix to go from edge currents to loop currents. By making the mesh cells, the triangles, small in comparison to the distance between the conducting surface and the pickup coil the field generated by the loop currents can be treated as arrays of magnetic dipoles. Through this the reciprocity principle can be used to derive an expression for the total flux through the pickup coil from the dipoles in the surface. The reciprocity principle allows one to replace calculation of the flux directly from its definition shown in (2)

$$\Phi = \oint_{\text{Area}} \vec{B} \left(\sum_i \vec{M}_i \right) \cdot d\vec{S} \quad (2)$$

with

$$\Phi = \sum_i \vec{B}_i^{\text{loop}} \cdot \vec{M}_i \quad (3)$$

where Φ is magnetic field flux generated by a magnetic dipole $\vec{M}_i$ through a surface limited by a contour C (the pick-up loop), $\vec{B}_i^{\text{loop}}$ is the magnetic field generated by unit current in the contour C at the position of the dipole $\vec{M}_i$. Further explanation of the reciprocity principle can be found in many textbooks on Electromagnetism. The calculation of $\vec{B}_i^{\text{loop}}$ can be done analytically for simple coil shapes. Using this method the algorithm becomes much more computationally efficient since it reduces a surface integral to a linear integral. In a system with multiple sensors the algorithm also gives the correlation for the noise between the sensors.

When going from edge currents to loop currents it is important to treat the mesh in the correct way. For this we use Euler's number as a check. The definition for Euler's number is number of vertices – number of edges + number of faces and it's a characteristic of the mesh. In a closed surface Euler's number is 2, for an open surface without any holes the Euler number is 1, and for a surface with holes the Euler number is less than 1. A cylinder for example will have Euler number 0, for its one hole in an “open mesh”. In our model we translate our mesh into the equivalent of an open surface, with an Euler number of 1. This means that if we have a sphere, Euler number 2, we have to take out one of the loop currents. If the surface is a cylinder we add a loop current around one of the edges of the cylinder. This gives the model the appearance of an open surface without holes and the Euler number will be 1.

The modeling setup dictates which of the two current systems you use. For a system where you are only interested in the field in certain points the edge current approach is the most computationally favorable. This is because the algorithm works with sparse matrices and it turns out that even though the matrix itself is smaller for the loop current, it's not as sparse as for the edge currents. If instead the system of interest requires knowing the flux through a

pick-up coil, then using loop currents is much more beneficial with the added advantage of being able to use the reciprocity principle and Green's law to go from calculating a surface integral to a contour integral.

III. Frequency Effects on Noise

Our model has been developed for low-frequency noise calculations, and it ignores the reduction in noise owing to impedance increase in the conducting surface with frequency, which is due to inductance and finite depth of field penetration limited by the conductor skin-depth. In [7], the frequency dependence of noise has been analyzed, and we can follow the analysis to estimate the range of frequencies to which our method is applicable. The skin-depth effect is straightforward to evaluate and can be easily incorporated in our model: essentially, the thickness, t , of the surface has to be replaced with the skin depth

$$\delta = (\pi f \mu_0 \sigma)^{-1/2} \quad (4)$$

where μ_0 is the permeability of free space, $4\pi \times 10^{-7} \text{ NA}^{-2}$, σ is the conductivity and f is frequency. According to analytical models of Table I, this would lead to $f^{-1/4}$ reduction in noise when

$$f > f_{\text{skin}} = 1/(\pi \mu_0 \sigma t^2) \quad (5)$$

where t is the thickness of the shield. In case of 1 mm aluminum shield $f_{\text{skin}} = 7 \text{ kHz}$, while 80 nm Mylar film has $f_{\text{skin}} \approx 1 \text{ THz}$. When the condition $\omega L \ll R$ is not satisfied, at frequencies greater than $f_{\text{ind}} = 1/2\pi\tau$, where $\tau = L/R$ is the decay time of eddy currents in the shield, the effect of the shield inductance L is important and the noise will fall off as f^{-1} . In general, the inductance of the surface is not easy to calculate, but it can be roughly estimated, which should be sufficient for finding the frequency range of applicability of our model. According to [9] for a disk, $\tau = L/R \approx 3 \times 10^{-7} A/\rho$, where ρ is the resistivity and A is the cross section of the disk; A can be roughly evaluated for various geometries of shields as $A = td$, where d is the characteristic size of the shield if the distance from the shield surface to the noise evaluation point is larger or this distance otherwise. In case of 1 mm aluminum infinite shield at a distance 0.2 m, $f_{\text{ind}} \sim 3 \text{ kHz}$, and in case of Mylar of 80 nm thickness at the same distance, $f_{\text{ind}} \sim 30$. Thus we can conclude that our theory will be applicable to Mylar films up to very high frequencies, but for 1 mm aluminum shields below 1 kHz.

It is possible to incorporate inductances into our network model to account for frequency effects. However, there is a complication owing to mutual coupling between inductances and requirement for some normalization scheme. Work is in capable of accurate prediction of frequency effects for complicated surface topologies.

IV. Validation of Our Model

In this section we present results for three different geometries. We calculate noise for a thin non magnetic shield a distance from a single pickup loop of 2.5 mm diameter. The geometries of the shield are an infinite plate, a spherical shell and a closed cylindrical shield as seen in Fig. 2. The three geometries are the same as in [7]. The equations for the analytical value of the field noise due to Johnson noise current, δB_{curr} , are shown in Table I where k is Boltzmann's constant, T is temperature, 293 K. σ is $3.8 \times 10^7 \Omega^{-1}\text{m}^{-1}$, t is 1 mm, and a is the distance from the shield to the pickup loop, 0.2 m. The pickup loop has a

diameter of 2.5 mm and is oriented horizontally in the x/y plane. With these geometries we get agreement with both the analytical and the computational results presented by S.-K. Lee and M. V. Romalis in [7] to within less than 2%, see Table II. B_{curr} is the magnetic field noise from eddy current loss.

V. Practical Examples

One primary benefit of our approach is that it can handle any shield geometry and any sensor configuration. We are not limited to calculating one sensor at a time. The added benefit is that we not only get the noise value in each sensor but also the correlation between different sensors. Fig. 3 shows a sensor setup that we use in one of our system setups. Fig. 4 shows the modeled noise correlation in the system while Fig. 5 shows the experimental data. The only input values to the model are shield thickness, 80 nm, and conductivity, $4.5 \times 10^7 \Omega^{-1}\text{m}^{-1}$ for the gold Mylar film. The data shows differences between simulation and the experimental data. This can be explained by the presence of additional noise sources other than the shield in the experimental data. The data do show us the expected correlation between channels.

VI. Conclusion

We have used Kirchhoff's equations to create an algorithm to calculate the conductive noise from a non-magnetic shield. The results agree to within 2% with analytical results and numerical results from other published algorithms. We also show the strength of the algorithm by presenting data in agreement with experimental data for the correlation matrix of conductive noise with multiple sensors in a gradiometer setup. To our knowledge this has not been demonstrated before. The algorithm also allows for shields of more general geometry, which has not been the case for prior algorithms that we know of. The other benefit of the algorithm is its memory efficiency and the low processing power needed. All results were acquired on a PC in a time scale of minutes.

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Fig. 1. (Left) Triangular mesh of a Dewar model. (Right) The simulated components, ε (voltage source) and R (resistive load) of each leg in the mesh.

Fig. 2. Geometrical forms for validation of our model. (Top left) Closed/In-finite cylindrical shell. (Top right) Spherical shell. (Bottom left) Infinite plate. The pickup loop is oriented to be sensitive in the z direction.

Fig. 3. Sensor setup in a LTS SQUID Dewar. (Left) Dewar with pick up coils and polarization coil visible. (Right) Sensor geometry and numbering.

Fig. 4.
Simulated noise covariance with the same system setup as our experimental setup.

Fig. 5.
Experimental noise covariance from our seven sensor setup.

TABLE I

Analytical Equations

Geometry	Non Magnetic Shield
Infinite plate	$\delta B_{curr} = \frac{1}{\sqrt{8\pi}} \frac{\mu_0 \sqrt{kT\sigma t}}{a}$
Spherical shell	$\delta B_{curr} = \sqrt{\frac{2}{3\pi}} \frac{\mu_0 \sqrt{kT\sigma t}}{a}$
Infinite cylindrical shell	$\delta B_{curr} = \sqrt{\frac{3}{16}} \frac{\mu_0 \sqrt{kT\sigma t}}{a}$

TABLE II

Results

Geometry	$\delta B_{\text{curr},z}$ (fT/Hz ^{1/2})		
	Our model	Analytical	Numerical*
Infinite plate	15.6	15.5	15.4
Spherical shell	35.8	35.9	35.7
Infinite cylindrical shell	33.8	33.7	
Closed cylindrical shell			
$L/2a = 0.5$	44.7	44.9	45.0
$L/2a = 1$	33.5	34.2	34.1
$L/2a = 2$	33.0	33.7	33.6

* The numerical values are taken from [7].