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Mechanisms of Bimaterial Attachment at the Interface of Tendon to Bone

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Abstract

The material mismatch at the attachment of tendon to bone is amongst the most severe for any tensile connection in nature. Attaching dissimilar materials is a major challenge in engineering, and has proven to be a challenge in surgical practice as well. Here, we examine the material attachment schemes employed at this connection through the lens of solid mechanics. We identify four strategies to that the body adopts to achieve effective load transfer between tendon and bone: 1) a shallow attachment angle at the insertion of transitional tissue and bone, 2) shaping of gross tissue morphology of the transitional tissue, 3) interdigitation of bone with the transitional tissue, and 4) functional grading of transitional tissue between tendon and bone. We provide solutions to model problems that highlight the first two mechanisms, discuss the third qualitatively in the context of engineering practice, and provide a review of our earlier work on the fourth. We study these strategies both in terms of ways that biomimetic attachment might benefit engineering practice, and of ways that engineering experience might serve to improve surgical healing outcomes.

Keywords

Stress concentrations; biomimetics; material optimization; fiber-reinforced laminate; attachment of dissimilar materials; bi-material interfaces

INTRODUCTION

A stressed connection between two materials with differing mechanical properties can lead to stress concentrations and failure at a level of applied stress which would not fail either material individually [1–4]. Amongst the most severe structural material mismatches in all of nature is the attachment of tendon to bone (i.e., the tendon-to-bone insertion site), the

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modulus mismatch between these two materials being approximately two orders of magnitude. At the natural, uninjured tendon-to-bone insertion site, the stress concentration problem is alleviated through material and morphological grading [5–8]. A comparison between uninjured and surgically repaired tendon-to-bone insertions reveals that this natural grading is highly optimized for load transfer. The natural grading between tendon and bone is not regenerated during tendon-to-bone healing, even following surgical intervention, resulting in an inferior connection between the two tissues [9–11]. Unlike the natural tendon-to-bone insertion, characterized by gradual variations in the collagen fiber orientation as well as a varying mineral content from bone to tendon, the scar tissue forming after healing is predominantly isotropic and homogeneous (Figure 1). While the natural insertion rarely fails under physiologic loads, surgical repairs of rotator cuff tendon-to-bone insertions have re-tear rates ranging from 20% to 94% depending on the severity of the initial injury [12, 13].

Our hypothesis is that these poor surgical outcomes are due to the inability of the body to regenerate the natural tendon-to-bone insertion site. If this is the case then, from the perspective of engineering materials, the natural tendon-to-bone insertion site might serve as a model for bimaterial attachment. From the perspective of rotator cuff reattachment procedures, this suggests that surgical strategies might benefit from the examination of natural tendon-to-bone attachment through the lens of basic engineering principles.

Here, we discuss four strategies that appear to influence effective load transfer between tendon and bone: 1) functional grading of transitional tissue between tendon and bone, 2) a shallow attachment angle at the insertion of transitional tissue and bone, 3) shaping of gross tissue morphology of the transitional tissue, and 4) interdigitation of bone with the transitional tissue. Our objective is to describe these four features from the perspective of macroscale engineering adaptations evident at the tendon-to-bone insertion site. While the focus is a qualitative understanding of these strategies, illustrative model problems are presented here for two of the four strategies. We study each of these in the context of fundamental mechanics challenges that must be overcome at the tendon-to-bone insertion site: avoiding an elastic singularity that can arise at a bimaterial attachment, reducing stress concentrations, and providing resistance to failure.

FOUR MECHANISMS OF BI-MATERIAL ATTACHMENT

1. Tailoring to avoid elastic singularities

We begin by exploring the circumstances under which a stress singularity arises in the case of bimaterial attachment under conditions of generalized plane strain. Our results demonstrate that a stress singularity between tendon and an isotropic material such as bone or scar tissue can appear or disappear as a function of gross morphology and material mismatch.

A central challenge in bi-material attachment is preventing a stress singularity at the interface. Consider, for example, a tendon-to-bone attachment (Figure 1). In the healthy attachment, an orthotropic tendon consisting of aligned collagen fibers embedded in water is connected to a much stiffer isotropic bone through functionally graded insertion site. The healthy interface includes collagen fibers with variable orientation from tendon to bone and mineral inclusions with gradually increasing density from tendon to bone. However, this graded insertion is not reproduced upon healing, being replaced with a so-called scar tissue that is isotropic and significantly less stiff than either tendon or bone. Under certain conditions, the asymptotic stress field at the free edges of an attachment between an orthotropic tendon and an isotropic scar tissue material can include a singularity (origin, Figure 2). Here, we specialize an existing solution in the literature [1, 3] to this specific case

and study the effects of mechanical property and local morphology variations on the nature of this asymptotic field. While others have studied this problem for a host of material mismatches [14, 15], we specialize to the case of a tendon attaching to an isotropic material with mechanical properties ranging from a relatively compliant scar tissue to a relatively stiff bone. The results shed light on two strategies that are evident in the body that eliminate a stress singularity: a grading of material properties and a shaping of the local morphology of the attachment points.

Asymptotic stress field—Consider two anisotropic materials perfectly bonded along the x -axis undergoing generalized planar deformation in which the out-of-plane normal component ε_{zz} of the linearized strain tensor ε is zero [16]. For each of the two anisotropic materials ($m=1, 2$), the stress-strain relations can be written as:

$$\varepsilon_i = S_{ij}^{(m)} \sigma_j, \quad (1)$$

where i and j range from 1 through 6. The engineering strains, ε_i in (1) are defined in a Cartesian system by:

$$\varepsilon_1 = \varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_2 = \varepsilon_{yy} = \frac{\partial v}{\partial y}, \varepsilon_3 = \varepsilon_{zz} = \frac{\partial w}{\partial z}, \varepsilon_4 = \varepsilon_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \varepsilon_5 = \varepsilon_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \varepsilon_6 = \varepsilon_{xy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial x} \quad (2)$$

where u , v and w are the components of displacements. The stresses σ_j are also defined in the Cartesian coordinate system. $S_{ij}^{(m)}$ is the associated matrix form of the compliance tensor for material m , and the repeated subscript indicates summation [16]. For $\varepsilon_{zz} = \varepsilon_3 = 0$, the third equation in (1) gives:

$$\sigma_3 = - \left(S_{13}^{(m)} \sigma_1 + S_{23}^{(m)} \sigma_2 + S_{34}^{(m)} \sigma_4 + S_{35}^{(m)} \sigma_5 + S_{36}^{(m)} \sigma_6 \right) / S_{33}^{(m)}. \quad (3)$$

Substituting these expressions into equation (1), we obtain:

$$\varepsilon_i = \bar{S}_{ij}^{(m)} \sigma_j, \quad (4)$$

where $\bar{S}_{ij}^{(m)} = S_{ij}^{(m)} - S_{i3}^{(m)} S_{j3}^{(m)} / S_{33}^{(m)}$, ($i, j=1, 2, 4, 5, 6$).

We summarize here the solution procedure using Lekhnitskii's stress potentials [16], $F(x,y)$ and $\Psi(x,y)$ defined as:

$$\sigma_x = \frac{\partial^2 F^{(m)}}{\partial y^2}, \sigma_y = \frac{\partial^2 F^{(m)}}{\partial x^2}, \tau_{xy} = \frac{\partial^2 F^{(m)}}{\partial x \partial y}, \tau_{xz} = \frac{\partial \Psi^{(m)}}{\partial y}, \tau_{yz} = \frac{\partial \Psi^{(m)}}{\partial x}. \quad (5)$$

Using these stress potentials, the equilibrium equations are satisfied identically and the compatibility equations reduce to

$$\begin{aligned} L_4^{(m)} F^{(m)} + L_3^{(m)} \Psi^{(m)} &= 0 \\ L_3^{(m)} F^{(m)} + L_2^{(m)} \Psi^{(m)} &= 0 \end{aligned} \quad (6)$$

in which the differential operators $L_2^{(m)}, L_3^{(m)}, L_4^{(m)}$ are defined by [16]:

$$\begin{aligned} L_2^{(m)} &= \bar{S}_{44}^{(m)} \frac{\partial^2}{\partial x^2} - 2\bar{S}_{45}^{(m)} \frac{\partial^2}{\partial x \partial y} + \bar{S}_{55}^{(m)} \frac{\partial^2}{\partial y^2} \\ L_3^{(m)} &= -\bar{S}_{24}^{(m)} \frac{\partial^3}{\partial x^3} + \left(\bar{S}_{25}^{(m)} + \bar{S}_{46}^{(m)} \right) \frac{\partial^3}{\partial x^2 \partial y} + \left(\bar{S}_{14}^{(m)} + \bar{S}_{56}^{(m)} \right) \frac{\partial^3}{\partial x \partial y^2} + \bar{S}_{15}^{(m)} \frac{\partial^3}{\partial y^3} \\ L_4^{(m)} &= \bar{S}_{22}^{(m)} \frac{\partial^4}{\partial x^4} - 2\bar{S}_{26}^{(m)} \frac{\partial^4}{\partial x^3 \partial y} + \left(2\bar{S}_{12}^{(m)} + \bar{S}_{66}^{(m)} \right) \frac{\partial^4}{\partial x^2 \partial y^2} - 2\bar{S}_{16}^{(m)} \frac{\partial^4}{\partial x \partial y^3} + \bar{S}_{11}^{(m)} \frac{\partial^4}{\partial y^4} \end{aligned} \quad (7)$$

For orthotropic materials which have principal axes coincident with the x, y, z frame of the problem (Figure 2), the $F^{(m)}(x, y)$ and $\Psi^{(m)}(x, y)$ terms are uncoupled in equations (6). Accordingly, the partial differential compatibility equation involving in-plane stresses may be written [3]:

$$L_4^{(m)} F^{(m)}(x, y) = 0, \quad (8)$$

where now:

$$L_4^{(m)} = \bar{S}_{22}^{(m)} \frac{\partial^4}{\partial x^4} + \left(2\bar{S}_{12}^{(m)} + \bar{S}_{66}^{(m)} \right) \frac{\partial^4}{\partial x^2 \partial y^2} + \bar{S}_{11}^{(m)} \frac{\partial^4}{\partial y^4} \quad (9)$$

The homogeneous solution for the governing partial differential equation (8) has the general form [3]:

$$F^{(m)}(x, y) = \sum_{k=1}^4 F_k^{(m)} \left(x + \mu_k^{(m)} y \right), \quad (10)$$

where μ_k are the roots of the following algebraic equation:

$$I_4(\mu) = \bar{S}_{11}^{(m)} \left(\mu^{(m)} \right)^4 + \left(2\bar{S}_{12}^{(m)} + \bar{S}_{66}^{(m)} \right) \left(\mu^{(m)} \right)^2 + \bar{S}_{22}^{(m)} = 0. \quad (11)$$

Following Delale [1] and transforming the field quantities to cylindrical coordinates, the stresses are obtained as:

$$\begin{aligned} \sigma_r &= \sum_{k=1}^4 (\lambda+2)(\lambda+1) A_k r^\lambda (-\sin \theta + \mu_k \cos \theta)^2 (\cos \theta + \mu_k \sin \theta)^\lambda \\ \sigma_\theta &= \sum_{k=1}^4 (\lambda+2)(\lambda+1) A_k r^\lambda (\cos \theta + \mu_k \sin \theta)^{\lambda+2} \\ \tau_{r\theta} &= - \sum_{k=1}^4 (\lambda+2)(\lambda+1) A_k r^\lambda (-\sin \theta + \mu_k \cos \theta) (\cos \theta + \mu_k \sin \theta)^{\lambda+1} \end{aligned} \quad (12)$$

where λ , the exponent of the leading term, defines the order of stress singularity.

The components of the displacements are given by:

$$\begin{aligned} u_r &= \sum_{k=1}^4 (\lambda+2) A_k^{(m)} \left(p_k^{(m)} \cos \theta + q_k^{(m)} \sin \theta \right) r^{\lambda+1} \left(\cos \theta + \mu_k^{(m)} \sin \theta \right)^{\lambda+1} \\ u_\theta &= \sum_{k=1}^4 (\lambda+2) A_k^{(m)} \left(-p_k^{(m)} \sin \theta + q_k^{(m)} \cos \theta \right) r^{\lambda+1} \left(\cos \theta + \mu_k^{(m)} \sin \theta \right)^{\lambda+1} \end{aligned} \quad (13)$$

where [3]:

$$p_k^{(m)} = \overline{S}_{11}^{(m)} \left(\mu_k^{(m)} \right)^2 + \overline{S}_{12}^{(m)}, \quad q_k^{(m)} = \overline{S}_{21}^{(m)} \mu_k^{(m)} + \overline{S}_{22}^{(m)} / \mu_k^{(m)} \quad (14)$$

The solution is found from both the continuity conditions at the bonded interface,

$$\begin{aligned} \sigma_\theta^{(1)}(r, 0) &= \sigma_\theta^{(2)}(r, 0) & \tau_{r\theta}^{(1)}(r, 0) &= \tau_{r\theta}^{(2)}(r, 0) \\ u_r^{(1)}(r, 0) &= u_r^{(2)}(r, 0) & u_\theta^{(1)}(r, 0) &= u_\theta^{(2)}(r, 0) \end{aligned} \quad (15)$$

and the traction-free boundary conditions along the free boundaries,

$$\begin{aligned} \sigma_\theta^{(1)}(r, \alpha) &= 0 & \tau_{r\theta}^{(1)}(r, \alpha) &= 0 \\ \sigma_\theta^{(2)}(r, -\beta) &= 0 & \tau_{r\theta}^{(2)}(r, -\beta) &= 0 \end{aligned} \quad (16)$$

where the superscripts 1 and 2 refer to materials 1 and 2, respectively. Using expressions (12) for the stresses and (13) for the displacements, the boundary and continuity conditions (15) and (16) yield:

$$\begin{aligned}
\sum_{k=1}^4 A_k^{(1)} - \sum_{k=1}^4 A_k^{(2)} &= 0 \\
\sum_{k=1}^4 A_k^{(1)} \mu_k^{(1)} - \sum_{k=1}^4 A_k^{(2)} \mu_k^{(2)} &= 0 \\
\sum_{k=1}^4 A_k^{(1)} p_k^{(1)} - \sum_{k=1}^4 A_k^{(2)} p_k^{(2)} &= 0 \\
\sum_{k=1}^4 A_k^{(1)} q_k^{(1)} - \sum_{k=1}^4 A_k^{(2)} q_k^{(2)} &= 0 \\
\sum_{k=1}^4 A_k^{(1)} (\cos \alpha + \mu_k^{(1)} \sin \alpha)^{\lambda+2} &= 0 \\
\sum_{k=1}^4 A_k^{(1)} (-\sin \alpha + \mu_k^{(1)} \cos \alpha) (\cos \alpha + \mu_k^{(1)} \sin \alpha)^{\lambda+1} &= 0 \\
\sum_{k=1}^4 A_k^{(2)} (\cos \beta + \mu_k^{(2)} \sin \beta)^{\lambda+2} &= 0 \\
\sum_{k=1}^4 A_k^{(2)} (-\sin \beta + \mu_k^{(2)} \cos \beta) (\cos \beta + \mu_k^{(2)} \sin \beta)^{\lambda+1} &= 0
\end{aligned} \tag{17}$$

The above equations constitute a system of homogeneous linear algebraic equations in $A_k^{(1)}$ and $A_k^{(2)}$. The existence of a nontrivial solution requires that the determinant of the coefficient matrix vanish:

$$|\Delta(\lambda)| = 0. \tag{18}$$

Equation (18) is transcendental in λ and has an infinite number of roots. These can be determined numerically. To ensure the finiteness of displacement components, the roots of Equation (18) should satisfy the condition that $\text{Re}(\lambda) > -1$. A singular stress-state will prevail only if $\text{Re}(\lambda) < 0$. Therefore, it is sufficient to search for roots whose real part is within the interval $-1 < \text{Re}(\lambda) < 0$. The root on this interval with the largest $|\text{Re}(\lambda)|$ dominates the asymptotic solution as r approaches 0.

Numerical examples—To model the attachment of tendon to scar, we studied the case in which material 1 was transversely isotropic, with the stiffest principal direction (axial direction) following the y -axis (Figure 2). Reasonable engineering constants for tendon are [17–19]:

$$\begin{aligned}
E_y &= 450 \text{ MPa} \\
E_x &= E_z = 45 \text{ MPa}, \\
G_{xy} &= G_{yz} = 0.75 E_1 / 1000 \\
G_{xz} &= \frac{E_x}{2(1+\nu_{xz})}
\end{aligned}$$

$\nu_{xz} = 0$ was assumed for simplicity. To maintain the positive-definiteness of the compliance matrix $S_{ij}^{(1)}$, ν_{yx} must satisfy the condition $\nu_{yx}^2 < \frac{E_y}{2E_x} = 5$. The values of Poisson's ratios used

in the examples were chosen $\nu_{yx} = \nu_{yz} = 2$, thermodynamic considerations underlying symmetry of the stiffness tensor then require that, $\nu_{xy} = \nu_{zy} = 0.2$ and $\nu_{zx} = 0$.

The degree of singularity at the free edge of the attachment line was assessed for a broad range of isotropic materials (material 2 in Figure 2) attaching to tendon (material 1 in Figure 2). We studied the effect of varying the insertion site interfacial angle β for a specific set of material properties. The tendon interfacial angle α was fixed as $\pi/2$ for all cases considered.

The solution of Equation (18) was non-trivial because of problems inherent to finding zeros in a two dimensional space and due to greatly differing magnitudes in the terms that appear in Equations (17). We employed a quasi-graphical approach in which initial guesses were found using an increasingly refined grid of test points, followed by local minimization.

Results—The stress singularity at the interface between dissimilar materials can be eliminated in two ways. For the case studied here, the singularity can disappear in the case of a butt joint ($\alpha = \beta = \pi/2$) between a tendon-like orthotropic material and an isotropic material of elastic modulus E for a range of moduli mismatches that depends strongly on the Poisson's ratio ν of the isotropic material (Figure 3). In all cases, the singularity approaches $\lambda \approx -0.26$ for attachment to an isotropic material that is stiff compared to tendon. For E smaller than the axial elastic modulus of tendon, the order of the singularity decreases with the ratio of E to the elastic modulus of tendon, and eventually disappears ($\lambda = 0$). For $\nu > 0$, the singularity returns for still smaller values of E over the range tested.

Discussion—While the amelioration of the stress singularity through selection of material properties is limited to special cases of modulus and Poisson's ratio mismatches, the stress field can be tailored through shape optimization for all material mismatches to reduce or eliminate the stress singularity. The case of a tendon with parallel faces ($\alpha = \pi/2$) was studied with $\nu = 0.3$ for material 2 (Figure 4). As above, the order of the stress singularity approaches $\lambda \approx -0.26$ for the case of an isotropic material that is much stiffer than tendon; however, this is the case only for very “steep” interfaces. A sufficiently shallow value of the interface angle ($\beta \leq \sim 0.05 \pi$) can be chosen such that the stress singularity disappears even for this case. $\lambda \geq 0$ for these cases; however, since the asymptotic solution is not a useful approximation in the absence of the singularity, these values of λ are not plotted. Even for cases in which the singularity is not eliminated entirely, its reduction can lead to a more favorable stress distribution; the spatial dimension associated with high stresses is reduced by a reduction in the order of the singularity in the same way that the spatial dimension associated with the Paris-Irwin plastic zone size in elastic-plastic fracture mechanics is reduced by a reduction in the linear elastic stress intensity factor [20]. Finally, for the case of an isotropic material less stiff than tendon, the behavior is more complicated, but an interface angle can again be chosen to annihilate the stress singularity.

2. Reduction of peak stress by optimizing gross morphology

Even after eliminating the stress singularity, the stress field in a bimaterial attachment can be refined to improve load transfer by adjusting the shape of the interface region. While the problem of optimizing the gross shape of an engineering component to reduce stresses is well-established [21–23], this more subtle problem of optimizing attachments and interfaces is poorly understood. The key difference between the two approaches is that efficient simulations designed to remove material from a bulk solid to optimize, for example, the stiffness or strength to weight ratio of a load bearing connection do so by assumption that all connections are joined and filleted in a way that does not introduce stress concentrations [23]. However, in the case of an attachment between two dissimilar materials, these omitted

details are the important part of the problem since fine details of the region close to the interface can influence the stress field significantly.

As a model of shape optimization for improved load transfer, we study here a coarse model of the supraspinatus tendon inserting into the humeral head. The underlying premise is that when tendon-to-bone insertion site tears, the insertion site tissue is replaced by scar tissue over the course of healing; this scar tissue is relatively disorganized, and can be modeled as isotropic [9]. We ask the question: what is the optimum shape of the outward splay of the scar-like insertion site to minimize principal stress?

Model problem—A rough, plane stress continuum approximation of the human supraspinatus tendon-to-bone insertion of the rotator cuff [24–30] was studied (Figure 5). The structure was loaded by a uniform traction in the vertical direction at the upper tendon boundary. The bottom boundary was prescribed symmetric boundary conditions (traction free parallel to the boundary, and zero displacement perpendicular to the boundary); all remaining boundaries were traction free. Because of the symmetry of the model about the symmetric plane (Figure 5), only half of the geometry was considered. The tendon was assigned the following mechanical properties: $E_1 = 450\text{MPa}$, $E_2 = 45\text{MPa}$, $G_{12} = 0.75E_1/1000$, $\nu_{12} = 3$ [17–19], where the 1-direction corresponds to the vertical direction on the diagram. Bone was taken to be isotropic with $E_{\text{bone}} = 20\text{GPa}$ and $\nu_{\text{bone}} = 0.3$ [31]. The Young's modulus of the scar tissue in this case was 1/10 of that of the peak modulus of tendon: $E_{\text{scar}} = 0.1E_{1\text{tendon}} = 45\text{MPa}$; Poisson's ratio was taken as $\nu_{\text{scar}} = 0.3$.

Optimization procedure—The model described above did not yield a stress singularity either at the tendon-scar or the scar-bone interfaces. However, the stress field was non-uniform, and, following the presumption that stress concentrations are undesirable, we modulated the outward splay of the scar region (red solid line, Figure 5) to minimize the peak stress. The optimization is sensitive to the choice of objective function. The goal of this shape optimization example is to illustrate that stress concentrations can be eliminated by changing the morphology of the attachment in a way that is sensitive to the material composition of the tendon-to-bone insertion site. Peak principal stress is considered because there exists a general assumption that tendon-to-bone insertion site is optimized for effective load transfer [5, 6, 32]. Optimizations with other objective functions are a focus of ongoing research.

The geometry of the outward splay was simulated with a 3rd degree Bezier curve, with the two end points fixed at the tendon and bone interfaces, leaving two unknown control points and, in total, four design variables. An optimization was performed to choose values of these four design variables that minimized the peak principal stress in the model. At each step, a coarse finite element approximation to the stress field was obtained using the commercial software Comsol (Comsol, Inc., Burlington MA) [33]. To reduce computational time, only 64 quadratic elements were used in most steps. Fine simulations with approximately 10,000 elements were performed following each optimization to ensure that the optimal solutions did not contain a stress concentration that was missed by the coarse analyses.

Standard gradient-based minimization algorithms implemented in the Matlab (Natick, MA) [34] environment were employed for the optimization [35]. This gradient-based algorithm stalled periodically in local minima. In such cases, a genetic approach was applied until the local minimum had been exited, and gradient-based minimization was resumed [36]. The genetic algorithm used here was the built-in function in Matlab [34]; the optimization was sufficiently straightforward that default parameter values for mutations could be used. The genetic optimizations ended either when they improved upon the results of the gradient-

based approach by 1% or after 20 generations. While we could not be certain that this criterion provided a global minimum, the results and progression proved instructive.

Results—The stress field evolved through the optimization from one in which the peak stress occurred at the free boundary of tendon-scar interface to one in which the stress concentration was nearly eliminated, and in which the peak stress occurred on the interior of the domain (Figure 6). The study of eight intermediate morphologies was undertaken on the way to the optimum (panel 9, Figure 6). It was found that following elimination of either elevated or singular stresses in the vicinity of the scar-tendon interface (narrow white regions at the outer edge of the scar-tendon interface, Figure 6, panels 1–3), the optimum stress field was reached by removing material from the scar region and transforming the scar-bone interface to an increasingly shallow attachment angle. The peak stress in the model dropped to only 1.05 times the applied load through this optimization. The most interesting aspect of the beginning phases of the analysis is what appears to be a misstep in which a bulge is generated at the scar-bone interface (panels 1–3, Figure 6). This might indicate that the high stresses at the tendon/scar interface are the dominant driving force in the problem at these initial optimization steps. The low stresses at the attachment point make the gradient of the objective function insensitive to the shape in this region (e.g. [22]). Note that no constraint on total material volume was enforced in the simulations; thus, no penalty was associated with adding material to unstressed regions.

As the optimization progressed, the scar region gradually tapered into a fillet (panels 4–6, Figure 6). The stress levels at the scar-bone interface were still low, meaning that the elimination of the bulge was more likely due to the optimality of extending the slender, neck-like region with a width identical to that of the tendon, rather than a penalty associated with the details of scar morphology in that region.

At the final steps of the simulation (panels 7–9, Figure 6), the optimal shape of the outward splay was found to be a smooth neck with a nearly circular fillet. Although the singularity was absent from the scar-tendon interface in the model studied, significant reduction in stresses was possible through shape optimization.

Discussion—The shape optimization for minimization of stress concentrations presented here was straightforward, but in other cases much more complicated procedures might be required. The elimination of the stress singularity in the plane stress problem occurs under conditions close to those for which the stress singularity disappears in plane strain (Figure 4). However, the angle of contact can certainly be modified to introduce singularities (cf. Figure 4). Similarly, for scar tissue with material properties differing from those tested, the optimal shape would be expected to differ, and a straight connection between tendon and scar tissue may result in a singularity. To address situations in which a stress singularity is a common occurrence over the course of an optimization, a refined optimization incorporating both an analytical solution for the singularity and a parametric analysis such as that performed here would be needed.

The final result of the shape optimization, a connection that is nominally filleted, is not surprising from an engineering standpoint. Two aspects are surprising, however. First, the optimum that the body reaches for a healthy tendon-to-bone insertion site, which was modeled by the initial conditions of the optimization, is far from optimal for scar-to-bone attachment. Second, the optimum achieved at the proximal end, at the attachment of scar to tendon, is the same as the optimal shape for attachment of tendon to the healthy insertion site.

The optimal stiffness at the tendon-to-bone insertion site is unknown. Indeed, in the natural tendon-to-bone insertion site, a zone exists that is more compliant than either tendon or bone [5, 7]. In the optimum found, the scar tissue in the place of the naturally functionally graded insertion site tissue was not permitted to change its elastic modulus. However, the shape optimization led to a more compliant region through structural remodeling. The reduction of the width of the region over the course of the simulation down to the width of the tendon led to a region whose stiffness was reduced despite the fixed modulus.

Shape optimization has been used by others to reduce stress concentrations. For example, Lipton studied the general problem of coupled shape and material optimization in the presence of stress constraints [37], and Huang et al. [36] studied strength of a plate of a porous solid that contains a central hole. A key difference between this earlier work and the problem studied here is that the stress concentration at a bimaterial interface can rise to a stress singularity. The ability to perform optimization in the presence of a singularity is an evolving science [38], and extensions of the example presented here will necessarily rely on these developments.

From the perspective of application to engineering materials, the lesson we can draw from human physiology is that a change in material properties can require a radical change in the attachment morphology for optimal load transfer. The representation of the supraspinatus tendon-to-bone insertion used in the analysis was coarse, missing important features of the three dimensional physiology of the shoulder. However, the change in shape required for optimal load transfer when replacing the normal graded mechanical properties of the region between tendon and bone with an isotropic scar mass highlights the important interplay between mechanical properties and optimal morphology of an attachment region. From the perspective of reparative surgery, the results offer some further insight. In even the most modern methods for the surgical reattachment of tendon to bone at the humeral head of the rotator cuff, the standard of care is to suture tendon directly to bone [39]. The simple morphology found for optimal load transfer in the idealized case studied here suggests that simple modifications of existing surgical techniques might show promise for improvement to patient outcomes.

3. Toughening through interdigitation

The final attachment strategy at the tendon-to-bone insertion site that we will discuss is the interdigitation of mineralized fibrocartilage and bone (Figure 7). This strategy differs fundamentally from those discussed above. Whereas the above methods seek to improve resistance to mechanical loads by reducing stress concentrations, the interdigitation of mineralized fibrocartilage and bone may have a greater impact on toughness than on the stress field. That is, this physiologic feature may serve to improve load transfer by increasing the resistance to load from a structural perspective, while the optimization of geometry and/or material grading may alleviate stress concentrations at a local level. Here, we discuss this physiologic feature in the context of a technology for composite attachment known as “z-pinning”, in which short fibers are inserted perpendicular to the direction of the long fibers of an engineering composite at a bond between components to enhance the integrity of the attachment (Figure 8).

A mechanical role for interdigitation has been long suspected. Schneider [40] suggested that the irregular surface between mineralized fibrocartilage and bone results in an enhanced resistance against shear, while unmineralized fibrocartilage transmits axial loads. Although this model differs from our modern picture of the tendon-to-bone insertion site as a smooth, functionally-graded material at the microscopic level [5, 41], the gross anatomic suggestions do provide useful insight. Milz et al. [42] showed that a complex 3-D interlocking between mineralized fibrocartilage and bone is paramount for the tendon-to-bone anchoring. They

also suggested that during the early phase of tendon repair the insertion mechanism is different from that in the healthy tendon-to-bone insertion site. This may be understood as an acknowledgement that a lack of interlocking results in a vulnerable tendon-to-bone insertion site during the recovery and reattachment process, and suggests that the existence of interdigitation is important to the toughness of the attachment.

While the mechanics of interdigitation in the tendon-to-bone insertion site have yet to be elucidated, the paradigm has important implications for attachment of composites through z-pinning. The z-pinning concept originated in the aerospace industry where z-pins represent high-strength small-diameter cylindrical rods oriented perpendicular or at an angle to the interface between the layers arresting cracks and preventing delamination of composite structures [43–45]. For example, Freitas et al. [46, 47] illustrated that adding just 1.9% volume fraction of carbon z-pin fibers can increase the fracture toughness of a composite laminate by a factor of 18, without a noticeable reduction in in-plane tensile strength. The effect of z-pins on fracture toughness of composite laminates was further studied by Byrd and Birman [43–45] using the example of a standard double cantilever (DCB) test. It was shown that adding z-pins constituting between 1% and 1.5% volume fraction of the DCB can result in the arrest of delamination cracks. Furthermore, it was suggested that the effectiveness of z-pins could be enhanced by designing their surface in a manner improving the “grasp” between the z-pin and adjacent material (e.g., woven z-pins with a naturally rough surface). Although quantitative guidance of engineering design based upon the physiologic analog of z-pinning at the tendon-to-bone insertion sites is premature, the existence of an analog to z-pinning in nature suggests that the strategy is well grounded.

Engineering success with z-pinning and its existence in the uninjured tendon-to-bone insertion site has important implications for surgical practice. The previous discussion illustrates that “z-pinning” at the tendon-to-bone insertion site may be beneficial during the reattachment process. Such z-pinning could result in a reduction of the stress concentration at the vulnerable corner of the insertion-to-bone interface surface and/or reduce the tendency to crack propagation along this surface. While z-pins of geometries referred to in the aerospace industry are not feasible, the search of potential candidates, including their materials, geometry and biocompatibility aspects, may improve the healing process.

4. Modulation of the stress field through functional grading

Functionally graded material systems are in common engineering usage for the purpose of providing a tough interface between two different materials, with a host of applications ranging from semiconductor thin films to prosthetic joints and limbs [48–50]. The typical strategy involves an interpolation in mechanical properties between two dissimilar materials with the aim of avoiding the sharp interface discussed in section 1. In earlier work, we and others have identified a unique functional grading that exists between tendon and bone at insertion sites such as the shoulder’s rotator cuff tendon-to-bone attachment [7] (Figure 1). Here, we summarize this functional grading scheme and literature that speculates upon its role.

Between tendon and bone lies a fibrocartilaginous transition zone [32]. The transition zone provides a connection between two fibrous materials: tendon, which consists of relatively aligned collagen fibers, and bone, which consists of mineralized collagen fibers that are much less aligned. The fibrocartilaginous region is graded in the degree of mineralization [8], in the density and organization of the underlying collagen fibers [7]. The combination of these three factors leads to a grading in mechanical properties that is highly unusual in the context of engineering attachment: rather than interpolating between the mechanical properties of the tendon and bone, the intermediate zone is more compliant than either tendon or bone [7].

The physical mechanisms underlying the creation of this compliant zone are clear. The first set of factors involves the stiffness of the underlying collagen network. The stiffness of this network drops between tendon and bone because of the decrease in fiber alignment and the reduction in collagen fiber volume fraction [5]. The second factor is the mineral volume fraction, which increases linearly from tendon to bone [8]. Mineralization of collagen fibers causes tremendous stiffening, but this stiffening outweighs the reduction of stiffness of the underlying collagen network only at degrees of mineralization above the percolation threshold [5]. The combined effect is a zone of lightly mineralized tissue, with a mineral volume fraction that lies below the percolation threshold, with stiffness lower than that of either tendon or bone.

The reasons for the existence of this grading are less clear. The compliant zone leads to a decrease in peak stresses and an increase in peak strains across the insertion site relative to a linear interpolation of moduli [6]. However, the reasons why this might be optimal are not known. One possibility is that the toughness of the compliant band might exceed that of tendon and bone, resulting in energy localization within a tough band, but further research is needed.

CONCLUSIONS

The natural tendon-to-bone insertion site incorporates several different strategies that alleviate difficulties associated with attaching two very different materials. These include functional grading of material content, adaptation of local morphology to avoid configurations that admit elastic singularities, adaptation of gross morphology to provide effective and efficient load transfer, and interdigitation analogous to z-pinning, presumably to enhance material toughness. As evidenced by the examples presented in this article, the study of these phenomena in terms of engineering principles can lead to new insight into design of bimaterial attachments. Furthermore, the generalization of engineering experience to the attachment of biomaterials has much to offer in terms of improved procedures for surgically assisted healing.

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Figure 1.

The natural tendon-to-bone insertion site (left) and the site after healing (right). The tendon, which is nominally orthotropic, attaches to bone, here presented as nominally isotropic, though a tissue region that presents gradients in mineral volume fraction (represented by blue shading) and in collagen fiber orientation (represented by black lines). Scar tissue at the tendon-to-bone interface following healing presents neither of these features, resulting in a highly vulnerable attachment.

Figure 2. Geometry in the vicinity of the free edge at the attachment between two dissimilar anisotropic materials (the z-axis is perpendicular to the plane of the drawing).

3a

3b

Figure 3.

The order of the singularity drops near zero for specific elastic property mismatches. Pictured here is the order of the singularity for the case of attachment of an isotropic material of elastic modulus E to an orthotropic material whose mechanical properties approximate those of tendon. (a) Cases for which Poisson's ratio of the isotropic material $v \geq 0$; (b) cases for which $v \leq 0$.

Figure 4.

The singularity disappears only for very shallow attachment angles between the orthotropic tendon and an isotropic material ($\nu = 0.3$) that is relatively stiff. The relationship is non-monotonic for attachment to an isotropic material that is relatively compliant.

Figure 5. Idealized plane stress model of the attachment of tendon to bone at the healing supraspinatus insertion site. This configuration served as a starting configuration for the shape optimization studies performed.

Figure 6. Selected geometries representing the evolution of the outer boundary of a scar region during optimization to reduce the peak principal stress associated with uniform axial loading of the tendon. Color contours represent peak principal stress normalized by the applied load.

Figure 7. Interdigitation between tendon/mineralized fibrocartilage and bone at the rat tendon-to-bone insertion site. This image comes from a paraformaldehyde fixed, paraffin embedded, 5 μm thin section of a rat supraspinatus tendon-humeral head bone insertion stained with Toluidine blue.

Figure 8.
The interdigitation in the natural tendon-to-bone insertion site is reminiscent of z-pinning of composite plates.