The effect of aspect ratio on adhesion and stiffness for soft elastic fibres

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The effect of aspect ratio on the pull-off stress and stiffness of soft elastic fibres is studied using elasticity and numerical analysis. The adhesive interface between a soft fibre and a smooth rigid surface is modelled using the Dugdale–Barenblatt model. Numerical simulations show that, while pull-off stress increases with decreasing aspect ratio, fibres get stiffer. Also, for sufficiently low aspect ratio fibres, failure occurs via the growth of internal cracks and pull-off stress approaches the intrinsic adhesive strength. Experiments carried out with various aspect ratio polyurethane elastomer fibres are consistent with the numerical simulations.

Keywords: elastomer; fibrillar adhesion; fibre stiffness; aspect ratio

1. INTRODUCTION

Fibrillar adhesion has attracted much attention in recent years. The remarkable attachment ability of some of the species that bear fibrillar footpads leads researchers to be inspired from and mimic these fibrillar adhesive pads [1,2]. In conjunction with fabrication attempts, many works were conducted to analyse the mechanism behind this unusual attachment ability [1,3,4]. Experimentally, it has been found that single-level structures consisting of long fibres with nominally flat ends generally fail to achieve theoretical predicted enhancement in strength owing to loss of contact area, lateral collapse and buckling of fibres [5–7]. In practice, enhanced adhesion is usually accomplished by fabricating an additional terminal element at the end of a fibre [8–13]. How the shape of these terminal elements affects adhesion has been studied theoretically by Gao & Yao [14], Greiner *et al.* [15], Spolenak *et al.* [16] and Soto *et al.* [17].

The mechanics of contact and adhesion of fibrillar surfaces can be studied using a cohesive zone model that describes the adhesive interaction between two contacting surfaces. The simplest of such models is the Dugdale–Barenblatt (DB) model [18,19]. In this model, the interface separates when the normal interfacial stress reaches the theoretical strength of the interface, denoted by σ_0 . The interface continues to separate at this stress until the separation reaches a critical distance δ_c , after which the interface can no longer support any stress, resulting in a crack initiation. The region where the separation of interface occurs is referred to as the cohesive zone. In this model, the work of adhesion of the surfaces in contact is given by $W_{ad} = \sigma_0 \delta_c$. Based on this model, Tang *et al.* [20] as

well as Gao & Yao [14] have established that a reduction in the characteristic lateral dimension of fibres forces the entire interface of an individual fibre to fail simultaneously rather than via propagation of the crack. This phenomenon increases adhesion strength and complements contact splitting [21], explaining why larger animals have finer diameter fibrils. Works by Arzt *et al.* [21] and Tang *et al.* [20] include detailed analysis on fibrillar interfaces and how it could enhance adhesion compared with flat unstructured surfaces.

While animals such as geckos favour long fibrils, some animals such as bush crickets use short pillars as adhesive pads [22]. Long fibres are advantageous over short ones in rough surface applications where fibre array compliance is crucial for roughness adaptation and high adhesion strength. On the other hand, smooth surface applications may not require long fibres. Short microfibre arrays are often used as adhesive footpads for climbing/walking robots for smooth surfaces [23–26]. In addition to short microfibre arrays, thin unstructured polymer footpads are also used as adhesive footpads in climbing robots [27]. The effect of aspect ratio (i.e. the ratio of the length of the fibre to its effective lateral dimension) on adhesion strength and compliance is crucial to be understood in order to design effective adhesives for these climbing robots. Previous analyses were carried out on high aspect ratio fibres,¹ that is, the fibres are long in comparison with their diameter. The contact mechanics of short fibres are quite different from those of long ones. Indeed, because fibres are made of incompressible materials, the compliance of the system decreases dramatically with increasing fibre aspect ratio. The

¹The term ‘fibre’ usually refers to structures with very high aspect ratios. In the text, the term fibre is used for both low and high aspect ratio structures for consistency throughout the paper.

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geometry for the adhesion of short fibres is practically identical to the so-called ‘poker-chip test’ [28], where a thin layer of elastomer is sandwiched between two rigid plates, which are pulled apart under tension. Another closely related test is the ‘probe-tack test’ [29]. In a typical ‘tack’ test, a rigid flat cylindrical punch is brought into intimate contact with a uniform thin layer of elastomer (a pressure-sensitive adhesive); the probe is then withdrawn and the force required for pull-off as well as the energy expended for the pull-off force is measured by finding the area under the load versus the displacement curve. When the interface between the punch and the elastic layer is sufficiently weak, failure occurs by the propagation of an interfacial edge crack. However, for high-strength adhesives, the layer fails by the initiation and growth of the *internal* interfacial cracks. These internal interfacial cracks are pinned along their fronts; as a result, they grow into large cavities with the long axis in the loading direction. The elastomeric material between these cavities acts as long fibrils to support the applied load.

In this work, the effect of aspect ratio on pull-off stress and stiffness of soft elastomeric fibres in contact with a rigid smooth surface is studied. Since the Poisson ratio of elastomers is very close to 0.5, fibres are assumed to be incompressible in this work. In §2, we present the elasticity formulation for the pull-off stress and the stiffness of a soft elastic fibre followed by numerical simulations in §3. Experiments carried out with various aspect ratio fibres and comparison with the simulation results are shown in §4. Discussions and conclusions are given in §5.

2. FORMULATION

2.1. Elastic fibre problem

In this section, we study how the fibril aspect ratio affects the force required to pull a fibre off a rigid substrate as shown in figure 1. Let us assume that a soft incompressible elastic fibre of length h and radius a is perfectly bonded to a rigid flat substrate and a uniform displacement Δ is applied to the free end of the fibre. Let P be the force exerted on the fibre owing to the displacement at the free end. For very long fibres, the applied displacement will cause a uniaxial stress state $P/\pi a^2$ in the fibre except in a region very close to the fibre edge. For these fibres, $P/\pi a^2$ is directly proportional to the strain $\varepsilon = \Delta/h$,

$$P = E \frac{\Delta}{h} \times \pi a^2, \quad (2.1)$$

where E is the Young modulus of the fibre. As the fibre becomes shorter relative to its radius, deviation from uniaxial behaviour becomes significant; as a result, equation (2.1) must be modified to reflect this deviation. Dimensional analysis and the linearity of the governing equation imply that

$$P = \pi E \frac{\Delta}{h} a^2 p\left(\frac{a}{h}\right), \quad (2.2)$$

where p is a dimensionless function of a single parameter a/h . According to equation (2.2), $p(a/h)$ can be

Figure 1. Soft elastic fibre in contact with a rigid smooth substrate. (Online version in colour.)

interpreted as the stiffness ratio of a fibre of aspect ratio h/a to one that has aspect ratio of infinity (i.e. very long fibre) and $p(a/h \rightarrow 0) \rightarrow 1$.

For an elastic fibre in contact with a rigid substrate, the normal stress σ_{zz} near the fibre edge has the form

$$\sigma \equiv \sigma_{zz}(z = 0, r \rightarrow a) = A(r - a)^{-\alpha}, \quad (2.3)$$

where $0 < \alpha < 1$ is a numerical factor that depends on the Poisson ratio, e.g. $\alpha \approx 0.4$ for $\nu = 0.5$ [30]. Equation (2.3) predicts that the normal stress becomes infinite as the edge is approached. The factor A in equation (2.3) represents the amplitude of the singular stress field. Dimensional analysis and the linearity of the governing equations imply that the amplitude A is given by

$$A = E \frac{\Delta}{h} a^\alpha g\left(\frac{a}{h}\right), \quad (2.4)$$

where g is a dimensionless function that depends only on a/h . We do not know g in general; however, in the limit of $a/h \rightarrow 0$,

$$A = E \varepsilon a^\alpha g(0). \quad (2.5)$$

In this limit, g approaches $g(0)$, which is a numerical constant.

2.2. Pull-off stress

It is assumed that the interfacial failure initiates when the amplitude of the singular field A reaches a critical value A_c , which is assumed to be a system constant independent of geometry,

$$A = E \frac{\Delta}{h} a^\alpha g\left(\frac{a}{h}\right) = A_c. \quad (2.6)$$

Here, we assume that the initiation of interfacial failure corresponds to pull-off. Whether pull-off (peak load) occurs at the instant of crack initiation or at a later

time will be addressed in §3. Using equations (2.2), (2.4) and (2.6), one can determine the pull-off load as

$$P_f = \frac{\pi A_c a^2}{g(a/h)} p\left(\frac{a}{h}\right) a^{-\alpha} = \pi A_c a^{2-\alpha} H\left(\frac{a}{h}\right), \quad H = \frac{p}{g}. \quad (2.7)$$

The value of A_c could be extracted from the previous work by Tang *et al.* [20], in which they estimated the pull-off force of a soft elastic long cylindrical fibre in contact with a rigid flat surface, whose height is much larger than its radius. In their study, they used the DB model to model the interfacial adhesion between the tip of the fibre and the adhering substrate. Define σ_s as the average stress the fibre bears at pull-off, $\sigma_s = P_f/\pi a^2$. They found that normalized pull-off stress σ_s/σ_o depends on a single dimensionless parameter χ such that

$$\frac{\sigma_s}{\sigma_o} = \begin{cases} 1, & \chi < 1 \\ B\chi^{-\alpha}, & \chi \gg 1 \end{cases} \quad (2.8a)$$

$$(2.8b)$$

where

$$\chi = \frac{\sigma_o^2 a(1-\nu^2)}{2\pi E W_{ad}} = \frac{\sigma_o a(1-\nu^2)}{2\pi E \delta_c}. \quad (2.9)$$

The dimensionless parameter is the ratio of two length scales: a and $\sigma_o/E\delta_c$. The former is the radius of the fibre and the latter the size of dominance of the interfacial adhesive forces. Here, $\alpha \approx 0.4$ for incompressible materials and B is a numerical constant of order 1 and was determined numerically to be $B = 0.83$.

Our assumption that the interface fails when $A = A_c$ is valid in the regime where $\chi \gg 1$, called the ‘flaw-sensitive’ regime. In this regime, the cohesive zone is very small in comparison with the fibre’s lateral dimension; its breakdown results in failure via the unstable growth of an interface crack at the edge of the fibre. In this case, $\sigma_s < \sigma_o$. In the ‘flaw-insensitive’ regime, where $\chi < 1$, the cohesive zone occupies the entire interface and, therefore, $\sigma_s = \sigma_o$. In the flaw-sensitive regime, for $a/h \rightarrow 0$, A_c can be obtained from equating (2.7) and (2.8b),

$$A_c = \frac{B\sigma_o}{H(0)} \left(\frac{\chi}{a}\right)^{-\alpha}. \quad (2.10)$$

As pointed out earlier, A_c is a system constant and is independent of geometry. Using equation (2.10), equation (2.7) can be rewritten to obtain the pull-off force of the elastic fibril and the pull-off stress (adhesive strength) in the flaw-sensitive regime, respectively, as

$$P_f = B\chi^{-\alpha} \pi a^2 \sigma_o \frac{H(a/h)}{H(0)} \quad (2.11a)$$

and

$$\sigma_s = B\chi^{-\alpha} \sigma_o \frac{H(a/h)}{H(0)}. \quad (2.11b)$$

3. NUMERICAL SIMULATIONS

Finite-element method (FEM) simulations were carried out using COMSOL MultiPhysics 3.4 to simulate the

stiffness and pull-off stress. The DB cohesive zone model is used in the pull-off stress calculations. Simulation details can be found in appendix A. Unless stated otherwise, the simulation parameters are $a = 1 \mu\text{m}$, $E = 2 \text{ MPa}$ and $\sigma_o = 100 \text{ kPa}$. The height of the fibre h is varied from 0.1 to 10 μm . χ is varied by changing δ_c from 0.2 to 3 nm for fixed a , E , ν and σ_o . Corresponding χ and W_{ad} range from 2 to 30 and from 20 to 300 $\mu\text{J m}^{-2}$, respectively.

3.1. Crack initiation and pull-off

For high aspect ratio fibres, because the length of the fibre is much larger than its radius, crack growth occurs under load control and is expected to be unstable. Therefore, pull-off is expected to occur once a crack initiates at the edge of the fibre. As the length of the fibre decreases, the detachment occurs under displacement control. In this case, crack growth can be stable, which could lead to pull-off after the crack initiation (i.e. during crack propagation). FEM simulations were carried out beyond the instant of crack initiation to determine whether pull-off occurs either at the instant of crack initiation or later during crack propagation. Figure 2 plots the simulation results for the average stress the fibre bears, $\sigma = P/\pi a^2$, as a function of the applied displacement Δ (figure 2a) and normal interfacial displacement δ_m (figure 2b). Both plots show that the peak load is reached either at the instant of or soon after the crack initiation. These results indicate that the load the fibre bears at the instant of crack initiation is a good approximation for the maximum attainable tensile load before interfacial failure and will be used as the pull-off load in this study.

3.2. Fibre stiffness and pull-off stress

The stiffness (i.e. spring constant) of the fibre is defined by rearranging equation (2.1) as

$$k_f = \frac{P}{\Delta} = \frac{\pi a^2 E p(a/h)}{h}. \quad (3.1)$$

The function $p(a/h)$ in equation (3.1) can be interpreted as the stiffness of a fibre normalized by the stiffness of a fibre with very high aspect ratio. It is calculated by fixing a and changing h in our FEM simulations. Figure 3a shows the FEM results for $p(a/h)$. Results for k_f are plotted along with the stiffness predictions for uniform strain across the cross section (i.e. $p(a/h) = 1$) in figure 3b. As expected, for small a/h values, the actual stiffness deviates very little from the uniform-strain approximation. However, the difference continues to grow for larger a/h , reaching close to an order of magnitude for $a/h = 10$.

Figure 4a shows the simulation results for the a/h dependence of pull-off stress for varying χ values (see equation (2.11b)). Pull-off stress increases for increasing a/h for all χ and approaches the intrinsic adhesive strength σ_o for sufficiently large a/h . As expected for larger χ , pull-off stress approaches the intrinsic adhesive strength at smaller a/h . For each χ value, normalized pull-off stress increases initially with increasing a/h , reaches an inflection point and then keeps increasing until it reaches the

Figure 2. FEM results for average normal stress on the fibre as a function of (a) applied displacement and (b) maximum normal interfacial separation. The instant maximum interfacial separation δ_m reaches the critical separation δ_c marks the initiation of crack. For cases where peak load is reached after the crack initiation, peak stress is only 1.0–3.9% larger than the stress at the instant of crack initiation. $\chi = 12$ in the simulations.

Figure 3. FEM results for (a) dimensionless stiffness function $p(a/h)$, and (b) the stiffness of the elastic fibre k_f . Dashed line is the stiffness estimated using uniform strain assumption, $p(a/h) = 1$. Squares with solid line, FEM; dashed line, uniform strain.

Figure 4. FEM results for (a) normalized pull-off stress and (b) $(B\chi^{-\alpha})^{-1} \sigma_s/\sigma_0$ as a function of a/h for varying χ . $(B\chi^{-\alpha})^{-1} \sigma_s/\sigma_0 = H(a/h)/H(0)$ for each χ before the inflection point, i.e. when the crack initiates at the edge of the fibre. Here $H(a/h)/H(0)$ is included (solid line) for reference. (a,b) Diamonds with solid line, $\chi = 30$; circles with solid line, $\chi = 12$; squares with solid line, $\chi = 6$; triangles with solid line, $\chi = 4$; inverted triangles with solid line, $\chi = 2$.

intrinsic adhesive strength (figure 4a). The trend observed in figure 4a is consistent with the theoretical expectations. Let us rearrange equation (2.11b) to obtain the normalized pull-off stress as

$$\frac{\sigma_s}{\sigma_o} = B\chi^{-\alpha} \frac{H(a/h)}{H(0)}. \quad (3.2)$$

Numerical simulations indicate that, as a/h increases, the value of $H(a/h)$ also increases, which according to equation (3.2) results in a higher pull-off stress. However, the maximum possible value of pull-off stress is the intrinsic adhesive strength, which implies that equation (3.2) has an upper bound of 1. Recall that equation (3.2) is valid only in the flaw-sensitive regime (that is, the cohesive zone is small in comparison with the fibre dimensions); therefore, despite the fact that equation (3.2) could theoretically be greater than 1 for sufficiently large a/h , the value of σ_s/σ_o cannot exceed 1, as required by the DB model. Simulation results plotted in figure 4a confirm this result.

Crack initiation behaviour is affected by the fact that σ_s/σ_o cannot exceed 1. In figure 4a, the inflection point marks the a/h value where the crack initiation switches from the edge of the fibre to the centre of the fibre. For a given χ that is larger than 1, the critical separation δ_c is reached at the edge of the fibre that initiates a crack from the edge. Beyond a critical a/h , the critical separation δ_c is reached *at the centre of the fibre first*, which leads to the crack initiation at the centre of the fibre.

The function $H(a/h)/H(0)$ can be calculated by rearranging equation (3.2) as

$$\frac{H(a/h)}{H(0)} = (B\chi^{-\alpha})^{-1} \frac{\sigma_s}{\sigma_o}, \quad (3.3)$$

where normalized pull-off stress σ_s/σ_o is calculated using FEM simulations, χ is calculated using equation (2.9), $B = 0.83$ and $\alpha = 0.4$. The equality in equation (3.3) holds with the given parameters only when the crack is initiated from the edge of the fibre. Results for equation (3.3) are included in figure 4b. As in figure 4a, inflection points in figure 4b also represent the transition of crack initiation from the edge to the centre of the fibre. Note that in the regime before the inflection points where the crack initiates from the edge, equation (3.3) holds and all the curves coincide and correspond to $H(a/h)/H(0)$. After the inflection point, crack initiation transitions to the centre of the fibre and, in this regime, equations (3.2) and (3.3) are no longer valid. Therefore, the regime after the inflection point in figure 4b is not equal to $H(a/h)/H(0)$ and thus all the curves in this regime follow different paths for increasing a/h values. The transition of the crack initiation from the edge to the centre of the fibre is shown in figure 5, which plots the normal separation of the tip of the fibre from the opposing rigid surface for $\chi = 12$ and varying a/h . For small a/h , critical separation is reached at the edge of the fibre ($r/a = 1$). As a/h increases, crack initiation transitions from the edge to the centre of the fibre ($r/a = 0$).

Figure 5. Normal displacement of the tip with respect to radial position at pull-off for different a/h values. Crack initiation transitions from the edge to the centre of the fibre tip with increasing a/h .

4. EXPERIMENTS

4.1. Material and methods

Experiments were performed with polyurethane elastomer (ST-1060; BJB Enterprises) cylinders, which were fabricated using a moulding technique. Fabrication details can be found in appendix B. Fabricated cylinders are 0.75 mm in radius and the length ranges from 0.155 to 3 mm. A glass slide (Microscope Slides; Fisher Scientific) was used as the adhering surface. Prior to testing, the glass slide was cleaned with acetone, ethyl alcohol and di-ionized water followed by nitrogen treatment for 2 min.

A custom adhesion measurement set-up was used to conduct the experiments (figure 6a). This system consists of an inverted optical microscope (Eclipse TE200; Nikon) with an automated high-precision stage (MFA-CC; Newport) that holds a high-resolution load cell (GSO-50; Transducer Techniques Inc.). In order to ensure proper alignment, an acrylic disc attached to a cylinder was first placed on the adhering surface (glass slide). Then, the acrylic disc–polyurethane cylinder assembly was glued to the load cell by lowering the load cell in contact with the acrylic disc while the polyurethane cylinder was in intimate contact with the adhering surface to ensure alignment between the adhering surface of the cylinder and the glass slide. Alignment was also checked using the optical microscope and necessary corrections were made with a two-axis rotational stage (GON 40-L and GON 40-U; Newport, UK). The automated stage, which was controlled by custom real-time software, lowered the cylinder into contact with the glass at a fixed velocity of $1 \mu\text{m s}^{-1}$ until a preload of 100 mN was reached. The cylinder was then retracted at a speed of $1 \mu\text{m s}^{-1}$ until it detached from the sample. The measurement system continuously captured the force data, which was used to produce the force–distance plot, as seen in figure 6b. Effective stiffness k_e was estimated from the slope of the retract curve away from the pull-off region. Pull-off force is the minimum force

Figure 6. (a) Illustration of the experimental set-up. (b) Sample force–distance curve obtained from experiments with a polyurethane fibre of 0.75 mm in radius and 1.68 mm in height. Grey line, approach; black line, retract. (Online version in colour.)

Figure 7. Experimental and simulation results for (a) effective stiffness k_e , and (b) pull-off force P_f . Pull-off force results include both cohesive zone and fixed length pre-existing circumferential crack simulations. (a) Squares, experiments; thin line, FEM. (b) Circles, experiments; thin line, FEM (cohesive zone); dashed line, FEM (pre-existing crack).

(maximum tensile force) on the force–distance plot. Measurements were repeated three times for each sample.

4.2. Experimental results

Experiments were performed to measure the pull-off force and the stiffness of the soft fibres. Figure 7a plots the experimental results for the stiffness of soft fibres along with simulations. The stiffness value in figure 7a is the effective spring constant k_e from the serial connection of the soft fibre and the measurement system. The effective spring constant for our system is

$$k_e = \frac{k_d k_f}{k_d + k_f}, \quad (4.1)$$

where k_d is the stiffness of the measuring device and is measured to be $k_d = 3.48 \text{ kN m}^{-1}$. The FEM data in figure 7a are obtained by calculating k_f from FEM simulations and using equation (4.1) to estimate the effective stiffness k_e .

The cohesive zone simulations presented in this study assume that the fibre is free of defects and in intimate contact with the smooth substrate. In reality, it is very challenging to fabricate fibres with perfectly vertical, defect-free edges. In order to determine the effect of a

potential defect at the edge of the fibre-adhering substrate interface, the defect is modelled as a fixed length circumferential crack. Simulation results for an edge crack are presented as a reference along with cohesive zone simulations and experimental results for pull-off force in figure 7b. Parameters used in the cohesive zone simulations are: $W_{ad} = 93 \text{ mJ m}^{-2}$ [6], $\sigma_o = 10 \text{ MPa}$ (fitting parameter), $E = 2 \text{ MPa}$ [31], $a = 0.75 \text{ mm}$ and h is measured using a calibrated optical microscope. While an intrinsic adhesive strength of 10 MPa might seem high, it is attainable considering that a typical van der Waals interaction could reach much higher adhesive stresses [32]. Some explanation of why van der Waals forces are much weaker than the theory predicts can be found in Hui *et al.* [33] and Tang *et al.* [34]. Using these values, χ is calculated from equation (2.9) to be 48 130. For fixed length circumferential crack simulations, the length of the crack measured from the edge of the fibre is $1 \text{ } \mu\text{m}$. Please refer to appendix A for simulation details.

5. DISCUSSIONS AND CONCLUSIONS

The measured stiffness and simulation results in figure 7a exhibit a close match. It is, however,

important to emphasize the limitations of the stiffness measurement performed in this work. In general, the stiffness of a structure is measured using a measuring device with a spring constant that is much larger than that of the structure whose spring constant is being measured. While, for small a/h values, stiffness can be measured accurately with the measuring device used for experiments in this work, it is prone to large errors for larger a/h values. The measured effective stiffness in figure 7a is very close to the stiffness of the measuring device for a/h values larger than 2, which implies that the fabricated fibres with $a/h > 2$ are much stiffer than the measuring device.

It was found that the results from the fixed length circumferential crack simulations were very close to those of cohesive zone simulations, neither of which demonstrated an exact fit to the experimental data, as shown in figure 7b. The discrepancy between the experimental data and the simulations suggests larger edge cracks with increasing a/h , as opposed to a fixed length edge crack for all a/h . It is possible that the size of the crack at the edge increases with thinner samples owing to variations in the fabrication technique, which could explain the plateau for the experimental data at higher a/h values. Nonetheless, both experimental and simulated pull-off stress increase with increasing a/h . It is important to note that the inflection point in figure 4a is not present in the FEM results plotted in figure 7b. The absence of an inflection point indicates that the crack initiates from the edge of the fibre for all a/h used in the experiments. $\chi = 48\,130$ is much larger than 1 for our experimental system. Even our relatively large a/h cannot force the system to be in the flaw-insensitive regime (i.e. $\sigma_s/\sigma_o \sim 1$). Thus, pull-off occurs by crack initiation and growth from the edge for all the experimental a/h values.

The interfacial strength σ_o used in the FEM simulations is larger than the Young modulus of the fibre material, which implies that, at pull-off, the region around the cohesive zone experiences nonlinear deformations. In addition, polyurethanes are viscoelastic materials. Although experiments were performed at $1\,\mu\text{m s}^{-1}$, rate-dependent behaviour could still be present. In our simulations, fibre material is assumed linearly elastic and incompressible.

In the light of the simulations and the experimental results, the effect of aspect ratio becomes evident particularly when the length of the fibre gets smaller than the radius of the fibre (i.e. $a/h > 1$). Existing literature suggests that, from a theoretical standpoint, an ideal design for fibrillar adhesives on the footpad of a climbing robot would be composed of long fibres with sufficiently small diameters to ensure flaw-insensitive adhesion [14,20]. However, for most elastomers, a flaw-insensitive regime could only be achieved with fibre diameters smaller than $1\,\mu\text{m}$, which might be very challenging to fabricate. Our study suggests that similar enhancement is possible with larger diameter, smaller aspect ratio fibres. The loss in compliance owing to lowering the aspect ratio could be compensated by adding a compliant element to the backing of the fibres, such as foam tapes, soft polymers [25], or by incorporating hierarchical fibre designs [35–38].

It is worth noting that this work is concerned with smooth surface adhesion and proposes enhancement in adhesive strength via lowering the aspect ratio of fibres. For climbing robot applications, the design of the footpads cannot solely be based on the adhesive strength. There are several other vital design aspects such as shear and peel strength of adhesive footpads as well as their ability to conform to roughness. An effective design should consider the effect of lowering the aspect ratio on all of the design parameters.

In summary, we investigated the effect of aspect ratio on the adhesion and stiffness of an elastic fibre in contact with a rigid smooth surface. Functions affecting pull-off stress and stiffness which only depend on the aspect ratio were determined using finite-element simulations. Results suggest that lower aspect ratios stiffen the fibre drastically and well beyond the stiffness calculated from beam equations. FEM analyses showed that pull-off stress increases with decreasing aspect ratio. In addition, it was found that, for fibres with sufficiently low aspect ratios, the crack initiates at the centre of the fibre and lowering the aspect ratio further results in flaw-insensitive adhesion as expected. Experiments performed with various aspect ratio polyurethane fibres showed an increase in adhesive strength and stiffness for lower aspect ratio fibres, which are consistent with numerical simulations.

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APPENDIX A. COMSOL IMPLEMENTATION DETAILS

A.1. Cohesive zone simulations

In the simulations, χ is varied by changing δ_c from 0.2 to 3 nm for fixed a , E , ν and σ_o . Aspect ratio is varied by fixing a and varying h . In COMSOL, Lagrange quadratic elements are used in two-dimensional axial symmetry mode to mesh elastic fibre with $a = 1\,\mu\text{m}$. Height h is varied from 0.1 to $10\,\mu\text{m}$ to adjust the aspect ratio. The elastic fibre's Young modulus is 2 MPa. For stiffness simulations, the fibre tip is assumed to have full friction contact with the rigid backing and the substrate (i.e. the fibre is perfectly bonded to the rigid backing and the substrate). While the rigid substrate is fixed, a displacement of $\Delta = h/20$ is applied to the rigid backing of the fibre for each h to calculate the stiffness of the fibre. Load p is calculated by integrating the normal stress distribution at the midsection of the fibre over the cross-sectional area. Effective stiffness k_e is determined using equation (4.1).

Pull-off simulations are performed by implementing the DB cohesive zone model at the tip of the fibre (i.e. interface between the fibre and the rigid substrate). Displacement is constrained to have no nodal displacement in the radial direction. The DB cohesive zone model is modified to incorporate into the FEM software. In the DB cohesive zone model, the interface is not allowed to separate until the stress reaches σ_o (i.e. infinite stiffness interfacial interaction). When the stress reaches σ_o , the interface is allowed to separate

Figure 8. (a) DB cohesive zone model, and (b) modified DB cohesive zone model for FEM simulations. (Online version in colour.)

and pull-off takes place when normal separation reaches the critical separation distance δ_c (figure 8a). In the modified DB model (figure 8b), the infinite stiffness interface is modified to a high stiffness interface where the interface is allowed to separate a small amount (δ_1) compared with δ_c until interfacial stress reaches σ_0 . The switch from σ_0 to 0 for δ larger than δ_c is approximated using the sigmoid function. The equation for the modified DB model is

$$P_f = \begin{cases} \sigma_0 \frac{\delta}{\delta_1}, & \delta < \delta_1, \\ \sigma_0 [1 + e^{50(\delta/\delta_c - 1)}]^{-1}, & \delta > \delta_1. \end{cases}$$

Such modifications were found necessary in order to ensure a convergent solution and avoid oscillations owing to the step functions involved in the DB model. $\delta_1 = 0.01$ nm for all simulations. The deviation of work of adhesion from the DB model owing to the modification is negligible provided that the sigmoid function has a steep decline and $\delta_1 \ll \delta_c$.

A.2. Circumferential crack simulations

The energy release rate for a circumferential crack of length c (measured from the edge of the fibre) at the interface between a long cylindrical fibre of radius a and a smooth rigid adhering surface is [39]

$$G = \frac{p^2 c}{2\pi E^* a^4}, \quad (\text{A } 1)$$

for $c \ll a$ where p is the applied load and $E^* = E/(1-\nu^2)$. Linearity and dimensional analysis imply that, for varying a/h , G becomes

$$G = \frac{p^2 c}{2\pi E^* a^4} f\left(\frac{a}{h}\right), \quad (\text{A } 2)$$

where $f(a/h)$ depends only on a/h and the boundary conditions at the interface. Pull-off occurs when the energy release rate G is equal to the critical energy release rate for fracture G_c . Rearranging equation (A 2), pull-off load can be calculated from

$$P_f = \sqrt{\frac{2\pi E^* a^4}{f(a/h)} \left(\frac{G_c}{c}\right)}. \quad (\text{A } 3)$$

Figure 9. (a) A steel rod is press fitted into a hole drilled on a flat piece of acrylic and spacers are placed on the edge of the acrylic piece. Liquid silicone rubber is poured around the steel rod. (b) Another acrylic piece is inserted from one end of the steel rod and pressed against the spacers. (c) After curing for 24 h at room temperature, the silicone rubber is removed from the steel rod and placed on a smooth substrate. The hole is filled with polyurethane and an acrylic disc is placed on top. (d) Upon curing the polyurethane for 24 h at room temperature, it is mechanically removed from the mould and the substrate to obtain a polyurethane fibre attached to an acrylic disc. (Online version in colour.)

The FEM simulations were performed for a fixed circumferential crack length of $c = 1 \mu\text{m}$ and $a = 750 \mu\text{m}$. The Young modulus $E = 2$ MPa and fibre material is assumed incompressible. The interface from the centre of the fibre to the tip of the crack is assumed to be perfectly bonded to the substrate. Displacement Δ is applied incrementally for $a/h \ll 1$ until a desired load is reached. At this load, which is taken to be the pull-off load at $a/h \ll 1$, G_c is found using a J -integral routine around the crack tip [40]. Then, pull-off load for the rest of the a/h values was found by increasing Δ until $G = G_c$. Equation (A 3) implies that, for constant G_c/c , pull-off load depends only on $f(a/h)$. Therefore, these simulations effectively give us $f(a/h)$, which can be multiplied by an appropriate constant to fit the experimental pull-off data.

APPENDIX B. FIBRE FABRICATION

The fabrication method is designed to form a replica of a smooth cylindrical steel rod from polyurethane elastomer. The steel rod is first press fitted into an acrylic plate with spacers placed on two sides. Liquid silicone rubber (HS II, Dow Corning) is poured around the steel rod onto an acrylic plate on the bottom (figure 9a). A second acrylic plate with a circular through hole, the same size as the diameter of the steel rod, is placed through the rod and pressed onto the spacers (figure 9b). The silicone rubber is allowed to cure for 24 h at room temperature and removed from the rod to form the mould for fibre fabrication. Then, this mould is placed onto a smooth flat silicone rubber layer. This way, the mould adheres to the substrate leaving no opening in between. Polyurethane (ST-1060; BJB enterprises) is poured into the mould and an acrylic cap is placed on top of it (figure 9c).

After curing at room temperature for 24 h, the polyurethane is peeled off from the mould to obtain a soft cylinder with a rigid acrylic backing (figure 9d). Scotch tape is used as a spacer to fabricate thin moulds for very low aspect ratio fibres. Fabricated fibres are 0.75 mm in diameter with lengths ranging from 0.155 to 3 mm.

REFERENCES

- 1 Autumn, K. et al. 2002 Evidence for van der Waals adhesion in gecko setae. *Proc. Natl Acad. Sci. USA* **99**, 12 252–12 256. (doi:10.1073/pnas.192252799)
- 2 Irschick, D. J., Austin, C. C., Petren, K., Fisher, R. N., Losos, J. B. & Ellers, O. 1996 A comparative analysis of clinging ability among pad-bearing lizards. *Biol. J. Linn. Soc.* **59**, 21–35. (doi:10.1111/j.1095-8312.1996.tb01451.x)
- 3 Sitti, M. & Fearing, R. S. 2003 Synthetic gecko foot-hair micro/nano-structures as dry adhesives. *J. Adhes. Sci. Technol.* **17**, 1055–1074. (doi:10.1163/156856103322113788)
- 4 Sun, W., Neuzil, P., Kustandi, T. S., Oh, S. & Samper, V. D. 2005 The nature of the gecko lizard adhesive force. *Biophys. J.* **89**, L14–L17. (doi:10.1529/biophysj.105.065268)
- 5 Glassmaker, N. J., Hui, C.-Y. & Kim, J. 2004 Design of biomimetic fibrillar interfaces. 1. Making contact. *J. R. Soc. Interface* **1**, 23–33. (doi:10.1098/rsif.2004.0004)
- 6 Aksak, B., Murphy, M. & Sitti, M. 2007 Adhesion of biologically inspired vertical and angled polymer micro-fiber arrays. *Langmuir* **23**, 3322–3332. (doi:10.1021/la062697t)
- 7 Jagota, A. & Bennison, S. J. 2002 Mechanics of adhesion through a fibrillar microstructure. *Integr. Comp. Biol.* **42**, 1140–1145. (doi:10.1093/icb/42.6.1140)
- 8 del Campo, A., Greiner, C. & Arzt, E. 2007 Contact shape controls adhesion of bioinspired fibrillar surfaces. *Langmuir* **23**, 10 235–10 243. (doi:10.1021/la7010502)
- 9 Glassmaker, N. J., Jagota, A., Hui, C.-Y., Noderer, W. L. & Chaudhury, M. K. 2007 Biologically inspired crack trapping for enhanced adhesion. *Proc. Natl Acad. Sci. USA* **104**, 10 786–10 791. (doi:10.1073/pnas.0703762104)
- 10 Gorb, S., Varenberg, M., Peressadko, A. & Tuma, J. 2006 Biomimetic mushroom-shaped fibrillar adhesive micro-structure. *J. R. Soc. Interface* **4**, 271–275. (doi:10.1098/rsif.2006.0164)
- 11 Kim, S. & Sitti, M. 2006 Biologically inspired polymer microfibers with spatulate tips as repeatable fibrillar adhesives. *Appl. Phys. Lett.* **89**, 261 911. (doi:10.1063/1.2424442)
- 12 Murphy, M. P., Aksak, B. & Sitti, M. 2007 Adhesion and anisotropic friction enhancements of angled heterogeneous micro-fiber arrays with spherical and spatula tips. *J. Adhes. Sci. Technol.* **21**, 1281–1296. (doi:10.1163/156856107782328380)
- 13 Yao, H., Della Rocca, G., Guduru, P. R. & Gao, H. 2008 Adhesion and sliding response of a biologically inspired fibrillar surface: experimental observations. *J. R. Soc. Interface* **5**, 723–733. (doi:10.1098/rsif.2007.1225)
- 14 Gao, H. & Yao, H. 2004 Shape insensitive optimal adhesion of nanoscale fibrillar structures. *Proc. Natl Acad. Sci. USA* **101**, 7851–7856. (doi:10.1073/pnas.0400757101)
- 15 Greiner, C., Spolenak, R. & Arzt, E. 2008 Adhesion design maps for fibrillar adhesives: the effect of shape. *Acta Biomater.* **5**, 597–606. (doi:10.1016/j.actbio.2008.09.006)
- 16 Spolenak, R., Gorb, S., Gao, H. & Arzt, E. 2005 Effects of contact shape on the scaling of biological attachments. *Proc. R. Soc. A* **461**, 305–319. (doi:10.1098/rspa.2004.1326)
- 17 Soto, D., Hill, G., Parness, A., Esparza, N., Cutkosky, M. & Kenny, T. 2010 Effect of fibril shape on adhesive properties. *Appl. Phys. Lett.* **97**, 053701. (doi:10.1063/1.3464553)
- 18 Barenblatt, G. I. 1962 Mathematical theory of equilibrium cracks in brittle fracture. *Adv. Appl. Mech.* **7**, 55–129.
- 19 Dugdale, D. S. 1960 Yielding of steel sheets containing slits. *J. Mech. Phys. Solids* **8**, 100–104. (doi:10.1016/0022-5096(60)90013-2)
- 20 Tang, T., Hui, C.-Y. & Glassmaker, N. J. 2005 Can a fibrillar interface be stronger and tougher than a non-fibrillar one? *J. R. Soc. Interface* **2**, 505–516. (doi:10.1098/rsif.2005.0070)
- 21 Arzt, E., Gorb, S. & Spolenak, R. 2003 From micro to nano contacts in biological attachment devices. *Proc. Natl Acad. Sci. USA* **100**, 10 603–10 606. (doi:10.1073/pnas.1534701100)
- 22 Varenberg, M. & Gorb, S. 2009 Hexagonal surface micro-pattern for dry and wet friction. *Adv. Mater.* **21**, 483–486. (doi:10.1002/adma.200802734)
- 23 Aksak, B., Murphy, M. P. & Sitti, M. 2008 Gecko inspired micro-fibrillar adhesives for wall climbing robots on micro/nanoscale rough surfaces. In *Proc. IEEE Int. Conf. Robotics and Automation, IEEE, Pasadena, CA, 19th May 2008*, pp. 3058–3063. Piscataway, NJ: IEEE. (doi:10.1109/ROBOT.2008.4543675)
- 24 Daltorio, K. A., Gorb, S., Peressadko, A., Horschler, A. D., Ritzmann, R. E. & Quinn, R. D. 2005 A robot that climbs walls using micro-structured polymer feet. In *Proc. of the 8th Int. Conf. on Climbing and Walking Robots and the Support Technologies for Mobile Machines (CLAWAR 2005)*, London, UK, 12–15 September 2005, pp. 131–138. Berlin, Germany: Springer.
- 25 Murphy, M. P. & Sitti, M. 2007 Waalbot: an agile small-scale wall-climbing robot utilizing dry elastomer adhesives. *IEEE/ASME Trans. Mechatron* **12**, 330–338. (doi:10.1109/TMECH.2007.897277)
- 26 Santos, D., Sangbae, K., Spenko, M., Parness, A. & Cutkosky, M. 2007 Directional adhesive structures for controlled climbing on smooth vertical surfaces. In *Proc. IEEE Int. Conf. Robotics and Automation, Rome, Italy, 10–14 April 2007*, pp. 1262–1267. Piscataway, NJ: IEEE. (doi:10.1109/ROBOT.2007.363158)
- 27 Unver, O. & Sitti, M. 2010 Flat dry elastomer adhesives as attachment materials for climbing robots. *IEEE Trans. Robot.* **26**, 131–141. (doi:10.1109/TRO.2009.2033628)
- 28 Lindsey, G. H. 1967 Triaxial fracture studies. *J. Appl. Phys.* **38**, 4843–4852. (doi:10.1063/1.1709232)
- 29 Lakrout, H., Sergot, P. & Creton, C. 1999 Direct observation of cavitation and fibrillation in a probetack experiment on model acrylic pressure-sensitive adhesives. *J. Adhes.* **69**, 307–359. (doi:10.1080/00218469908017233)
- 30 Boggy, D. B. 1971 Two edge-bonded elastic wedges of different materials and wedge angles under surface tractions. *Appl. Mech.* **38**, 377–386. (doi:10.1115/1.3408786)
- 31 BJB Enterprises, Inc. 2009 Shore A Polyurethanes. See <http://www.bjbenterprises.com/pdf/ST-1060.pdf>.
- 32 Israelachvili, J. N. 1992 *Intermolecular and surface forces*, 2nd edn. London, UK: Academic Press.
- 33 Hui, C.-Y., Jagota, A., Bennison, S. J. & Londono, J. D. 2003 Crack blunting and the strength of soft elastic solids. *Proc. R. Soc. Lond. A* **459**, 1489–1516. (doi:10.1098/rspa.2002.1057)
- 34 Tang, T., Jagota, A., Hui, C.-Y. & Chaudhury, M. K. 2006 Thermal fluctuations limit the cohesive strength of compliant solids. *J. Adhes.* **82**, 671–696. (doi:10.1080/00218460600775781)

- 35 Asbeck, A., Dastoor, S., Parness, A., Fullerton, L., Esparza, N., Soto, D., Heyneman, B. & Cutkosky, M. 2009 Climbing rough vertical surfaces with hierarchical directional adhesion. In *Proc. IEEE Int. Conf. Robotics and Automation, Kobe, Japan, 12–17 May 2009*, pp. 2675–2680. Piscataway, NJ: IEEE. (doi:10.1109/ROBOT.2009.5152864)
- 36 Jeong, H. E., Lee, J.-K., Kim, H. N., Moon, S. H. & Suh, K. Y. 2009 A nontransferring dry adhesive with hierarchical polymer nanohairs. *Proc. Natl Acad. Sci. USA* **106**, 5639–5644. (doi:10.1073/pnas.0900323106)
- 37 Lee, J., Bush, B., Maboudian, R. & Fearing, R. S. 2009 Gecko-inspired combined lamellar and nanofibrillar array for adhesion on nonplanar surface. *Langmuir* **25**, 12 449–12 453. (doi:10.1021/la9029672)
- 38 Murphy, M. P., Kim, S. & Sitti, M. 2009 Enhanced adhesion by gecko-inspired hierarchical fibrillar adhesives. *ACS Appl. Mater. Interfaces* **1**, 849–855. (doi:10.1021/am8002439)
- 39 Tada, H., Paris, P. C. & Irwin, G. R. 2000 *The stress analysis of cracks handbook*, 3rd edn. New York, NY: The American Society of Mechanical Engineers.
- 40 Rice, J. R. 1968 A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Appl. Mech.* **35**, 379–386.