Fabric dependence of quasi-waves in anisotropic porous media

Luis Cardoso^{a)} and Stephen C. Cowin

The Department of Biomedical Engineering, The City University of New York, New York, New York 10031

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Assessment of bone loss and osteoporosis by ultrasound systems is based on the speed of sound and broadband ultrasound attenuation of a single wave. However, the existence of a second wave in cancellous bone has been reported and its existence is an unequivocal signature of poroelastic media. To account for the fact that ultrasound is sensitive to microarchitecture as well as bone mineral density (BMD), a fabric-dependent anisotropic poroelastic wave propagation theory was recently developed for pure wave modes propagating along a plane of symmetry in an anisotropic medium. Key to this development was the inclusion of the fabric tensor—a quantitative stereological measure of the degree of structural anisotropy of bone—into the linear poroelasticity theory. In the present study, this framework is extended to the propagation of mixed wave modes along an arbitrary direction in anisotropic porous media called quasi-waves. It was found that differences between phase and group velocities are due to the anisotropy of the bone microarchitecture, and that the experimental wave velocities are more accurately predicted by the poroelastic model when the fabric tensor variable is taken into account. This poroelastic wave propagation theory represents an alternative for bone quality assessment beyond BMD. © 2011 Acoustical Society of America. [DOI: 10.1121/1.3557032]

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I. INTRODUCTION

Ultrasound waves have a broad range of clinical applications as a non-destructive testing approach to image and diagnose medical conditions. In particular, quantitative ultrasound has been considered as an attractive alternative to the use of Dual Energy X-Ray Absorptiometry (DEXA) to diagnose osteoporosis (Siffert and Kaufman, 2006; Hans *et al.*, 1996; Grimm and Williams, 1997a) because it is non-ionizing, inexpensive, and non-invasive. Ultrasound waves are elastic vibrations that can provide direct information on the mechanical properties of the medium in which they propagate. Unfortunately, ultrasound wave propagation phenomena in anisotropic poroelastic medium are highly complex. This complexity has limited the development and application of ultrasound approaches to assess bone status and osteoporosis when compared to the development and application of DEXA systems. A major limitation associated with current clinical ultrasound systems (Grigorian *et al.*, 2002)—often called ultrasound densitometers—consist of determining bone mass density as DEXA does, without taking advantage of the fact that ultrasound is sensitive to microarchitecture and tissue composition (Sakata *et al.*, 2004; Xia *et al.*, 2007; Mizuno *et al.*, 2008; Sasso *et al.*, 2008; Lin *et al.*, 2009).

In most clinical ultrasound densitometer systems, only one wave arriving to the ultrasound probe is identified. If only one wave is measured, the analysis is limited to an “equivalent medium approach” in which the solid trabecular structure cannot be distinguished from the fluid within the pores. However, the existence of a second wave in cancellous bone has been reported *in vitro* (Hosokawa and Otani, 1997, 1998; Cardoso *et al.*, 2001, 2003; Mizuno *et al.*, 2009). These

two waves propagate with different velocities (Wear *et al.*, 2005; Anderson *et al.*, 2008, 2009; Wear, 2009; Nguyen *et al.*, 2010; Wear, 2010) and have been shown to correspond to the fast and slow waves predicted by Biot’s (Biot, 1941, 1955, 1956a,b, 1962a,b) poroelastic wave propagation theory. Therefore, a poroelastic wave propagation theory is conceptually more appropriate than an equivalent media approach to characterize the properties of the porous medium.

Isotropic poroelasticity theory has been used for many years to analyze wave propagation in cancellous bone (Williams, 1992; Hosokawa and Otani, 1997, 1998; Haire and Langton, 1999; Kaczmarek *et al.*, 2002; Fellah *et al.*, 2004; Wear *et al.*, 2005; Pakula *et al.*, 2008; Fellah *et al.*, 2008; Sebaa *et al.*, 2008; Cardoso *et al.*, 2008), and just recently the role of microarchitecture has been included in poroelasticity theory through the fabric tensor (Cowin and Cardoso, 2011). Most clinical ultrasound densitometers depend on empirical relationships between the speed of sound (SOS), broadband ultrasound attenuation (BUA), and bone density in order to assess bone loss. In contrast, the fabric-dependent anisotropic poroelastic approach proposed by Cowin and Cardoso (2011) has the advantage of providing a theoretical framework to describe the relationship between measurable wave properties (i.e., wave velocity and attenuation) and the elastic constants of the trabecular bone structure. Key to the development of such a theory was the incorporation of the fabric tensor into the governing equations for wave motion in the linear theory of anisotropic poroelastic materials (Cowin, 1985, 1986, 1999, 2004; Cowin and Mehrabadi, 2007). Fabric is a quantitative stereological measure of the degree of structural anisotropy in the pore architecture of a porous medium (Hilliard, 1967; Whitehouse, 1974; Whitehouse and Dyson, 1974; Cowin and Satake, 1978; Satake, 1982; Kanatani, 1983, 1984a,b, 1985; Harrigan and Mann, 1984; Odgaard, 1997, 2001; Odgaard *et al.*, 1997; Matsuura

^{a)}Author to whom correspondence should be addressed. Electronic mail: cardoso@engr.cuny.cuny.edu

et al., 2008). This new approach resulted in a poroelastic Christoffel equation for anisotropic poroelastic media represented by an eigenvalue problem with a characteristic polynomial equation of order 6. Four of those six roots are nonzero, and correspond to the four wave modes of propagation in porous media, two of which are longitudinal and two are shear wave modes. Analytical expressions were given in Cowin and Cardoso (2011) for the velocity and attenuation of each wave mode for the case in which the direction of wave propagation coincides with the normal to a plane of symmetry of the anisotropic medium. Since this poroelastic wave propagation theory depends on anisotropy of the structure and tissue composition in addition to bone mass density, it represents an alternative for bone quality assessment beyond bone mineral density (BMD).

In the present study, the theoretical framework of Cowin and Cardoso (2011) is extended to the propagation of waves along an arbitrary direction in orthotropic porous media. The plane wave equation in an anisotropic fluid-saturated poroelastic medium developed in Cowin and Cardoso (2011) is reviewed in Sec. II. The fabric dependence of tensors appearing in the poroelastic model of wave propagation is summarized in Sec. III. The propagation of plane waves in an anisotropic, fabric-dependent, saturated porous medium along an arbitrary direction is derived in Sec. IV, and the practical application of these results to the wave propagation in trabecular bone is presented in Sec. V. The final section, Sec. VI, contains our discussion and concluding remarks.

II. PLANE WAVE EQUATION IN AN ANISOTROPIC FLUID-SATURATED POROELASTIC MEDIUM

The field equations of motion in an anisotropic porous medium (Biot, 1941) were obtained from the conservation of linear momentum and the conservation of mass by substituting constitutive equations for stress and fluid flux (Cowin and Cardoso, 2011). The field equations describe the solid displacement field $\mathbf{u}$ and the displacement field $\mathbf{w}$ of the fluid relative to the solid,

$$Z_{ijk} \frac{\partial^2 u_k}{\partial x_m \partial x_j} + M_{ij} \frac{\partial^2 w_k}{\partial x_k \partial x_j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i, \quad (1)$$

$$M_{km} \frac{\partial^2 u_k}{\partial x_m \partial x_i} + M \frac{\partial^2 w_k}{\partial x_k \partial x_i} = \rho_f (\ddot{u}_i + J_{ij} \ddot{w}_j) + \mu R_{ij} \dot{w}_j, \quad (2)$$

where ρ is the bulk density of the solid matrix, ρ_f is the density of the pore fluid, μ is the viscosity of the pore fluid, and M is the constant of proportionality between the fluid pore pressure, p , and the variation in fluid content, ζ . The variation in fluid content, ζ , a traditional Biot variable, is defined as the divergence of the displacement vector $\mathbf{w}$ of the fluid relative to the solid, $\zeta = -\nabla \cdot \mathbf{w}$. Also, the Biot effective stress coefficient tensor $\mathbf{A}$ represents the proportionality factor between the stress tensor $\mathbf{T}$ and the pore fluid pressure p , $T_{ij} = C_{ijk}^d E_{km} - A_{ij} p$, where T_{ij} are the components of the stress tensor and C_{ijk}^d represents the components of the

drained elasticity tensor. The four constitutive tensors, Z_{ijk} , M_{ij} , J_{ij} , and R_{ij} , appearing in Eqs. (1) and (2) are identified as follows: Z_{ijk} is Biot's elasticity tensor— Z_{ijk} differs from the drained elasticity tensor C_{ijk}^d by the term $MA_{ij}A_{km}$, which is the open product of the Biot effective stress coefficient tensor $\mathbf{A}$ with itself; M_{ij} is directly related to the Biot effective stress coefficient tensor $\mathbf{A}$ and the scalar M by $M_{ij} = MA_{ij}$; the constant M is related to the effective drained elastic stiffness tensor C_{ijk}^d , the drained compliance tensor S_{ijk}^d , and the Biot's effective stress tensor A_{ij} by

$$M = (C_{eff}^d - A_{ij} S_{ijk}^d A_{km})^{-1}. \quad (3)$$

J_{ij} is the micro-macro velocity average tensor—it functions as a density distribution function that relates the relative micro-solid-fluid velocity to its bulk volume average $\dot{\mathbf{w}}$, and R_{ij} is the flow-resistivity tensor, the reciprocal of the permeability tensor K_{ij} . Throughout this study, Einstein's convention of summing over repeated indices was adopted.

The propagation of plane waves in an anisotropic fluid-saturated porous medium is represented kinematically by a direction of propagation, denoted by $\mathbf{n} = (n_1, n_2, n_3)^T$ a unit normal to the wave front, and $\mathbf{a}$ or $\mathbf{b}$, which are the directions of displacement for the wave fronts associated with $\mathbf{u}$ and $\mathbf{w}$, respectively. These two plane waves are represented by

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{a} e^{i\omega(\mathbf{n} \cdot \mathbf{x}/v - t)} \quad \text{and} \quad \mathbf{w}(\mathbf{x}, t) = \mathbf{b} e^{i\omega(\mathbf{n} \cdot \mathbf{x}/v - t)}, \quad (4)$$

where v is the wave phase velocity in the direction $\mathbf{n}$, $\mathbf{x}$ is the position vector, ω is the frequency, and t is the time. The relationship between the complex phase velocity v and frequency ω of attenuating waves is represented by

$$v = v_{\text{Re}} + i v_{\text{Im}} = \frac{\omega}{k_{\text{Re}} + i \alpha_{\text{Im}}}. \quad (5)$$

The imaginary part α is related to the wave attenuation as a function of traveled distance ($e^{-\alpha \mathbf{n} \cdot \mathbf{x}}$) and the real part k describes the wave number associated to the wave propagating in the direction $\mathbf{n}$. A transverse wave is characterized by $\mathbf{a} \cdot \mathbf{n} = 0$, a longitudinal wave by $\mathbf{a} \cdot \mathbf{n} = 1$. Substituting the relations (4) for the plane waves into the field equations (1) and (2) leads to the *poroelastic Christoffel equation* (6)

$$\begin{bmatrix} \mathbf{Q} - \rho v^2 \mathbf{1} & \mathbf{C} - \rho_f v^2 \mathbf{1} \\ \mathbf{C}^T - \rho_f v^2 \mathbf{1} & M \mathbf{n} \otimes \mathbf{n} - \rho_f v^2 \left(\mathbf{J} + \frac{i\mu}{\rho_f \omega} \mathbf{R} \right) \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = 0, \quad (6)$$

where the notation $Q_{ik} = Z_{ijk} n_m n_j$ and $C_{ik} = M_{ij} n_j n_k$ has been introduced.

Equation (6) represents an eigenvalue problem, the squares of the wave speeds v^2 representing the eigenvalues, and the vectors $\mathbf{a}$ and $\mathbf{b}$ representing the eigenvectors. Since the right hand side of this linear system of equations is a zero 6D vector, it follows from Cramer's rule that, in order to avoid the trivial solution, it is necessary to set the determinant of the 6 by 6 matrix equal to zero, thus

$$\begin{vmatrix} \mathbf{Q} - \rho v^2 \mathbf{1} & \mathbf{C} - \rho_f v^2 \mathbf{1} \\ \mathbf{C}^T - \rho_f v^2 \mathbf{1} & \mathbf{M} \mathbf{n} \otimes \mathbf{n} - \rho_f v^2 \mathbf{S} \end{vmatrix} = 0, \quad (7)$$

where the notation $S_{ik} = J_{ik} + \frac{i\mu}{\rho_f \omega} R_{ik}$ has been introduced.

III. FABRIC DEPENDENCE OF TENSORS APPEARING IN THE POROELASTIC MODEL OF WAVE PROPAGATION

The second rank fabric tensor $\mathbf{F}$ is a non-dimensional quantitative stereological measure of the degree of structural anisotropy in the pore architecture of a porous medium. The experimental procedure for the surface area orientation measurement of cancellous bone is described by Whitehouse (1974), Whitehouse and Dyson (1974), Harrigan and Mann (1984), and Turner *et al.* (1987, 1990). The work of these authors, and Odgaard (1997, 2001), Odgaard *et al.* (1997), van Rietbergen *et al.* (1996, 1998), Matsuura *et al.* (2008) and others, has shown that the fabric tensor is a good measure of the structural anisotropy in cancellous bone tissue (Cowin, 1997).

The second rank fabric tensor $\mathbf{F}$ is symmetric, therefore its invariants I_F , II_F , and III_F are related to the traces of $\mathbf{F}$, $\mathbf{F}^2$, and $\mathbf{F}^3$ by the formulas recorded, for example, in Erickson (1960),

$$\begin{aligned} I_F &= \text{tr} \mathbf{F}, II_F = \frac{1}{2}[(\text{tr} \mathbf{F})^2 - \text{tr} \mathbf{F}^2], \\ III_F &= \frac{1}{6}(\text{tr} \mathbf{F}^3 - 3\text{tr} \mathbf{F}^2 + 2\text{tr} \mathbf{F}). \end{aligned} \quad (8)$$

The fact that a matrix satisfies its own characteristic equation, the Cayley–Hamilton theorem, is then written in the form

$$\mathbf{F}^3 - I_F \mathbf{F}^2 + II_F \mathbf{F} - III_F \mathbf{1} = 0. \quad (9)$$

The significance of this result is that any power of $\mathbf{F}$ of the order three or higher may be eliminated by repetitive use of this result. From the first and second equations of Eq. (8) one can see that $\text{tr} \mathbf{F}^2 = I_F^2 - 2II_F$. Using the Cayley–Hamilton theorem it is easy to show that

$$\begin{aligned} \text{tr} \mathbf{F}^3 &= I_F^3 - 3I_F \cdot II_F + 3III_F \quad \text{and} \\ \text{tr} \mathbf{F}^4 &= I_F^4 - 3I_F^2 \cdot II_F + 2II_F^2 + 4I_F \cdot III_F. \end{aligned} \quad (10)$$

These results will be used below. Finally, we normalize the fabric tensor by setting $I = \text{tr} \mathbf{F} = \mathbf{1}$. Thus in the applications of the formula $\text{tr} \mathbf{F}^2 = I_F^2 - 2II_F$ and Eq. (10), I is replaced by $\mathbf{1}$. The Cayley–Hamilton theorem is here used not only on the fabric tensor but also on other second rank tensors such as $\mathbf{R}$, $\mathbf{K}$, $\mathbf{M}$, and $\mathbf{J}$. Formulas relating the acoustic tensor $\mathbf{Z}$, the flow-resistivity tensor $\mathbf{R}$, and the tensor $\mathbf{M}$, representing the interaction of the velocity fields $\mathbf{u}$ and $\mathbf{w}$, to the fabric tensor $\mathbf{F}$ were obtained in Cowin and Cardoso (2011).

Briefly, the dependence of the elastic acoustic tensor $\mathbf{Z}$ upon the fabric tensor $\mathbf{F}$ is described by the following relationships:

$$\begin{aligned} Z_{ijkl} &= \left[a_1^{cd} + M \left(\frac{3K^m - a_o^{cd}}{3K^m} \right)^2 \right] \delta_{ij} \delta_{kl} \\ &+ \left[a_2^{cd} - \frac{M(3K^m - a_o^{cd})a_l^{cd}}{(3K^m)^2} \right] (F_{ij} \delta_{kl} + \delta_{ij} F_{kl}) \\ &+ \left[a_3^{cd} - M \frac{(3K^m - a_o^{cd})a_{ll}^{cd}}{(3K^m)^2} \right] (\delta_{ij} F_{kl} F_{qm} + \delta_{kl} F_{ij} F_{qm}) \\ &+ \left[b_1^{cd} + M \left(\frac{a_l^{cd}}{3K^m} \right)^2 \right] F_{ij} F_{kl} \\ &+ \left[b_2^{cd} + M \frac{a_l^{cd} a_{ll}^{cd}}{(3K^m)^2} \right] (F_{ij} F_{kl} F_{qm} + F_{kl} F_{ij} F_{qm}) \\ &+ \left[b_3^{cd} + M \left(\frac{a_{ll}^{cd}}{3K^m} \right)^2 \right] F_{is} F_{sj} F_{kl} F_{qm} \\ &+ c_1^{cd} (\delta_{ki} \delta_{mj} + \delta_{mi} \delta_{kj}) \\ &+ c_2^{cd} (F_{ki} \delta_{mj} + F_{kj} \delta_{mi} + F_{im} \delta_{kj} + F_{mj} \delta_{ki}) \\ &+ c_3^{cd} (F_{ir} F_{rk} \delta_{mj} + F_{kr} F_{rj} \delta_{mi} + F_{ir} F_{rm} \delta_{kj} + F_{mr} F_{rj} \delta_{ik}). \end{aligned} \quad (11)$$

The tensor $\mathbf{M}$ represents the elastic coupling between the solid and the fluid phases,

$$M_{ij} = M \delta_{ij} - \frac{M}{3K^m} (a_o^{cd} \delta_{ij} + a_l^{cd} F_{ij} + a_{ll}^{cd} F_{ij} F_{ij}). \quad (12)$$

The micro–macro velocity average tensor $\mathbf{J}$ is related to the fabric by

$$J_{ij} = j_1 \delta_{ij} + j_2 F_{ij} + j_3 F_{ij} F_{ij}. \quad (13)$$

Similarly, the flow-resistivity tensor $\mathbf{R}$ is related to the fabric by

$$R_{ij} = r_1 \delta_{ij} + r_2 F_{ij} + r_3 F_{ij} F_{ij}, \quad (14)$$

where the quantities a_1^{cd} , a_2^{cd} , a_3^{cd} , b_1^{cd} , b_2^{cd} , b_3^{cd} , c_1^{cd} , c_2^{cd} , c_3^{cd} , a_o^{cd} , a_l^{cd} , a_{ll}^{cd} , j_1 , j_2 , j_3 , r_1 , r_2 , and r_3 are scalar-valued functions of φ , II_F , and III_F (Cowin, 1985; Cowin and Cardoso, 2011). $\mathbf{R}$ is the inverse of the second rank intrinsic permeability tensor $\mathbf{K}$ and it represents dissipation phenomena due to viscous losses at low frequencies of fluid motion. The conception of $\mathbf{K}$ was extended to take into account the change in fluid flow regime occurring between low and high frequencies of wave propagation (Johnson *et al.*, 1987)

$$K_{ij}(\omega) = \kappa_0 \left[1 - 2 \frac{J_1(d\chi)}{d\chi J_0(d\chi)} \right] (K_1 \delta_{ij} + K_2 F_{ij} + K_3 F_{ij} F_{ij}), \quad (15)$$

where the dynamic permeability tensor $\mathbf{K}$ is a function of the average intrinsic permeability κ_0 , the fabric tensor, and Bessel functions that characterize the dynamics of the oscillatory fluid flow inside a cylindrical channel. In this equation, J_1 and J_0 are, respectively, the first order and zeroth order Bessel functions of the first kind, and d corresponds to the average characteristic pore dimension. The inverse of the viscous skin depth

χ is defined as a function of the angular frequency ω , the fluid mass density ρ_f , and the dynamic viscosity of the fluid μ ,

$$\chi = \left(\frac{i\omega\rho_f}{\mu} \right)^{1/2}. \quad (16)$$

IV. PROPAGATION OF WAVES ALONG AN ARBITRARY DIRECTION IN ORTHOTROPIC POROUS MEDIA

A. Phase velocity and phase direction

In the principal coordinate system of the fabric tensor, the poroelastic Christoffel equation (7) for wave propagation in an arbitrary direction $\mathbf{n} = (n_1, n_2, n_3)^T$ can be written as

$$\left| \begin{bmatrix} \mathbf{Q} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{T} \end{bmatrix} - v^2 \begin{bmatrix} \rho \cdot 1 & \rho_f \cdot 1 \\ \rho_f \cdot 1 & \rho_f \cdot \mathbf{S} \end{bmatrix} \right| = 0, \quad (17)$$

where the detailed algebraic structure of the tensors $\mathbf{Q}$, $\mathbf{C}$, and $\mathbf{S}$ as functions of the general direction of wave propagation $\mathbf{n}$ are recorded in the Appendix. The characteristic equation of system (17) is represented by a sixth order polynomial in $z = v^2$, given by

$$c_6 z^6 + c_5 z^5 + c_4 z^4 + c_3 z^3 + c_2 z^2 + c_1 z + c_0 = 0, \quad (18)$$

where the $c_0(\mathbf{n})$ to $c_6(\mathbf{n})$ coefficients are:

$$\begin{aligned} c_0(\mathbf{n}) = & III_Q III_T + 2Q_4 Q_6 C_5 C_8 T_4 T_6 + 2Q_5 C_4 C_6 C_7 C_9 T_5 + III_C^2 \\ & + \dots - C_2^2 C_3^2 C_8^2 - C_4^2 C_7^2 T_1 Q_1 - Q_6^2 C_3^2 T_6^2 + \dots \\ & - Q_5^2 T_5^2 T_2 Q_2 - Q_4^2 C_1^2 T_4^2 - C_6^2 C_9^2 Q_3 T_3, \end{aligned}$$

$$\begin{aligned} c_1(\mathbf{n}) = & -(\rho III_Q III_T + \rho_f III_Q TTS) \\ & - 2\rho_f Q_4 Q_6 C_5 C_8 (T_4 S_6 + T_6 S_4) \\ & + \dots - 2\rho_f Q_5 C_4 C_6 C_7 C_9 S_5 - 2\rho_f (III_C III_C + III_C^2) + \dots \\ & + 2\rho_f C_2 C_3^2 C_8^2 + C_4^2 C_7^2 (\rho T_1 + \rho_f S_1 Q_1) \\ & + 2\rho_f Q_6^2 C_3 T_6 P + \dots \\ & + Q_5^2 T_5 (2\rho_f T_2 Q_2 S_5 + \rho T_5 T_2 + \rho_f T_5 Q_2 S_2) + \dots \\ & + 2\rho_f Q_4^2 (C_1 T_4^2 + C_1^2 T_4 S_4) + C_6^2 C_9^2 (\rho T_3 + \rho_f Q_3 S_3), \end{aligned}$$

$$\begin{aligned} c_2(\mathbf{n}) = & (\rho^2 I_Q III_T + \rho \rho_f II_Q TS + III_Q ST \rho_f^2) \\ & + 2\rho_f^2 Q_4 Q_6 C_5 C_8 S_4 S_6 + \rho_f^2 (II_C^2 + 2III_C I_C) + \dots \\ & - \rho_f^2 C_5^2 C_8^2 - \rho \rho_f C_4^2 C_7^2 S_1 - \rho_f^2 Q_6^2 (P^2 + 2T_6 S_6 C_3) + \dots \\ & - \rho_f Q_5^2 [T_2 S_5 (2\rho_f Q_2 S_5 + 2\rho T_5) \\ & + S_2 T_5 (2\rho_f Q_2 S_5 + \rho T_5)] + \dots \\ & - \rho_f^2 Q_4^2 (T_4^2 + 4C_1 T_4 S_4 + C_1^2 S_4^2) - \rho \rho_f C_6^2 C_9^2 S_3, \end{aligned}$$

$$\begin{aligned} c_3(\mathbf{n}) = & -(\rho^3 III_T + \rho^2 \rho_f I_Q TS + \rho \rho_f^2 II_Q ST + \rho_f^3 III_Q III_S) \\ & - 2\rho_f^3 (I_C II_C + III_C) + 2\rho_f^3 Q_6^2 S_6 P + \dots \\ & + Q_5^2 [\rho_f^3 S_5^2 Q_2 S_2 + \rho \rho_f^2 (S_5^2 T_2 + 2S_5 T_5 S_2)] \\ & + 2\rho_f^3 Q_4^2 (T_4 S_4 + C_1 S_4^2), \end{aligned}$$

$$\begin{aligned} c_4(\mathbf{n}) = & (\rho^3 \rho_f TS + \rho^2 \rho_f^2 I_Q ST + \rho \rho_f^3 II_Q III_S) \\ & + \rho_f^4 (I_C^2 + 2II_C) + \dots - \rho_f^4 Q_6^2 S_6^2 - \rho \rho_f^3 Q_5^2 S_2 S_2^2 \\ & - \rho_f^4 Q_4^2 S_4^2, \end{aligned}$$

$$c_5(\mathbf{n}) = -\rho^2 (\rho \rho_f^2 ST + \rho_f^3 I_Q III_S) - 2\rho_f^5 I_C \quad \text{and}$$

$$c_6(\mathbf{n}) = \rho_f^3 (\rho^3 III_S + \rho_f^3), \quad (19)$$

where

$$I_Q = Q_1 + Q_2 + Q_3, \quad I_C = C_1 + C_2 + C_3,$$

$$I_T = T_1 + T_2 + T_3, \quad II_Q = Q_1 Q_3 + Q_2 Q_3 + Q_1 Q_2,$$

$$II_C = C_1 C_3 + C_2 C_3 + C_1 C_2,$$

$$II_T = T_1 T_3 + T_2 T_3 + T_1 T_2, \quad III_Q = Q_1 Q_2 Q_3,$$

$$III_C = C_1 C_2 C_3, \quad III_T = T_1 T_2 T_3,$$

$$I_S = S_1 + S_2 + S_3, \quad TS = T_1 T_2 S_3 + T_1 T_3 S_2 + T_2 T_3 S_1,$$

$$II_S = S_1 S_3 + S_2 S_3 + S_1 S_2,$$

$$ST = S_1 S_2 T_3 + S_1 S_3 T_2 + S_2 S_3 T_1, \quad III_S = S_1 S_2 S_3,$$

$$P = T_4 + S_4 C_3. \quad (20)$$

The eigenvalues z_m ($m = 1, 2, 3, 4, 5, 6$) of the poroelastic Christoffel equation (17) provides the values of the possible squares of the wave phase speeds v_m^2 in a fluid-saturated porous medium. The phase velocity is defined as the propagation velocity of a point of constant phase ($\omega t - kx = \text{const}$), thus $v = (dx/dt) = \omega/k$. Each phase velocity v_m is related to the common angular frequency ω and its own wave vector k_m ,

$$v_m = \omega/k_m, \quad m = 1, 2, 3, 4, 5, 6. \quad (21)$$

In a general direction, $\mathbf{n}$, there will be six roots of which four are non-trivial wave speeds, two shear waves and two longitudinal modes, representing Biot's fast and slow waves. Only waves propagating along the axes of symmetry are considered as pure wave modes ($m = \text{P1, P2, S1, and S2}$), while waves propagating off axes of symmetry are composed of mixed modes and called quasi-waves ($m = \text{qP1, qP2, qS1, and qS2}$). For each value of a squared wave speed v_m^2 substituted back into Eq. (7), two three-dimensional (3D) vectors $\mathbf{a}$ and $\mathbf{b}$ will be determined subject to the condition that they are both unit vectors. For an isotropic medium, the wave number is a scalar quantity and the phase velocity v is constant for any direction $\mathbf{n}$. For an anisotropic medium, the wave number k becomes a vector $\mathbf{k}(k_1, k_2, k_3)$ and the associated phase velocity vector $\mathbf{v}(v_1, v_2, v_3)$ varies with the direction of wave propagation $\mathbf{n}$. An alternate approach to the solution of Eq. (6), for the squares of the wave speeds v^2 and the two 3D vectors $\mathbf{a}$ and $\mathbf{b}$, has been undertaken by Sharma (2005, 2008) who uses Eq. (6) to solve the relationship between the two 3D vectors $\mathbf{a}$ and $\mathbf{b}$ and then obtains a 3 by 3 matrix equation equivalent to Eq. (7) for one of the 3D vectors $\mathbf{a}$ or $\mathbf{b}$. This approach provides a 3 by 3 matrix eigenvalue problem but for a more complicated matrix. The results must, of course, be the same.

FIG. 1. (a) Spherical coordinate system, where the radial distance from a fixed origin, is the magnitude of v , θ is the inclination angle measured from a fixed zenith direction (n_3), and φ is the azimuth angle of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction (n_1) on that plane. (b) Phase (v) and group (V^{gr}) wave velocities of quasi-waves propagating in a dispersive anisotropic medium, exhibiting different magnitude and direction ψ .

Using a spherical coordinate system, the phase velocity vector $\mathbf{v}(v, \theta, \varphi)$ along the direction $\mathbf{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ is expressed by the magnitude (radial distance from a fixed origin, v), its inclination angle (θ) measured from a fixed zenith direction (n_3), and the azimuth angle (φ) of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction (n_1) on that plane [Fig. 1(a)].

B. Group velocity and ray direction

Plane waves in an anisotropic medium generally travel as a waveform containing two or more waves with slightly different ω and k

$$\begin{aligned} & \cos(\omega t - kx) + \cos[(\omega + \delta\omega)t - (k + \delta k)x] \\ &= 2 \cos \left[\left(\omega + \frac{\delta\omega}{2} \right) t - \left(k + \frac{\delta k}{2} \right) x \right] \cos \left(t \frac{\delta\omega}{2} - x \frac{\delta k}{2} \right), \end{aligned} \quad (22)$$

where the term $2 \cos\{(\omega + \delta\omega/2)t - (k + \delta k/2)x\}$ corresponds to the carrier wave, and the term $\cos(t \delta\omega/2 - x \delta k/2)$ represents the modulation envelope of the waveform. The propagation velocity of the carrier is the phase velocity,

$$v = \frac{\omega + \delta\omega/2}{k + \delta k/2} \rightarrow \frac{\omega}{k}, \quad (23)$$

and the propagation velocity of the modulation envelope is the group velocity (Auld, 1973),

$$V^{gr} = \frac{\delta\omega}{\delta k} \rightarrow \frac{\partial\omega}{\partial k}. \quad (24)$$

For the 3D case, the modulation envelope or group velocity is given in spherical coordinates by

$$\mathbf{V}^{gr} = \nabla_{\mathbf{v}}(v, \theta, \varphi) = \frac{\partial\mathbf{v}}{\partial v} \hat{\mathbf{v}} + \frac{1}{v} \frac{\partial\mathbf{v}}{\partial\theta} \hat{\boldsymbol{\theta}} + \frac{1}{v \sin\theta} \frac{\partial\mathbf{v}}{\partial\varphi} \hat{\boldsymbol{\varphi}}, \quad (25)$$

where the two angular components of the group velocity in Eq. (25) can be obtained by differentiation of the dispersion equation (7)

$$\mathbf{V}_\theta^{gr}(v, \theta, \varphi) = \frac{\partial\mathbf{v}(v, \theta, \varphi)}{\partial\theta}, \quad \mathbf{V}_\varphi^{gr}(v, \theta, \varphi) = \frac{\partial\mathbf{v}(v, \theta, \varphi)}{\partial\varphi}, \quad (26)$$

and the rectangular components of group velocity ($V_1^{gr}, V_2^{gr}, V_3^{gr}$) are given by

$$\begin{aligned} V_1^{gr} &= v \cos \varphi \sin \theta + \cos \varphi \cos \theta \frac{\partial v}{\partial \theta} - \frac{\sin \varphi}{\sin \theta} \frac{\partial v}{\partial \varphi}, \\ V_2^{gr} &= v \cos \varphi \sin \theta + \sin \varphi \cos \theta \frac{\partial v}{\partial \theta} + \frac{\cos \varphi}{\sin \theta} \frac{\partial v}{\partial \varphi}, \\ V_3^{gr} &= v \cos \theta - \sin \theta \frac{\partial v}{\partial \theta}. \end{aligned} \quad (27)$$

Therefore, the magnitude for the group velocity for each wave mode is

$$V^{gr} = \sqrt{(v)^2 + \left(\frac{\partial v}{\partial \theta} \right)^2 + \frac{1}{\sin^2 \theta} \left(\frac{\partial v}{\partial \varphi} \right)^2}, \quad (28)$$

and the ray (energy) direction ($\theta^{gr}, \varphi^{gr}$) is obtained as

$$\theta^{gr} = \tan^{-1} \left(\frac{\sqrt{(V_1^{gr})^2 + (V_2^{gr})^2}}{V_3^{gr}} \right), \quad \varphi^{gr} = \tan^{-1} \left(\frac{V_2^{gr}}{V_1^{gr}} \right). \quad (29)$$

Thus, the group velocity propagates along a ray at an angle $\mathbf{V}_m^{gr}$ to the phase propagation direction ($\theta_m^{gr}, \varphi_m^{gr}$) for each mode of wave propagation ($m = \text{qP1, qP2, qS1, qS2}$). Different phase and group velocities may exist for each of the qP1, qP2, qS1, and qS2 wave modes. In isotropic, non-dispersive media, the phase and group velocities are the same. However, phase and group velocities may be different due to dispersion or anisotropy or both. The wave vector ($\mathbf{k}$) magnitude is proportional to the ratio of the frequency and the wave phase velocity, and its direction is perpendicular to the wavefront. If the difference between the phase and group velocities depends only on the magnitude of $\mathbf{k}$, the difference $\mathbf{V}^{gr} - \mathbf{v}$ is caused by dispersion. In a dispersive medium, the phase and group velocities have the same direction but

different magnitudes. If the difference between phase and group velocities depends only on the direction of $\mathbf{k}$, the difference $\mathbf{V}^{gr} - \mathbf{v}$ is caused by anisotropy [Fig. 1(b)]. The magnitude and direction of phase and group velocities are both different in dispersive anisotropic media. In this case, the phase velocity is the projection of the velocity of energy transport in the direction of the wave normal [Fig. 1(b)]. The phase and group velocities are thus characterized by the phase direction (θ, φ) and ray direction $(\theta^{gr}, \varphi^{gr})$, respectively. The difference between phase and ray directions is shown in Fig. 1(b) by the angle ψ . Anisotropy and dispersion may co-exist in biological tissues such as cancellous bone.

C. Attenuation coefficient

The ultrasonic attenuation coefficient α_m represents the amount of energy absorption lost by the ultrasonic beam during its propagation through the medium along a path of length l for each wave mode ($m = \text{qP1, qP2, qS1, qS2}$). In porous media, the solution of the poroelastic Christoffel equation (17) gives complex roots since absorption is considered in the model. The complex wave number (5) defines the attenuation coefficient α for the corresponding wave mode

$$\alpha = -\frac{\omega v_{\text{Im}}}{v_{\text{Re}}^2 + v_{\text{Im}}^2}, \quad (30)$$

and

$$\text{Att}_{dB} \equiv -20 \log_{10}(e) l \alpha_m \approx -8.686 l \alpha_m \quad (m = \text{qP1, qP2, qS1, qS2}). \quad (31)$$

The factor $-20 \log(e)$ in the last expression is a constant that represents -20 times the base-10 logarithm of Euler's number e and is used to translate the attenuation from nepers into

decibels. It is important to notice that the attenuation of each wave mode depends on the frequency ω and the direction of propagation via the dependence of v_{Re} and v_{Im} in the angles θ and φ .

V. RESULTS

A. Numerical example

The anisotropic poroelastic model of wave propagation is now applied to the case of cancellous bone using the values of the fluid and solid constituents of bone obtained from the literature, summarized in Table I, and for 0.5 and 1.6 MHz frequency of wave propagation, accordingly to frequency values used in our previous experimental measurements (Cardoso *et al.*, 2003).

B. Phase velocity and phase direction

Figure 2 shows the phase velocity for the four possible wave modes generated in a porous medium as a function of phase direction (θ, φ) at a constant frequency in a dispersive and anisotropic medium ($F_{11} = 2/9$, $F_{22} = 1/3$, and $F_{33} = 4/9$) with 83% porosity. Figure 2(a) shows a continuous variation on the velocity of the fast qP1 wave as a function of the inclination θ and azimuthal φ angles, ranging between 1470 and 1860 m/s. The slow quasi-wave mode qP2 is shown in Fig. 2(b), demonstrating a much milder variation in its velocity, with values ranging between 1260 and 1410 m/s, always smaller than the fast qP1 wave velocities at any phase direction. The two quasi-shear wave modes qS1 and qS2 are shown in Figs. 2(c) and 2(d), respectively. The qS1 wave has a smaller range of velocities when compared to qS2 at any analyzed phase direction (θ, φ) . The phase velocity of each wave mode was calculated using three different frequency values, 0.5, 1.6, and 2.25 MHz, resulting in a

TABLE I. Material properties of bone used in the theoretical model.

Parameter	Symbol	Value	Units
Mass density of the fluid	ρ_f	1000	Kg/m ³
Fluid bulk modulus	K_f	2.25	GPa
Fluid viscosity	μ	1×10^{-3}	Pa s
Young's elastic modulus the solid	E_s	18 (Turner <i>et al.</i> , 1999; Jorgensen and Kundu, 2002; Rho <i>et al.</i> , 1997, 1999; Roy <i>et al.</i> , 1999; Zysset <i>et al.</i> , 1999; Hoffer <i>et al.</i> , 2000a, Hoffer <i>et al.</i> , 2000b; Hengsberger <i>et al.</i> , 2001, 2002; Pakula <i>et al.</i> , 2008)	GPa
Shear modulus of the solid	G	7.2	GPa
Poisson ratio	ν	0.25	
Mass density of the solid	ρ_s	2000 (Ashman and Rho, 1988; Nicholson <i>et al.</i> , 1997; Morgan <i>et al.</i> , 2003; Pakula <i>et al.</i> , 2008)	Kg/m ³
Porosity	ϕ	80	%
Frequency	f	0.5, 1.6, and 2.25	MHz
Fabric tensor	$\mathbf{F}$	Isotropy case, $F_{11} = 1/3$, $F_{22} = 1/3$, and $F_{33} = 1/3$; orthotropy case, $F_{11} = 2/9$, $F_{22} = 1/3$, and $F_{33} = 4/9$. In both cases, $\text{tr}\mathbf{F} = F_{11} + F_{22} + F_{33} = 1.0$	
Pore diameter	d	$d_1 = 310$, $d_2 = 353$, $d_3 = 400$ (Parfitt <i>et al.</i> , 1983; Rehman <i>et al.</i> , 1994; Hildebrand <i>et al.</i> , 1999; Glorieux <i>et al.</i> , 2000)	μm
Intrinsic permeability	κ_0	$k_1 = 2.16 \times 10^{-7}$, $k_2 = 2.44 \times 10^{-7}$, $k_3 = 2.80 \times 10^{-7}$ (Lim and Hong, 2000; Grimm and Williams, 1997b; Nauman <i>et al.</i> , 1999; Kohles <i>et al.</i> , 2001; Kohles and Roberts, 2002; Baroud <i>et al.</i> , 2004; Beaudoin <i>et al.</i> , 1991; Li <i>et al.</i> , 1987; Pakula <i>et al.</i> , 2008)	m ²

FIG. 2. (Color online) Phase velocity as a function of phase direction (θ, φ) at a constant frequency for an anisotropic porous medium with 80% porosity.

negligible difference in phase velocity within this range of frequency.

C. Group velocity and group direction

Absorption due to viscous losses generated between the solid and fluid phases of the porous medium give rise to differences between the magnitude of group and phase velocities of the several wave modes propagating in the porous medium. Generally, the effect of absorption due to dispersion is analyzed as the difference between the magnitude of group and phase velocities as a function of the frequency, along the direction of the principal axes of symmetry. In Fig. 3, we have followed a different approach by illustrating the difference in magnitude between $\mathbf{V}_m^{gr} - \mathbf{v}_m$, $\sqrt{(\mathbf{V}_m^{gr} - \mathbf{v}_m) \cdot (\mathbf{V}_m^{gr} - \mathbf{v}_m)}$, at a constant frequency as a function of the phase direction (θ, φ) in an isotropic medium. The difference between group and phase velocities is shown for the quasi-fast and quasi-slow wave modes in Figs. 3(a) and 3(b), respectively. It should be noticed that the difference is almost negligible ($O 1 \times 10^{-5}$ m/s) at 0.5, 1.6, and 2.25 MHz, regardless of the phase direction being analyzed. The difference between the magnitude of the group and phase velocities for qS1 and qS2 waves is shown in Figs. 3(c) and 3(d), respectively. The difference is equally small ($O 1 \times 10^{-5}$ m/s) as for quasi-longitudinal wave modes at 0.5, 1.6, or 2.25 MHz, regardless of the phase direction (θ, φ). This result indicates that the group velocities of all the quasi-transverse and quasi-longitudinal wave modes are not affected by absorption processes at 0.5 – 2.25 MHz in our model.

Figure 4 shows the difference in magnitude between the group and phase velocities ($\mathbf{V}_m^{gr} - \mathbf{v}_m$) as a function of the phase direction (φ, θ) in a dispersive and orthotropic medium ($F_{11} = 2/9$, $F_{22} = 1/3$, and $F_{33} = 4/9$) as opposed to the isotropic medium ($F_{11} = 1/3$, $F_{22} = 1/3$, and $F_{33} = 1/3$) presented in Fig. 3. The magnitude difference between group and phase velocities as a function of phase direction for the longitudinal quasi-waves varies up to 40 m/s, and up to 10 m/s for the quasi-shear wave modes at 83% porosity. The largest magnitude difference in ($\mathbf{V}_m^{gr} - \mathbf{v}_m$) is observed in the fast quasi-wave, as shown in Fig. 4(a); however, this velocity difference depends on the porosity, and the slow quasi-wave mode may exhibit the largest magnitude difference between group and phase velocities at higher porosities (data not shown). It was also observed that the magnitude of the group velocity for all quasi-wave modes is always higher than the magnitude of its corresponding phase velocity.

The combined results shown in Figs. 3 and 4 indicate that the magnitudes of the differences between group and phase velocities for each quasi-wave mode in a dispersive–orthotropic porous medium are different, but not in a dispersive–isotropic medium, clearly indicating the effect of the fabric anisotropy on such differences. These results also show a difference between the ray ($\theta_m^{gr}, \varphi_m^{gr}$) and phase (θ_m, φ_m) directions of each wave mode in an orthotropic medium. The difference between ray and phase directions is described by the angle ψ_m and is shown in Fig. 5 for all four quasi-wave modes as a function of phase direction (θ_m, φ_m). In this orthotropic medium with 83% porosity, the angle ψ_m varies

FIG. 3. (Color online) Difference between the magnitude of group and phase velocities as a function of phase direction (θ , ϕ) at a constant frequency for an isotropic porous medium with 80% porosity.

by approximately 15° for the fast quasi-wave mode and 10° for the other three quasi-waves. The difference between angles ψ_1 and ψ_2 implies that the fast and slow quasi-wave modes propagate at slightly different directions when propagating off axes of material symmetry.

D. Attenuation

The attenuation of longitudinal and shear waves is generally analyzed at a given direction of wave propagation as a function of the frequency. However, that is not done here; Fig. 6 displays the dependence of the attenuation at a constant frequency as a function of the phase direction (θ , ϕ) in which the quasi-wave propagates in an orthotropic medium with 83% porosity. The attenuation exhibits small variations of the order of 1×10^{-1} to 1×10^{-2} dB/cm for both quasi-longitudinal and quasi-shear wave modes. However, there exists a dependence of the attenuation for all four quasi-wave modes on the phase direction (θ , ϕ), indicating that the attenuation due to absorption processes in an anisotropic porous medium is dependent upon the inclination and azimuthal angles. When considering the results for the quasi-longitudinal waves qP1 and qP2 together, it can be observed that the highest attenuation for qP1 occurs at $\theta = 0^\circ$, and the lowest occurs at $\theta = 90^\circ$. The opposite, however, is shown for the qP2 wave mode, indicating that at $\theta = 0^\circ$, the qP1 wave has higher attenuation than qP2, but qP2 has higher attenuation than qP1 at $\theta = 90^\circ$. This transition in the dominating attenuation

between qP1 and qP2 agrees with our previous results for pure wave modes (P1 and P2) obtained along a principal axis of fabric in an orthotropic medium (Cowin and Cardoso, 2011). This transition seems to be specific to longitudinal waves and is absent in quasi-shear waves, as shown in our numerical example, where the attenuation of qS1 is higher than the one for qS2 at any phase direction (θ , ϕ). The attenuation of quasi-wave modes was calculated using a frequency value of 0.5, 1.6, and 2.25 MHz; however, the attenuation within this range of frequency was found to be practically the same.

E. Comparison between experimental measurements and theoretical predictions

Predictions of the fast and slow wave velocities done by this model will now be compared with experimental measurements previously reported (Cardoso *et al.*, 2003). Briefly, 14 bovine and 60 human trabecular bone samples were retrieved from bovine femoral heads, human femoral heads, and femoral and tibial condyles. Samples of approximately $1 \times 1 \times 1$ cm in size were prepared, followed by removal of fat and marrow, and saturated with water under vacuum for 30 min. Group velocities of quasi-wave modes were measured in immersion with distilled water at room temperature, using two broadband ultrasound transducers (Panametrics V323-SU, Waltham, MA, USA) at a central frequency of 2.25 MHz (0.25 in diameter). The emitter was excited by a damped single pulse generated by an ultrasonic

FIG. 4. (Color online) Difference between group and phase velocities as a function of phase direction (φ , θ) at a constant frequency for an anisotropic porous medium with 80% porosity.

source (Panametrics 5052 UA, Waltham, MA, USA) operated in a transmission mode. The signal was amplified in 40 dB, digitized by a 100 MHz Digital Oscilloscope (Tektronik 2430, Beaverton, OR, USA) and analyzed in MATLAB. Importantly, bone cubes were cut without aligning its orthogonal faces with the planes of symmetry of the sample. Since bone samples were interrogated along the normal direction to each face of the sample (e.g., arbitrary directions labeled as A, B, and C), which do not coincide with the normal to the planes of symmetry of the sample, it was considered that propagated signals correspond to quasi-wave modes for each of the three interrogated directions. It was shown in that study that the wavelength of the fast and slow waves are significantly different and that this property can be used to separate the two waves using digital filtering. Therefore, acquired signals were filtered out using bandpass filters with a frequency bandwidth of 0.5 ± 0.25 MHz for the fast wave and a frequency bandwidth of 1.6 ± 0.6 MHz for the slow wave. The fabric tensor describing the microarchitecture on each sample was measured using the mean intercept length as described in Whitehouse (1974), Whitehouse and Dyson (1974), Harrigan and Mann (1984), and Turner *et al.* (1987, 1990). The phase angles θ , φ were determined as the relative orientation of the principal axes of fabric in respect to the measurement directions A, B, and C of the cube samples. The porosity, pore diameter, and apparent density of the sample were obtained through analysis of bone images and weight of the sample using Archimedes principle. The intrinsic

permeability was estimated on each sample using the relationship $k_0 = d^2$ (Bear, 1988). Values of the mass densities and elastic modulus of the solid and fluid phases, as well as the viscosity of the fluid, were taken from Table I.

In order to investigate the effect of fabric on the fast and slow wave velocity measurements, two cases were analyzed by comparing experimental values with theoretical predictions. First, the theoretical phase velocities for qP1 and qP2 were computed using the experimental values of porosity, tissue density, and without taking into account the fabric anisotropy, but considering the fabric as isotropic ($F_{11} = F_{22} = F_{33} = 1/3$). Then, the ray direction and group velocity were calculated for each quasi-longitudinal mode. Figure 7(a) shows a comparison between the group velocities predicted from the model (under the assumption of isotropy) and the experimental wave velocity measurements obtained on three orthogonal directions on each sample. In the second case, the theoretical phase velocities for qP1 and qP2 were computed as in the previous case, but this time using the images-derived fabric anisotropy values for each sample. Again, the ray direction and group velocity were calculated for each quasi-longitudinal mode. Figure 7(b) shows the comparison between experimental group wave velocities, for all three directions A, B, and C and predicted wave velocity values when the fabric was taken into account. The correlation coefficient between experimental and theoretical predictions when the fabric anisotropy is not taken into account was $R^2 = 0.53$ and when the fabric anisotropy was included in the theoretical model was $R^2 = 0.86$. This analysis

FIG. 5. (Color online) Difference between group and phase angle ψ as a function of phase direction (ϕ, θ) at a constant frequency for an anisotropic porous medium with 80% porosity.

indicates a much higher *quantitative* agreement between experimental and theoretical values of group quasi-wave velocity when the fabric anisotropy is included in the model.

VI. DISCUSSION

A theoretical framework for analysis of wave propagation along a principal axis of fabric in an anisotropic poroelastic medium was recently developed (Cowin and Cardoso, 2011). That model was extended in this study to the case of an arbitrary phase direction (θ, ϕ) of wave propagation. The advantage of the present development is the ability to distinguish the role of fabric anisotropy in the predictions of phase and group velocities of the four wave modes propagating in porous media at any phase or ray direction.

The fabric anisotropy plays a critical role in the phase and group velocity of quasi-wave modes. The results from Fig. 2 indicate that the phase velocity of quasi-wave modes is strongly dependent on the phase direction (θ, ϕ) in anisotropic porous media. This acoustic anisotropy was included into the model using the fabric tensor measurement. When the fabric tensor was isotropic, the resultant wave propagation velocities are reduced to those of an isotropic medium. Importantly, the results shown in Figs. 3 and 4 indicate that the dispersion due to the viscous friction between the solid and fluid in bone within the 0.5–2.25 MHz frequency range is practically negligible, while the difference between phase

and group velocities of quasi-wave modes was due to fabric anisotropy. The effect of anisotropy on the difference between group and phase velocities was found more significant in longitudinal than in shear wave modes. However, the sensitivity of the quasi-fast or quasi-slow wave to fabric anisotropy varies according to the porosity of the medium as shown in Cowin and Cardoso (2011). Moreover, the results presented in Fig. 5 confirmed a significant difference between the phase and ray directions analyzed in anisotropic media. The results in Fig. 6 predict a dependence of the attenuation for all four quasi-wave modes on the phase direction (θ, ϕ) at 0.5–2.25 MHz. However, this attenuation predicted is very small when compared to experimental values reported in the literature (Haïat *et al.*, 2006; Sasso *et al.*, 2008; Pakula *et al.*, 2009). This result indicates that the role of absorption and anisotropy on the attenuation of quasi-wave modes is minimal and cannot explain the experimental attenuation recorded in those experimental studies. The possible mechanism explaining such measurements is likely to be scattering of waves, for which the fabric dependence would need to be developed. It is also important to notice that the wavelengths generated in the porous medium by waves traveling at any given frequency should be larger than the pore diameter (the inhomogeneity) in the porous medium. Otherwise, the applicability of the theory cannot be guaranteed. Figure 8 shows the comparison between the size of the wavelength of the fast and slow waves generated at

FIG. 6. (Color online) Attenuation coefficients as a function of phase direction (ϕ , θ) at a constant frequency in a dispersive–anisotropic porous medium with 80% porosity.

either 0.5 or 1.6 MHz and the size of the pore diameter (the inhomogeneity of the pore medium). In this figure, it can be seen that the wavelength of the fast wave at 0.5 MHz is larger than the pore diameter from 0% to 97% porosity, and that the wavelength of the slow wave is larger than the pore diameter from 0% to 90% porosity, when the 1.6 MHz fre-

quency is used. The pore diameter vs porosity relationship was previously introduced (Cowin and Cardoso, 2011) based on histomorphometrical studies on cancellous bone that have reported pore sizes (trabecular spacing) ranging from 300 to 2200 μm for samples between 52% and 96% porosity (Parfitt *et al.*, 1983; Rehman *et al.*, 1994; Hildebrand *et al.*, 1999;

FIG. 7. (Color online) Comparison of experimental ultrasound wave velocities against theoretical model predictions when considering (a) isotropic fabric and (b) anisotropic fabric measurements. The predictability of experimental values by the poroelastic theory is much increased when the fabric anisotropy is taken into account.

FIG. 8. Comparison between the size of the wavelength of the fast and slow waves generated at 0.5 and 1.6 MHz, respectively, and the size of the pore diameter (porous medium inhomogeneity). The wavelength of the fast wave at 0.5 MHz (solid squares) is larger than the pore diameter from 0% to 97% porosity, and that the wavelength of the slow wave at 1.6 MHz (solid diamonds) is larger than the pore diameter (circles) from 0% to 90% porosity, when this 1.6 MHz frequency is used. For comparison, we also included in this figure the wavelengths of the fast wave at 1.6 MHz (empty diamonds) and the wavelengths of the slow wave at 0.5 MHz (empty squares).

Glorieux *et al.*, 2000). Furthermore, the pore size in 5% – 10% porosity cortical bone tissue is considered to vary around 20 to 60 μm , which corresponds to the pore size of Haversian canals (Jones *et al.*, 2004; Basillais *et al.*, 2007).

The comparison between experiments and theoretical predictions was performed in samples that were not cut aligned to their planes of symmetry. Therefore, the measured waves on those samples are not pure wave modes but quasi-waves. The development of the fabric-dependent anisotropic theory of propagation of quasi-waves in porous media was presented in this study, and a quantitative analysis of these experimental results was performed. The theoretical model was able to predict the high variability of fast and slow wave velocities observed in bovine and human bone in our experimental study. Figure 7 indicates that the fabric tensor measurement alone is able to drastically increase the model predictability on wave velocities from 53% to 86%. In other words, directional variability within a sample was effectively explained by the theoretical model after inclusion of the fabric; this directional variability could not be explained by the porosity only. The agreement between experimental and theoretical values indicates that despite the complexity added to the poroelastic theory, a tensorial variable describing the bone microstructure is required to explain the directional variability of the wave propagation with bone architecture. Therefore, the fabric tensor—a measure of bone microarchitecture—exhibited a role as important as the mass density in determining the acoustic properties of anisotropic porous bone samples.

A major goal of ultrasound wave propagation measurements is to determine non-invasively the elastic constants of the porous medium using an inverse problem approach. However, measurements of wave velocities in anisotropic—

dispersive media correspond to the group velocities of each wave mode, not to the phase velocities. The latter are, in fact, directly related to the elastic constants of the porous medium. Therefore, it is necessary to better understand the mechanisms (absorption and anisotropy) that are at the origin of differences between phase and group velocities in a porous medium. The analysis presented in this study allowed identifying fabric anisotropy, and not the solid–fluid viscous losses, as the dominating factor affecting the difference between phase and group velocities in anisotropic–dispersive porous media at high frequencies (0.5 – 1.6 MHz). The opposite may happen, however, at low frequencies (Hz to KHz range). Importantly, this study suggests that the distinction between group velocity and phase velocity is important when the estimation of anisotropic elastic constants in a porous medium is sought. Moreover, it indicates the possibility of using such variations between phase and group velocities as a function of the phase direction (θ , ϕ) to estimate the fabric tensor in porous media using ultrasound.

Ultrasound wave propagation is sensitive to directional changes in microarchitecture of porous media. Unfortunately, most clinical ultrasound densitometers perform their measurements in a single direction, not taking advantage of the ability of ultrasound to distinguish microarchitecture anisotropy. This has also been a consequence of the lack of anisotropic theoretical models of wave propagation permitting to describe the wave velocity variability in anisotropic cancellous bone. The proposed approach provides a rigorous theoretical framework to interpret and analyze measurements of wave velocities in porous media along a general direction. Efforts in clinical ultrasound, which are made toward developing experimental approaches to measure ultrasound wave propagation in multiple directions at appropriate skeletal sites, such as the wrist or the femur, major sites of osteoporosis-related fractures, may benefit of this anisotropic poroelastic wave propagation theory.

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APPENDIX

Tensors $\mathbf{Q}$, $\mathbf{C}$, and $\mathbf{S}$ are expressed as a function of the general direction of wave propagation $\mathbf{n}$

$$\mathbf{Q} = \begin{bmatrix} Q_1(\mathbf{n}) & Q_6(\mathbf{n}) & Q_5(\mathbf{n}) \\ Q_6(\mathbf{n}) & Q_2(\mathbf{n}) & Q_4(\mathbf{n}) \\ Q_5(\mathbf{n}) & Q_4(\mathbf{n}) & Q_3(\mathbf{n}) \end{bmatrix} = \begin{bmatrix} Q_{11}(\mathbf{n}) & Q_{12}(\mathbf{n}) & Q_{13}(\mathbf{n}) \\ Q_{12}(\mathbf{n}) & Q_{22}(\mathbf{n}) & Q_{23}(\mathbf{n}) \\ Q_{13}(\mathbf{n}) & Q_{23}(\mathbf{n}) & Q_{33}(\mathbf{n}) \end{bmatrix}, \quad (\text{A1})$$

where

$$\begin{aligned}
Q_{11}(\mathbf{n}) &= A_{111}n_1^2 + A_{121}n_2^2 + A_{131}n_3^2 + 2A_{112}n_1n_2 \\
&\quad + 2A_{113}n_1n_3 + 2A_{123}n_2n_3, \\
Q_{22}(\mathbf{n}) &= A_{212}n_1^2 + A_{222}n_2^2 + A_{232}n_3^2 + 2A_{212}n_1n_2 \\
&\quad + 2A_{213}n_1n_3 + 2A_{223}n_2n_3, \\
Q_{33}(\mathbf{n}) &= A_{313}n_1^2 + A_{323}n_2^2 + A_{333}n_3^2 + 2A_{313}n_1n_2 \\
&\quad + 2A_{313}n_1n_3 + 2A_{323}n_2n_3, \\
Q_{12}(\mathbf{n}) &= A_{112}n_1^2 + A_{122}n_2^2 + A_{132}n_3^2 + \dots \\
&\quad \times (A_{112} + A_{122})n_1n_2 + (A_{112} + A_{132})n_1n_3 \\
&\quad + (A_{122} + A_{132})n_2n_3, \\
Q_{13}(\mathbf{n}) &= A_{113}n_1^2 + A_{123}n_2^2 + A_{133}n_3^2 + \dots \\
&\quad \times (A_{113} + A_{123})n_1n_2 + (A_{113} + A_{133})n_1n_3 \\
&\quad + (A_{123} + A_{133})n_2n_3, \\
Q_{23}(\mathbf{n}) &= A_{213}n_1^2 + A_{223}n_2^2 + A_{233}n_3^2 + \dots \\
&\quad \times (A_{213} + A_{223})n_1n_2 + (A_{213} + A_{233})n_1n_3 \\
&\quad + (A_{223} + A_{233})n_2n_3, \tag{A2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{C} &= \begin{bmatrix} C_1(\mathbf{n}) & C_6(\mathbf{n}) & C_5(\mathbf{n}) \\ C_9(\mathbf{n}) & C_2(\mathbf{n}) & C_4(\mathbf{n}) \\ C_8(\mathbf{n}) & C_7(\mathbf{n}) & C_3(\mathbf{n}) \end{bmatrix} \\
&= \begin{bmatrix} C_{11}n_1^2 & C_{12}n_2n_1 & C_{13}n_3n_1 \\ C_{21}n_2n_1 & C_{22}n_2^2 & C_{23}n_3n_2 \\ C_{31}n_3n_1 & C_{32}n_3n_2 & C_{33}n_3^2 \end{bmatrix}, \tag{A3}
\end{aligned}$$

$$\begin{aligned}
\mathbf{T} &= \begin{bmatrix} T_1(\mathbf{n}) & T_6(\mathbf{n}) & T_5(\mathbf{n}) \\ T_6(\mathbf{n}) & T_2(\mathbf{n}) & T_4(\mathbf{n}) \\ T_5(\mathbf{n}) & T_4(\mathbf{n}) & T_3(\mathbf{n}) \end{bmatrix} \\
&= M \begin{bmatrix} n_1^2 & n_2n_1 & n_3n_1 \\ n_2n_1 & n_2^2 & n_3n_2 \\ n_3n_1 & n_3n_2 & n_3^2 \end{bmatrix}, \tag{A4}
\end{aligned}$$

where the constant M was previously defined in Eq. (3),

$$\mathbf{S} = \begin{bmatrix} S_1(\mathbf{n}) & S_6(\mathbf{n}) & S_5(\mathbf{n}) \\ S_6(\mathbf{n}) & S_2(\mathbf{n}) & S_4(\mathbf{n}) \\ S_5(\mathbf{n}) & S_4(\mathbf{n}) & S_3(\mathbf{n}) \end{bmatrix} = \begin{bmatrix} \left(J_{11} + \frac{i\mu}{\rho_f\omega}R_{11}\right)n_1^2 & \left(J_{12} + \frac{i\mu}{\rho_f\omega}R_{12}\right)n_2n_1 & \left(J_{13} + \frac{i\mu}{\rho_f\omega}R_{13}\right)n_3n_1 \\ \left(J_{12} + \frac{i\mu}{\rho_f\omega}R_{12}\right)n_2n_1 & \left(J_{22} + \frac{i\mu}{\rho_f\omega}R_{22}\right)n_2^2 & \left(J_{23} + \frac{i\mu}{\rho_f\omega}R_{23}\right)n_3n_2 \\ \left(J_{13} + \frac{i\mu}{\rho_f\omega}R_{13}\right)n_3n_1 & \left(J_{23} + \frac{i\mu}{\rho_f\omega}R_{23}\right)n_3n_2 & \left(J_{33} + \frac{i\mu}{\rho_f\omega}R_{33}\right)n_3^2 \end{bmatrix}. \tag{A5}$$

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