Material parameter computation for multi-layered vocal fold models

Bastian Schmidt,^{a)} Michael Stingl, and Günter Leugering

Department Mathematics, Applied Mathematics II, University of Erlangen, Martensstr. 3, 91058 Erlangen, Germany

David A. Berry

Laryngeal Dynamics Laboratory, Division of Head and Neck Surgery, Medical School, University of California Los Angeles, 1000 Veteran Avenue, Los Angeles, California 90095-1794

Michael Döllinger

Department of Phoniatrics and Pediatric Audiology, University Hospital Erlangen, Medical School, Bohlenplatz 21, 91054 Erlangen, Germany

(Received 8 April 2010; revised 22 December 2010; accepted 28 December 2010)

Today, the prevention and treatment of voice disorders is an ever-increasing health concern. Since many occupations rely on verbal communication, vocal health is necessary just to maintain one's livelihood. Commonly applied models to study vocal fold vibrations and air flow distributions are self sustained physical models of the larynx composed of artificial silicone vocal folds. Choosing appropriate mechanical parameters for these vocal fold models while considering simplifications due to manufacturing restrictions is difficult but crucial for achieving realistic behavior. In the present work, a combination of experimental and numerical approaches to compute material parameters for synthetic vocal fold models is presented. The material parameters are derived from deformation behaviors of excised human larynges. The resulting deformations are used as reference displacements for a tracking functional to be optimized. Material optimization was applied to three-dimensional vocal fold models based on isotropic and transverse-isotropic material laws, considering both a layered model with homogeneous material properties on each layer and an inhomogeneous model. The best results exhibited a transversal-isotropic inhomogeneous (i.e., not producible) model. For the homogeneous model (three layers), the transversal-isotropic material parameters were also computed for each layer yielding deformations similar to the measured human vocal fold deformations. © 2011 Acoustical Society of America. [DOI: 10.1121/1.3543988]

PACS number(s): 43.70.Bk, 43.70.Gr [AL]

Pages: 2168–2180

I. INTRODUCTION

A high number of communication and speech disorders are caused by disturbances in vocal fold dynamics. The human voice, which we perceive as the acoustic signal emitted from the mouth after modulation by the vocal tract (i.e., nasal and oral cavities), is generated by laryngeal dynamics. The primary voice signal is generated within the larynx by two oscillating vocal folds (100–400 Hz), Fig. 1. This is the sound source of the carrier signal which transfers information via speech.

The vocal folds possess a three-dimensional (3D) geometry and exhibit 3D vibration patterns, Fig. 1. In normal voices, the dynamics originate in the inferior part of the vocal folds, e.g., as a mucosal wave traveling in a superior direction along the medial surface of the vocal folds. In terms of its influence on voice production, the most critical region of mucosal wave propagation occurs on the medial surface of the vocal folds. There, it generates relatively large tissue vibrations in lateral (up to 2.2 mm) and vertical (up to 1.3 mm) directions.^{1,2} Mucosal wave propagation along the vocal fold surface is governed by a complex set of dynamics that is only beginning to be understood.³

Numerical⁴ and physical models^{5,6} are applied to analyze vocal fold dynamics and laryngeal air flow. However, because of the complex multi-layered tissue structure of vocal folds, numerical and physical models usually represent a considerable simplification of the actual physical system. Commonly the suggested physical models are air driven whereby the vocal folds are built of silicone⁷ or polyurethane⁸ mixtures. The utilized vocal fold bodywork varies between one layered⁶ and multi-layered⁹ frameworks. However, so far biomechanical parameters for physical models are commonly derived from experimental data obtained from both the laboratory and the clinic^{10–13} and are then one-to-one transferred to synthetic models. Laboratory experiments have included animal models^{14–19} and human *in-vitro* models.^{20–22} However, the problem hereby is, that indeed the built synthetic models exhibit the same material parameters (e.g., Young's modulus, Poisson's ratio, shear modulus) but the assignment of such parameter values to an idealized model hardly guarantees that the resultant synthetic model will yield dynamics which are similar to the original larynx.

Therefore, we propose a combination of experimental and numerical techniques to build synthetic vocal fold models which exhibit the same behavior as real vocal folds: The static mechanical characteristics of excised human larynges are transferred to a numerical model being

^{a)}Author to whom correspondence should be addressed. Electronic mail: Bastian.Schmidt@am.uni-erlangen.de

FIG. 1. An endoscopic view onto the vocal folds.

equal to the synthetic multi-layer vocal fold model. We use optimization techniques to minimize any differences between the dynamics of the resultant theoretical model and the laboratory model. In other words, the dynamic characteristics of human vocal folds with complex geometry and anatomy are transferred toward synthetic vocal folds with less complicated geometry and a pre-defined layer framework. The material parameters of the synthetic vocal folds do then not necessarily coincide with human vocal folds, but the dynamics do.

The underlying mathematics, namely material parameter optimization procedures, is still in their infancies in voice research. In order to compute optimal material parameters for a numerical model, an inverse optimization problem must be formulated and solved. This optimization problem belongs to the broad class of structural optimization problems. Structural optimization has been applied successfully in many fields, especially within the aircraft or automotive industries. Thousands of scientific investigations of structural optimization have been reported, e.g., Refs. 23 and 24. For an overview, the interested reader is referred to the monographs.^{25,26} Typically, the goal in

structural optimization is to find a design which is as light as possible and can simultaneously withstand a specified load. In this work, the objective is different: here, a deformation associated with a certain load is prescribed. Then material parameters are identified for which the deviation from this prescribed deformations is as small is possible. This is similar to what is reported in Ref. 27. However, in that study, a so-called “free material” is investigated, while in the present study more specific geometries and material laws (e.g., isotropy, transverse isotropy) were assumed. This is similar to what is reported in Ref. 28 for acoustics.

In this work, we present the generally valid and applicable mathematical methods and apply them to measurements on an excised human larynx.

In future, analysis methods need to be developed to objectively quantify severity of disease and to document therapy. Finally, understanding causalities within the human larynx may help to guide surgical interventions necessitated by anatomical alterations or cancer.

II. METHODS

A. Hemilarynx experiments

The deformation behavior along the entire surface of an excised human hemilarynx serves as template for the resulting physical model. The excised human larynx was obtained from the autopsy unit of the University of California Los Angeles Medical Center. The larynx was obtained from a 31 year old male. The larynx was quick-frozen with liquid nitrogen to maintain tissue properties,²⁹ stored in -20°C freezer for 6 months, and defrosted the day before the experiment. The hemilarynx methodology was originally developed for investigating vocal fold vibrations.^{30–32} However, the methodology is also suitable for imaging static displacements of the vocal folds. A detailed description of the setup was given previously,¹ therefore, only a brief overview of the imaging methods and mathematical techniques for computing the 3D physical coordinates of the vocal fold geometry along the medial surface of the folds will be given. The creation of a hemilarynx required the removal of one (i.e., right) vocal fold (Fig. 2). The trachea was mounted over a cylindrical tube. A glass plate

FIG. 2. On the left, the view through the prism on the vocal fold is depicted. The two induced views enable the 3D computation of the suture positions. Here from the 3D coordinates of the sutures are computed. On the right, the reconstructed 3D vocal fold can be seen. Here, on suture S2 it was pulled applying 10 g.

was attached at the top of the tube. Images with a digital camera at a pixel resolution of 1024×1024 were taken. A right-angle prism was placed at the glass plate, opposite the vocal fold, to yield two distinct camera views of the vocal folds—a necessary condition to compute 3D coordinates from a two-dimensional (2D) recording.³³ For later calibration, a brass cube (5 mm^3) was glued to the glass plate superior to the vocal fold.^{32,34} The tracking of suture positions and the computation of the physical coordinates was performed using a linear transformation method.³² The resulting root-mean square linearization error³² was computed at 4.9%. To track the displacements of the entire surface of the vocal fold, 30 surgical microsutures were mounted, see Fig. 2. They were positioned in five vertical rows with six sutures per row. The rows were placed along the medial surface. The top-most row was placed above the vocal fold edge. The vertical and horizontal distances between the sutures in rest position were approximately 2 mm. The sutures were positioned to penetrate only the mucosal epithelium and not the superficial layer.³¹ Deformations of the vocal fold were induced by approximately vertically pulling sutures of the most superior suture line with different weights (10–100 g). The computed displacements of the sutures were calculated relative to the rest positions. Figure 3 shows the displacements of S2, S3, and S4, being pulled with weights ranging from 10 to 100 g. From that, we assume, that the tissue shows approximately linear properties up to 20 g. Therefore, a linear stress–strain relation could be assumed (Sec. II B 2) for numerical modeling of the deformations induced by up to 20 g weights. In detail, pulling deformations induced by pulling S2, S3, and S4 and S2S3S4 together using 10 g and pulling S1S2S3S4S5 (10, 10, 20, 10, and 10 g) together were applied for the multiple-load case in the optimization algorithm. For the optimization algorithm, the deformations of all 30 sutures along the vocal fold surface were considered.

FIG. 3. Magnitude of displacement of sutures S2, S3, and S4 and the corresponding weights were applied.

B. Mathematic modeling and optimization

1. List of symbols

$\Omega \in \mathbb{R}^d$	Elastic body with Lipschitz boundary $\Gamma := \partial\Omega$
$\Gamma_0 \subseteq \Gamma$	Dirichlet boundary
$\Gamma_T \subseteq \Gamma$	Tracking part of the boundary
$L^2(\Omega; \mathbb{R}^d)$	Hilbert space of quadratically integrable functions
$\mathcal{H} := H^1(\Omega; \mathbb{R}^d)$	Sobolev space of quadratically integrable functions with weak derivatives
$L^\infty(\Omega; \mathbb{R}^d)$	Banach space of essentially bounded functions
$u \in \mathcal{H}$	Displacement vector field
$f \in L^\infty(\Gamma; \mathbb{R}^d)$	Boundary traction
$e(u) \in L^2(\Omega; \mathbb{R}^d)$	Strain tensor (small)
$\sigma \in L^2(\Omega; \mathbb{R}^d)$	Stress tensor
$\mathcal{P} \in L^\infty(\Omega; \mathbb{R}^p)$	Set of admissible material parameters
$\alpha \in \mathcal{P}$	Material parameter vector
$C(\alpha) \in L^\infty(\Omega; \mathbb{R}^{d \times d \times d})$	Material tensor associated with parameter vector α
$\gamma \in \mathbb{R}$	Regularization parameter
$E, \nu \in \mathbb{R}$	Young's modulus and Poisson's ratio ³⁹ characterizing isotropic material
$\lambda, \mu \in \mathbb{R}$	Lamé parameters ³⁹ characterizing isotropic material
$G \in \mathbb{R}$	Shear modulus

2. The physical model

The result of the hemilarynx experiments as described in Sec. II A are measured displacement fields for given surface tractions. Accordingly, the physical model of our choice is of static nature. It is described as follows: Let $\Omega \subset \mathbb{R}^d$ with $d \in \{2, 3\}$ denote an elastic body¹⁶ with Lipschitz boundary $\Gamma = \partial\Omega$. On Ω , we consider the following boundary value problem of linear elasticity:

Find a displacement field $u \in H^1(\Omega; \mathbb{R}^d)$, s.t.,

$$\begin{aligned}
 -\operatorname{div}(\sigma) &= 0 & \text{in } \Omega \\
 \sigma \cdot n &= f & \text{on } \Gamma_F \\
 u &= 0 & \text{on } \Gamma_0 \\
 \sigma &= C \cdot e(u) & \text{in } \Omega.
 \end{aligned} \tag{1}$$

Here $f \in L^2(\Gamma_F; \mathbb{R}^d)$, $\Gamma_F \subset \Gamma$ is a given surface force (the traction force in the experiments, for instance) and $L^2(\Omega; \mathbb{R}^d)$ denotes the space of quadratically integrable functions on Ω , which takes values in $\mathbb{R}^d$. When loaded by f the elastic body is deformed; the corresponding displacement field is denoted by $u \in \mathcal{H} := H^1(\Omega; \mathbb{R}^d)$, where $H^1 \subset L^2(\Omega)$ is the Sobolev space of quadratically integrable functions with weak derivatives. Associated with the displacement field is the (small) strain tensor given as $e(u) = \frac{1}{2}(\nabla u + \nabla u^T) \in L^2(\Omega; \mathbb{R}^{d \times d})$. The elastic properties of the material are characterized by a material tensor $C \in L^\infty(\Omega; \mathbb{R}^{d \times d \times d \times d})$. In the most general setting, the tensor C is a function of the space variable, i.e., it varies from point to point. Finally the strain tensor e is related to the stress tensor σ by Hooke's law $\sigma = C \cdot e(u) \in L^\infty(\Omega; \mathbb{R}^{d \times d})$.

As the material tensor C as well as the strain and stress tensors satisfy the following symmetry properties,

$$C_{ijkl} = C_{jikl} = C_{klij}, \quad i, j, k, l = 1, \dots, d.$$

$$e_{ij} = e_{ji}, \sigma_{ij} = \sigma_{ji}, \quad i, j = 1, \dots, d,$$

we can define $d' := \frac{1}{2}d(d+1)$ and rewrite $e \in L^2(\Omega; \mathbb{R}^{d'})$, $\sigma \in L^\infty(\Omega; \mathbb{R}^{d'})$, as well as $C \in L^\infty(\Omega; \mathcal{S}^{d'})$, where $\mathcal{S}^{d'}$ is the space of all symmetric $d' \times d'$ -matrices. The rewritten tensors are explicitly given by

$$\begin{aligned} e &= (e_{11}, e_{22}, \sqrt{2}e_{12})^\top, \\ \sigma &= (\sigma_{11}, \sigma_{22}, \sqrt{2}\sigma_{12})^\top, \\ C &= \begin{pmatrix} C_{1111} & C_{1122} & \sqrt{2}C_{1112} \\ & C_{2222} & \sqrt{2}C_{2212} \\ \text{sym.} & & 2C_{1212} \end{pmatrix} \in L^\infty(\Omega; \mathcal{S}^{d'}), \end{aligned}$$

for the 2D case. Analogous vectors and matrices can be derived for the 3D case. In the remaining part of this work e , σ , and C will always denote the rewritten tensors.

In the context of optimization it has certain advantages to work with the weak formulation of partial differential equations. Accordingly, we state the weak form of system (1): find $u \in \mathcal{H}_0 := \{u \in \mathcal{H} | u|_{\Gamma_0} = 0\}$, s.t.,

$$\int_{\Omega} \langle C(x)e(u)(x), e(v)(x) \rangle dx = \int_{\Gamma_F} f(x)^\top v(x) dx \quad \forall v \in \mathcal{H}_0. \quad (2)$$

3. Formulation of the optimization problem

On the basis of the physical model stated in Sec. II B 2, we consider the following problem:

Given a deformation $u_0 \in L^2(\Gamma_T)$ on a part of the boundary and given a force f applied to the body Ω , find a material C^* , which generates a deformation u^* as close as possible to the given u_0 .

In our particular application, the given boundary deformation arises from the hemilarynx experiments described in Sec. I as well as the load f . The formulation above is stated for the *single-load* case. However multiple reference deformations $u_{i,0}$ and corresponding forces f_i , $i = 1, \dots, M$ can be combined in a natural way using different weighting factors w_i . The associated *multiple-load* optimization problem reads as follows:

$$\min_{\alpha \in \mathcal{P}} J(\alpha) := \sum_{i=1}^N w_{i,2} \|u_i(\alpha) - u_{i,0}\|_{\Gamma_T}^2 + \gamma \|\alpha\|^2, \quad (3)$$

s.t.,

$u_i(\alpha)$ is a solution of (2) with force f_i , $i = 1, \dots, N$,

$$\underline{V} \leq \text{vol}(C(\alpha(x))) \leq \bar{V}.$$

Here the “volume” of the material $C(\alpha)$ is defined as

$$\text{vol}(\alpha) := \int_{\Omega} \text{Tr}(C(\alpha(x))) dx, \quad (4)$$

a measure for the over-all stiffness of the material. The set of *admissible material parameters* $\mathcal{P}$ in its most general form consists of all material parameters given for every point in the domain Ω , i.e.,

$$\mathcal{P} \subset L^\infty(\Omega; \mathbb{R}^p). \quad (5)$$

Using a given Hookes’s law, described by a function $C : \mathbb{R}^p \rightarrow \mathcal{S}^{d'}$; $\alpha \mapsto C(\alpha)$ (e.g., isotropic or transversal-isotropic; cf. Sec. II B 4), this p -tuple of parameters is mapped to a material tensor at each point of the domain Ω . Subsequently, the parameter set $\mathcal{P}$, will be chosen such that C contains only material, which is physically attainable; for instance we have to guarantee that $C(\alpha)$ is positive definite for all $\alpha \in \mathcal{P}$.

Problem (3) with the set of admissible material parameters $\mathcal{P}$ given by Eq. (5) is an infinite dimensional optimization problem, where we consider material parameters independent for every point of the domain. In addition to this problem, we investigate simpler optimization problems, in which the domain Ω consists of a given number of pair-wise disjoint subdomains Ω_i , $i = 1, \dots, M$ with

$$\bigcup_{i=1}^M \overline{\Omega_i} = \overline{\Omega},$$

where the material parameters are assumed to be constant. In other words, we constrict the parameter function α to have a constant value $\alpha_i \in \mathbb{R}^p$ on every subdomain Ω_i . The corresponding set of admissible material parameters is then given as

$$\mathcal{P}' = \{\alpha \in \mathcal{P}, \alpha \equiv \alpha_i \in \mathbb{R}^p \text{ a.e. in } \Omega_i, \quad i = 1, 2, \dots, M\}. \quad (6)$$

Problem (3) falls into the general class of design problems; for a survey see Refs. 25, 26, 35. and the references therein. Moreover due to the tracking functional $J(\alpha)$ problem (3) is a so-called inverse problem.³⁶ Inverse problems are known to be *ill-posed* in many cases. Hence, we add the tychonoff³⁷ type regularization term $\gamma \|\alpha\|^2$. It is further known that in case of non-uniqueness of the solution, the smallest norm solution can be retrieved by successively solving the optimization problem (3) with decreasing values of the regularization parameter γ . Additionally the volume constraint [third equation in Eq. (1)] can act as a regularization term as will be outlined in Sec. III A. If the volume of the optimal solution is known or at least can be estimated, one might not be interested in the solution with minimal norm. In this case the bounds $\underline{V}$ and $\bar{V}$ can be chosen tight around the estimated volume V and the regularization parameter γ is small or even zero.

4. Material laws

In the framework of this section we describe the material classes considered in problem (3). In particular we give precise formulas for possible parameter functions α and corresponding mappings C as well as the set of admissible materials $\mathcal{P}$ (5).

When choosing a material law to implement, one has to find a way to satisfy two opposing goals. Finding a material,

described by parameters, that is easily manufacturable and resembling the natural human vocal fold tissue as good as possible. Therefore we present different material laws (e.g., isotropic, transversal-isotropic) characterizing the material properties itself and two different approaches for the material distribution, either having several layers of homogeneous material or having a material varying from point to point (which could also be seen as having an infinite number of infinitesimal small layers of material). In Ref. 6 a model consisting of only one layer of silicon, an isotropic material, is described. We believe an isotropic model with three layers can also be manufactured with necessary precision. However, a vocal fold model with transversal-isotropic material has up to now not been produced. And a model with material parameters varying from point to point can easily resemble the natural material but will not be manufacturable in the near future. As transversal-isotropic material is very suitable for approximating laminates as well as fibrous materials, it can get very close to the natural human vocal fold tissue and we consider it the most general material law in this work. The results obtained using inhomogeneous transversal-isotropic material can only be seen as intermediate results, as approximations would have to be used in the manufacturing process.

First we consider *isotropic material*, which is characterized by the following symmetric material matrix:³⁸

$$C_{\text{iso}}(E, \nu) = \begin{pmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ & & \lambda + 2\mu & 0 & 0 & 0 \\ S & & & \mu & 0 & 0 \\ & Y & & & \mu & 0 \\ & & M & & & \mu \end{pmatrix}.$$

Here the Lamé parameters λ and μ are given in terms of the Young's modulus and Poisson ratio:³⁹

$$\lambda(E, \nu) := \frac{\nu E}{(1 + \nu)(1 - 2\nu)},$$

$$\mu(E, \nu) := \frac{E}{2(1 + \nu)} = G.$$

In order to guarantee positive definiteness of $C_{\text{iso}}(E, \nu)$, we assume $E > 0$, $-1 < \nu < \frac{1}{2}$. Consequently α , C , and $\mathcal{P}$ are given as

$$\alpha : \Omega \rightarrow \mathbb{R}^2, \quad x \mapsto (E(x), \nu(x)),$$

$$C : \mathbb{R}^2 \rightarrow \mathcal{S}^6, \quad (E, \nu) \mapsto C_{\text{iso}}(E, \nu),$$

and

$$\mathcal{P}_{\text{iso}} = \{\alpha = (E, \nu) \in L^\infty(\Omega; \mathbb{R}^2) \mid E > 0, -1 < \nu < \frac{1}{2} \text{ a. e. in } \Omega\}. \quad (7)$$

Moreover we work with *transversal-isotropic* material, which is aligned to the coordinate axes. It can be interpreted as an orthotropic material aligned to the coordinate axes,

for which additionally isotropy is imposed in the x - y -plane. Following Ref. 38 we define $D(E_{\text{iso}}, \nu_{\text{iso}}, E_o, \nu_{\text{oi}}) := \frac{1 + \nu_{\text{iso}}}{E_{\text{iso}}} (1 - \nu_{\text{iso}} - 2\nu_{\text{oi}}\nu_{\text{oi}})$ and write the transversal-isotropic tensor as

$$C_{\text{trans-iso}}(E_{\text{iso}}, \nu_{\text{iso}}, E_o, \nu_{\text{oi}}, G_{\text{io}}) = \begin{pmatrix} \frac{1 - \nu_{\text{oi}}\nu_{\text{oi}}}{D} & \frac{\nu_{\text{iso}} + \nu_{\text{oi}}\nu_{\text{oi}}}{D} & \frac{(1 + \nu_{\text{iso}})\nu_{\text{oi}}}{D} & 0 & 0 & 0 \\ & \frac{1 - \nu_{\text{oi}}\nu_{\text{oi}}}{D} & \frac{(1 + \nu_{\text{iso}})\nu_{\text{oi}}}{D} & 0 & 0 & 0 \\ & & \frac{(1 - \nu_{\text{iso}})E_o}{1 - \nu_{\text{iso}} - 2\nu_{\text{oi}}\nu_{\text{oi}}} & 0 & 0 & 0 \\ S & & & G_{\text{io}} & 0 & 0 \\ & Y & & & G_{\text{io}} & 0 \\ & & M & & & G_{\text{iso}} \end{pmatrix}, \quad (8)$$

where the parameters are bounded by

$$E_{\text{iso}} > 0, E_o > 0, G_{\text{io}} > 0, \quad (9)$$

$$-1 < \nu_{\text{iso}}, \nu_{\text{oi}}\nu_{\text{oi}} < 1, 1 - \nu_{\text{iso}} - 2\nu_{\text{oi}}\nu_{\text{oi}} > 0, \quad (10)$$

to ensure feasible materials. These constraints correspond to positive definiteness of the tensor. Note that equations $\nu_{\text{oi}}E_o = \nu_{\text{oi}}E_{\text{iso}}$ and $G_{\text{iso}} = E_{\text{iso}}/2(1 + \nu_{\text{iso}})$ hold. In order to get rid of the non-linear bounds Eq. (10) the non-physical quantities,

$$\theta := \nu_{\text{oi}} \sqrt{\frac{2E_o}{E_{\text{iso}}(1 - \nu_{\text{iso}})}} = \nu_{\text{oi}} \sqrt{\frac{2E_{\text{iso}}}{E_o(1 - \nu_{\text{iso}})}}, \quad (11)$$

are introduced. Then matrix Eq. (8), can be reformulated as

$$C_{\text{trans-iso}}(E_{\text{iso}}, \nu_{\text{iso}}, E_o, \theta, G_{\text{io}}) = \begin{pmatrix} \frac{2 - \theta^2(1 - \nu_{\text{iso}})}{2D} & \frac{\theta^2 + (2 - \theta^2)\nu_{\text{iso}}}{2D} & \frac{(1 + \nu_{\text{iso}})\nu_{\text{oi}}}{D} & 0 & 0 & 0 \\ & \frac{2 - \theta^2(1 - \nu_{\text{iso}})}{2D} & \frac{(1 + \nu_{\text{iso}})\nu_{\text{oi}}}{D} & 0 & 0 & 0 \\ & & \frac{E_o}{1 - \theta^2} & 0 & 0 & 0 \\ S & & & G_{\text{io}} & 0 & 0 \\ & Y & & & G_{\text{io}} & 0 \\ & & M & & & G_{\text{iso}} \end{pmatrix}, \quad (12)$$

with $D(E_{\text{iso}}, \nu_{\text{iso}}, \theta) = (1 - \nu_{\text{iso}}^2)/E_{\text{iso}}(1 - \theta^2)$. Now conditions [Eqs. (9) and (10)] simplify to box constraints,

$$E_{\text{iso}} > 0, E_o > 0, G_{\text{io}} > 0,$$

$$-1 < \nu_{\text{iso}} < 1, -1 < \theta < 1,$$

which can be treated more directly by our optimization approach. Finally α and C are given in the transversal-isotropic case by the formulas,

$$\begin{aligned}\alpha: \Omega \rightarrow \mathbb{R}^5, x \mapsto (E_{\text{iso}}(x), v_{\text{iso}}(x), E_o(x), \theta(x), G_{\text{io}}(x)), \\ C: \mathbb{R}^5 \rightarrow \mathcal{S}^6, \\ (E_{\text{iso}}, v_{\text{iso}}, E_o, \theta, G_{\text{io}}) \mapsto C_{\text{trans-iso}}(E_{\text{iso}}, v_{\text{iso}}, E_o, \theta, G_{\text{io}}).\end{aligned}$$

Moreover the parameter set $\mathcal{P}$ is given by

$$\begin{aligned}\mathcal{P}_{\text{trans-iso}} = \{ \alpha = (E_{\text{iso}}, v_{\text{iso}}, E_o, \theta, G_{\text{io}}) \in L^\infty(\Omega; \mathbb{R}^5) \mid \\ E_{\text{iso}} > 0, E_o > 0, -1 < v_{\text{iso}} < 1, -1 < \theta < 1, G_{\text{io}} > 0 \text{ a. e. in } \Omega \}.\end{aligned}\quad (13)$$

Though these material tensors are defined for material with Poisson number $\nu < 0$, we generally use restrictions $\nu \geq 0$ for isotropic materials and $v_{\text{iso}} \geq 0$ in the transversal-isotropic case in order to avoid problems in the manufacturing process.

Finally note that precise definitions for $\mathcal{P}'$ in Eq. (6) in the isotropic, respectively, transversal-isotropic case can be deduced from Eqs. (7) and (13) in a straight forward manner.

5. Discretization

In order to solve the infinite dimensional problem (3), we discretize it using the finite-element method.⁴⁰ Supposing Ω can be decomposed into elements $\omega_1, \omega_2, \dots, \omega_m$, we approximate the parameter function α by a function, which is constant on each element ω_i . This allows us to identify the function $\alpha \in L^\infty(\Omega; \mathbb{R}^p)$ with a matrix $\alpha = (\alpha_1, \dots, \alpha_m)$, where $\alpha_i \in \mathbb{R}^p, i = 1, \dots, m$, and the corresponding material matrix function $C_\alpha = C \circ \alpha: \Omega \rightarrow \mathcal{S}^d$ as m -tuple of (constant) matrices $C_\alpha = (C_{\alpha,1}, \dots, C_{\alpha,m})$ (note that we keep the same symbols for the discretized objects). Denoting the finite-element basis associated with the decomposition $(\omega_1, \omega_2, \dots, \omega_m)$ by $\{\theta_j \mid j = 1, 2, \dots, n\}$ we further approximate the displacement field u and the force f by

$$u(x) = \sum_{j=1}^n u_j \theta_j(x) \quad \text{and} \quad f(x) = \sum_{j=1}^n f_j \theta_j(x),$$

respectively. As usual, u_j and f_j are the values of the functions u and f evaluated at the j -th node associated with the basis function θ_j . As $u(x)$ has d components, $u \in \mathbb{R}^{n \cdot d}$. Using this, we are able to write the global stiffness matrix $K(\alpha)$ as the sum over-all element stiffness matrices $K_i(\alpha)$,

$$K(\alpha) = \sum_{i=0}^n K_i(\alpha), \quad K_i(\alpha) = \sum_k w_k B_{i,k}^\top C_{\alpha,i} B_{i,k}.$$

Here the matrices $B_{i,k}$ are the discretized strain tensors evaluated at Gauss points and w_k are the corresponding integration weights. Now the discrete counterpart of the partial differential Eq. (2) becomes

$$K(\alpha)u = f. \quad (14)$$

Using the same technique as above we discretize the cost functional J in Eq. (3) and obtain

$$J(\alpha) = \frac{1}{2} \sum_{j=0}^n \sum_{\ell=0}^d \tilde{w}_{j,\ell} (u_{j,\ell} - \bar{u}_{j,\ell})^2 + \gamma \|\alpha\|^2,$$

where $\bar{u} \in \mathbb{R}^{n \cdot d}$ is the given reference displacement and $\tilde{w}_{j,\ell}$ are real weighting factors taking into account whether the node j is element of the boundary Γ_T or not.

Finally, the discretized counterpart of the optimization problem (3) reads as follows:

$$\min_{\alpha \in \hat{\mathcal{P}}} \frac{1}{2} \sum_{j,\ell} \tilde{w}_{j,\ell} (u_{j,\ell} - \bar{u}_{j,\ell})^2 + \gamma \|\alpha\|^2, \quad (15)$$

s.t.,

$$K(\alpha)u = f,$$

$$\underline{V} \leq \sum_{i=1}^m |\omega_i| \text{Tr}(C_\alpha) \leq \bar{V},$$

where $\hat{\mathcal{P}}$ is the discretized version of $\mathcal{P}_{\text{iso}}$ or $\mathcal{P}_{\text{trans-iso}}$, respectively, which are given by the formulas,

$$\hat{\mathcal{P}}_{\text{iso}} = \{ \alpha = (\alpha_1, \dots, \alpha_m)^\top \in \mathbb{R}^{m \times 2} \mid$$

$$\alpha_i = (E_i, v_i) \in \mathbb{R}^2, E_i > 0, -1 < v_i < \frac{1}{2}, i = 1, \dots, m \},$$

$$\hat{\mathcal{P}}_{\text{trans-iso}} = \{ \alpha = (\alpha_1, \dots, \alpha_m)^\top \in \mathbb{R}^{m \times 5} \mid$$

$$\alpha_i = (E_{\text{iso},i}, v_{\text{iso},i}, E_o,i, \theta_i, G_{\text{io},i}) \in \mathbb{R}^5, E_{\text{iso},i} > 0, E_o,i > 0, \\ -1 < v_{\text{iso},i} < 1, -1 < \theta_i < 1, G_{\text{io},i} > 0, i = 1, \dots, m \}.$$

In case of $\mathcal{P}_{\text{iso}'}$ or $\mathcal{P}'_{\text{trans-iso}}$ no further discretization is necessary, as these sets already describe materials that are constant on a given region.

6. Sensitivity analysis

Generally, Eq. (15) is a large-scale smooth optimization problem involving linear and simple box constraints. Optimization problems of this type are known to be solved efficiently by so-called first (or even second) order methods, i.e., optimization methods exploiting derivatives of the cost functional. For this purpose, we need to calculate the sensitivities of the cost functional J with respect to the optimization parameter α .

Using methods described in Ref. 25 it is possible to derive the so-called adjoint state equation,

$$K(\alpha)p = \text{diag}(\tilde{w})(u - \bar{u}) =: z, \quad (16)$$

associated with the tracking functional J . Denoting the solution of problem (16) by p , we obtain the following explicit formulas for the partial derivatives of J with respect to the optimization parameters $\alpha_{i,r}, i = 1, 2, \dots, m, r = 1, 2, \dots, p$,

FIG. 4. Flowchart of the optimization process. Actions from CFS++ and SCIP (shaded) are shown.

$$\frac{\partial J}{\partial \alpha_{i,r}} = -p^\top \frac{\partial K(\alpha)}{\partial \alpha_{i,r}} u,$$

$$\frac{\partial K(\alpha)}{\partial \alpha_{i,r}} = \frac{\partial K_i(\alpha)}{\partial \alpha_{i,r}} = \sum_k w_k B_{i,k}^\top \frac{\partial C_{\alpha,i}}{\partial \alpha_{i,r}} B_{i,k}.$$

(17)

FIG. 5. Geometry and boundary conditions used to validate the method.

Here $\frac{\partial C_{\alpha,i}}{\partial \alpha_{i,r}}$ denotes the sensitivity of a single material matrix $C_{i,\alpha}$ w.r.t. the (i,r) -th component of α .

7. Optimization method

In this paragraph, we describe the numerical optimization concept used to solve problem (15). We combine two existing software packages. For the simulation, i.e., the definition and the solution of the state problems (14) and (16), we employ the finite-element software CFS++.⁴¹ CFS++ is combined with the optimization solver sequential convex programming with interior point method (SCIP),^{42,43} which has been recently applied in many areas of design and structural optimization, see e.g., Ref. 44. SCIP is based on the so-called sequential convex programming concept. The main idea in sequential convex programming is as follows. Instead of solving an optimization problem directly, a sequence of simpler model problems is solved. The model problems are convex local first-order approximations of the original problem.

The flow of the optimization process is shown in Fig. 4 and works as follows. In a pre-processing step, we define geometry, boundary conditions, and forces for the state equation in CFS++. Then we choose an initial design α_0 and start the optimization loop. In the optimization loop, we repeat the following steps until a convergence criterion is met:

- (1) Solve Eqs. (14) and (16) (CFS++).
- (2) Evaluate the cost functional J and compute the sensitivities of J w.r.t. the design according to Eq. (17).
- (3) Build and solve a new (local) modal problem (SCIP). The solution of the model problem serves as search direction d .
- (4) Update the design by the formula,

$$\alpha^{\text{new}} = \alpha + t \cdot d,$$

where t is computed by a step size algorithm in SCIP.

In order to enable the communication between CFS++ and SCIP, several extensions have been made in CFS++. Among them are

- a new software layer for support of constrained first-order optimization solvers;
- I/O routines enabling flow control of an optimization process.

TABLE I. Number of iterations required to identify the parameter values $E = 5 \cdot 10^4 \pm 5 \cdot 10^{-2}$, $\nu = 0.3 \pm 5 \cdot 10^{-7}$ for isotropic material.

Initial values		Number of iterations for	
E	ν	2D	3D
$1 \cdot 10^4$	0.00	21	24
$1 \cdot 10^4$	0.45	22	36
$1 \cdot 10^7$	0.00	29	27
$1 \cdot 10^7$	0.45	29	33

The software is capable of solving the optimization problem with material matrices that are varied in every finite element, as well as the problem with matrices, which are assumed to be constant on each layer.

8. Regularization and multiple-load case

The optimization method described above works for one single load as well as for multiple-load cases. Using multiple-load cases, the amount of data in the optimization problem (3) is increased. We start with solving a single-load problem and then successively add more load cases, using the result from the previous optimization run as start point for the new optimization problem. We iterate this procedure until the solutions stabilize, i.e., the difference of two subsequent solutions becomes sufficiently small in the sense that the change in cost functional J is below a certain threshold (e.g., 3%). This indicates that the significance of data cannot be further increased by adding additional load cases.

In order to solve the problem for the material parameters with the smallest norm, which makes the solution to the optimization problem unique, it has to be solved for a decreasing sequence of regularization parameters γ . We do this in the following way:

- Step 1: Choose $\gamma > 0$.
- Step 2: Solve Eq. (15) until convergence or until a prescribed iteration limit is exceeded.
- Step 3: Decrease γ .
- Step 4: Repeat steps 2 and 3 until γ is sufficiently small.

TABLE II. Parameter values for 3D transversal-isotropic material with corresponding functional values and tracking errors. (1) Original parameter set used in 3D simulation. Optimized parameter and tracking results obtained from different starting values: (2)–(4) no volume constrained applied, (5)–(6) with added regularization term, (7) with applied volume constraint. (8) For comparison: one of the starting points.

	Material parameters					Tracking errors		
	E_{iso} (k Pa)	ν_{iso}	E_o (kPa)	ν_{oi}	G_{io} (kPa)	J (mm ²)	Max. err (mm)	Avg. err (mm)
(1)	50.00	0.050	1000	0.200	10.00	$4.8 \cdot 10^{-6}$	$2.7 \cdot 10^{-12}$	$5.6 \cdot 10^{-14}$
(2)	45.73	0.142	834.7	0.174	10.43	$5.7 \cdot 10^7$	1.2	$7.2 \cdot 10^{-1}$
(3)	44.42	0.169	845.6	0.174	10.43	$9.6 \cdot 10^1$	1.6	$9.1 \cdot 10^{-1}$
(4)	45.84	0.138	815.5	0.172	10.49	$5.4 \cdot 10^1$	1.0	$7.1 \cdot 10^{-1}$
(5)	46.97	0.113	925.9	0.186	10.20	$2.4 \cdot 10^1$	$7.5 \cdot 10^{-1}$	$4.6 \cdot 10^{-1}$
(6)	46.93	0.114	922.0	0.185	10.21	$2.5 \cdot 10^1$	$7.7 \cdot 10^{-1}$	$4.7 \cdot 10^{-1}$
(7)	50.00	0.050	1002	0.201	9.996	$2.4 \cdot 10^{-3}$	$7.5 \cdot 10^{-3}$	$4.3 \cdot 10^{-3}$
(8)	10 000	0.000	10 000	0.000	10 000	$1.6 \cdot 10^6$	$1.7 \cdot 10^2$	$1.3 \cdot 10^2$

FIG. 6. 3D model of the vocal fold showing the boundary conditions. Black: pressure, light gray: free boundaries, and dark gray: fixed boundaries.

III. RESULTS

A. Validation of the optimization method

The purpose of our first experiment is to test, if our algorithm (see Sec. II B 7) is able to reproduce given material parameters (identification property). We use two test cases: In the first case (Fig. 5, left), we consider a rectangular domain, resting on two rails with a pressure load applied from above. The second test case is modeled with similar boundary conditions for a 3D cuboid; see Fig. 5, right. We start with a simulation using given material parameters (first isotropic, then transversal-isotropic) and extract the displacement of the design bodies in a neighborhood of the edges. Then, we perform the optimization, i.e., we solve problem (15), for the given geometry, displacement field $\bar{u}$, boundary conditions, and forces. In this experiment we use only one layer of material, i.e., we assume the material parameters to be homogeneous in the entire body. In order to obtain reliable results, we repeat the experiment with different starting points and compare the results. Moreover, in the case of transversal-isotropic material, we test the effect of adding a volume constraint. Table I shows that—in case of isotropic material—the original material parameters $E = 5 \cdot 10^4$ and $\nu = 0.3$ were identified after at most 36 iterations. As it is clear, that in 2D several transversal-isotropic material parameter sets correspond to one and the same tensor, we perform experiments with transversal-isotropic material in 3D only. Results of the 3D experiment are shown in Table II: One can easily see that several parameter sets [cf. lines (2)–(4)], which are not necessarily close to the parameters used in the forward simulation—lead to a displacement with maximum error $e_{\text{max}} < 1.6$ mm, reduced by over 99% compared to a typical

FIG. 7. 3D model with mid-plane (left) that was used for generating a reference displacement using a forward simulation and corresponding 2D vocal fold model (right) used for the optimization process.

start point [line (8)] with $e_{\max} = 1.7 \cdot 10^2$ mm (the model is a cuboid with dimensions $1 \text{ m} \times 1 \text{ m} \times 0.5 \text{ m}$). Such a behavior is well-known for ill-posed inverse problems. As already mentioned at the end of Sec. II B 2, regularization of the problem is expected to improve the situation, making the solution in the best case unique. Lines (5)–(6) of Table II show the effect of regularization. The solution is almost equal for different starting points. However these results show the parameters with minimal norm, which are clearly different from the original parameters used in the forward simulation. Line (7) of Table II shows the optimization result after adding a volume constraint, fixing the total volume in the design body to the corresponding value obtained from the forward simulation: we are able to identify the correct parameter set with rather high accuracy. Thus the volume constraint acts indeed as a regularization and improves the stability of our approach.

B. Comparison of material laws

The goal of this paragraph is to study the influence of the particular choice of material models introduced in Sec. II B 4.

FIG. 8. Comparison of optimal boundary displacement achieved with different material laws. The transversally isotropic optimization solution is clearly superior to the isotropic solution as can be seen in the close up.

We use the following experiment as a basis for a comparison. We consider two numerical vocal fold models. The first one is a symmetric 3D model consisting of three layers, where each of the layers is made of a fixed isotropic material (Fig. 6). Moreover, the 3D model is loaded by a given pressure load from the left. The second model is a layered 2D model, whose geometry coincides with the geometry of the mid-plane in the 3D model (Fig. 7). We perform a simulation step with the 3D model and extract the resulting boundary displacement of the mid-plane. The optimization task is now to identify material properties (in each layer) of the 2D model, such that the boundary of the 2D model is displaced in the same way as the boundary of the 3D mid-plane; i.e., the displacement curve extracted from the 3D simulation takes the role of $\bar{u}$ in the cost functional J in Eq. (15). Note, that even though the 3D model used for simulation is isotropic the behavior of the 2D middle plane cannot be reproduced precisely using isotropic material assumptions due to the influence of the boundary conditions applied to the 3D body.

Figure 8 visualizes the qualitative results of our experiments and provides a comparison of the *optimal* deformations obtained from different material laws. As we look at a vertical–lateral slice of the vocal fold in this 2D setup, transversal-isotropic material with plane of isotropy in this vertical–lateral direction reduces to purely isotropic material. Other possible choices for the plane of isotropy lead precisely to the same material law in 2D; thus, we present only results for transversal-isotropic material with plane of isotropy in vertical–longitudinal direction for this setup. We

TABLE III. Comparison of different material laws. Optimization was done for a 2D vertical–lateral plane. The table shows that transversally isotropic material optimization with isotropic plane chosen in a reasonable direction (vertical–longitudinal, lateral–longitudinal) produces a result, which is a about 33% better than in the isotropic case.

Material law	Tracking J (mm ²)	Tracking node error	
		Maximum (mm)	Average (mm)
Isotropic	$5.65 \cdot 10^{-2}$	$9.02 \cdot 10^{-2}$	$3.96 \cdot 10^{-2}$
Transversally iso.	$3.40 \cdot 10^{-2}$	$6.00 \cdot 10^{-2}$	$3.37 \cdot 10^{-2}$

FIG. 9. Comparison of optimization results for different material laws to the reference displacement. The optimization was done in 3D combining five different load cases. The visualized curves show the deformation corresponding to one of these load cases at a cut in the vertical–lateral plane at the middle (left picture) and 3.4 mm left of the middle (right picture). The transversal-isotropic optimization is much more close to the reference displacement. Free transversal-isotropic material optimization, where a completely inhomogeneous material with transversal-isotropic material parameters varying from point to point is optimized, is just shown as a benchmark.

clearly see that the transversally isotropic material law is superior to the isotropic material, see also Table III.

Table III shows the corresponding quantitative results: the value of the cost functional for the optimal material and the corresponding nodal errors. Note that the model is of size 10 mm. Summarizing Table III, the transversally isotropic material provides a 33% decrease of the maximum nodal error and 15% decrease of the average nodal error compared to the isotropic material law. Thus transversally isotropic material should be the preferred, if it can be manufactured.

C. Application: Results with data from experiments

In our third experiment we use boundary displacements, boundary conditions, and forces from the hemilarynx experiments described in Sec. II A. We test our method by means of one arbitrarily chosen vocal fold. In accordance with observations made in Sec. II A we use only load cases with traction forces up to 20 g, resulting in a total of five different load cases. In each of these load cases, either one or several sutures were loaded. In Fig. 9, one of these load cases is visualized. As our model geometry does not precisely match the geometry in the real experiment, we map the given data

TABLE IV. Optimal transversal-isotropic material parameters resulting from the hemilarynx experiments. Note that isotropy is assumed orthogonal to the longitudinal direction.

	E_{iso} (kPa)	ν_{iso}	G_{iso} (kPa)	E_o (kPa)	ν_{io}	ν_{oi}	G_{io} (kPa)
Cover	44.0	0.41	217	796	0.09	1.63	15.6
Ligament	4.09	0.22	3.73	1937	0.00	2.20	1.67
Body	2.68	0.04	0.07	50.8	0.01	0.22	1.29

TABLE V. Optimal isotropic material parameters resulting from the hemilarynx experiments.

	E (kPa)	ν	G (kPa)
Cover	387	0.11	174
Ligament	10.2	0.01	5.04
Body	4.50	0.08	20.9

set into our model geometry (see Fig. 6). This is done in the following way: The coordinates of the sutures in the reference, i.e., unloaded configuration, are rotated and scaled such that the average distance of the transformed suture coordinates to the vocal fold surface of the numerical model become minimal. The suture coordinates in the loaded situation are then recomputed using the same transformation. After these adjustments the optimization method proposed in Sec. II B 7 is applied.

Figure 9 shows the simulated result and Table IV shows the corresponding material parameters. The tracking functional at the minimum has a value of $J = 3.7 \cdot 10^{-4}$. As isotropic material is more easy to manufacture, also the optimized isotropic material parameters are given in Table V. The tracking functional only achieved a value of $J = 4.4 \cdot 10^{-4}$. The improvement by using transversal-isotropic material has to be considered against the process of manufacturing. Also, Table VI shows material parameters where the isotropic material parameters were additionally constrained to values that could easily be manufactured using silicone. The tracking functional for these parameters achieved a value of $J = 5.4 \cdot 10^{-4}$.

On the other hand, the value of the tracking functional of $J = 3.7 \cdot 10^{-4}$ for transversal-isotropic material shows that the reference displacements cannot be matched with high accuracy. This indicates that the model geometry with the given three layers (Fig. 7) and the assumption that the material is constant on each of these layers is too restrictive.

The situation improves significantly, if we allow for the use of inhomogeneous material within the layers, i.e., we allow the material to vary from element to element (compare Sec. II B 3). Here the functional achieved a value of $J = 1.5 \cdot 10^{-4}$ corresponding to a significantly better matching but with a material that would be very difficult if not impossible to manufacture. The simulated displacement is shown in Fig. 10.

IV. DISCUSSION

In this work, we presented a widely applicable material optimization methodology for identification of material parameters in a numerical vocal fold model using experimental

TABLE VI. Optimal material parameters for a silicone model resulting from the hemilarynx experiments.

	E (kPa)	ν	G (kPa)	E_{max} (kPa)	ν_{min}	ν_{extmax}
Cover	30.0	0.499	10.0	30.0	0.400	0.499
Ligament	91.8	0.400	32.8	100.0	0.400	0.499
Body	56.6	0.499	18.9	100.0	0.400	0.499

FIG. 10. The surface displacement (solid vectors) at the sutures as extracted from the hemilarynx-experiments used as reference deformation for one load case with applied force (dashed vector). The load case corresponds to the experiment shown in Fig. 2.

data. The computed material parameters can now be transferred to a physical vocal fold model with given geometry and layer structure of the numerical model.

We gave a formulation of the optimization problem in function space and a discretized version of the problem using finite elements, which led to an implementation in the software package *CFS++*. We discussed enhancements for improving stability of the basic problem, including the treatment of multiple-load cases, as well as regularization techniques. The optimization method was successfully validated using some academic test cases. Finally material parameters for the numerical vocal fold model were computed.

Moreover, we presented numerical results. First some academic examples, where the reference displacements were simulated and later tracked. The stability of our method was demonstrated, i.e., the capability of retrieving the original material parameters used in the simulation. Using these examples we also show the regularization, either using a tychonoff type regularization term or a volume equality constraint, is needed with certain material laws and examples to guarantee uniqueness of the solution.

Most of the computed material parameters lead to fair agreement with previously reported data. However it must be emphasized that previously reported data have not been consistent, and even intersubject differences are typically as large as an order of magnitude (e.g., a factor of 10), or greater.⁴⁵ On the other hand we conclude that a purely isotropic choice of material parameters is too restrictive. It was observed that using a less conservative approach for the choice of the material parameters as a benchmark, still much better agreement could be achieved. Unfortunately, the less conservative approach resulted in an artificial material, which is currently impossible to manufacture. In order to obtain a manufacturable material, but still less restrictive as the one discussed in this work, we plan to vary the geometry of the layers in the multi-layer model in addition to the material parameters in forthcoming studies. In the framework of this approach the shape of the layers are parameterized by spline curves and subjected to optimization

using a cost function similar to the one described in this article. The main mathematical ideas are already described in Ref. 46. On the other hand the finite-element model used in this work is not capable of exactly reconstructing the measured displacement even when using the most general material. This shows that improvements to the model itself might be needed, e.g., changes in the geometry. Anyway, so far the basic behavior could be reproduced (Fig. 9). To solve that problem, we also plan to incorporate a shape optimization approach in the future, which has been successfully applied in many different disciplines as, for instance, light weight design recently.²⁶ In any case, the presented approach and results are promising and provide a foundation for building synthetic vocal fold models which behave more realistically. Theoretical topics like the existence of the solutions to the problem (3) and convergence of the discretizations were not discussed in this work but will be studied in a forthcoming paper by the authors. Also, the optimization model will be further extended to investigate transient oscillations so that displacement fields measured over an interval of time can be tracked and frequency dependent effects can be observed.

Previously, in order to manufacture a model of the vocal folds, one had to rely on elastic properties that were measured directly. This was done by dissecting tissue from the vocal folds and performing tensile tests^{16,17} or applying shear forces.^{21,45} However these methods are not suitable for identifying more than one parameter simultaneously. Additionally obtaining parameters for different layers of tissue is rather difficult. Other measurements have been performed using rheometers.^{20,47,48} These measurements can be performed *in-vivo*. However, they are constricted to retrieving material parameters at several local points situated at the surface only. Another problem is the lack of consistent measurements between research laboratories. For example, different investigations report results which vary from 1 kPa²¹ up to 40 kPa.¹⁶ Even using the same technique, intersubject differences often vary by as much as an order of magnitude, or greater.⁴⁵ Our approach is completely

different starting from the idea of manufacturing a model that behaves as similar as possible to the excised vocal fold. This allows manufacturing constraints or other limitations to the model to be considered. Also, depending on various parameters, a given model can easily be improved. However, the method is not very suitable for identifying the material parameters in the original vocal fold tissue as can be seen by the great variation of the optimal parameters using different manufacturing constraints. This is common in ill-posed inverse problems. However, our approach simulates the static behavior of the model as a function of applied forces, and the objective function takes the distance of these simulated results into account. Thus, we do not directly measure material parameters, but instead determine a set of such parameters for a theoretical model which produces a prescribed deformation. The parameters can then be utilized in physical models which exhibit more realistic dynamical behavior.

From a medical point of view, the ultimate goal is to manufacture laryngeal physical models capable of producing normal phonation and to identify material and geometric properties which can appropriately simulate pathological dynamics and surgical interventions.

ACKNOWLEDGMENTS

This work was supported by Deutsche Forschungsgemeinschaft (DFG, German Research Foundation), Grant No. FOR 894/1, "Strömungsphysikalische Grundlagen der menschlichen Stimmgebung" (fluid physical basis of human vocalization). Dr. Berry's contributions to this investigation were supported by National Institute on Deafness and Other Communication Disorders (NIDCD), Grant no. R01 DC03072.

- ¹M. Doellinger and D. A. Berry, "Visualization and quantification of the medial surface dynamics of an excised human vocal fold during phonation," *J. Voice* **20**, 401–413 (2006).
- ²A. Bößenecker, D. A. Berry, J. Lohscheller, U. Eysholdt, and M. Döllinger, "Mucosal wave properties of a human vocal fold," *Acta. Acust. Acust.* **93**, 815–23 (2007).
- ³I. R. Titze, *The Myoelastic Aerodynamic Theory of Phonation* (The University of Iowa and the Denver Center for performing arts, National Center for voice and speech, Denver CO and Iowa City, IA, 2006), pp. 1–232.
- ⁴D. D. Cook and L. Mongeau, "Sensitivity of a continuum vocal fold model to geometric parameters, constraints, and boundary conditions," *J. Acoust. Soc. Am.* **121**, 2247–2253 (2007).
- ⁵M. Spencer, T. Siegmund, and L. Mongeau, "Determination of superior surface strains and stresses, and vocal fold contact pressure in a synthetic larynx model using digital image correlation," *J. Acoust. Soc. Am.* **123**, 1089–1103 (2008).
- ⁶S. Becker, S. Kniesburges, S. Müller, A. Delgado, G. Link, M. Kaltenbacher, and M. Döllinger, "Flow-structure-acoustic interaction in a human voice model," *J. Acoust. Soc. Am.* **125**, 1351–1361 (2009).
- ⁷L.-J. Chen, M. Zanartu, D. Cook, and L. Mongeau, "Effects of acoustic loading on the self-oscillations of a synthetic model of the vocal folds," in *Proc. Ninth Intl. Conf. on Flow Induced Vibration*, 30 June–3 July 2008, I. Zolotarev and J. Horacek, Eds., Prague, Czech Republic.
- ⁸S. L. Thomson, L. Mongeau, and S. H. Frankel, "Aerodynamic transfer of energy to the vocal folds," *J. Acoust. Soc. Am.* **118**, 1689–1700 (2005).
- ⁹B. A. Pickup and S. L. Thomson, "Influence of asymmetric stiffness on the structural and aerodynamic response of synthetic vocal fold models," *J. Biomech.* **42**, 2219–2225 (2009).
- ¹⁰G. S. Berke and M. E. Smith, "Intraoperative measurement of the elastic modulus of the vocal fold. Part 2. Preliminary results," *Laryngoscope* **102**, 770–778 (1992).
- ¹¹G. S. Berke, "Intraoperative measurement of the elastic modulus of the vocal fold. Part 1. Device development," *Laryngoscope* **102**, 760–769 (1992).
- ¹²Q. T. Tran, G. S. Berke, B. R. Gerratt, and J. Kreiman, "Measurement of Young's modulus in the in vivo human vocal folds," *Ann. Otol. Rhinol. Laryngol.* **102**, 584–591 (1993).
- ¹³E. Goodyer, F. Müller, K. Licht, and M. Hess, "In vivo measurement of the shear modulus of the human vocal fold: Interim results from eight patients," *Eur. Arch. Otorhinolaryngol.* **264**, 631–635 (2007).
- ¹⁴F. Alipour-Haghighi, I. R. Titze, and P. Durham, "Twitch response in the canine vocalis muscle," *J. Speech Hear. Res.* **30**, 290–294 (1987).
- ¹⁵F. Alipour-Haghighi, I. R. Titze, and A. L. Perlman, "Tetanic contraction in vocal fold muscle," *J. Speech Hear. Res.* **32**, 226–231 (1989).
- ¹⁶F. Alipour-Haghighi and I. R. Titze, "Elastic models of vocal fold tissues," *J. Acoust. Soc. Am.* **90**, 1326–1331 (1991).
- ¹⁷Y. B. Min, I. R. Titze, and F. Alipour-Haghighi, "Stress-strain response of the human vocal ligament," *Ann. Otol. Rhinol. Laryngol.* **104**, 563–569 (1995).
- ¹⁸T. Riede and I. R. Titze, "Vocal fold elasticity of the rocky mountain elk (*cervus elaphus nelsoni*)—Producing high fundamental frequency vocalization with a very long vocal fold," *J. Exp. Biol.* **211**, 2144–2154 (2008).
- ¹⁹S. L. Thibeault, S. A. Klemuk, M. E. Smith, C. Leugers, and G. Prestwich, "In vivo comparison of biomimetic approaches for tissue regeneration of the scarred vocal fold," *Tissue Eng., A*, 1481–1487 (2008).
- ²⁰R. W. Chan and M. L. Rodriguez, "A simple-shear rheometer for linear viscoelastic characterization of vocal fold tissues at phonatory frequencies," *J. Acoust. Soc. Am.* **124**, 1207–1219 (2008).
- ²¹E. Goodyer, S. Hemmerich, F. Müller, J. B. Kobler, and M. Hess, "The shear modulus of the human vocal fold, preliminary results from 20 larynxes," *Eur. Arch. Otorhinolaryngol.* **264**, 45–50 (2007).
- ²²M. Döllinger, N. Tayama, and D. A. Berry, "Empirical eigenfunctions and medial surface dynamics of a human vocal fold," *Methods Inf. Med.* **44**, 384–391 (2005).
- ²³J. Luo, Z. Luo, L. Chen, L. Tong, and M. Y. Wang, "A semi-implicit level set method for structural shape and topology optimization," *J. Comput. Phys.* **227**, 5561–5581 (2008).
- ²⁴S. Wang, K. Lim, B. Khoo, and M. Wang, "An extended level set method for shape and topology optimization," *J. Comput. Phys.* **221**, 395–421 (2007).
- ²⁵M. P. Bendsøe and O. Sigmund, *Topology Optimization. Theory, Methods and Applications* (Springer-Verlag, Heidelberg, 2002), pp. 1–370.
- ²⁶J. Haslinger and R. Mäkinen, *Introduction to Shape Optimization* (SIAM, Philadelphia, PA, 2002), pp. 1–273.
- ²⁷M. Kočvara, M. Stingl, and J. Zowe, "Free material optimization: Recent progress," *Optimization* **57**, 79–100 (2008).
- ²⁸L. He, S. Kindermann, and M. Sini, "Reconstruction of shapes and impedance functions using few far-field measurements," *J. Comput. Phys.* **228**, 717–730 (2009).
- ²⁹R. W. Chan and I. R. Tize, "Effect of postmortem changes and freezing on the viscoelastic properties of vocal fold tissues," *Ann. Biomed. Eng.* **31**, 482–491 (2003).
- ³⁰J. J. Jiang and I. R. Titze, "A methodological study of hemilaryngeal phonation," *Laryngoscope* **103**, 872–882 (1993).
- ³¹D. A. Berry, D. W. Montequin, and N. Tayama, "High-speed digital imaging of the medial surface of the vocal folds," *J. Acoust. Soc. Am.* **110**, 2539–2547 (2001).
- ³²M. Döllinger and D. A. Berry, "Computation of the three-dimensional medial surface dynamics of the vocal folds," *J. Biomech.* **39**, 369–374 (2006).
- ³³Y. I. Abdel-Aziz and H. M. Karara, "Direct linear transformation from comparator coordinates into object space coordinates in close-range photogrammetry," in *Proceedings of the Symposium on Close-Range Photogrammetry*, American Society of Photogrammetry, Falls Church, VA (1971), pp. 1–18.
- ³⁴M. Döllinger, D. A. Berry, and G. S. Berke, "Medial surface dynamics of an in vivo canine vocal fold during phonation," *J. Acoust. Soc. Am.* **117**, 3174–3183 (2005).
- ³⁵F. Allaire, *Shape Optimization by the Homogenization Method* (Springer-Verlag, New-York, 2002), pp. 1–456.
- ³⁶H. W. Engl, M. Hanke, and A. Neubauer, *Regularization of Inverse Problems* (Springer, Netherland, Berlin, 1996), pp. 1–321.
- ³⁷A. N. Tychonoff and V. Y. Arsenin, *Solution of Ill-posed Problems* (Winston and Sons, New York, 1977), pp. 1–258.
- ³⁸J. Altenbach and H. Altenbach, *Einführung in die Kontinuums-Mechanik* (Teubner, Stuttgart, 1994), pp. 281–283.

- ³⁹F. Kang and S. Zhong-Ci, *Mathematical Theory of Elastic Structures* (Springer, New York, 1981), pp. 1–395.
- ⁴⁰P. G. Ciarlet, *The Finite Element Method for Elliptic Problems* (North-Holland, Amsterdam, New York, Oxford, 1978), pp. 1–530.
- ⁴¹M. Kaltenbacher, “Advanced simulation tool for the design of sensors and actuators,” in *Proceedings of the Eurosensors XXIV*, Linz, Austria (2010), pp. 1757–1760.
- ⁴²C. Zillober, “A globally convergent version of the method of moving asymptotes,” *Struct. Optim.* **6**, 166–174 (1993).
- ⁴³C. Zillober, Software Manual for SCIP 3.0. Technical Report, Informatik, Universität Bayreuth (2004).
- ⁴⁴J.-A. Désidéri, B. A. E. Majd, and A. Janka, “Nested and self-adaptive bézier parameterizations for shape optimization,” *J. Comput. Phys.* **224**, 117–131 (2007).
- ⁴⁵R. W. Chan and I. R. Titze, “Viscoelastic shear properties of human vocal fold mucosa: Measurement methodology and empirical results,” *J. Acoust. Soc. Am.* **106**, 2008–2021 (1999).
- ⁴⁶B. Schmidt, G. Leugering, M. Döllinger, R. Lerch, M. Kaltenbacher, and M. Stingl, “Optimizing material properties and geometry of a physical multi-layered vocal fold model,” in *Proceedings of the International Conference on Acoustics NAG/DAGA* (2009), pp. 1757–1760.
- ⁴⁷M. M. Hess, F. Mueller, J. B. Kobler, S. M. Zeitels, and E. Goodyer, “Measurements of vocal fold elasticity using the linear skin rheometer,” *Folia Phoniatr Logop* **58**, 207–216 (2006).
- ⁴⁸D. K. Chhetri, G. S. Berke, A. Lotfizadeh, and E. Goodyer, “Control of vocal fold cover stiffness by laryngeal muscles: A preliminary study,” *Laryngoscope* **119**, 222–227 (2009).