Estimation of CT cone-beam geometry using a novel method insensitive to phantom fabrication inaccuracy: Implications for isocenter localization accuracy

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Purpose: Mechanical instabilities that occur during gantry rotation of on-board cone-beam computed tomography (CBCT) imaging systems limit the efficacy of image-guided radiotherapy. Various methods for calibrating the CBCT geometry and correcting errors have been proposed, including some that utilize dedicated fiducial phantoms. The purpose of this work was to investigate the role of phantom fabrication imprecision on the accuracy of a particular CT cone-beam geometry estimate and to test a new method to mitigate errors in beam geometry arising from imperfectly fabricated phantoms.

Methods: The authors implemented a fiducial phantom-based beam geometry estimation following the one described by Cho *et al.* [Med Phys **32**(4), 968–983 (2005)]. The algorithm utilizes as input projection images of the phantom at various gantry angles and provides a full nine parameter beam geometry characterization of the source and detector position and detector orientation versus gantry angle. A method was developed for recalculating the beam geometry in a coordinate system with origin at the source trajectory center and aligned with the axis of gantry rotation, thus making the beam geometry estimation independent of the placement of the phantom. A second CBCT scan with the phantom rotated 180° about its long axis was averaged with the first scan to mitigate errors from phantom imprecision. Computer simulations were performed to assess the effect of 2D fiducial marker positional error on the projections due to image discretization, as well as 3D fiducial marker position error due to phantom fabrication imprecision. Experimental CBCT images of a fiducial phantom were obtained and the algorithm used to measure beam geometry for a Varian Trilogy with an on-board CBCT.

Results: Both simulations and experimental results reveal large sinusoidal oscillations in the calculated beam geometry parameters with gantry angle due to displacement of the phantom from CBCT isocenter and misalignment with the gantry axis, which are eliminated by recalculating the beam geometry in the source coordinate system. Simulations and experiments also reveal an additional source of oscillations arising from fiducial marker position error due to phantom fabrication imprecision that are mitigated by averaging the results with those of a second CBCT scan with phantom rotated. With a typical fiducial marker position error of 0.020 mm (0.001 in.), source and detector position are found in simulations to be within 250 μm of the true values, and detector and gantry angles less than 0.2°. Detector offsets are within 100 μm of the known value. Experimental results verify the efficacy of the second scan in mitigating beam geometry errors, as well as large apparent source/detector isocenter offsets arising from phantom fabrication imprecision.

Conclusions: The authors have developed and validated a novel fiducial phantom-based CBCT beam geometry estimation algorithm that does not require precise positioning of the phantom at machine isocenter and is insensitive to positional imprecision of fiducial markers within the phantom due to fabrication errors. The method can accurately locate source and detector isocenters even when using an imprecise phantom, which is very important for measurement of isocenter coincidence of the therapy and on-board imaging systems. © 2011 American Association of Physicists in Medicine. [DOI: 10.1118/1.3589130]

Key words: cone-beam computed tomography, geometric calibration, fiducial phantom, image-guidance, radiation therapy, isocenter

I. INTRODUCTION

On-board kilovoltage (kV) cone-beam computed tomography (CBCT) imaging systems are becoming an increasingly important component of image-guided radiation therapy (IGRT), providing patient setup verification via image matching with

conventional planning CT.^{1–11} Accuracy of the geometric information provided by an on-board CBCT is limited by the geometric stability of the CBCT imaging system and its precise position and alignment in relation to the LINAC, to which the CBCT is mounted. Flexing or sagging of the

imaging system under its own weight during gantry rotation results in nonideal kV source and detector positions during CBCT image acquisition.^{4,12,13} Numerous methods have been developed for estimating these nonidealities, a procedure often referred to as geometric calibration or beam geometry estimation, defined as estimation of a set of parameters that describe the system geometry.^{12–24} Many of these methods, including the one described herein, rely on imaging of a dedicated phantom containing one or more radio-opaque fiducial markers. As noted by Mao *et al.*, estimates of the CBCT beam geometry are limited by the mechanical imprecision of the positions of the fiducial markers in the phantom.¹³ The current work was motivated by a desire to understand the effect of phantom fabrication imprecision on the accuracy of a particular beam geometry estimate, based on the one described by Cho *et al.*,¹⁴ which we have implemented. Several investigators have added Gaussian noise to the fiducial marker positions on simulated CBCT projections to simultaneously address both the error associated with determining the exact center of each marker on each projection (due to image discretization and possibly to a lesser extent, image noise), and marker positional error in the phantom due to fabrication imprecision.^{14,19,22–24} In this work, we set out to separate these two sources of error and investigate their relative importance in generating uncertainty in the beam geometry estimate. We present a novel procedure for accounting for positional imprecision of the fiducial markers, thus providing an accurate estimate of CBCT beam geometry. We note that the procedure can also be applied to the LINAC megavoltage source and EPID detector, providing geometric calibration of the radiotherapy system, but we focus in this paper on the kV CBCT imaging system.

II. METHODS AND MATERIALS

II.A. Phantom and data acquisition procedure

The phantom used for computer simulations and experimental validation is one suggested by Cho *et al.*¹⁴ It consists of two circular rings of regularly spaced steel ball bearings (BBs), 3 mm in diameter, encased in a hollow acrylic cylinder. Each ring contains 8 BBs and has radius of 51 mm. The rings are separated by 162 mm. The phantom is placed on the couch with its long axis approximately parallel to the axis of gantry rotation, and centered via room lasers. X-ray projections are acquired via a conventional full-fan CBCT scan of the phantom, typically acquiring approximately 660 projections over 360° of gantry rotation (Fig. 1).

A novel component of our method is the addition of a second CBCT scan with the phantom rotated 180° about its long axis for the purpose of minimizing the effects of phantom fabrication errors. As described below, the method does not require precise repositioning of the phantom for the second scan.

II.B. CBCT geometry

Our beam geometry estimation algorithm is based on the geometric calibration method described by Cho *et al.*,¹⁴ and we use similar nomenclature and a nearly identical set of

FIG. 1. CBCT projection of the calibration phantom, showing the two rings of eight BBs at either end of the phantom.

coordinate systems [Fig. 2(a)]. The nomenclature is summarized in Table I. Position is denoted as $P = (X, Y, Z)$; subscripts D and S specify the detector and source position (P_D, P_S), and superscripts specify the coordinate system. For example, the position of the source in the phantom coordinate system is denoted $P_S^p = (X_S^p, Y_S^p, Z_S^p)$. For clarity, we rename from “world” to “phantom,” the coordinate system (x^p, y^p, z^p) fixed to the phantom with origin at phantom center and z^p axis parallel to the long axis of the phantom cylinder. We define a set of coordinates (x^s, y^s, z^s) fixed to the room with origin at the nominal center of the x-ray source circular trajectory; the z^s axis (the axis of gantry rotation) is normal to the nominal source trajectory plane, and x^s points to the source at gantry angle 0°. The virtual detector coordinate system (x^i, y^i, z^i) has its origin at the piercing point, i.e., the projection of the phantom center on the detector; the y^i axis is antiparallel to z^p , and the x^i axis is perpendicular to the ray joining the source and the piercing point. The virtual detector is coplanar with the x^i - y^i plane and its pixel rows and columns are parallel and normal, respectively, to the x^i axis. Imperfect orientation of the real detector, defined by in-plane coordinates (x^j, y^j) and normal axis z^j are specified by the Euler angles (η, θ, ϕ), also known as yaw, pitch, and roll, that describe a general rotation with respect to the ideal virtual detector plane [Fig. 2(b)]. Finally, detector offset, or translation of the real detector’s central pixel with respect to the piercing point, is denoted Uo and Vo for displacement in the x^j and y^j directions, respectively. Cho *et al.*¹⁴ provide useful equations for transforming spatial coordinates between the coordinate systems. We have introduced a new source trajectory coordinate system (x^s, y^s, z^s), and therefore introduce a translation vector T_s^p , that specifies the position of the phantom with respect to the source coordinate system, and angles (α, β) that specify the angle of the phantom with respect to the gantry axis, as shown in Fig. 2(c).

II.C. Estimation of beam geometry

Beam geometry is specified for each projection by nine parameters: detector offsets (Uo, Vo), detector orientation

FIG. 2. Coordinate systems used in the beam geometry estimation: phantom (p), source (s), virtual (i), real (l). Phantom, virtual, and real coordinate systems are defined as in Cho *et al.* (Ref. 14). Source coordinate system has its origin (source isocenter) at the center of the nominal circular source trajectory, which also defines its orientation. Angles α , β define the orientation of the phantom relative to the source coordinate system.

TABLE I. Nomenclature convention.

$P = (X, Y, Z)$	Position in 3D Cartesian coordinates
p	Phantom coordinate system (superscript) ^a
s	Source coordinate system (superscript)
i	Virtual detector coordinate system (superscript)
l	Real detector coordinate system (superscript)
S	Source (subscript)
D	Detector (subscript)
$\mathbf{T}_a^b$	Translation vector from a to b coordinate system
$\mathbf{R}_a^b$	Rotation matrix from a to b coordinate system
η, θ, ϕ	Detector rotation Euler angles
Uo, Vo	Detector offsets
α, β	Phantom orientation
r	Known radius of phantom BB circle
l	Known distance between BB end rings
M	Magnification factor
SAD	Source to gantry axis distance
DAD	Detector to gantry axis distance
DEV	Out-of-plane source deviation, parallel to gantry axis

^aused as superscript on an algebraic symbol; e.g., Z_D^p is z -coordinate of detector in phantom coordinate system

(η, θ, ϕ), source to detector distance (SDD), source to axis distance (SAD), gantry angle γ (i.e., angle of x-ray source), and out-of-plane deviation (DEV) of the source position from its nominal circular trajectory plane; i.e., deviation in a direction parallel to the axis of gantry rotation. Estimation of these parameters is based on the method described by Cho *et al.*,¹⁴ which calculates all parameters in the “phantom” coordinate system (x^p, y^p, z^p). Unless the phantom is perfectly centered and aligned with the gantry axis, determination of the beam geometry parameters in the more useful “source” coordinate system (x^s, y^s, z^s) requires calculation of the position $\mathbf{T}_p^s$, and orientation (α, β) of the phantom. Transformation to the source coordinate system provides beam geometry independent of the placement of the phantom.

II.C.1. BB positions

An algorithm was implemented in MATLAB (Mathworks, Natick, MA) for automatically locating the 16 BBs in each projection. The algorithm for the first projection is as follows: for every pixel in the image, the mean intensity in a 3 pixel radius circular neighborhood is compared to the mean intensity of a circular rim of pixels at a radius of 6 pixels from the central pixel. If the ratio of two intensities is below a given threshold, then the pixel is identified as being within a BB. Thus is formed a cluster of pixels for each BB. The center of the cluster is taken as the (x^l, y^l) position of the BB. The BB positions in each end ring are fit to an ellipse allowing enumeration of the BBs in order of increasing angle around the circular ring, and matching of BBs in opposite end rings. The thresholding process is repeated for subsequent projections but is much accelerated by only searching in the neighborhood of the BB from the previous projection. This provides tracking of individual BBs through the CBCT; the output is a text file containing the (x^l, y^l) position of each of the 16 BBs in all projections. The entire process takes about a minute.

II.C.2. Detector offsets

Following the method described in Ref. 14, the first step is to fit the BB positions in each end ring to an ellipse for each projection. For this purpose we use a robust algorithm described by Halir and Flusser,²⁵ and the spatial position of the intersection of each ellipse with its major axis (i.e., the ends of each ellipse) is calculated. Opposite ends of the two ellipses in each projection are connected by rays. The intersection of the two rays is the piercing point. The displacement between the center of the projection (i.e., the geometric midpoint of the real detector) and the piercing point provides the detector offset (Uo, Vo). The measured BB positions are then offset by this amount, and the ellipses refitted to the new BB positions before further processing.

II.C.3. Detector orientation

Detector rotation about its normal axis, η , or yaw, is calculated from the positions and orientations of the fitted ellipses for each projection exactly as described in Ref. 14. BB

positions are then counter-rotated by this angle to remove the effect of yaw, and the ellipses refitted to the new BB positions before further processing.

The method of Cho *et al.* describes a nonlinear root-finding method for solving a pair of transcendental equations for simultaneous determination of the detector angles θ, ϕ , or pitch and roll, that is applicable even for large angles.¹⁴ However, we have found the method to be unstable in the presence of BB positional errors. Therefore, we have replaced this part of the algorithm with a more stable method that is suitable for small detector angles. It is described in detail in Appendix 1.

II.C.4. Source and detector position

The z-component of x-ray source position Z_S^i is calculated as in Appendix 1. Y_S^i is calculated according to Eq. (15) in Cho *et al.*¹⁴

$$Y_S^i = P_0^a \cos(\theta)$$

where P_0^a is the distance from the real detector origin to the point P_a , which is calculated¹⁴ as an intermediate result for determination of the detector angle η . The detector position $(0, Y_D^i, Z_D^i)$ is calculated exactly as according to Cho *et al.*¹⁴ We clarify a potential source of confusion in the formalism: “detector position in the virtual coordinate system” would appear to be zero, since the detector is at its origin, but what is meant by detector position is actually the translation vector $\mathbf{T}_p^i = (X_D^i, Y_D^i, Z_D^i)$ and thus is actually the position of the phantom origin in the virtual detector system, as in Eq. (3) of Cho *et al.*¹⁴ X_S^i and X_D^i are zero by definition. Source and detector positions in the real detector coordinate system, P_S^i and P_D^i are calculated via a rotation matrix formed from the detector orientation angles (η, θ, ϕ) , and serve as input to the estimation of gantry angle γ as described previously.¹⁴ High frequency variations of the nine beam geometry parameters versus gantry angle are then removed using the built-in MATLAB routine “smooth,” which utilizes a robust local regression low-pass filter, with filter span = 101. Finally the source and detector positions in the phantom coordinate system, P_S^p and P_D^p , are determined by a straightforward rotation and translation from the detector to phantom frame of reference.

II.C.5. Phantom position and orientation

Recalculation of the beam geometry parameters in the source coordinate system requires determination of the translation vector $\mathbf{T}_p^s$ and rotation matrix $\mathbf{R}_p^s$ relating the phantom and source coordinate systems; i.e., finding the phantom position relative to the source trajectory center and finding the phantom orientation relative to the axis of gantry (source) rotation. This is accomplished by an iterative optimization routine that recalculates the beam geometry in the source coordinate system for given values of $\mathbf{T}_p^s$ and $\mathbf{R}_p^s$ and finding those values that minimize variations in beam geometry parameters that arise from imperfect placement of the phantom relative to the imaging system; i.e., by finding phantom pose parameters that minimize the apparent sinusoidal oscillations in magnification and detector offset, and

minimize the apparent offset and tilt of the source trajectory plane. The method is described in detail in Appendixes 2 and 3. Finally, the beam geometry parameters are recalculated in the source coordinate system via the approximate method described in Appendix 4. Converting to the source trajectory coordinate system provides the CBCT beam geometry as if the phantom had been placed exactly at the center of the imaging system and aligned perfectly with the gantry axis.

II.C.6. Phantom rotation

Inaccuracies in BB positions in real fiducial phantoms occur due to limited machining precision of the phantom fabrication process estimated to be on the order of 20 μm . In order to alleviate errors in beam geometry estimation that arise from positional inaccuracies of the BBs, the phantom is rotated 180° about its longitudinal axis and a second CBCT scan is obtained. Accurate beam geometry parameters are obtained by taking the mean of the beam geometry parameters, calculated in the source coordinate system, from the two scans. As a check, the phantom position and orientation is recalculated using the averaged results of the two scans; this calculation should ideally evaluate to phantom displacement $\mathbf{T}_p^s = (0, 0, 0)$ and orientation $(\alpha, \beta) = (0, 0)$, since the beam geometry parameters from the two scans were each calculated in the source coordinate system.

II.D. Computer simulations

The beam geometry method described above was implemented in MATLAB. The method was tested using an *in silico* phantom consisting of two rings of 8 BBs each as previously described. The imaging system in the simulation possessed the same geometry as the Varian Trilogy on-board CBCT system used experimentally: SAD = 100 cm, DAD = 50 cm, 1024×768 detector pixels spaced 0.388 mm in both dimensions. Positions (X^i, Y^i) of forward projections of the 16 BBs onto the detector are calculated as in Cho *et al.*¹⁴ for 660 gantry angles ranging from 0 to 360°, and served as input to the beam geometry estimation algorithm. To simulate the effects of image discretization and partial volume effects, which limits the precision with which the exact center of a BB can be determined, the calculated BB positions were varied from projection to projection by adding to X^i and Y^i values drawn from a normal distribution with zero mean and constant rms amplitude (i.e., standard deviation) typically on order of 10% of a pixel. This amplitude provides noise in the beam geometry parameters of approximately the same magnitude as seen in the experimental results. Other input to the simulations included the position $\mathbf{T}_p^s$ and orientation (α, β) of the phantom as well as nonidealities of the beam geometry including detector offset (U_o, V_o) and rotation (η, θ, ϕ) . For all computer simulations, we utilized a set of beam geometry input values of the same order as those found experimentally: SAD and DAD as above, $U_o = V_o = 1$ mm, $\eta = \theta = \phi = 0.2^\circ$, DEV = 0, and zero source displacement perpendicular to the gantry axis (i.e., gantry angle deviation). These values were constant with gantry angle. Phantom

position and orientation for all simulations was $\mathbf{T}_p^s = (t_1, t_2, t_3) = (2 \text{ mm}, 3 \text{ mm}, 4 \text{ mm})$ and $\alpha = 0.3^\circ$, $\beta = 50^\circ$.

The effects of BB position inaccuracy that can arise from the phantom fabrication process were modeled by adding a positional offset to the BB positions that was constant for all gantry angles. Both random-type BB positional errors and systematic-type errors were investigated. Random-type BB positional error is intended to mimic imprecise placement of the machining tool when drilling the BB holes in the phantom. The magnitude of geometric fabrication error depends on the machining method used and size of the piece;²⁶ for BB placement of the current fiducial phantom this error is on order of tens of microns, or 1/1000th of an inch. Random-type BB positional errors were generated by adding to the BB positions in all three axes an offset drawn from a normal distribution characterized by zero mean and a given rms amplitude. Simulations were performed with 21 values of rms amplitude varying from 0 to 0.1 mm. We also investigated a systematic-type of error, in which the BB positions are not randomly displaced from their ideal positions, but are displaced systematically in groups. One example of this type of fabrication error would be the displacements in BBs that occur when the machinist has drilled holes for the BBs in a perfect circle, but there is a slight misalignment in placing the work in the machine, causing the end ring to be tilted. We simulated this type of error by offsetting four contiguous BBs in an end ring along the axis of the phantom, and moving the other four BBs in the opposite direction (Fig. 3). The same offsets were applied to the other end ring but in the opposite direction such that the two end rings tilted away from one another. The offset was again varied from 0 to 0.1 mm. This sort of end ring tilt was chosen over other systematic-type errors (such as squashing of the end rings) because it resulted in large beam geometry deviations. Uncertainty analyses of the effect of BB position error on the projections due to image noise or in the phantom were performed by plotting the uncertainty in beam geometry parameters versus BB position error. Beam geometry parameter uncertainty is defined as the rms deviation from the true, constant value, when averaged over all gantry angles. Uncertainty in the

Fig. 3. Displacements of BB positions in z -direction (along the longitudinal axis of the cylinder) used in computer simulations of systematic-type fabrication imperfections. The modeled fabrication error causes the BB endrings to tilt toward one another. Displacements are exaggerated and not to scale.

phantom position and orientation is defined simply as the differences from the known values. For studies of imperfect phantoms, a second simulation with a rotated phantom was also performed as described above. Finally, the effect of imperfect phantom rotation for the second scan was investigated by performing the simulations with phantom rotation angles varying from 180° to 190° , in this experiment utilizing random 3D BB position errors of rms amplitude $25 \mu\text{m}$.

Output consisted of estimates of the nine beam geometry parameters in both the phantom and the source coordinate systems for every gantry angle, as well as the position and orientation of the phantom. Each simulation required approximately 20 s on an ordinary personal computer.

II.E. Experimental testing

Experimental validation of the method was performed by scanning the fiducial phantom described above using the on-board CBCT imaging system of a Varian Trilogy with an On-Board Imager CBCT (Varian Medical Systems, Palo Alto, CA). The phantom was placed on a piece of styrofoam overhanging the couch to avoid interference of the couch in the x-ray beam. The phantom was positioned using the room lasers. A full-fan acquisition of 660 projections with no bow-tie filter was obtained. The phantom was then rotated 180° about its longitudinal axis and a second CBCT scan was obtained. The projections from the two scans served as input to the beam geometry estimation method described above. Output consisted of estimates of the nine beam geometry parameters versus gantry angle and the position and orientation of the phantom for each of the two scans, as well as the accurate beam geometry parameters obtained by taking the mean of the beam geometry parameters from the two scans. CBCT acquisition required approximately 1 min, and the BB position finding and beam geometry code required approximately 1 min.

III. RESULTS

III.A. Sensitivity analysis via computer simulations

We first present results of the computer simulation assuming a phantom with perfectly accurate spatial location of BBs. Figure 4 shows the beam geometry parameters versus gantry angle, calculated in the virtual coordinate system prior to phantom location. Some of the parameters exhibit variations with changing gantry angle due to the phantom's positional offset and nonideal orientation with the gantry axis of rotation. The beam geometry parameters recalculated in the source coordinate system are shown in Fig. 5. Calculated position and orientation of the phantom was $\mathbf{T}_p^s = (1.97, 2.96, 3.96 \text{ mm})$ and $\alpha = 0.29^\circ$, $\beta = 49.9^\circ$. The effect of adding 2D random error to the BB positions on each projection, intended to mimic BB position uncertainty due to image discretization and partial volume effect, is shown in Fig. 6 (dotted lines). We note that adding random error with amplitude of 7% of the detector pixel size (i.e., 0.027 mm) gives rise to approximately the magnitude of error observed in parameters calculated from real

FIG. 4. Beam geometry parameters from computer simulation, assuming a perfect phantom. Parameters are calculated in the virtual coordinate system, prior to transformation to the source coordinate system. The horizontal line in each plot is the true value. Deviations from the true value are caused by nonideal placement and orientation of the phantom from isocenter.

experimental projections; therefore, all of the following results included the addition of that amplitude of error to the BB positions on each projection.

Next we present results of the computer simulation using a phantom with inaccurate 3D spatial location of BBs. Displayed in Fig. 6 (dotted-dashed lines) is the uncertainty in the calculated beam parameters as a function of the amplitude of the random BB position error in x, y , and z for the 16 BBs. This three-dimensional geometric fabrication positional error is constant for all projections, as distinct from the two-dimensional error added to each projection to mimic the effects of image discretization and partial volume. The uncertainty analysis of systematic-type BB positional errors is shown in Fig. 6 (solid lines). The range of the uncertainty analysis, 0–0.1 mm, or up to 4/1000ths of an inch encompasses the approximate range of fabrication errors expected for milling and boring a piece the size of the phantom we used.²⁶

Shown in Fig. 7 is the result of beam geometry calculations simulating a second CBCT with the phantom rotated 180° about its longitudinal axis. In each figure are plotted the beam geometry versus gantry angle for the first simulation, the second simulation with the phantom rotated 180° about its long axis, and the averaged result of the two scans. Shown are the SAD, DAD, and detector roll and pitch parameters; the other beam geometry parameters exhibited similar improvements, some to a lesser degree. Results for systematic-type errors (not shown) are very similar to those for the random error.

FIG. 5. Beam geometry parameters from computer simulation, assuming a perfect phantom. Parameters are calculated in the source coordinate system following phantom location. The horizontal line in each plot is the true value. The vertical axes are expanded compared to Fig. 4.

Figure 8 displays the uncertainty in the averaged beam parameters as a function of the amplitude of 3D systematic and random fabrication BB position errors as well as 2D projection positional errors. The beam geometry parameter uncertainties using random and systematic BB position errors with a typical rms amplitude (25 μm) for a single scan (from Fig. 6), and then averaged with a repeat scan with rotated phantom, (from Fig. 8), are listed in Table II. Finally, no trends in beam geometry parameters were seen in varying the rotation of the phantom about its long axis for the second scan from 180° to 190°, with the exception of the SAD and DAD, uncertainties of which increased from approximately 0.2 and 0.1 mm, to 0.4 and 0.2 mm, respectively.

III.B. Validation via phantom experiments

We present the results of application of the method to experimentally acquired CBCT projections of the phantom. Figure 9 shows plots of several of the CBCT beam geometry parameters versus projection angle, as calculated using our modified algorithm, for the original CBCT scan, a repeat scan with the phantom rotated 180°, and the averaged result. The phantom position and orientation with respect to the gantry axis changed between the two scans: $\mathbf{T}_p^s = (0.42, 1.06, 2.51 \text{ mm})$, $\alpha = 0.28^\circ$, $\beta = 8.0^\circ$, and $\mathbf{T}_p^s = (0.25, 1.22, 1.90 \text{ mm})$, $\alpha = 0.16^\circ$, $\beta = 35.9^\circ$, for the first and second scans, respectively. Plotted in Fig. 10 are the source and detector (central pixel) trajectories in both the phantom and source coordinate systems. The

trajectories appear tilted in the phantom coordinate system due to the imperfect alignment of the phantom with the gantry axis. A large in-plane offset in source/detector trajectory centers (isocenters) is apparent for the first scan, and the offset is mitigated by averaging the two scans. The source/detector isocenter offset $(x,y,z) = (-0.9, -7.1, -1.4 \text{ mm})$ for first scan, $(1.3, 7.1, -1.4 \text{ mm})$ for second scan, and $(0.2, 0.0, -1.4 \text{ mm})$ after averaging the two scans.

IV. DISCUSSION

Several authors have described CBCT beam geometry estimation and correction schemes that do not require dedicated phantoms.^{17,20,21} We desired a method that: (1) would provide full nine parameter estimation of beam geometry; (2) would be applicable to both the CBCT imaging system and the MV treatment system; and (3) would provide isocenter coincidence between the imaging and treatment systems. To fulfill these requirements, we have implemented a fiducial-phantom-based method for estimating CBCT beam geometry based on one previously described.¹⁴

Cho *et al.* noted that the beam parameters can be recalculated in a coordinate system to make geometric calibration independent of the placement of the phantom.¹⁴ We have provided an explicit method for achieving this; i.e., transforming from the phantom to the source coordinate system. Failure to do so can result in large apparent sinusoidal variations in the beam geometry parameters (Fig. 4). These variations due to phantom placement are eliminated by recalculating the beam geometry parameters in the source coordinate system, as seen in Fig. 5; however, as discussed below, similar variations are introduced due to phantom fiducial marker position errors due to fabrication imprecision. Following the recalculation, the beam geometry estimation performed *in silico* is seen to be very accurate, with source and detector position within $100 \mu\text{m}$ of the true values, and detector and gantry angles less than 0.05° . Detector offsets are within $150 \mu\text{m}$ of the known value. Phantom position relative to the source isocenter and its orientation with respect to the gantry axis is within $50 \mu\text{m}$ and 0.1° of the true values, respectively.

Several investigators have added Gaussian noise, typically on order of 10% of pixel pitch, to the fiducial marker positions on simulated CBCT projections to address both the error associated with determining the exact center of each BB on each 2D projection, as well as 3D marker positional error in the phantom, on beam geometry estimation accuracy.^{14,19,22–24} In this work, we have separated the effects of 2D projection BB positional error and 3D geometric fabrication error. The beam geometry errors arising from the former are seen to be up to an order of magnitude smaller than those arising from phantom fabrication errors (Fig. 6). This large difference is due to the smoothing filter applied to each beam parameter over gantry angle. Beam parameter uncertainty due to 2D projection BB positional error vary randomly from projection to projection and are filtered out, whereas errors from 3D BB position imprecision in the

phantom vary sinusoidally with gantry angle. We note that in applications where sudden, jerky changes in the beam geometry parameters are suspected to occur, the smoothing filter may need to be adjusted accordingly to allow higher frequency changes with gantry angle to be detected, with some concomitant loss of precision. Although there are some differences, comparison of random and systematic-type phantom BB positional errors due to fabrication imperfections indicate similar magnitude and trend of beam geometry uncertainties (Fig. 6). We attribute the similarity to the fact that the beam geometry calculations are all based on the fitted ellipses of the two BB endrings. The beam geometry calculation does not depend on whether the ellipses are tilted due to random or systematic errors in BB position; i.e., it is independent of the distance of individual BBs from the fitted ellipse.

Simulations indicate that our innovation for dealing with BB positional errors works well (Fig. 7). Averaging the results of a second scan (phantom rotated 180°) with the original scan (nonrotated phantom) provides more accurate beam geometry parameters that do not vary sinusoidally with gantry angle. Comparison of the uncertainty analyses with the second scan (Fig. 8) and without the second scan (Fig. 6), reveal a dramatic improvement in beam geometry accuracy for most of the parameters, showing an order of magnitude increase in accuracy for source and detector position as well as phantom location and orientation. At a typical BB random position error of 0.025 mm (0.001 in.), source

FIG. 6. Uncertainty of calculated beam geometry parameters in the source coordinate system versus amplitude of added error: (dotted lines) random error to the two-dimensional BB positions on the projections in the computer simulation; (dot-dashed lines) three-dimensional random error to the BB positions in the phantom; (solid lines) systematic error to the BB positions in the phantom.

FIG. 7. Simulated beam geometry parameters for phantom with random BB position errors of amplitude 0.02 mm, for a normal scan, a second scan with phantom rotated 180° about its longitudinal axis, and the averaged result of the two scans.

and detector position are found to be within 250 μm of the true values, and detector and gantry angles less than 0.2°. Detector offsets are within 100 μm of the known value. Phantom position relative to the source isocenter and its

FIG. 8. Uncertainty of calculated beam geometry parameters in the source coordinate system following a second simulated CBCT scan with rotated phantom, versus amplitude of added error: (dotted lines) random error to the two-dimensional BB positions on the projections in the computer simulation; (dot-dashed lines) three-dimensional random error to the BB positions in the phantom; (solid lines) systematic error to the BB positions in the phantom.

orientation with respect to the gantry axis is within 20 μm and 0.001° of the true values, respectively.

Comparison of the uncertainty analyses with the second scan and without the second scan for 2D projection and 3D random and systematic BB position error of 0.025 mm are shown in Table II. Uncertainties for 2D projection errors are comparable with the ones in the original paper.¹⁴ The two-scan method is seen to have little effect for 2D BB positional errors; this is not surprising as these positional errors randomly vary from projection to projection, and rotating the phantom has no effect. The beam geometry uncertainties are virtually identical for 3D random and 3D systematic BB positional errors, both exhibiting a factor of from 3x to 15x reduction in uncertainty for source and detector distance and detector angles using the two-scan method. Systematic errors in the endrings of shape different than the tilting one chosen (Fig. 3), may produce different results; however, the one chosen gave the largest geometric errors, so we surmise that systematic 3D errors do not result in greater beam geometry errors than random 3D BB positional errors. We note that the method does not substantially reduce uncertainty in the detector piercing point offsets (Uo, Vo). The most important parameters for CBCT image quality are detector offsets, detector angle η (yaw), and magnification.¹⁴ Since the errors in the source and detector distances, SAD and DAD, vary similarly as a function of gantry angle (c.f. Figs. 7 and 9), the net result is little error in magnification resulting from BB position error, as mentioned in Appendix 2. Therefore, while the innovative method has important implications for improving system geometric calibration, including isocenter localization, it may have less impact on improving CBCT image quality.

Experimental CBCT acquisitions appear to validate the two-scan method for mitigating BB positional errors (Fig. 9). Large sinusoidal oscillations in beam geometry parameters versus gantry angle due to phantom imperfections are substantially reduced by averaging the two scans, even though the phantom position and orientation with respect to the gantry axis changed. Beam geometry parameters are for the most part insensitive to the exact angle that the phantom is rotated for the second scan, with the largest uncertainties in SAD and DAD being under 400 μm for up to a 10° error in phantom rotation (i.e., 190° vs 180°). Thus the second scan can be done without the need for precise positioning and rotation of the phantom. Sinusoidal variations in beam parameters with gantry angle arise from BB positional errors, and also from nonideal placement of the phantom at isocenter. The top row of Fig. 10, showing the calculated trajectories of the source and detector, illustrates how the transformation from phantom to source coordinate systems removes their apparent motions along the z-axis (the gantry axis); i.e., the apparent tilt of the trajectories due to nonideal phantom placement is removed by the transformation.

At the initial stages of this project, sinusoidal oscillations observed in the beam geometry parameters calculated using experimental CBCT projection data (Fig. 9), were assumed to arise solely from phantom misalignment and displacement from isocenter. However, we were unable to find a physically realistic coordinate system transformation that

TABLE II. Comparison between single scan and repeat scan method of CBCT beam geometry rms uncertainties, from simulations using two-dimensional (2D), three-dimensional random (3Dr) and three-dimensional systematic (3Ds) BB position error of $25\text{ }\mu\text{m}$.

rms uncertainties	SAD (mm)	DAD (mm)	DEV (mm)	η (deg)	ϕ (deg)	θ (deg)	Uo (mm)	Vo (mm)	γ (deg)
Single scan:									
2D	0.17	0.08	0.01	0.003	0.013	0.055	0.02	0.08	0.042
3Dr	3.33	1.65	0.07	0.010	0.219	0.649	0.04	0.16	0.052
3Ds	3.33	1.65	0.07	0.010	0.219	0.649	0.04	0.16	0.052
Averaged with second scan with rotated phantom:									
2D	0.16	0.09	0.02	0.003	0.010	0.028	0.02	0.06	0.040
3Dr	0.21	0.11	0.02	0.003	0.022	0.132	0.04	0.10	0.057
3Ds	0.21	0.12	0.02	0.003	0.022	0.132	0.04	0.10	0.057

completely and simultaneously accounted for both source and detector position oscillations. The manifestation of the problem is illustrated in the bottom row of Fig. 10, which exhibits a large ($>7\text{ mm}$) offset between the centers of the source and detector trajectories in the x - y plane. This isocenter offset is understood by noting that, even after transformation to the source coordinate system, the SAD and DAD oscillate in phase with one another for each scan (Fig. 9). We did not observe this problem with simulated data, assuming a perfect phantom, but were able to reproduce the problem by displacing BBs in the *in silico* phantom. Thus, we were led to conclude that phantom fabrication imprecision is a significant source of error in our beam geometry estimation algorithm, and invented a means for mitigating the error by using a second scan with a rotated phantom. We note that the source/detector isocenter offset (1.4 mm) in the z -direction (parallel to the gantry axis) is a real offset that remains consistent with gantry angle. It does not affect CBCT image quality, but must be taken into account for accurate image-guided radiotherapy, and is corrected for by the built-in Varian geometric calibration. We emphasize that the CBCT source/detector isocenter offset discussed here is relative to one another, not to the machine mechanical isocenter; how-

ever, eventual application of this technique to the MV treatment system to provide treatment system/ imaging system isocenter coincidence would be confounded if not for the ability of our algorithm to mitigate isocenter localization errors arising from phantom fabrication imprecision.

A simple interpretation for how the method works is provided by considering Fig. 3, which shows tilted ellipses fitted to the BB endrings in a phantom with fabrication errors. When the phantom is rotated 180° about the z -axis, the ellipses turn upside-down and the tilt is now opposite. By rescanning the phantom, and averaging the beam geometry results with the original scan, the ellipse tilting errors cancel. A more principled approach would be to first average the BB positions from the two scans before input into the beam geometry algorithm. However, this approach would require that the phantom position and orientation be exactly maintained between the two scans, a requirement not demanded by the current implementation. Finally, we note that a further additional scan following rotation of the phantom about the y -axis may be beneficial in improving accuracy of the detector offset parallel to the gantry axis.

FIG. 9. Beam geometry parameters for experimentally acquired CBCT projections, showing improvement in accuracy by averaging the two scans.

FIG. 10. CBCT source (outer orbit) and detector (inner orbit) trajectories, calculated in the phantom coordinate system (top left) and source coordinate system (top right), for experimentally acquired CBCT data, for the first scan, prior to rotating the phantom. Source/detector trajectory centers are non-coincident for the first scan (bottom left), and overlap after averaging the results with the second scan using the rotated phantom (bottom right).

V. CONCLUSION

We have validated, via computer simulation and experimentally, a fiducial phantom-based CBCT beam geometry estimation algorithm, based on a published one¹⁴ that does not require precise positioning of the phantom at machine isocenter, and is insensitive to positional imprecision of fiducial markers within the phantom due to fabrication errors. We present modifications to the published algorithm that provide stable detector rotation values in the presence of fiducial marker positional errors and present an explicit method for calculating the position and orientation of the phantom with respect to the CBCT imaging system isocenter. An innovative method for mitigating errors in the estimated beam geometry due to phantom fiducial marker imprecision, using a second CBCT scan of the phantom after rotation about its long axis, has also been presented. In particular, we have established that phantom fiducial marker imprecision can cause large apparent offsets in source and detector trajectory centers, and this has important implications for utilization of the method in quality assurance of IGRT systems.

Future work will include application of the method to the LINAC megavoltage source and EPID detector, providing geometric calibration of the radiotherapy system. Since the CBCT and radiotherapy systems will scan a common phantom, isocenter coincidence of the therapy and on-board imaging systems can also be measured. The method has the potential to replace the Winston–Lutz test, providing a single phantom and methodology for performing quality assurance of CBCT and MV image quality as well as therapy/imaging system isocenter coincidence, similar to the method described by Mao *et al.*¹³ Finally, we note that the rotated phantom innovation for mitigating phantom fabrication errors may be applicable to other fiducial phantom-based geometric calibration algorithms.

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APPENDIX

1. Replacement algorithm for estimation of detector rotation angles θ and ϕ

We follow the nomenclature convention used by Cho *et al.*¹⁴ summarized in Table I. The modified algorithm separates the calculation of detector angles θ and ϕ , which is an approximation appropriate for small angles. The algorithm first calculates the “converging points” for the two angles, P_θ and P_ϕ as described in Ref. 14. The converging point $P_\theta(\mu_\theta, \nu_\theta)$, and $P_\phi(\mu_\phi, \nu_\phi)$, are the (x^I, y^I) coordinates in the real detector plane where the lines joining the endpoints of the two ellipses, and the major axes of the two fitted ellipses intersect, respectively (Fig. 11). If P_θ or P_ϕ exceeds a threshold distance from the detector origin, the corresponding

angle is vanishing and is set to zero. Otherwise, ϕ is calculated via the root finding routine “fzero” in MATLAB applied to the following equations¹⁹:

$$\sin(\phi) = -c_1 \xi_1 / (2a_1) - c_2 \xi_2 / (2a_2)$$

$$\xi_k = Z_S^I a_k \sqrt{a_k} / \sqrt{a_k b_k + a_k^2 b_k (Z_S^I)^2 - c_k^2}, \quad k = 1, 2$$

$$Z_S^I = \mu_\phi \sin(\phi) \cos(\phi)$$

where the subscript k refers to the upper or the lower fitted ellipse in the projection, and a, b, c are the coefficients of the equation of the ellipse

$$a(x - x_0)^2 + b(y - y_0)^2 + 2c(x - x_0)(y - y_0) = 1$$

Finally, the detector angle θ is calculated from

$$\theta = \sin^{-1}(Z_S^I \cos(\phi) / \nu_\theta)$$

where the source to detector distance, Z_S^I is calculated according to Ref. 14

$$Z_S^I = 2rL_1L_2 / (l(L_2 - L_1))$$

where r is the radius of the ring of BBs and l is the distance between the two rings (51 and 162 mm, respectively), and L_1 and L_2 are inner and outer distances between the projections of the BB rings, as shown in Fig. 11.

2. Algorithm for estimation of phantom position

Phantom position in the source coordinate system is estimated in the following manner. The apparent displacement of the phantom as the gantry rotates is given by

$$dx = t_1 \cos(\gamma) + t_2 \sin(\gamma)$$

$$dy = t_1 \sin(\gamma) - t_2 \cos(\gamma)$$

where $\mathbf{T}_p^s = (t_1, t_2, t_3)$. We transform the z -component of detector position Z_D^I magnification factor M , detector offset Uo , and y -component of source position Y_S^I to source coordinates with the following approximation:

FIG. 11. Rays in real detector plane used to calculate convergence points P_θ P_ϕ as described by Cho *et al.* (Ref. 14).

$$\text{DAD} \equiv \sqrt{(X_D^s)^2 + (Y_D^s)^2} = Z_D^i - dx$$

$$\text{SAD} = Z_S^i - \text{DAD}$$

$$M^s = Z_S^i / \text{SAD}$$

$$Uo^s = Uo^i - M^s dy$$

$$\text{DEV} \equiv Z_S^i = Y_S^i - Y_D^i - t_3 + \text{SAD} \sin(\alpha \cos(\gamma - \beta))$$

noting that Z_S^i , the source to detector distance, is independent of phantom position. The following objective function is then minimized by varying $\mathbf{T}_p^s = (t_1, t_2, t_3)$ using the routine “fmincon” in MATLAB:

$$f(t_1, t_2, t_3) = (\sigma_M + \sigma_{Uo} + \overline{Z_S^s})$$

where σ_M and σ_{Uo} are the standard deviation of M^s and Uo^s over all gantry angles, and $\overline{Z_S^s}$ indicates the mean. Thus the intent of this optimization is to minimize the apparent sinusoidal oscillations in magnification and detector offset, and the apparent offset of the source trajectory plane, that arise from the phantom center being displaced by $\mathbf{T}_p^s = (t_1, t_2, t_3)$ from the center of the source trajectory. Although the magnification is not one of the nine beam geometry parameters, it is used in the optimization objective function because it is much less sensitive to phantom fabrication error than is SAD and DAD.

3. Algorithm for estimation of phantom orientation

Estimation of the phantom orientation relative to the source coordinate system (α, β) is facilitated by transforming the source position in the virtual coordinate system P_S^i to the source position in the source coordinate system P_S^s . This requires calculation of several rotation matrices. One matrix characterizes the rotation from phantom to source coordinate system, $\mathbf{R}_p^s$ and is given by

$$\mathbf{R}_p^s = R_z(-\beta)R_y(\alpha)R_z(\beta)$$

$$\text{where } R_z(-\beta) = \begin{pmatrix} C\beta & -S\beta & 0 \\ S\beta & C\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_y(\alpha) = \begin{pmatrix} C\alpha & 0 & -S\alpha \\ 0 & 1 & 0 \\ S\alpha & 0 & C\alpha \end{pmatrix},$$

$$R_z(\beta) = \begin{pmatrix} C\beta & S\beta & 0 \\ -S\beta & C\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and $C\beta = \cos(\beta)$, $S\beta = \sin(\beta)$, $C\alpha = \cos(\alpha)$, $S\alpha = \sin(\alpha)$. A second matrix characterizes the gantry rotation from virtual detector to the phantom coordinate system, $\mathbf{R}_i^p$ and is given by

$$\mathbf{R}_i^p = \begin{pmatrix} S\gamma & 0 & C\gamma \\ -C\gamma & 0 & S\gamma \\ 0 & -1 & 0 \end{pmatrix}$$

where $C\gamma = \cos(\gamma)$, $S\gamma = \sin(\gamma)$. The final matrix characterizes an apparent rotation caused by the displacement of the phantom from source trajectory center

$$\mathbf{R}_d = \begin{pmatrix} Cb & -Sb & 0 \\ Sb & Cb & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $Cb = \cos(b)$, $Sb = \sin(b)$, and $b = \tan^{-1}((-t_2 Cg + t_1 Sg)/Z_S^i)$. The source position in the source coordinate system is given by

$$P_S^s = \mathbf{R}_d \mathbf{R}_p^s \mathbf{R}_i^p (P_S^i - T_p^i) + T_p^s$$

The following objective function is then minimized by varying (α, β) using the routine “fmincon” in MATLAB:

$$f(\alpha, \beta) = \sigma_z$$

where σ_z is the standard deviation of the z -component of source position Z_S^s over all gantry angles. This optimization minimizes the apparent tilt of the source trajectory plane due to the misalignment of the phantom with the axis of gantry rotation.

4. Transformation of beam parameters to source coordinate system

Once the phantom position and orientation are known, the remaining beam geometry parameters (c.f. Appendix 2) may be transformed to the source coordinate system via the following approximation:

$$Vo^s = Vo^i + M^s t_3$$

$$\eta^s = \eta^p + \alpha \sin(\gamma - \beta)$$

$$\theta^s = \theta^p + \alpha \cos(\gamma - \beta)$$

$$\phi^s = \phi^p - \tan^{-1}(dy/\text{SAD})$$

$$\gamma^s = \gamma^p$$

This set of equations has its basis in a heuristic approach to removing the important contributors to beam geometry deviation caused by misplacement of the phantom and ignores higher-order terms in the coordinate transformation, the validity of which is demonstrated via computer simulation by the close agreement between calculated and true beam geometry parameters.

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